Constraining *CP* violating phases of the minimal supersymmetric standard model

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Possible CP violation in supersymmetric extensions of the standard model is discussed. The consequences of CP violating phases in the gaugino masses, trilinear soft supersymmetry-breaking terms, and the μ parameter are explored. Utilizing the constraints on these parameters from electron and neutron electric dipole moments, possible CP violating effects in B physics are shown. A set of measurements from the B system, which would overconstrain the above CP violating phases, is illustrated.

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I. INTRODUCTION

The peculiar flavor structure of the standard model (SM) currently has no generally accepted explanation. Included in this structure is the prediction that fundamental physics is not invariant under the operations of parity (P), charge (C), or their combination, CP. More precisely, it is the distribution of complex numbers in the SM Lagrangian which leads to CP violation at an extremely small[1] but observable level [1]. Currently experiments [2–8] are taking data, or are under construction, and will very precisely measure the Cabibbo-Maskawa-Kobayashi (CKM) [9] matrix elements believed responsible for CP violation in the SM. Any observed discrepancy with the SM prediction indicates that the SM is incomplete in the flavor sector and new physics must appear in the fundamental theory. An excellent candidate for new physics is supersymmetry (SUSY) [10]. As SUSY introduces many more potentially large sources of CP violation in the mass matrices and various field couplings, its inclusion requires that there be some suitable relationships among the parameters to reproduce the agreement of the existing levels of CP violation in K decay and electric dipole moment (EDM) data with the SM predictions [11]. The understanding of what CP violating parameters are allowed in SUSY models, therefore, provides a constraint on such models.

The simplest SUSY models cannot satisfy the experimental EDM bounds without either setting all SUSY CP violating phases to zero or raising SUSY particle masses above 1 TeV [12–14]. Recent works have noted, however, that the supergravity-broken minimal supersymmetric standard model (MSSM) with $O(1)$ phases for the gaugino masses M_i, triscalar coupling A, and Higgs coupling parameter μ, and no flavor-mixing beyond the standard CKM matrix, can be consistent with existing limits on the electric dipole moments (EDMs) of the electron and neutron [15,16]. Given this, we may now investigate how the phases in this model will contribute to other CP violating observables and how new measurements can constrain the values of these phases. We choose to investigate several CP violating observables of the B meson system, as B-processes occupy both a favorable theoretical and experimental position. On the theoretical side, uncertainties from nonperturbative QCD are low due to the large mass of the b quark (≈ 4 GeV) [17] relative to the energy scale $\Lambda \approx 200$ MeV [18], which is both reliable and characteristic of strong interactions and perturbation theory in α_s. Furthermore, many SM CP violating asymmetries in the B system are small due to the suppression of CKM matrix elements involving the third generation; new CP violating physics may be easily detectable. Experimentally, many dedicated facilities are already or will soon be generating large amounts of high-precision B-physics data [19]. From our analysis we will see that these experiments will be able to either precisely determine the above set of five phases or reject this particular model of CP violation altogether.

We first briefly discuss the features of the MSSM in Sec. II and review the EDM constraints in Sec. III. In Sec. IV we begin discussion of how the above phases enter observables in various sectors of B-meson physics through CP violation in pure mixing, mixing and decay, and pure decay effects in the processes $b \to s\gamma$, $B_s^0 \to \phi\phi$, $J/\psi\phi$, and $B^- \to \phi K^-$. Finally, we collect results and compare them with the expected experimental sensitivities in Sec. V. Throughout this discussion we reserve most of the more complicated formulas and analyses for the Appendix, to which we will refer the reader at the appropriate points.

II. THE MODEL

The setting for our calculations is the MSSM superpotential

$$\mathbf{W} = \mathbf{y_u}\overline{u}QH_2 + \mathbf{y_d}\overline{d}QH_1 + \mathbf{y_e}\overline{e}LH_1 + \mu H_1 H_2, \tag{1}$$

with the chiral matter superfields for quarks and leptons $\overline{u}$, $\overline{d}$, Q, $\overline{e}$, L, and Higgs boson $H_{1,2}$ transforming under $SU(3)_C \times SU(2)_L \times U(1)_Y$ as

[1] Jarlskog [1] has expressed this as follows: considering the interaction-basis up- and down-quark mass matrices m and m', one can form a rephase-invariant measure of CP violation equal to $a \equiv 3\sqrt{6}\mathrm{Det}(C)/[\mathrm{Tr}(C^2)]^{3/2}$, where $iC \equiv [mm^\dagger, m'm'^\dagger]$. Then $-1 \leq a \leq +1$ and CP violation$\Leftrightarrow a \neq 0$. In the SM one finds $a \approx 10^{-7}$.

