Constraints on two-body lepton flavor violating decay processes

S. Nussinov

Department of Physics, Tel-Aviv University, Ramat-Aviv, Tel-Aviv, Israel

R. D. Peccei

Department of Physics and Astronomy, University of California, Los Angeles, California 90095-1547

X. M. Zhang

Institute of High Energy Physics, Beijing 100039, People's Republic of China
(Received 19 April 2000; published 30 November 2000)

Simple ''unitarity inspired'' relations between two- and three-body lepton flavor violating decays are noted and discussed. In the absence of cancellations, the existing strong bounds on $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma\gamma$ severely constrain two-body lepton flavor violating decays.

Lepton flavor violating (LFV) processes are strongly suppressed in the standard model by powers of (small) neutrino masses. Such decays signal therefore new physics. At present we have stringent bounds for μ decays [$\mathrm{BR}(\mu \rightarrow 3e) \leq 10^{-12}$, $\mathrm{BR}(\mu \rightarrow e\gamma\gamma) \leq 10^{-10}$] and somewhat weaker $O(10^{-6})$ bounds on LFV τ decays [1].

The availability of large samples of decaying vector bosons [$V = J/\psi$, Υ, and Z^0] or pseudoscalars [π^0, η] and the clear signature provided by $\mu^\pm e^\mp$ final states suggests searching for LFV two-body decays $V \rightarrow \mu^\pm e^\mp$ or $\pi^0/\eta \rightarrow \mu^\pm e^\mp$. In this note we show that rather simple considerations, based on unitarity, provide rather strong constraints on two-body LFV processes. Hence, most three-body μ and τ LFV decays are likely to remain more sensitive tests of lepton flavor violation, rather than the corresponding two-body decays.

BASIC CONSIDERATIONS

Let us assume that a vector boson V_i (here V_i could be either a fundamental state, such as the Z^0, or a quark-antiquark bound state such as the $\phi, J/\psi$, or Υ) couples to $\mu^\pm e^\mp$. If it couples also to $e^+ e^-$, as all the states above do, then by unitarity its exchange contributes also to $\mu \rightarrow 3e$. Let us write the effective coupling between the vector boson V_i and $\mu^\pm e^\mp$ as

$$\mathcal{L}_{\mathrm{eff}} = \tilde{g}_{V\mu e} \bar{\mu} \gamma_\alpha e V^\alpha + \mathrm{H.c.} \qquad (1)$$

This coupling, through the diagram of Fig. 1, contributes to the $A(\mu \rightarrow 3e)$ amplitude a term

$$A(\mu \rightarrow e) = \bar{u}_\mu(p) \gamma^\alpha u_e(k_3) \bar{v}_e(k_1) \gamma_\alpha u_e(k_2) \cdot \frac{\tilde{g}_{V\mu e} g_{Vee}}{M_V^2 - s}. \qquad (2)$$

Here g_{Vee} is the effective coupling of the vector boson V_i to

$e^+ e^-$, while $s \equiv (k_1 + k_2)^2 \leq m_\mu^2$.[1] As a first approximation, it is sensible to neglect s in comparison with M_V^2. Then comparing the above contribution to the $\mu \rightarrow 3e$ process to that of ordinary muon decay, $\mu \rightarrow e\nu\bar{\nu}$, which proceeds via W exchange and (almost) identical kinematics, gives the relation

$$\frac{[\Gamma(\mu \rightarrow 3e)]_{V-\mathrm{exch.}}}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} \approx \frac{\tilde{g}_{V\mu e}^2 g_{Vee}^2}{M_V^4} \bigg/ \frac{g_W^4}{M_W^4}. \qquad (3)$$

Since $\Gamma(V \rightarrow e^+ e^-) \sim g_{Vee}^2 M_V$ and $\Gamma(V \rightarrow \mu e) \sim \tilde{g}_{V\mu e}^2 M_V$, while $\Gamma(W \rightarrow e\nu) \sim g_W^2 M_W$, we can rewrite the last expression as

$$[\mathrm{BR}(\mu \rightarrow 3e)]_{V-\mathrm{exch.}}$$
$$\approx \frac{\Gamma(V \rightarrow e^+ e^-) \Gamma(V \rightarrow \mu^\pm e^\mp)}{\Gamma^2(W \rightarrow e\nu)} \left(\frac{M_W}{M_V}\right)^6. \qquad (4)$$

