Minimal composite Higgs model with light bosons

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We analyze a composite Higgs model with the minimal content that allows a light standard-model-like Higgs boson, potentially just above the current CERN LEP limit. The Higgs boson is a bound state made up of the top quark and a heavy vector-like quark. The model predicts that only one other bound state may be lighter than the electroweak scale, namely a CP-odd neutral scalar. Several other composite scalars are expected to have masses in the TeV range. If the Higgs boson decay into a pair of CP-odd scalars is kinematically open, then this decay mode is dominant, with important implications for Higgs boson searches. The lower bound on the CP-odd scalar mass is loose, in some cases as low as $\sim$100 MeV, being set only by astrophysical constraints.

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I. INTRODUCTION

The standard model is phenomenologically successful as an effective theory below some energy scale where new degrees of freedom (other than the yet-to-be-discovered Higgs boson) should become relevant. Generic evidence for new physics is provided by the unphysical Landau poles for the quartic, hypercharge and Yukawa couplings within the standard model, and by the existence of the gravitational interactions. Barring an unlikely tuning of the parameters, the scale M_c of new physics that has an impact on the Higgs self-energy should be in the TeV range or below. The nature of this new physics remains unknown, and until experimental evidence for physics beyond the Standard Model will emerge, we should seek plausible explanations or alternatives to the less compelling aspects of the standard model. One such aspect is that the Higgs doublet is an *ad hoc* part of the standard model, which fits well the data but does not have an intrinsic motivation. This remains true for supersymmetric or grand unified extensions of the Standard Model. By contrast, the fermion content of the Standard Model is better motivated, due to the anomaly cancellations and chiral symmetries.

It is therefore useful to investigate the possibility that the Higgs doublet is not a fundamental degree of freedom but rather a bound state that appears only in the effective theory below the scale M_c. Composite models in which the Higgs doublet is made up of some new fermions which belong to chiral representations of the electroweak gauge group have been known for a long time [1]. Currently, the electroweak precision measurements constrain tightly the number of new chiral fermions, so that this type of models is disfavored unless there are non-perturbative effects or other phenomena that reduce the deviations of the electroweak observables.

An economical way of satisfying the constraints from the electroweak data is to bind a Higgs doublet out of the known fermions. The top quark, having the mass close to the electroweak scale, is a prime candidate for a Higgs constituent. However, if the Higgs doublet were a $\overline{t_R} t_L$ bound state, then a fairly reliable relation between the top quark mass, m_t, and the electroweak scale, $v \approx 246$ GeV, can be derived [2–4]. Given the measured value $m_t \approx 175$ GeV, models of this type could produce sufficiently large W and Z masses only if the compositeness scale is exponentially larger than the electroweak scale, and therefore they require fine-tuning.

Thus, it appears necessary that some new states play the role of Higgs constituents. A minimal choice is to introduce a vector-like quark, χ, and non-perturbative four-quark interactions that involve the χ and t. Consequently, the vacuum becomes populated with $\overline{\chi_R} t_L$ virtual pairs which make it opaque to the W and Z, so that the electroweak symmetry is broken. Furthermore, the t and χ mix, allowing $m_t \approx 175$ GeV and a χ mass in the TeV range. This is the top condensation seesaw mechanism [5]. Below the scale of the four-quark operators, the effective theory contains a number of composite scalars, including a CP-even neutral Higgs boson which is mainly a $\overline{\chi_R} t_L$ bound state [6]. This theory has a decoupling limit in which at low energy it behaves as the standard model, and therefore is phenomenologically viable.

The four-quark interactions should be softened at high energy within a renormalizable or finite theory. Examples of this type involve new spontaneously broken gauge symmetries [5–7] or extra dimensions accessible to the gluons [8,9].

In this paper we study in detail a minimal composite model which allows a light standard-model-like Higgs boson. This model is based on the top condensation seesaw mechanism, and the groundwork for its analysis is the effective potential formalism presented in Ref. [6]. Here we focus on the low-energy effective theory and its phenomenological implications.

In Sec. II we discuss the compositeness condition, and we identify a minimal set of ingredients necessary for the existence of a light composite Higgs boson.

In Sec. III we write down the effective potential, and we discuss the Higgs boson spectrum. We establish that, besides the lightest neutral CP-even scalar, the composite Higgs sector may include only one physical state below a scale of order 1 TeV. This is a CP-odd scalar, which we will generically call the composite axion.

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The properties of the composite axion and lightest neutral CP-even Higgs boson are presented in Sec. IV. In Sec. V we turn to the Higgs couplings to the quarks and leptons.

In Sec. VI we study the lower bounds on the composite axion mass. In Sec. VII we compare the minimal composite Higgs model with the minimal supersymmetric standard model, and we make some final remarks on phenomenology. In the Appendix we list the extremization conditions for the effective potential.

II. INGREDIENTS OF A MINIMAL COMPOSITE HIGGS MODEL

The Higgs sector of the standard model depends mainly on three parameters: the Higgs doublet squared-mass, M_H^2, the quartic coupling, λ, and the top Yukawa coupling, $y_t = m_t\sqrt{2}/v \approx 1$. The relevant piece of the Lagrangian at the electroweak scale is given by

$$\mathcal{L}_{\rm SM}(v) = (D^\nu H^\dagger)(D_\nu H) - M_H^2(v) H^\dagger H - \frac{\lambda(v)}{2}(H^\dagger H)^2 - y_t(\overline{\psi_L^3} t_R H + \text{H.c.}), \tag{2.1}$$

where the Lagrangian is defined at the electroweak scale, ψ_L^3 is the top-bottom left-handed doublet, and we have chosen H to have hypercharge $+1$ for convenience. The other couplings are of little relevance for the renormalization group evolution of these three parameters (one possible exception would be large neutrino Yukawa couplings, which in the presence of large Majorana masses yield acceptable neutrino masses; we will not consider this possibility here).

If the Higgs doublet is a bound state with a compositeness scale M_c, then at scales above M_c the Higgs doublet is no longer a physical degree of freedom. Therefore, its kinetic term should vanish at M_c. We will refer to this requirement as the compositeness condition [2,10]. Note that this is equivalent with the statement that all the Higgs parameters blow up at M_c if the kinetic term normalization is fixed.

A. The vector-like quark

The top quark loop correction to the Higgs kinetic term is negative, diminishing the wave function renormalization, suggestive of the possibility that the Higgs doublet is a $\overline{t_R} t_L$ bound state [2–4]. However, the top Yukawa coupling is perturbative, $y_t \approx 1$, and the kinetic term may vanish only if the M_c scale is exponentially higher than the electroweak scale, such that the logarithm overcomes the loop factor. Although a large hierarchy between M_c and the electroweak scale cannot be ruled out, we will ignore this possibility due to a lack of explanation of the exponential fine-tuning required in that case.

Therefore, the compositeness condition requires new physics above the electroweak scale in order to speed up the running of the Higgs parameters. A simple choice is to include a vector-like quark, χ, which has the same transformation properties under the standard model gauge group as t_R, and a mass $m_\chi > v$. This introduces a new Yukawa coupling:

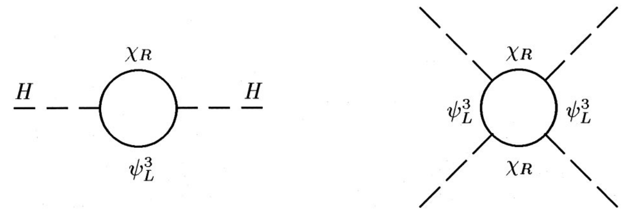

FIG. 1. Large-N_c contributions to the Higgs doublet self-energy and quartic coupling.

$$-\xi(\overline{\psi_L^3}\chi_R H + \text{H.c.}). \tag{2.2}$$

If ξ is sufficiently large, then the χ contribution to the Higgs self-energy may lead to the cancellation of the Higgs kinetic term at a scale M_c which is not hierarchically bigger than m_χ. However, in this case the renormalization group evolution is nonperturbative (the cancellation of the Higgs kinetic term requires the loop expansion parameter to be of order one), and a very precise computation is not within reach. Fortunately, the χ is a color triplet, so that we can use an expansion in $1/N_c$, where N_c is the number of colors. In this case the leading effects of χ on the Higgs parameters are the same as the perturbative one-loop contributions. Although it is hard to estimate precisely how large are the corrections from the non-leading-N_c terms, trading the physical problem of fine-tuning $M_c/m_\chi \gg 1$ for the computational problem at $M_c \sim m_\chi$ seems justified. In practice, these two problems may be balanced by considering a small hierarchy between M_c and m_χ, such that the fine-tuning is not excessive while the ξ Yukawa coupling is not much larger than one.