$$\overline{u} \equiv (\overline{3},1,-\tfrac{2}{3}), \quad \overline{d} \equiv (\overline{3},1,\tfrac{1}{3}), \quad Q \equiv (3,2,\tfrac{1}{6}),$$
$$\overline{e} \equiv (1,1,1), \quad L \equiv (1,2,-\tfrac{1}{2}). \tag{2}$$
$$H_1 \equiv (1,2,-\tfrac{1}{2}), \quad H_2 \equiv (1,2,\tfrac{1}{2}),$$

The 3×3 Yukawa matrices $\mathbf{y_{u,d,e}}$ couple the three generations of quarks and leptons, and the μ parameter couples the two Higgs bosons. In this discussion all flavor and gauge indices are implicit. The superpotential (1) and general considerations of gauge invariance give the minimally supersymmetric $SU(3)_C \times SU(2)_L \times U(1)_Y$ Lagrangian

$$\mathcal{L}_{SUSY} = \mathcal{L}_{kinetic} - \sum_i \frac{dW}{d\phi^i}\frac{dW^*}{d\phi^i} - \frac{1}{2}\sum_a g_a^2(\phi^* T^a \phi)^2, \tag{3}$$

where the kinetic terms have the canonical forms

$$\mathcal{L}_{kinetic} = -D^\mu \phi^{\dagger i} D_\mu \phi_i - i\psi^{\dagger i}\overline{\sigma}^\mu D_\mu \psi_i - \frac{1}{4}F^{\mu\nu a}F_{\mu\nu a}, \tag{4}$$

with the indices i and a running over the scalar fields and gauge group representations, respectively. To this Lagrangian we add a soft supersymmetry-breaking (SUSY B) piece

$$\mathcal{L}_{SUSY\,B} = (M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B}) - (\mathbf{a_u}\overline{u}QH_2 + \mathbf{a_d}\overline{d}QH_1 + \mathbf{a_e}\overline{e}LH_1)_{scalar} - \left(\sum_{X_i = Q,\overline{u},\overline{d},\overline{e}} \mathbf{m_i^2} X_i X_i^\dagger - \sum_{i=1,2} m_{H_i}^2 H_i^* H_i - b\mu H_1 H_2\right)_{scalar} + \text{H.c.}, \tag{5}$$

where $(\tilde{g},\ \tilde{W},\ \tilde{B})$ are the fermionic partners of the gauge bosons of $SU(3)_C, SU(2)_L, U(1)_Y$, respectively, and where *scalar* implies that only the scalar component of each superfield is used. SUSY breaking of this type is characteristic of supergravity [20,21].

An additional simplification is to take the matrices $\mathbf{m_i^2}$ and $\mathbf{a_{u,d,e}}$ in Eq. (5), along with the Yukawa matrices $\mathbf{y_{u,d,e}}$ in Eq. (1), to be simultaneously diagonal in the mass basis after electroweak symmetry breaking. This implies

$$\mathbf{m_i^2} \equiv m_0^2 \mathbf{1},$$
$$\mathbf{a_i} \equiv A\mathbf{y_i} (i = u,d,e). \tag{6}$$

Thus the only mixing between families is the usual CKM matrix, since quark and squark mass matrices are diagonalized by the same rotation.

The full Lagrangian $\mathcal{L}_{SUSY} + \mathcal{L}_{SUSY\,B}$ has six arbitrary phases imbedded in the parameters $M_i (i=1,2,3)$, μ, b, and A. The phase of b can immediately be defined to be zero by appropriate field redefinitions of $H_{1,2}$. A mechanism to set another phase to zero utilizes the $U(1)_R$ symmetry of the Lagrangian, which is broken by the supersymmetry-breaking terms, in particular, the gaugino masses in Eq. (5). We may perform a $U(1)_R$ rotation on the gaugino fields to remove one of the phases M_i. For consistency with [15] we choose M_2 real ($\phi_2 \equiv 0$). Note that this $U(1)_R$ transformation affects neither the phase of A, since having the Yukawa matrices $\mathbf{y_i}$ in Eq. (1) be real fixes the phases of the same fields that couple to A, nor the phase of μ, as having chosen $\phi_b \equiv 0$ fixes the phases of $H_{1,2}$. Therefore the final set of physical phases we study is $\{\phi_{1,3}, \phi_A, \phi_\mu\}$.