Using $\mathrm{BR}(\mu \rightarrow 3e) \leq 10^{-12}$ and other data pertaining to the $e^+ e^-$ widths of the various vector mesons V_i, we find a set of bounds for the two-body LFV branching ratios of these vector bosons. These bounds are

$$\mathrm{BR}(Z^0 \rightarrow \mu e) \leq 5 \times 10^{-13}, \qquad (5)$$

$$\mathrm{BR}(J/\psi \rightarrow \mu e) \leq 4 \times 10^{-13}, \qquad (6)$$

$$\mathrm{BR}(\Upsilon \rightarrow \mu e) \leq 2 \times 10^{-9}, \qquad (7)$$

$$\mathrm{BR}(\phi \rightarrow \mu e) \leq 4 \times 10^{-17}. \qquad (8)$$

Likewise, the generic upper bounds on LFV tau decays $\mathrm{BR}(\tau \rightarrow ll'\bar{l}') \leq 10^{-6}$ yields

---

[1]There are, of course, also potential axial vector couplings of V to $e^+ e^-$, which contribute to this process. These have not been included in the above, since they do not change our qualitative discussion. These couplings are, however, taken into account in the Z bounds given in Eqs. (5) and (9) below.
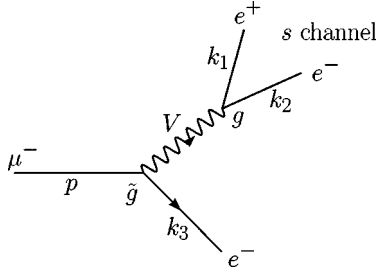
FIG. 1. A vector exchange diagram contributing to $\mu \to 3e$.

$$\text{BR}(Z^0 \to \tau \bar{l}) \lesssim 3 \times 10^{-6}, \tag{9}$$

$$\text{BR}(J/\psi \to \tau \bar{l}) \lesssim 6 \times 10^{-7}, \tag{10}$$

$$\text{BR}(\Upsilon \to \tau \bar{l}) \lesssim 10^{-2}, \tag{11}$$

with $l/l' = e/\mu$. Except for Eq. (11), these inferred bounds are unlikely to be improved by future experimental data on two-body decays.

One can use similar considerations to obtain bounds on the LFV decays of pseudoscalar states. For these purposes, one considers instead of the $\mu \to 3e$ process the $\mu^- \to e^- \gamma \gamma$ decay, which has a LFV bound $\text{BR}(\mu \to e \gamma \gamma) \lesssim 10^{-10}$. For this latter process the LFV couplings of the π^0/η^0 contribute, due to the exchange of these particles in the s-channel. We can again utilize this fact to infer upper bounds on $\pi^0/\eta^0 \to \mu^{\pm} e^{\mp}$. The $\pi^0/\eta^0 \gamma \gamma$ vertex, because of gauge invariance, involves two derivatives:

$$\mathcal{L}_{\text{eff}} = \frac{\phi}{f_\phi} F^{\mu\nu} \widetilde{F}_{\mu\nu}, \tag{12}$$

where $\phi = \pi^0, \eta^0$. This derivative coupling, in contrast to the $V \to \mu e$ non-derivative coupling encountered earlier, kinematically suppresses the off-shell $\pi^0/\eta^0 \to \gamma \gamma$ contribution at $s = (k_1 + k_2)^2 \lesssim m_\mu^2$ relative to what it would be for on-shell π^0/η^0 decay. Consequently the analog to Eq. (4) for the present case,

$$[\text{BR}(\mu \to e \gamma \gamma)]_{\pi^0/\eta^0-\text{exch.}}$$

$$\approx \frac{\Gamma(\pi^0/\eta^0 \to \gamma \gamma) \cdot \Gamma(\pi^0/\eta^0 \to \mu^{\pm} e^{\mp})}{\Gamma^2(W \to e\nu)}$$