At a scale $\mu > m_\chi$, the Higgs sector takes the form

$$\mathcal{L}_{\rm SM+\chi}(\mu) = Z_H(\mu)(D^\nu H^\dagger)(D_\nu H) - M_H^2(\mu) H^\dagger H - \frac{\lambda(\mu)}{2}(H^\dagger H)^2 - [\overline{\psi_L^3}(y_t t_R + \xi\chi_R)H + m_\chi \overline{\chi_L}\chi_R + \text{H.c.}], \tag{2.3}$$

while below m_χ the χ is integrated out and we recover the standard model.

A straightforward computation of the one-loop Higgs self-energy and quartic coupling (see Fig. 1) gives

$$Z_H(\mu) = 1 - \frac{N_c \xi^2}{16\pi^2}\ln\left(\frac{\mu^2}{m_\chi^2}\right),$$

$$\lambda(\mu) = \lambda(v) + 2\xi^2[Z_H(\mu) - 1],$$

$$M_H^2(\mu) = M_H^2(v) + \frac{N_c\xi^2}{8\pi^2}(\mu^2 - v^2), \tag{2.4}$$

where we neglected the top-quark contributions. The compositeness condition, $Z_H(M_c) = 0$, yields

$$\xi^2 = \frac{8\pi^2}{N_c \ln(M_c/m_\chi)}. \tag{2.5}$$

Since the ratio M_c/m_χ is unlikely to be exponentially large, it follows that $\xi \gg 1$, suggesting that the Higgs doublet is mainly a $\overline{\chi_R}\psi_L^3$ bound state. But as stated before, keeping a reasonably small hierarchy between M_c and m_χ allows more control over the computation. For example, $M_c/m_\chi \sim 10-100$ gives $\xi \sim 3.4-2.4$.

If we impose $\lambda(\mu)>0$ at all scales below M_c, so that the scalar potential is bounded from below, then the quartic coupling at the electroweak scale,

$$\lambda(v)=\lambda(M_c)+2\xi^2, \tag{2.6}$$

is significantly larger than one, corresponding to a large Higgs boson mass, $v\sqrt{\lambda(v)}$. After the non-leading contributions (from finite-N_c, top quark, electroweak, and QCD effects) are taken into account, we expect the Higgs boson mass to be close to the unitarity bound of 0.7–0.9 TeV.

B. Extending the Higgs sector

So far we have shown that the compositeness condition, $Z_H(M_c)=0$, suffices to prove that the Higgs doublet cannot be a $\overline{t_R}\psi_L^3$ bound state without exponential fine-tuning, while it can be a $\overline{\chi_R}\psi_L^3$ bound state provided the Higgs boson is quite heavy.

Next we would like to identify the circumstances which allow the composite Higgs boson to be light, close to the current experimental bounds. The large quartic coupling is a rather generic feature of a composite Higgs sector. However, only in the standard model the Higgs boson mass is straightforwardly determined by the quartic coupling. For extended Higgs sectors, the mixing between different CP-even scalars may drive the lightest neutral Higgs boson significantly below the standard model unitarity bound. In order to allow a large scalar mixing, the constituents of the composite Higgs sector should mix themselves. For the minimal fermion content, i.e. three generations of quarks and leptons plus the vector-like quark χ, the only fields that may have large mixings with the χ_R and ψ_L^3 are the t_R and χ_L.

Therefore, we will consider a composite Higgs sector which involves four scalar fields: two weak-doublets, $H_\chi \sim \overline{\chi_R}\psi_L^3$ and $H_t \sim \overline{t_R}\psi_L^3$, and two weak-singlets, $\phi_{\chi t}\sim \overline{t_R}\chi_L$ and $\phi_{\chi\chi}\sim\overline{\chi_R}\chi_L$. Note that the case where one of the fermion fields is not a Higgs constituent can be recovered by taking the masses of the corresponding two scalars to infinity, but in that case the Higgs boson is heavy [9].

For an extended Higgs sector, a natural formulation of the compositeness condition is that all scalar kinetic terms vanish at the same scale. In the large-N_c limit, the only contribution to a scalar kinetic term comes from the fermions with large Yukawa couplings to that composite scalar, namely from its constituents. Hence, the chiral symmetry of the constituents, $U(3)_L\times U(2)_R$, is preserved by the Yukawa couplings of the composite scalars:

$$\xi(\overline{\psi_L^3},\ \overline{\chi_L})\Phi\begin{pmatrix} t_R \\ \chi_R\end{pmatrix}+\text{H.c.} \tag{2.7}$$

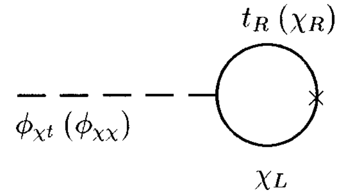

FIG. 2. Tadpole terms for the electroweak singlet scalars.

where the scalar Φ is a 3×2 complex matrix,

$$\Phi=\begin{pmatrix} H_t & -H_\chi \\ \phi_{\chi t} & \phi_{\chi\chi}\end{pmatrix}, \tag{2.8}$$

with the phase of H_χ chosen negative for later convenience. Note that the $SU(2)_W\times U(1)_Y$ electroweak symmetry is a gauged subgroup of this chiral symmetry.

Likewise, the leading-N_c contributions to the running of the quartic couplings between the scales v and μ is $U(3)_L\times U(2)_R$ symmetric:

$$\mathcal{L}_{\text{quartic}}(\mu)=\mathcal{L}_{\text{quartic}}(v)-\frac{\lambda(v)}{2}\text{Tr}[(\Phi^\dagger\Phi)^2]. \tag{2.9}$$

There are no other $U(3)_L\times U(2)_R$ symmetric terms in the scalar potential.

Since the χ quark is vector-like, and transforms under the standard model gauge group as the t_R, we can write two gauge invariant mass terms:

$$\mu_{\chi t}\overline{\chi_L}t_R+\mu_{\chi\chi}\overline{\chi_L}\chi_R+\text{H.c.} \tag{2.10}$$

These break explicitly the chiral symmetry down to $SU(2)_W\times U(1)_Y\times U(1)_B$, where the last group refers to a global baryon number. The effect of these explicit mass terms is to induce tadpole terms for the weak-singlet scalars in the effective potential (see Fig. 2):

$$-(C_{\chi t}\phi_{\chi t}+C_{\chi\chi}\phi_{\chi\chi}+\text{H.c.}). \tag{2.11}$$

The tadpole coefficients may be estimated by cutting off the loop integral at M_c. For $\mu_{\chi t,\,\chi\chi}\ll M_c$,

$$C_{\chi t,\,\chi\chi}\approx\frac{N_c\xi}{8\pi^2}\mu_{\chi t,\,\chi\chi}M_c^2. \tag{2.12}$$

Another effect of these explicit mass terms is to induce trilinear scalar terms proportional with $\mu_{\chi t,\,\chi\chi}$, due to the large-N_c running between the scales v and M_c.

A generic high energy theory at the scale M_c gives rise to the most general mass terms for the composite scalars, which also break explicitly the $U(3)_L\times U(2)_R$ chiral symmetry down to $SU(2)_W\times U(1)_Y\times U(1)_B$. Putting together all these terms, the scalar potential for the two-doublet-two-singlet composite Higgs sector has all possible gauge invariant terms and is hard to analyze. In order to progress we need to make some assumptions about the high energy theory that is responsible for binding together the χ, t_R and ψ_L^3 within the composite scalars.

First, we can invoke a small hierarchy between the compositeness scale and the masses of the composite scalars, as mentioned in Sec. II A. As a result, the trilinear, quartic and higher-dimensional Higgs couplings at the M_c scale are small, suppressed by powers of the M_c/m_χ ratio. Second, we will see in Sec. III C that the sector of the high-energy theory responsible for binding the composite Higgs sector is likely to preserve a global $U(1)_t\times U(1)_\chi\times U(1)_B$ subgroup of the chiral symmetry. This symmetry precludes the presence of mass terms that mix the doublets or the singlets in the effective potential. The quark charges under this symmetry are determined only up to a unitary transformation. A simple basis is that where only t_R and χ_R are charged under $U(1)_t$ and $U(1)_\chi$, respectively. One linear combination of these two $U(1)$'s has an axial QCD anomaly, but this effect may be neglected as we will argue in Sec. VI.

With these assumptions, one can easily integrate out the composite scalars at scales above M_c, where they can be treated as non-propagating (spurion) fields. This bottom-up approach results in the following four-quark operators at the scale M_c:

$$\mathcal{L}_{\rm eff}=\frac{g^2_{\psi\chi}}{M_c^2}(\overline{\psi}_L^3\chi_R)(\overline{\chi}_R\psi_L^3)+\frac{g^2_{\psi t}}{M_c^2}(\overline{\psi}_L^3 t_R)(\overline{t}_R\psi_L^3)+\frac{g^2_{\chi t}}{M_c^2}(\overline{\chi}_L t_R)\times(\overline{t}_R\chi_L)+\frac{g^2_{\chi\chi}}{M_c^2}(\overline{\chi}_L\chi_R)(\overline{\chi}_R\chi_L). \tag{2.13}$$

Altogether, there are seven parameters: the four coefficients of the above operators, the two masses ($\mu_{\chi\chi}$ and $\mu_{\chi t}$), and the overall scale M_c. The effective potential below the scale M_c is sufficiently simple to be analyzed analytically. Before doing so in Sec. III, we will argue in the remainder of this section that the assumptions made here are realistic.