$$\times \left(\frac{M_W}{m_{\pi'/\eta}} \right)^6 \left(\frac{\langle s_{\gamma\gamma} \rangle}{m_{\pi/\eta}^2} \right)^2, \tag{13}$$
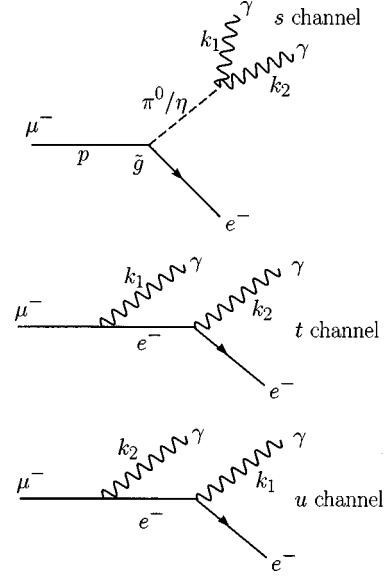
contains an extra factor

$$\frac{\langle s_{\gamma\gamma}^2 \rangle}{m_{\pi/\eta}^4} \approx \left(\frac{m_\mu}{2m_{\pi/\eta}} \right)^4, \tag{14}$$

which tends to weaken the bounds one can derive. One finds, for pseudoscalar LFV decays the bounds:

$$\text{BR}(\eta \to \mu e) \lesssim 10^{-8}, \tag{15}$$

$$\text{BR}(\pi^0 \to \mu e) \lesssim 10^{-10}. \tag{16}$$

FIG. 2. The π/η^0 (s channel) and e (t and u channel) exchange contributing to $\mu \to e \gamma \gamma$.

In the discussion above we have obtained the quoted bounds purely by concentrating on the contribution of the exchanged state in question to the LFV process. One can imagine, however, additional LFV contributions. For example, for the $\mu^- \to e^- \gamma \gamma$ decay, in addition to π^0/η^0 exchange in the s channel, we have also the contribution of electron exchange in the t and u channels (see Fig. 2). In this case, however, the stringent bound on the $\mu \to e \gamma$ vertex coming from experiment [BR $(\mu \to e \gamma) \lesssim 5 \times 10^{-11}$] strongly suppresses these additional diagrams and causes negligible modifications to the bounds (15), (16). Even in the absence of a strong bound on the $\mu \to e \gamma$ coupling, we would like to note that cancellations between s and t channel contributions are in general expected to be at best rather partial. Unless all particles, both external and exchanged, are spinless any specific s channel amplitude will have different $\cos \theta_s$ (or $\cos \theta_t$) dependence, and will contribute to different combinations of helicity amplitudes than the t channel exchange contributions.

By the same token, it is clear that cancellations among different angular momentum states exchanged in the s channel are also impossible. Indeed, for example, the total decay rate for $\mu \to 3e$ can be expressed as

$$\Gamma(\mu \to 3e) = \int_0^{\mu^2} ds \sum_{\alpha\beta,\gamma\delta} (s - 4m_e^2)^{-1/2} \left[\frac{\Delta(m_\mu^2, s, m_e^2)}{s} \right]^{-1/2}$$

$$\times \sum_J (2J+1) |A_{\alpha\beta,\gamma\delta}^J(s)|^2 \tag{17}$$

with Δ the triangular function expressing the initial c.m. momentum in the s channel, which here is that of $\mu \bar{e}_1$ or, equivalently, $\bar{e}_3 e_2$. The $A_{\alpha\beta,\gamma\delta}^J(s)$ are the partial waves in the Jacob-Wick expansion of the various s channel helicity amplitudes. Note that for the $\mu \to 3e$ case, adding the t channel amplitude amounts to enforcing the (anti) symmetrization

between the e_3 and e_2 fermions. Since Fermi statistics does not preclude the vectorial coupling contributing to $\mu \to 3e$ considered here [cf. Eq. (2)], no cancellation of s and t contributions should arise as well.