C. Candidates for physics above the compositeness scale

The basic assumption we are making for an extended composite Higgs sector is that all scalar kinetic terms vanish at the same scale, referred to as the compositeness scale M_c. Therefore, the composite scalars are no longer physical degrees of freedom and they should be integrated out above M_c. This gives rise to higher-dimensional operators which at high-energy should be replaced by a renormalizable or finite theory.

A conspicuous direction for seeking such a high-energy theory is to consider some new gauge dynamics which binds the t and χ within the composite scalars. Such dynamics cannot be confining because the top has already been observed by the CDF and DØ Collaborations. On the other hand, the new gauge interactions have to be rather strongly coupled at the compositeness scale in order to deeply bind the Higgs doublets and trigger the electroweak phase transition. Therefore, unless the new physics is very unconventional right above the compositeness scale, the new gauge interactions must be asymptotically free. These requirements single out spontaneously broken non-Abelian gauge theories.

The choice of a gauge group is further restricted if no new chiral generations of fermions are introduced. The representations of the new non-Abelian gauge group may coincide with those of $SU(3)_C$, as in top color [11], or may correspond to some flavor or family symmetry [7].

The compositeness scale is approximately given by the masses of the heavy gauge bosons. Below M_c, the gauge bosons are integrated out resulting in higher-dimensional operators, including those listed in Eq. (2.13). The four-quark operators with left-left or right-right current-current structure do not contribute to the effective potential in the large-N_c limit (though they do contribute to observables, most importantly to the ρ parameter [12,13], but these contributions are sufficiently small for M_c above a few TeV [14]). All other four-quark operators which are invariant under the standard model gauge group and involve only the ψ_L, t_R and χ fields violate the global $U(1)_t\times U(1)_\chi$ symmetry, and are not expected to be induced by heavy gauge boson exchanges.

Operators of dimension-8 or higher are also induced by the gauge dynamics. However, their effects are negligible at scales significantly below M_c. Therefore it is convenient to ignore them by arranging a small hierarchy between M_c and the composite scalar masses. Such a hierarchy arises if there is a second order phase transition in which a continuous variation of the gauge coupling induces a continuous variation of the scalar masses. There are various arguments, based in general on the large-N_c limit, indicating that a spontaneously broken gauge group leads indeed to a second order phase transition [13,15]. In practice, since we do not require an exponential hierarchy, it is sufficient to have a weakly first order phase transition.

The new gauge dynamics should be flavor dependent, so that only the top and χ acquire large masses. This can be realized in various ways. The strongly coupled gauge interaction may act only on the third generation quarks and on the χ, while the splitting between the χ, t, b masses may be given by some perturbative interactions. Examples of this type have been given in [5,6]. Alternatively, the strongly coupled gauge interaction may be flavor universal, with the flavor breaking provided by an extended vector-like quark sector [7].

Above the M_c scale there must be some additional physics that leads to the spontaneously breaking of the non-Abelian gauge symmetry responsible for Higgs compositeness. This may involve new gauge dynamics, or fundamental scalars and supersymmetry. Yet another alternative may be provided by quantum gravitational effects if gravity is modified at short distance [16–18] such that it becomes strong at a scale in the multi-TeV range, not far above M_c.

Instead of a new gauge symmetry, the binding of the Higgs sector may be produced by the standard model gauge bosons propagating in extra dimensions [8,9] of radius $1/M_c$. Basically, the exchange of Kaluza-Klein modes of the gluons induces four-quark operators of the type (2.13). Although the Kaluza-Klein modes are weakly coupled at a TeV scales, the combined effect of all modes is nonperturbative. This is also consistent with gauge coupling unification at a scale below 100 TeV [19], albeit the theoretical uncertainties are somewhat larger than in the minimal supersymmetric standard model.

Moreover, the extra dimensions allow new explanations for flavor symmetry breaking, such as flavor-dependent positions of the fermions in the extra dimensions [20], messengers of flavor-breaking propagating in the bulk [21], or exponential suppressions due to renormalization effects [22]. The extra dimensions could also provide a natural reason for the existence of the vector-like quark, because the χ has the same gauge quantum numbers as the Kaluza-Klein modes of the t_R. This idea however requires further study.

The gauge theories in extra dimensions are non-renormalizable, so that new physics should soften the interactions at a scale close to the compactification scale. This physics may be based on an underlying theory that includes quantum gravity, such as string or M theory. Alternatively, a physical cutoff to the interactions of the Kaluza-Klein modes could be set by the brane recoil [23], potentially allowing the fundamental (string) scale to be substantially higher than the compactification scale.

III. THE TWO-DOUBLET-TWO-SINGLET HIGGS SECTOR

In this section we study the composite Higgs sector which includes two weak-doublets, H_t and H_χ, and two weak-singlets, $\phi_{\chi t}$ and $\phi_{\chi\chi}$. The effective potential is determined based on the following four assumptions discussed in Sec. II B:

(1) The compositeness condition: the kinetic terms of all composite scalars vanish at the same scale M_c.

(2) There is a separation between the compositeness scale and the scalar masses.

(3) The interactions which bind the composite scalars preserve the $U(1)_t\times U(1)_\chi$ chiral symmetry of the t_R and χ_R quarks.

(4) The large-N_c limit is a reasonable approximation for computing the effects of the strong dynamics responsible for compositeness.

The effective potential below the compositeness scale is given by

$$\begin{aligned}V=&\frac{\lambda}{2}[(H_t^\dagger H_t+\phi_{\chi t}^\dagger\phi_{\chi t})^2+(H_\chi^\dagger H_\chi+\phi_{\chi\chi}^\dagger\phi_{\chi\chi})^2\\&+2|H_t^\dagger H_\chi-\phi_{\chi t}^\dagger\phi_{\chi\chi}|^2]+M_{tt}^2H_t^\dagger H_t+M_{t\chi}^2H_\chi^\dagger H_\chi\\&+M_{\chi t}^2\phi_{\chi t}^\dagger\phi_{\chi t}+M_{\chi\chi}^2\phi_{\chi\chi}^\dagger\phi_{\chi\chi}+(C_{\chi t}\phi_{\chi t}+C_{\chi\chi}\phi_{\chi\chi}+\text{H.c.}).\end{aligned}\tag{3.1}$$

The seven parameters listed at the end of Sec. II B have been replaced by four real squared-mass parameters, two tadpole coefficients (chosen positive), and the quartic coupling. With the exception of λ which can be computed in the large-N_c limit and depends only logarithmically on M_c, the other parameters are essentially free, and remain to be determined within the underlying theory above the compositeness scale.

The effective potential is $SU(2)_W\times U(1)_Y$ and CP invariant, and has a $U(1)_t\times U(1)_\chi$ global symmetry softly broken by the tadpole terms. The tadpole terms also force the $\phi_{\chi t}$ and $\phi_{\chi\chi}$ to have non-zero vacuum expectation values (VEVs). For $M_{t\chi}^2<0$, there is a range of parameters where the H_χ doublet has a non-zero VEV, breaking the electroweak symmetry. In that case, the third term of V provides a tadpole for H_t, which acquires a VEV too. Finally, the condition $M_{tt}^2>0$ is sufficient to keep the VEVs of the two doublets aligned, leaving the photon massless:

$$H_t=\begin{pmatrix}\frac{1}{\sqrt{2}}[v\cos\beta+h_{tt}^0+i(A^0\sin\beta-G^0\cos\beta)]\\ H^-\sin\beta-G^-\cos\beta\end{pmatrix},$$

$$H_\chi=\begin{pmatrix}\frac{1}{\sqrt{2}}[v\sin\beta+h_{t\chi}^0-i(A^0\cos\beta+G^0\sin\beta)]\\ -(H^-\cos\beta+G^-\sin\beta)\end{pmatrix},$$

$$\phi_{\chi t}=\frac{1}{\sqrt{2}}\left(-\frac{v\sin\beta}{\epsilon\tan\gamma}+h_{\chi t}^0+iA_{\chi t}^0\right),$$

$$\phi_{\chi\chi}=\frac{1}{\sqrt{2}}\left(-\frac{v}{\epsilon}\sin\beta+h_{\chi\chi}^0+iA_{\chi\chi}^0\right),\tag{3.2}$$

where we have written the VEVs in terms of the electroweak scale, fixed at $v\approx 246$ GeV, and three other parameters: $\beta,\gamma\in(0,\pi/2)$ and $\epsilon>0$. These VEVs are related to the parameters in the effective potential by the extremization conditions listed in the Appendix. Note that the phases of the VEVs for $\phi_{\chi t}$ and $\phi_{\chi\chi}$ are fixed by the tadpole terms, the relative phase of the VEVs for H_t and H_χ is fixed by the third term in Eq. (3.1), and the phase of H_χ has been chosen in the Yukawa couplings (2.7).

$G^\pm$ and G^0 are the Nambu-Goldstone bosons that become the longitudinal W and Z. Altogether there are nine massive degrees of freedom, which are characterized by their electric charge and CP-parity: two charged states $H^\pm$, three CP-odd neutral scalars A^0, $A_{\chi t}^0$, $A_{\chi\chi}^0$, and four CP-even neutral scalars, h_{tt}^0, $h_{t\chi}^0$, $h_{\chi t}^0$ and $h_{\chi\chi}^0$. Before dissecting their spectrum, let us discuss the constraints on the parameter space.

The H_χ doublet contributes more to the electroweak symmetry breaking than H_t (this is the motivation for introducing the vector-like quark), so that $\tan\beta>1$. Due to the Yukawa couplings of the scalars to their constituents [see Eq. (2.7)], the t and χ mix, with a mass matrix

$$\frac{\xi v\sin\beta}{\epsilon\sqrt{2}}(\overline{t_L},\overline{\chi_L})\begin{pmatrix}-\epsilon\cot\beta & \epsilon\\ \cot\gamma & 1\end{pmatrix}\begin{pmatrix}t_R\\ \chi_R\end{pmatrix},\tag{3.3}$$

where the Yukawa coupling ξ is given by Eq. (2.5) in the large-N_c limit. We are interested in the case where the χ is heavier than the top, so that the corrections to the electroweak observables are small. This implies $\epsilon<1$. In what follows we will often consider the limit in which χ decouples, i.e. $\epsilon\ll 1$. The physical top quark is the light mass eigenstate of the above matrix:

$$m_t \approx \frac{\xi v}{\sqrt{2}}\sin(\beta+\gamma)[1+\mathcal{O}(\epsilon^2)]. \tag{3.4}$$

Therefore, $\sin(\beta+\gamma)\approx 1/\xi$. For $\xi^2 \gg 1$ one has $\tan\beta, \tan\gamma \gg 1$. More generally, we allow $\beta,\gamma \in (\pi/4, \pi/2)$, which also satisfies the $\cos(\beta+\gamma)<0$ restriction imposed by the extremization condition written in the Appendix. Finally, the quartic coupling at the electroweak scale is related to the Yukawa coupling by $\lambda \approx 2\xi^2$ in the large-N_c limit, because the quartic coupling at the compositeness scale is assumed to be negligible [see Eq. (2.6)].

Let us proceed with the computation of the scalar spectrum. The charged Higgs boson, $H^\pm$, has a mass

$$M_{H^\pm}^2 = \frac{\lambda}{2} v^2 \left(\frac{\tan\beta}{\epsilon^2 \tan\gamma} - 1 \right). \tag{3.5}$$

This sets the scale for the heavy composite scalars. In addition, this is roughly the scale for the vector-like quark, whose mass is given by

$$m_\chi = M_{H^\pm} \sqrt{\frac{\sin 2\beta}{2\sin 2\gamma}}[1+\mathcal{O}(\epsilon^2)]. \tag{3.6}$$

A. CP-odd neutral scalars

From the effective potential one can find the squared-mass matrix for the three CP-odd neutral scalars, A^0, $A^0_{\chi t}$, and $A^0_{\chi\chi}$:

$$\left(M_{H^\pm}^2 + \frac{\lambda v^2}{2} \right)\left(1+\epsilon^2 \frac{\cos^2\beta}{\cos^2\gamma} \right) U_0 \mathrm{diag}(1,0,0) U_0^\dagger + \frac{\sqrt{2}\epsilon}{v\sin\beta}\mathrm{diag}(0, C_{\chi t}\tan\gamma, C_{\chi\chi}). \tag{3.7}$$

Up to an $SU(2)$ transformation, the matrix

$$U_0 = \begin{pmatrix} \cos\theta_1\cos\theta_2 & -\sin\theta_1 & \cos\theta_1\sin\theta_2 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1 & \sin\theta_1\sin\theta_2 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix}, \tag{3.8}$$

would define the mixing angles if the $C_{\chi t}$ and $C_{\chi\chi}$ were zero (we will see below that this would not be a viable case). The angles $\theta_1, \theta_2 \in (0, \pi/2)$ are given by

$$\tan\theta_1 = \epsilon\cos\beta\tan\gamma,$$

$$\tan\theta_2 = \epsilon\cos\beta\cos\theta_1. \tag{3.9}$$

The mass matrix of the CP-odd states is diagonal in the $\epsilon \to 0$ limit. Therefore, we can diagonalize it by expanding in ϵ, and we obtain the following squared-masses of the CP-odd neutral scalars:

$$M_{A_1^0}^2 = M_{H^\pm}^2 \left\{ 1+\epsilon^2 \left[\cos\beta(c_A\tan\gamma + c_A') + \frac{\tan\gamma}{\tan\beta} \right] + \mathcal{O}(\epsilon^4) \right\},$$

$$M_{A_t^0}^2 = C_{\chi t}\frac{\sqrt{2}\epsilon\tan\gamma}{v\sin\beta}[1-\epsilon^2 c_A \tan\gamma\cos\beta + \mathcal{O}(\epsilon^4)],$$

$$M_{A_\chi^0}^2 = C_{\chi\chi}\frac{\sqrt{2}\epsilon}{v\sin\beta}[1-\epsilon^2 c_A'\cos\beta + \mathcal{O}(\epsilon^4)]. \tag{3.10}$$

The two dimensionless coefficients

$$c_A \equiv \frac{\cos\beta\tan\gamma}{1-(C_{\chi t}/C_0)\tan\gamma},$$

$$c_A' \equiv \frac{\cos\beta}{1-(C_{\chi\chi}/C_0)}, \tag{3.11}$$

where

$$C_0 \equiv \frac{\lambda v^3}{2\sqrt{2}\epsilon^3}\frac{\tan\beta}{\tan\gamma}\sin\beta, \tag{3.12}$$

are defined such that the mass eigenstates of the CP-odd neutral scalars take a simple form:

$$A_1^0 = A^0 + \epsilon(c_A A^0_{\chi t} - c_A' A^0_{\chi\chi}) + \mathcal{O}(\epsilon^2)$$

$$A_t^0 = A^0_{\chi t} - \epsilon c_A A^0 + \mathcal{O}(\epsilon^2)$$

$$A_\chi^0 = A^0_{\chi\chi} + \epsilon c_A' A^0 + \mathcal{O}(\epsilon^2). \tag{3.13}$$

The A_1^0 state is included predominantly in the Higgs doublets. In the small-ϵ limit, it belongs to a linear combination of Higgs doublets, namely $(-H_t\sin\beta + H_\chi\cos\beta)$, whose VEV vanishes. The other states of this linear combination are the charged Higgs and a CP-even neutral scalar. The degeneracy of these states is lifted only by electroweak symmetry breaking effects. As a result, the mass splittings among these states are proportional with $v^2/M_{H^\pm}^2 \sim \epsilon^2$, which explains the first equation in Eq. (3.10).

The A_t^0 and A_χ^0 are predominantly the imaginary parts of the weak-singlet fields. They are the Nambu-Goldstone bosons associated with the $U(1)_t \times U(1)_\chi$ global symmetry of the effective potential, which is spontaneously broken at a scale of order v/ϵ. The tadpole terms from the effective potential break the $U(1)_t \times U(1)_\chi$ symmetry explicitly, so that the A_t^0 and A_χ^0 acquire squared-masses proportional to $C_{\chi t}$ and $C_{\chi\chi}$, respectively. This explains the second and third equations in Eq. (3.10).