POSSIBLE LIMITATIONS ON THE DERIVED LFV BOUNDS

Although we have called the bounds we obtained above unitarity bounds, in the strict sense the inferred bounds are not true unitarity bounds—as would be the case if the exchanged particle(s) were on mass shell. To illustrate this point, let us recall a well known example of a true unitarity bound arising in rare Kaon decays. This is the lower bound for the BR($K_L \to \mu^+ \mu^-$) derived from the measured branching ratio of $K_L \to \gamma\gamma$. The $K_L \to \gamma\gamma$ process, with an on-shell $\gamma\gamma$ intermediate state, contributes to Im $A(K_L \to \mu^+ \mu^-)$ via unitarity since Im $A(K_L \to \mu^+ \mu^-) \sim A(K_L \to \gamma\gamma)^* A(\gamma\gamma \to \mu^+ \mu^-)$. This contribution provides a strict lower bound to the BR($K_L \to \mu^+ \mu^-$), so the apparent violation of this bound in early $K_L \to \mu^+ \mu^-$ data was a source for much concern. Modern day data, as expected, agrees with this bound [2].

In the present context, an example of a ''pure'' unitarity bound for a LFV process is provided by the ''τ analog'' of the $\phi \to \mu e$ process. Because the decay $\phi \to \tau \bar{l}$ is kinematically forbidden, what one should consider instead is $\tau \to \phi\mu$ (or $\tau \to \phi e$). The ''on-shell'' ϕ emitted in this putative process propagates over a long distance, of order $1/\Gamma_\phi \simeq 30$ fm, before decaying into $K\bar{K}, \mu^+ \mu^-, e^+ e^-$ in a manner which is completely independent of its production. This will generate a distinct narrow contribution to the corresponding three-body processes $\tau \to K\bar{K}\mu$, $\tau \to \mu^+ \mu^- \mu$, $\tau \to e^+ e^- \mu$, contributing to the imaginary part of these amplitudes at $s = m_\phi^2$. Hence, for example, there is an attendant lower bound on BR$[\tau \to ll'\bar{l}']$ which is simply BR($\tau \to \phi\bar{l}$) $\cdot$ BR($\phi \to l'\bar{l}'$) $\simeq 3 \times 10^{-4} \times$ BR($\tau \to \phi\bar{l}$). The resulting rigorous upper bound one obtains,

$$\text{BR}(\tau \to \phi\bar{l}) \lesssim 3 \times 10^{-3}, \qquad (18)$$

unfortunately happens to be rather weak.

All the vector (or pseudoscalars) used as intermediaries in deriving the bounds in Eqs. (5)–(11) and Eqs. (15) and (16) are not on-shell. Thus we must entertain the possibility that their contribution to the three-body decays considered are reduced. This could void, or at least weaken the various strong bounds obtained above. In the rest of this note we will focus on possible mechanisms for such a reduction.

Kinematical suppression of the LFV bounds

The size of the boson exchange contribution to the three-body decay amplitude can be reduced if there are *kinematical* suppressions. These arise when the effective boson couplings are not minimal, involving derivatives (or momentum factors). We already encountered one such case above, when we discussed the π^0/η^0 contribution to $\mu \to e\gamma\gamma$. We want to

discuss here whether such kinematical suppressions may not also affect the vector exchange contributions.

It is clearly possible to imagine that the LFV $V_i \mu^\pm e^\mp$ vertex, instead of having the form of Eq. (1), involves an anomalous magnetic moment coupling:

$$\mathcal{L}_{\text{eff}}^{\text{Magnetic}} = \frac{1}{M_V} \bar{\mu} \sigma_{\alpha\beta} e (\partial^\alpha V^\beta - \partial^\beta V^\alpha) + \text{H.c.} \qquad (19)$$

In this case the contribution of the virtual V_i to $\mu \to 3e$ is reduced by

$$\frac{q^2}{M_V^2} \approx \frac{\langle s \rangle}{M_V^2} \approx \frac{m_\mu^2}{2M_V^2} = \begin{cases} 3 \times 10^{-3} & V = \phi \\ 3 \times 10^{-4} & V = J/\psi \\ 3 \times 10^{-5} & V = \Upsilon \\ 3 \times 10^{-7} & V = Z^0 \end{cases} . \qquad (20)$$

This would considerably weaken the bounds in Eqs. (6)–(8) and reduce the bound on Z decay to only BR($Z^0 \to e\mu$) $\lesssim 1.5 \times 10^{-6}$.