B. CP-even neutral scalars

The composite Higgs sector includes four CP-even neutral scalars. The two states belonging to the Higgs doublets, $h^0_{tt}, h^0_{t\chi}$, do not mix in the $\epsilon \to 0$ limit with the two states from the Higgs singlets, $h^0_{\chi\chi}$, $h^0_{\chi\chi}$. This allows us to identify immediately the mass eigenstates to leading order in ϵ. As a result we learn that it is convenient to write the squared-mass matrix for CP-even neutral scalars (without expanding in ϵ) in the basis $(-\sin\beta\, h^0_{tt} + \cos\beta\, h^0_{t\chi})$, $(\cos\beta\, h^0_{tt} + \sin\beta\, h^0_{t\chi})$, $h^0_{\chi t}$ and $h^0_{\chi\chi}$:

$$\mathcal{M}_h^2=\frac{\lambda v^2}{2\epsilon^2}\begin{pmatrix} \mathrm{diag}\left(\frac{\tan\beta}{\tan\gamma},2\epsilon^2\right) & \epsilon\mathcal{B} \\ \epsilon\mathcal{B}^\top & \Gamma\,\mathrm{diag}\left(\frac{\sin 2\beta}{\sin 2\gamma}\epsilon^2, 2\frac{\sin^2\beta}{\sin^2\gamma}\right)\Gamma^\top \end{pmatrix}+\frac{\sqrt{2}\epsilon}{v\sin\beta}\mathrm{diag}(0,0,C_{\chi t}\tan\gamma, C_{\chi\chi}) \tag{3.14}$$

where the 2×2 matrix $\mathcal{B}$ depends only on β and γ,

$$\mathcal{B}=\frac{\sin\beta}{\sin\gamma}\begin{pmatrix} \sin(2\beta+\gamma) & \cos(2\beta+\gamma) \\ -2\cos\beta\cos(\beta+\gamma) & 2\sin\beta\cos(\beta+\gamma)\end{pmatrix}, \tag{3.15}$$

and Γ is a unitary matrix,

$$\Gamma=\begin{pmatrix} \sin\gamma & \cos\gamma \\ -\cos\gamma & \sin\gamma \end{pmatrix}. \tag{3.16}$$

From the mass matrix it can be seen that one of the mass eigenstates, H_1^0, is predominantly the $(-\sin\beta\, h_{tt}^0+\cos\beta\, h_{t\chi}^0)$ state. Up to mixings of order ϵ, H_1^0 forms together with A_1^0 and $H^\pm$ a weak-doublet with a zero VEV. As discussed in the case of A_1^0, the mass splitting among these states are given by electroweak symmetry breaking effects which show up in the heavy scalar spectrum only at order ϵ^2:

$$M_{H_1^0}^2=M_{H^\pm}^2[1+\mathcal{O}(\epsilon^2)]. \tag{3.17}$$

This can also be checked directly from the expression for $\mathcal{M}_h^2$.

Two other scalars are linear combinations of the real parts of the weak-singlets and order ϵ admixtures of the neutral components of the weak-doublets. The third and fourth lines and rows of $\mathcal{M}_h^2$ give their masses to leading order in ϵ^2:

$$M_{H_{2,3}^0}^2=M_{H^\pm}^2\frac{\sin 2\beta}{\sin 2\gamma}\left[1+x_t+x_\chi\pm\sqrt{(1+x_t-x_\chi)^2-4(x_t-x_\chi)\sin^2\gamma}\,\right][1+\mathcal{O}(\epsilon^2)], \tag{3.18}$$

where we used the notation

$$x_{t,\chi}\equiv\frac{\sin 2\gamma}{2\sin 2\beta}\,\frac{M_{A_{t,\chi}^0}^2}{M_{H^\pm}^2}>0. \tag{3.19}$$

It is straightforward to check that $M_{H_{2,3}^0}^2$ are positive everywhere within the parameter space.

The only remaining mass eigenstate, which we label h^0, is predominantly $(\cos\beta\, h_{tt}^0+\sin\beta\, h_{t\chi}^0)$. Its mass cancels at leading order in ϵ^2. This is due to the fact that h^0 sits mainly in the only combination of weak-doublets which breaks the electroweak symmetry. Hence, for fixed v the h^0 mass does not depend on ϵ, whereas the other scalar masses scale as $1/\epsilon$. To compute M_{h^0} we have to go to the next-to-leading order in the ϵ expansion. Fortunately, we do not need to diagonalize the 4×4 $\mathcal{M}_h^2$ matrix. It is sufficient to compute the determinant of the mass matrix, and then to use

$$\mathrm{Det}\mathcal{M}_h^2=M_{h^0}^2M_{H_1^0}^2M_{H_2^0}^2M_{H_3^0}^2. \tag{3.20}$$

The result is fairly simple:

$$M_{h^0}^2=\lambda v^2\left[1-\cos^2(\beta+\gamma)\frac{x_t\sin^2\beta+x_\chi\cos^2\beta+\sin^2(\beta+\gamma)}{x_tx_\chi+x_t\sin^2\gamma+x_\chi\cos^2\gamma}+\mathcal{O}(\epsilon^2)\right]. \tag{3.21}$$

The mass eigenstate corresponding to this eigenvalue may also be derived by expanding in powers of ϵ:

$$h^0=\sin\beta\; h_{t\chi}^0+\cos\beta\; h_{tt}^0+\epsilon(c_h h_{\chi t}^0+c_h' h_{\chi\chi}^0)+\mathcal{O}(\epsilon^2), \tag{3.22}$$

where the two dimensionless coefficients are defined by

$$\begin{pmatrix} c_h \\ c_h' \end{pmatrix}=\frac{\sin\gamma\,\cos(\beta+\gamma)}{\sin\beta(x_tx_\chi+x_t\sin^2\gamma+x_\chi\cos^2\gamma)}\left[\begin{pmatrix} x_\chi\cos\beta \\ -x_t\sin\beta\end{pmatrix}+\sin(\beta+\gamma)\begin{pmatrix}\sin\gamma \\ -\cos\gamma\end{pmatrix}\right]. \tag{3.23}$$

We are now in a good position for discussing the vacuum stability. The extremum conditions written down in the Appendix are automatically satisfied because they have been used to replace the four mass-squared parameters from the effective potential with four new parameters. Therefore, the vacuum defined by Eq. (3.2) is a local minimum if and only if all four eigenvalues of $\mathcal{M}_h^2$ are positive. We have seen that three of them, $M^2_{H^0_1}$, $M^2_{H^0_2}$, and $M^2_{H^0_3}$ are always positive. The only remaining condition, $M^2_{h^0}>0$, is restrictive. For example, if both $M_{A^0_t}$ and $M_{A^0_\chi}$ were of order v or lighter, then x_t and x_χ were of order ϵ^2, and $M^2_{h^0}$ would be negative. Thus, at most one of $M_{A^0_t}$ and $M_{A^0_\chi}$ may be as light as the electroweak scale, the other one having a mass of order v/ϵ or larger.

Imposing $M^2_{h^0}>0$ ensures that the vacuum that we study is a local minimum of the potential, but not necessarily a global minimum. An inspection of the extremum conditions shows that there is only one other candidate for a global minimum, namely that obtained by taking $v\to 0$ and $v/\epsilon>0$. This is easy to understand, because the tadpole terms always give rise to VEVs for $\phi_{\chi t}$ and $\phi_{\chi\chi}$. It is clear that the $v>0$ minimum which we study here is deeper than the $v=0$ extremum for sufficiently large and negative values of $M^2_{t\chi}$. It seems hard to compute analytically the critical value for $M^2_{t\chi}$, so that we do not derive the condition for having a completely stable vacuum. Note however that even a local minimum is likely to be very long lived, barriers with sizes of order TeV implying lifetimes typically longer than the age of the universe [24].

IV. LIGHT BOSON SPECTRUM

In the previous section we have seen that the charged Higgs, three of the CP-even neutral scalars and one CP-odd neutral scalar are always heavy, with masses of order $\sqrt{\lambda}v/\epsilon$, in the TeV range. The only remaining physical states are the CP-even h^0, and the CP-odd A^0_t and A^0_χ. Furthermore, the vacuum stability condition implies that only one of A^0_t and A^0_χ may have a mass of order v or smaller. Therefore, there are three possible contents for the composite Higgs spectrum below a TeV scale:

(1) Only the h^0;
(2) the h^0 and A^0_t;
(3) the h^0 and A^0_χ.

In this section we analyze these cases in turn.

A. Standard model in the decoupling limit

If the only scalar lighter than a scale of order 1 TeV is the CP-even Higgs boson, h^0, then the low energy theory has precisely the standard model field content. The corrections due to the heavier states are of order ϵ^2. Therefore, the standard model is obtained in the decoupling limit where $\epsilon\ll 1$. However, in practice ϵ cannot be smaller than one by many orders of magnitude if we want to avoid an exponential fine-tuning. Note that the mass terms in the effective potential have coefficients of order $M^2_{H^\pm}$, which is larger than v^2 by a factor $1/\epsilon^2$. This means that the extremization conditions listed in the Appendix require a fine-tuning of order ϵ^2.

Due to the current agreement of the standard model to the experimental data, it follows that the minimal composite Higgs model discussed in this paper is viable for small ϵ. The strongest bound, $\epsilon\lesssim 0.2$ comes from the ρ parameter, which receives corrections due to the t-χ mixing [5,6]. This bound is loose enough to avoid worrisome fine-tuning, but sufficient to make the decoupling limit a reasonable approximation.

Since the standard model is the decoupling limit of an underlying theory with dynamical electroweak symmetry breaking, the Higgs boson mass is a function of the parameters of the high energy theory. Hence, one has to check whether there are restrictions on the Higgs boson mass in addition to the usual standard model upper bounds from unitarity and triviality, and the lower bounds from direct searches. Note that the indirect upper bound on M_h^2 from the electroweak data is not constraining unless the scale of new physics is very high [25]. Also, the constraint from vacuum stability at large field is easily relaxed in the presence of new physics [26].

From the expression for $M^2_{h^0}$ in Eq. (3.21) it is clear that the upper end of the standard model range can be reached when $x_t, x_\chi\gg 1$, which corresponds to large values for $\mu_{\chi t}$ and $\mu_{\chi\chi}$. By reducing x_t and x_χ continuously we can cover the whole mass range of the standard model Higgs boson.

It is useful to find out in more detail the situations in which M_{h^0} may be as light as $\mathcal{O}(100)$ GeV. To this end, we would like to express M_{h^0} in terms of the parameters of the effective potential. For simplicity we will consider the "seesaw limit," $\tan\beta\gg 1$, in which only the H_χ doublet is responsible for the bulk of electroweak symmetry breaking, and the top mass is produced almost entirely via the seesaw mechanism. To leading order in $1/\tan\beta$ and ϵ^2, the Higgs boson squared-mass takes the form

$$M^2_{h^0}=\frac{2v^2}{M^2_{\chi\chi}-3M^2_{t\chi}}\left[\xi^2(M^2_{\chi\chi}-M^2_{t\chi})+\frac{2M^4_{t\chi}}{M^2_{\chi\chi}-3M^2_{t\chi}}\right.$$
$$\left.\times\left(1+\frac{4M^4_{t\chi}}{M^2_{\chi t}-M^2_{t\chi}}\right)+\mathcal{O}\left(\frac{1}{\xi^2}\right)\right]. \quad (4.1)$$

In deriving this equation we have used $\cos\gamma\approx 1/\xi<1$, which follows from the expression (3.4) for the top quark mass, and $\lambda=2\xi^2$. The leading order in $1/\xi^2$ has been derived previously in [6], where it is argued that there may be natural situations in which the underlying theory above the compositeness scale dictates a partial cancellation between $M^2_{\chi\chi}$ and $M^2_{t\chi}$, making a light composite Higgs boson a distinct possibility. In practice it is sufficient that this cancellation is of order $1/\xi^2\sim 10\%$. To see this, let us define a parameter $d_M\sim\mathcal{O}(1)$ by

$$\frac{M^2_{\chi\chi}}{M^2_{t\chi}}=1+\frac{d_M}{\xi^2}, \quad (4.2)$$

and assume for simplicity that $M^2_{\chi t} \gg M^2_{t\chi}$. The $M^2_{h^0}$ dependence on d_M,

$$M^2_{h^0} \approx (1-d_M)v^2, \tag{4.3}$$

shows that the Higgs boson mass can easily be below the electroweak scale in this case.

One may wonder how large are the radiative corrections to the Higgs boson mass. In fact we have already included the leading large-N_c loop corrections when we derived the effective potential. The corrections from the quartic and trilinear scalar couplings are in general significant given that the quartic coupling in the effective potential is large. However, these contributions are of order $1/N_c$ compared to the ones we included, and we will assume that their effects do not change qualitatively our results.

Although in the decoupling limit the low energy effective theory looks like the standard model, the minimal composite Higgs model has a distinctive feature: the trilinear and quartic Higgs boson couplings are large and rather independent of the Higgs boson mass. The quartic coupling is given by $\lambda/8$ while the trilinear coupling is $\sim \lambda v/2$. If the Higgs boson will be discovered, it is conceivable that its trilinear coupling will be measured at futures colliders [27], and therefore the minimal composite Higgs model will be tested even if all other composite states happen to be heavier than the reach of those collider experiments.

B. Light top-axion

If the amount of $U(1)_t$ explicit symmetry breaking is small, namely $C_{\chi t} \ll |\langle \phi_{\chi t}\rangle|^3$, then the A^0_t is much lighter than the $H^\pm$. From Eq. (2.12) we find that the A^0_t has a mass of the order of the electroweak scale or below for

$$\mu_{\chi t} \lesssim \frac{v^3}{2\pi \epsilon M_c^2}. \tag{4.4}$$

In the limit where $\mu_{\chi t}=0$, the A^0_t receives a small mass only from the QCD anomaly. Although such an extreme case is ruled out (see Sec. VI), we will refer to A^0_t as the ''composite top-axion,'' because it couples to the right-handed top.

The h^0 has a large trilinear coupling to A^0_t pairs:

$$\mathcal{L}_3 \approx \frac{\lambda}{2} v a_t\, h^0 (A^0_t)^2. \tag{4.5}$$

At tree level, $a_t \sim \epsilon^2$ because to leading order in ϵ, the h^0 belongs to the Higgs doublets, whereas the A^0_t is part of the $\phi_{\chi t}$ singlet. There are however large one-loop contributions, as shown in Fig. 3, due to the $\lambda \gg 1$ quartic coupling. These give $a_t \sim 1/N_c$. The contributions from more loops which involve the quartic coupling are suppressed by more powers of $1/N_c$, so are unlikely to change the order of magnitude of a_t.

This large trilinear coupling is very important for Higgs boson searches if $M_{h^0} > 2M_{A^0_t}$. The width for the Higgs boson decay into top-axion pairs,

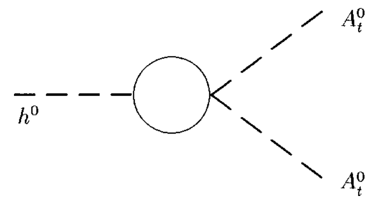

FIG. 3. One-loop contribution of the heavy scalars to the trilinear coupling of the Higgs boson to composite-axion pairs.

$$\Gamma(h^0 \to A^0_t A^0_t) = \frac{\lambda^2 v^2 a_t^2}{32\pi M_{h^0}} \sqrt{1 - \frac{4M^2_{A^0_t}}{M^2_{h^0}}}, \tag{4.6}$$

is of the order of the Higgs boson mass for a light Higgs boson, and decreases for larger M_{h^0}.

The Higgs boson mass has a simple form when the top-axion is light. To show this, we remark that $M_{A^0_t} \lesssim v \sim \epsilon M_{H^\pm}$ implies $x_t \lesssim \epsilon^2$. Then, using the expression for the top mass (3.4) with $y_t = m_t\sqrt{2}/v \approx 1$, we may write the Higgs boson squared-mass as

$$M^2_{h^0} = \lambda v^2 \left[1 - \frac{1}{\cos^2\gamma} \left(\cos^2\beta + \frac{1}{x_\chi \xi^2} \right) + \mathcal{O}(1/\xi^2) \right]. \tag{4.7}$$

It appears that the full standard model range is open for the Higgs boson mass, but a light h^0 requires $\cos\beta/\cos\gamma \sim 1$, or a fine-tuning of $x_\chi \approx 1$ (i.e., $M_{A^0_\chi} \approx 2m_\chi$). Therefore, the Higgs boson is generically a very broad resonance, which decays most of the time into top-axion pairs, or into W and Z pairs for large M_{h^0}.

C. Light χ-axion

The last possible light composite scalar is A^0_χ. This is similar with the light A^0_t case: for small $\mu_{\chi\chi}$ the amount of $U(1)_\chi$ explicit breaking is small and the A^0_χ may have a mass below the electroweak scale. We will call A^0_χ the ''composite χ-axion.''

Recall that there is no region of the parameter space in which both A^0_t and A^0_χ are light. Therefore the only scalar trilinear coupling relevant at current collider energies is

$$\mathcal{L}_3 = \frac{\lambda}{2} v a_\chi\, h^0 (A^0_\chi)^2, \tag{4.8}$$

where the renormalized value of a_χ is again $\sim 1/N_c$. If $M_{h^0} > 2M_{A^0_\chi}$, then the branching ratio for the $h^0 \to A^0_\chi A^0_\chi$ decay mode may be large. The width for the Higgs decay into a χ-axion pair is similar with that from the light-A^0_t case, and can be estimated using the value of λ from Eq. (2.6):

$$\Gamma(h^0 \to A^0_\chi A^0_\chi) \sim \frac{8\pi^4 v^2}{N_c^4 M_{h^0} \ln^2(M_c/m_\chi)} \sqrt{1 - \frac{4M^2_{A^0_\chi}}{M^2_{h^0}}}. \tag{4.9}$$

Since the h^0 has standard model couplings to the weak gauge bosons (because the other CP-even neutral scalars decouple up to ϵ^2), we can immediately compare its widths for the decays into χ-axions and into W or Z pairs:

$$\frac{\Gamma(h^0 \rightarrow A_\chi^0 A_\chi^0)}{\Gamma(h^0 \rightarrow WW,ZZ)} \approx \frac{\lambda^2 a_\chi^2 v^4}{3 M_{h^0}^4}, \tag{4.10}$$

where we neglected the χ-axion mass and the gauge boson masses. The dominant decay mode of a Higgs boson lighter than the electroweak scale is into χ-axion pairs.