It does not seem likely to us, however, that the strong suppression factors appearing in Eq. (20) obtain in practice. Indeed, various model calculations involving mixing among heavy neutrinos [3] lead to $Z^0 \to e\mu$ and effective $c\bar{c} \to \mu e$ flavor violating vertices, which involve non anomalous terms— terms like the $Z\mu e$ coupling of Eq. (1) and vectorial couplings like $\bar{c}\gamma_\alpha c \bar{\mu}\gamma^\alpha e$ [4]. Hence we believe that the kinematic suppression given by Eq. (20) probably should not be included in our bounds.

Dynamical suppression of the LFV bounds

There is another possible source of suppression which needs to be considered. This is connected to possible ''form factor'' effects due to the *dynamics* which would, for example, reduce the contribution of the various V_i states to $\mu \to 3e$ compared to the naive expectations. However, the effect of form factors should be minimal or controllable if the LFV is induced by physics at scales much higher than the EW scale or the Z mass. The effects of dynamics are nicely illustrated in a recent paper by Ilana, Jack, and Riemann [3]. These authors find, in fact, an apparent mild enhancement when the $Z^0 \to \mu e$ process is induced by relatively light ($m_{\nu_i} \lesssim 45$ GeV) neutrinos. Indeed, in this case the on-shell contribution of $\nu\bar{\nu}$ loops enhances the Z decay rates relative to the $s \approx 0$ contribution by factors of 10–100. However, such light active neutrinos would contribute to the Z^0 width and are hence ruled out. Thus such an enhancement is not physically expected.[2]

In terms of the dispersive approach adopted here, such a ''form factor'' suppression would result from cancellations in the corresponding partial wave amplitudes. Consider, for

---

[2]The BR considered in Ref. [3] for light neutrinos—i.e., neutrinos with masses in the eV range, as inferred from the SuperKamiokande [5] data—are very much below our bounds.
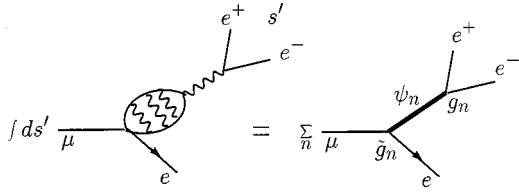
FIG. 3. The $\bar{c}c$ bubble and its equivalent description in terms of $\psi, \psi', \ldots, \psi^n$ exchanges.

example, the $A^{J=1}$ partial wave amplitude for the $\mu \to 3e$ process:

$$A^{J=1}_{\alpha\beta,\gamma\delta}(s) = \sum_i \frac{g_{V_i ee} \tilde{g}_{V_i \mu e}}{M_i^2 - s} + \int \frac{ds' \rho^{J=1}_{\text{LFV}}(s')}{s' - s}. \quad (21)$$

To get a ''form factor'' suppression, there must be a cancellation between the contributions of the various (V_i) particles among themselves, or between these contributions and those of the continuum. Let us examine these possibilities.

For the case of quarkonium intermediate states, besides the lowest energy bound state there are towers of states of the same spin and parity. Thus, for example, in Eq. (21) besides the contribution of the J/ψ one should also take into account the exchange of the $\psi', \psi'', \ldots, \psi^{(n)}$ charmonium bound resonances. Is it possible that these additional contributions largely cancel the J/Ψ term in Eq. (21)? This is unlikely for the following reason. To get J/ψ exchange to contribute to $\mu \to 3e$ in the first place, one needs to assume that the LFV physics at a high scale induces an effective four-Fermi coupling of the form:

$$\mathcal{L}_{\text{eff}} = \tilde{G}_{c\bar{c}\mu e} \bar{c} \gamma_\alpha c \bar{\mu} \gamma^\alpha e. \quad (22)$$

Such a coupling underlies all the other charmonium contributions. In fact, quark-hadron duality identifies the $J/\psi, \psi', \ldots, \psi^{(n)}$ contributions to $\mu \to 3e$ as arising from specific portions of the $s' \equiv M_{c\bar{c}}$ integration region where due to non-perturbative QCD effects 1^{--} $c\bar{c}$ bound states dominate, as shown schematically in Fig. 3. Both the $g_{\psi^{(n)}ee}$ and $\tilde{g}_{\psi^{(n)}\mu e}$ couplings appearing in Eq. (21) are proportional to the wave function of $\psi^{(n)}$ at the origin, $\Psi^{(n)}(0)$. Thus all the terms in the sum share a common sign— fixed by the sign of $\tilde{G}_{c\bar{c}\mu e} \cdot Q_c$, with $Q_c = 2/3$ being the charge of the charm quark— and cancellation cannot occur. Similar arguments apply against possible cancellations among the various states in the Υ sector.

The above discussion still leaves open the possibility of cancellations in the partial wave amplitude between different quarks-antiquark contributions ($c\bar{c}, b\bar{b}, s\bar{s}, \ldots$) or, equivalently, between the various resonant states ($J/\psi, \Upsilon, \phi, \ldots$). While possible this seems highly unlikely. For example, even if all the effective couplings $\tilde{G}_{q_i \bar{q}_i \mu e}$ were equal due to some universality, and the bubble kinematics were identical, the net contribution would not vanish since the total contribution would be proportional to $\Sigma Q_{q_i} \neq 0$. Furthermore, for the case of light quarks such cancellations cannot work even in principle. The s dependence for $0 \leq s \leq m_\mu^2$ neglected above implies, for example, that a ω/ρ and ϕ contribution to the total decay rate cancel only at the level of $(m_\mu^2/3)(m_\phi^2 - m_{\rho/\omega}^2)/(m_\phi^2)^2 \approx 10^{-3}$.

CONCLUDING REMARKS

In general, lepton flavor violating processes have been analyzed within a specific theoretical framework. In this context, the restrictive role played by the low energy bounds ($\mu \to 3e$, $\mu \to e$ conversion, etc.) has been noted by many authors [3,6]. In this note instead we tried to present in a relatively model-independent manner, the connections which unitarity implies between some two-body and three-body LFV decays. We have illustrated these connections by focusing on a few processes. Clearly, many other bounds can be obtained. Indeed, since the Particle Data Group [1] lists altogether about one hundred LFV processes, many additional results can come from a more comprehensive analysis.

We have noted that the bounds that we derived can be avoided if one can kinematically suppress the small s contributions (e.g. by having a purely anomalous magnetic $Z^0 \mu e$ coupling), or as a result of some (rather unlikely) cancellations. Because we cannot rule out these possibilities with absolute certainty, we hope that our discussion will not dissuade future efforts to improve the bounds on LFV decays of the $Z, J/\psi, \ldots$. Such decays would not only signal new LFV physics but, because of our considerations, this physics must also naturally give cancellations among terms so as to lead to a small $\mu \to 3e$ branching ratio. Thus searching for $V_i \to \mu^\pm e^\mp$ decays at levels considerably higher than our suggested bounds remains a worthwhile experimental challenge.

[1] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).

[2] E871 Collaboration, D. Ambrose *et al.*, Phys. Rev. Lett. **81**, 4309 (1998).

[3] J. Illana, M. Jack, and T. Riemann, hep-ph/0001273, and references therein.

[4] ''Probing lepton flavor violation in decays of charmonium and bottomonium systems,'' R. D. Peccei, J. X. Wang, and X. M. Zhang, 1998 note (unpublished).

[5] SuperKamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998).

[6] See, for example, P. Langacker and D. London, Phys. Rev. D **38**, 886 (1988); D. Tommasini *et al.*, Nucl. Phys. **B444**, 451 (1995).