The novel feature of the light χ-axion case is that it places an upper bound on the Higgs boson mass. The condition for a light χ-axion, $M_{A_\chi^0} \lesssim \epsilon M_{H^\pm}$, implies $x_\chi \gtrsim \epsilon^2$, and the Higgs boson squared-mass becomes

$$M_{h^0}^2 = v^2 \left[4\xi \cos\beta - \frac{2}{x_t} + \mathcal{O}(1/\xi^2) \right]. \tag{4.11}$$

Because $\cos\beta < 1/\xi$, we find that the upper bound on the Higgs boson mass is $2v$. This bound is not very stringent, but still relevant for searches at the CERN LHC.

V. COMPOSITE SCALAR COUPLINGS TO QUARKS AND LEPTONS

The couplings of the light bosons to the quarks and leptons are model dependent, as in a general two-doublet Higgs model. All quarks and leptons have to couple to at least one of the two Higgs doublets in order to acquire masses. Such couplings may arise in the low energy effective theory in various ways, depending on the structure of the underlying theory above the compositeness scale. A simple possibility is that there are four-fermion couplings between the t, χ and the light fermions:

$$\frac{1}{M_c^2} [(\overline{\psi}_L^j u_R^k)(\eta_{jk}^u \overline{t}_R \psi_L^3 + \eta_{jk}'^u \overline{\chi}_R \psi_L^3) + (\overline{\psi}_L^j d_R^l) i\sigma_2 (\eta_{jl}^d \overline{\psi}_L^3 t_R + \eta_{jl}'^d \overline{\psi}_L^3 \chi_R)] + \text{H.c.} \tag{5.1}$$

For brevity, we show here only the four-fermion operators involving quarks. The couplings of $\overline{\chi}_R \psi_L^3$ and $\overline{t}_R \psi_L^3$ to the leptons have the same form. The generational indices j and l run from 1 to 3, while k runs from 1 to 4 because the χ_R may mix with the u_R, c_R and t_R weak eigenstates. The above set of four-fermion operators may be viewed as a parametrization of the flavor symmetry breaking effects, whose origin could be explained in principle within a variety of high-energy theories, as discussed in Sec. II C. The four-fermion operators give rise in the low energy theory to Yukawa couplings of the H_t and H_χ doublets to the quarks and leptons:

$$-(\overline{\psi}_L^j u_R^k)(\lambda_{jk}^u H_t + \lambda_{jk}'^u H_\chi) + (\overline{\psi}_L^j d_R^l) i\sigma_2 (\lambda_{jl}^d H_t^\dagger + \lambda_{jl}'^d H_\chi^\dagger). \tag{5.2}$$

The Yukawa coupling constants, $\lambda^d, \lambda'^d, \lambda^u, \lambda'^u$, are proportional with the coefficients of the four-quark operators, $\eta^d, \eta'^d, \eta^u, \eta'^u$, respectively. The factor of proportionality

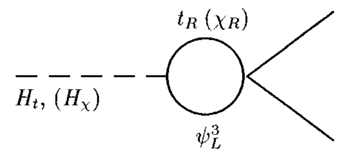

FIG. 4. Leading-N_c contribution to the Yukawa couplings of the standard model fermions.

may be estimated by computing the leading-N_c contribution shown in Fig. 4, with a physical cutoff at M_c, and the result is

$$(\lambda^d, \lambda'^d, \lambda^u, \lambda'^u) \approx \frac{\xi N_c}{8\pi^2} (-\eta^d, \eta'^d, \eta^u, -\eta'^u). \tag{5.3}$$

Note that six-fermion couplings and other higher-dimensional couplings can also contribute to the light quark and lepton masses. Below the compositeness scale, they give rise to terms in the effective Lagrangian involving several Higgs doublets and fermions. If they give the dominant contribution to some of the fermion masses, then the couplings of the Higgs boson to those fermions are non-standard [28]. Another possibility for fermion mass generation is to let all quarks and leptons to participate in a seesaw mechanism, by extending the vector-like quark sector [7,29]. In what follows we will ignore these possibilities, and study the couplings induced by the Yukawa couplings shown above.

Only the linear combination ($H_t \cos\beta + H_\chi \sin\beta$) has an electroweak asymmetric VEV, so that the down-type quark masses are given by

$$\text{diag}(m_d, m_s, m_b) = \frac{v}{\sqrt{2}} S_d^\dagger (\lambda^d \cos\beta + \lambda'^d \sin\beta) T_d, \tag{5.4}$$

where S_d and T_d are unitary matrices. A similar statement applies to the lepton sector. The only light scalar contained in this linear combination is the h^0, and its induced couplings to down-type quarks or leptons are standard-model-like up to corrections of order ϵ. Note that the b-quark mass requires $\eta_{33}^d \cot\beta + \eta_{33}'^d \sim 0.2$, which shows that generically the coefficients of the four-quark operators responsible for light fermion masses are indeed perturbative at the compositeness scale.

The other linear combination of Higgs doublets,

$$-H_t \sin\beta + H_\chi \cos\beta = \begin{pmatrix} \frac{1}{\sqrt{2}} (+h_{t\chi}^0 \cos\beta - h_{tt}^0 \sin\beta - iA^0) \\ -H^- \end{pmatrix}, \tag{5.5}$$

has couplings which may induce flavor changing neutral currents (FCNC's) in the down-type quark sector. The charged Higgs induces FCNC's at one loop level, but is sufficiently heavy to make these effects insignificant. The neutral states, however contribute to FCNC's at tree level, and we have to make sure that these contributions are not too large. The couplings of the neutral scalars to the down-type quark mass eigenstates are given by

$$\frac{1}{\sqrt{2}}\overline{d_L}S_d^\dagger(-\lambda^d\sin\beta+\lambda'^d\cos\beta)T_d d_R[H_1^0+iA^0 \\ +\epsilon(-ic_A A_t^0+ic_A' A_\chi^0+c_H H_2^0+c_H' H_3^0)+\mathcal{O}(\epsilon^2)], \quad (5.6)$$

where c_H and c_H' are parameters of order one in the ϵ expansion. Notice that the h^0 couplings are not affected by these terms up to order ϵ^2.

In general, these couplings may be flavor non-diagonal because the FCNC's induced by them are suppressed at order ϵ^2. However, in order to avoid too strong bounds on ϵ^2 (which would correspond to fine-tuning), it is preferable to assume that the matrix $S_d^\dagger(-\lambda^d\sin\beta+\lambda'^d\cos\beta)T_d$ is approximately flavor-diagonal. There are many situations in which this happens. For example, when the two matrices λ^d and λ'^d are approximately proportional, or when one of them vanishes.

The scalar couplings to up-type quarks are more complicated due to the mixing with the χ. The up-type quark mass matrix is 4×4, and has large elements corresponding to the χ and t weak eigenstates, given by Eq. (3.3). The other elements are given by the Yukawa couplings (5.2) and are typically small because they are produced by perturbative four-quark operators at the M_c scale or above. Since the χ is much heavier than the electroweak scale, its mixing with the quarks other than t is small. If we ignore this mixing altogether, we have a situation similar with that in the down-type sector: the h^0 has standard model couplings up to corrections of order ϵ^2, while the A_t^0 and A_χ^0 have couplings of order ϵ to the standard model fermions. On the other hand, the mixing of the up-type quarks with the χ could lead to certain flavor non-diagonal couplings of the h^0 which may be allowed by the FCNC constraints, while producing interesting phenomena such as single-top decays of the Higgs boson [30]. Note also that the h^0 has a large coupling, of $\sim\xi/\sqrt{2}$, to the $\overline{t_L}\chi_R$ quark mass eigenstates.

VI. BOUNDS ON THE COMPOSITE AXION

In this section we study the lower mass bounds on the composite axion. These are sensitive to the axion couplings to fermions, which are of order ϵ or smaller, and depend on its identity (A_t^0 or A_χ^0) only up to an overall constant, as can be seen from Eq. (5.6). The axion-fermion couplings are very model dependent and is beyond the scope of this paper to comprehensively analyze the mass bounds in all cases. We will rather concentrate on the cases which are most favorable for a light axion.

The tree level axion couplings to light quarks and leptons have two sources. One of them is the Yukawa couplings to light fermions of the doublet with no VEV, shown in Eq. (5.5). These may vanish or be very small because they are not restricted by the quark and lepton masses. The other source is the Yukawa couplings of the composite scalars to their constituents, Eq. (2.7). After transforming to the mass eigenstate basis, these Yukawa interactions induce axion couplings to all the up-type quarks. However, the mixings between the weak eigenstates of the t or χ with u and c are again unrestricted, and may vanish without affecting the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Notice that in this case the V_{ts} and V_{td} elements are fixed by the S_d unitary matrix [see Eq. (5.4)], while V_{ub} and V_{cb} are combinations of the transformations in both the up- and down-type sectors. Once a predictive and compelling theory of flavor is found, one can decide whether the aforementioned mixings and couplings are naturally small or vanishing.

Here we will assume they do, so that the only fermions that couple at tree level to the composite axion (A_t^0 or A_χ^0) are the t and χ mass eigenstates:

$$\frac{i\xi}{\sqrt{2}}(\overline{t_L},\overline{\chi_L})\left[A_t^0\begin{pmatrix}\mathcal{O}(\epsilon) & \mathcal{O}(\epsilon)\\ 1 & 0\end{pmatrix}+A_\chi^0\begin{pmatrix}\mathcal{O}(\epsilon) & \mathcal{O}(\epsilon)\\ 0 & 1\end{pmatrix} \\ +\mathcal{O}(\epsilon^2)\right]\Gamma\begin{pmatrix}-t_R\\ \chi_R\end{pmatrix}+\text{H.c.}, \quad (6.1)$$

where Γ is the unitary matrix given in Eq. (3.16). Therefore, the composite axion may be produced at colliders through a t or χ loop, but the production rate is too small for placing bounds even at the Z pole at the CERN e^+e^- collider LEP [31].

The quarkonium decays could in principle constrain the composite axion mass. However, the current limit on the branching ratio of the most promising decay mode, $\Upsilon(1S)\to A_{t,\chi}^0\gamma$, is at the level of 10^{-5} [32], which is not sufficient for constraining the composite axion. Note that this decay occurs through a top-loop, and a suppression factor of order ϵ^2 appears in the width.

The $K^+\to A_{t,\chi}^0\pi^+$ decay is another usual suspect for constraining the axions. This again involves a top-loop and is further suppressed by $V_{ts}V_{td}$, so that no useful mass bounds can be derived.

More generally, if the axion is coupled at tree level only to the χ and t, it is sufficiently insulated from the light fermions to avoid constraints from usual laboratory searches. The astrophysical constraints are harder to avoid. The composite axion may be produced in stars if it is light enough, leading to unacceptable cooling rates. At one-loop, the axion couples to gluon pairs and to photon pairs. Combined, these couplings rule out very light axions with a decay constant below $\sim10^{10}$ GeV [32]. In our model the axion decay constant is of order v/ϵ, implying that a very small value for ϵ is required, which leads to an exponential fine-tuning. Note that in the limit where the $\mu_{\chi t}$ or $\mu_{\chi\chi}$ mass parameter vanishes, the only contribution to the axion mass is given by the QCD anomaly, and it is tempting to solve the strong CP problem using the composite axion. However, the small value for ϵ is not encouraging. We therefore do not attempt to solve the strong CP problem, and assume that the Peccei-Quinn symmetry is explicitly broken by a non-zero $\mu_{\chi t}$ and $\mu_{\chi\chi}$, or by some higher dimensional operators.

The bound on the axion decay constant is avoided if the composite axion is heavier than the core temperature of the stars by an order of magnitude, because the axion production is Boltzmann suppressed and the cooling rate is not much affected. The red giant stars have a core temperature of order 10 keV which impose a lower mass bound of $\sim$200 keV on the axion [33].

The larger temperature of the supernova 1987A, about 30 MeV at the center, appears to yield the most stringent lower limit on the composite axion mass. However, the cooling rate of the supernova is not affected by our composite axion, because the very high density of the newly formed neutron star reduces the axion emission to acceptable levels for an axion decay constant below $\sim 10^6$ GeV [33]. Although the axion flux could not affect the supernova cooling, there are constraints due to the absence of an axion signal in water Cerenkov detectors during the SN 1987A [34]. These impose a lower bound of a few hundred TeV on the axion decay constant. We find more reasonable to evade this bound by imposing a lower limit on the composite axion mass of $\mathcal{O}(100)$ MeV, such that its production is substantially suppressed.

If A_t^0 or A_χ^0 has a mass between this lower bound and $2m_\pi \approx 270$ MeV, then the $\pi^0\pi^0$ decay channel is closed, and the composite axion decays predominantly to photon pairs (assuming that the tree level coupling to e^+e^- vanishes). In this case, the CP-even Higgs boson, which decays most of the time to axion pairs, will have a striking signature at future colliders: two pairs of almost collinear photons.

VII. DISCUSSION

It is instructive to make a comparison of the minimal composite Higgs model (MCHM) presented here with the minimal supersymmetric standard model (MSSM). Both these models have a decoupling limit in which they look like the standard model, and therefore are consistent with current electroweak precision data. Both models include two Higgs doublets, but the composite model requires also two gauge singlet fields resulting in a more complicated Higgs sector. The top quark plays an active role in electroweak symmetry breaking within both the MCHM and MSSM.

These two models may be viewed as effective theories whose parameters have to be determined by higher-energy physics. The MCHM includes four coefficients of the four-quark operators which are fixed by the gauge couplings and representations of the top and χ quarks, and possibly by their position in extra dimensions. The MSSM has soft supersymmetry breaking parameters which need to be determined within a theory of dynamical supersymmetry breaking. Similarly, the presence of the gauge invariant fermion mass terms that are present in the MCHM, and the μ term in the MSSM are hopefully accounted for by physics at higher energy.

From a more theoretical point of view, both the MCHM and the MSSM can be linked to low energy manifestations of certain features that are likely to occur in a more comprehensive theory which includes quantum gravity, such as string or M theory. Furthermore, in the presence of extra dimensions compactified at a scale in the TeV range, the composite Higgs model is compatible with gauge coupling unification [19], although this cannot be checked at the level of precision allowed by the perturbativity of the MSSM.

Despite these similar aspects, the MCHM and MSSM are conceptually different. In the MCHM there is no fundamental Higgs field. Therefore, the origin of electroweak symmetry breaking is found in dynamical phenomena, as opposed to the radiative corrections involved in the MSSM. Also, the phenomenology of the MCHM and MSSM is different. In the MCHM there are no superpartners at the electroweak scale, but there is a potentially light axion, a heavy vector-like quark, and interesting phenomena at scales in the TeV range, associated with the strong dynamics.

An important phenomenological aspect of the MCHM is the dominant branching ratio (if allowed kinematically) of the Higgs boson decay into composite axion pairs. The discovery of the Higgs boson in this decay mode would be a spectacular evidence for the MCHM. On the other hand, if only a light standard model Higgs boson will be discovered, it will probably be necessary to measure its trilinear coupling or to experiment with colliders at higher energies in order to distinguish between the MCHM, the MSSM, or other models with a decoupling limit.

Note added. A related study of a top quark seesaw model has appeared while this work was concluded [35]. The focus of that study is rather different than in this paper. For example, the models discussed there do not have a decoupling limit in which the standard model with a light Higgs boson is recovered.

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APPENDIX: EXTREMIZATION CONDITIONS FOR THE EFFECTIVE POTENTIAL

In this Appendix we list the extremization conditions for the effective potential studied in Sec. III. These can be read from Eq. (3.1) by imposing the cancellation of the tadpole terms:

$$\frac{\partial V(0)}{\partial h_{tt}^0} = v\cos\beta\left[M_{tt}^2 + \frac{\lambda v^2}{2\epsilon^2}\left(r_s\frac{\tan\beta}{\tan\gamma} + \epsilon^2\right)\right] = 0 \quad \text{(A1)}$$

$$\frac{\partial V(0)}{\partial h_{t\chi}^0} = v\sin\beta\left[M_{t\chi}^2 - \frac{\lambda v^2}{2\epsilon^2}(r_s - \epsilon^2)\right] = 0$$

$$\frac{\partial V(0)}{\partial h_{\chi t}^0} = -\frac{v\sin\beta}{\epsilon\tan\gamma}\left[M_{\chi t}^2 + \frac{\lambda v^2}{2\epsilon^2}\left(\frac{\sin^2\beta}{\sin^2\gamma} + \epsilon^2 r_s\frac{\tan\gamma}{\tan\beta}\right)\right] + \sqrt{2}\,C_{\chi t} = 0$$

$$\frac{\partial V(0)}{\partial h^0_{\chi\chi}} = -\frac{v}{\epsilon}\sin\beta\left[M^2_{\chi\chi} + \frac{\lambda v^2}{2\epsilon^2}\left(\frac{\sin^2\beta}{\sin^2\gamma} - \epsilon^2 r_s\right)\right] + \sqrt{2}\,C_{\chi\chi} = 0$$

where we introduced for convenience the following notation:

$$r_s \equiv \frac{\sin\beta}{\sin\gamma}\cos(\beta+\gamma). \tag{A2}$$

Note that $M^2_{t\chi}<0$ requires $\cos(\beta+\gamma)<0$, while the expression for the top mass (3.4) imposes

$$-1 < r_s < -1 + \frac{1}{\xi^2} + \mathcal{O}(1/\xi^4). \tag{A3}$$

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