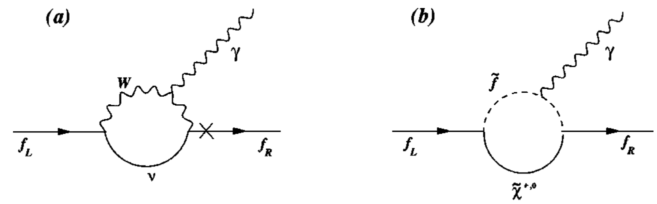

FIG. 1. One-loop contributions to the EDM in (a) the SM, where the graph vanishes since the complex phases at the vertices cancel and (b) in the MSSM, where the requisite helicity-flip may be placed on either the chargino ($\tilde{\chi}^\pm$), neutralino ($\tilde{\chi}^0$), or scalar ($\tilde{f}$) propagators to introduce $O(1)$ phases in the amplitude.

III. ELECTRIC DIPOLE MOMENT CONSTRAINTS

The electric dipole moment (EDM) of an elementary fermion, a manifestly CP violating quantity, is the coefficient d_f of the effective operator

$$O_{edm} = -(i/2)\overline{f}\gamma_5\sigma_{\mu\nu} f F^{\mu\nu},$$

where $F^{\mu\nu}$ is the electromagnetic field-strength tensor. The SM prediction of this coefficient vanishes at one loop [see Fig. 1(a)] since the CKM phases from the two vertices cancel each other. For the electron, d_e even vanishes at two loops; and the three-loop prediction is miniscule, of order $10^{-50} e$ cm [22]. For the neutron EDM, gluon interactions can give rise to a two-loop contribution to d_n, but the result is still tiny at $d_n \leq 10^{-33} e$ cm [23]. The above predictions no longer hold if the QCD Lagrangian contains the CP violating "θ-term" $\theta(g_{QCD}^2/32\pi^2)G^{\mu\nu}\tilde{G}_{\mu\nu}$, which is a potentially significant source of CP violation. But, then the theory is consistent with the EDM limits only if $\theta < 10^{-9}$ [24]; such a fine tuning is unnatural and, henceforth, we assume that $\theta = 0$.

Although the amplitude of a typical supersymmetric contribution to the dipole moment [see Fig. 1(b)] suffers relative to the SM contribution by a reduction factor of at least $(m_W/\tilde{m})^2$ from SUSY particles of mass $\tilde{m}$ propagating in the loop, the imaginary piece of the SUSY amplitude could very well dominate over that from the SM. As previous calculations have shown [12–14], the diagrams in Fig. 1(b) lead to EDMs in violation of the current limits of $d_e < 4.3 \times 10^{-27} e$ cm [25] and $d_n < 6.3 \times 10^{-25} e$ cm [26], unless SUSY particles are either heavier than $O(1)$ TeV or the CP violating phases are less than $O(10^{-2})$. However, small phases are not inevitable, for it has recently been pointed out [15,16] that a cancellation can occur among the various

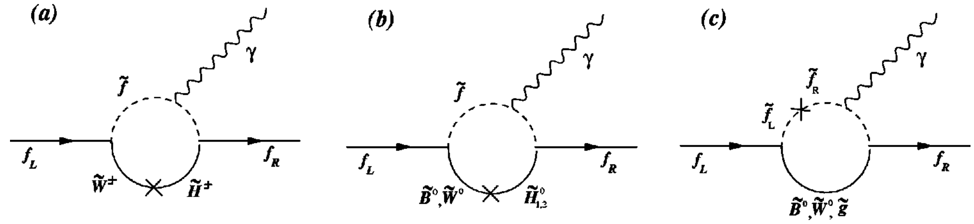

FIG. 2. Leading SUSY contributions to the EDM: (a) charged *W*-ino ($\tilde{W}^{\pm}$) and Higgsino ($\tilde{H}^{\pm}$) mixing provide the largest part of d_f, which (b) neutral *W*-ino ($\tilde{W}^0$), *B*-ino ($\tilde{B}$), and Higgsino ($\tilde{g}^0$) mixing partially cancels. (c) Exchange of a $\tilde{W}^0$, $\tilde{B}$, or gluino $\tilde{g}^0$, with mixing between the scalar superpartners ($\tilde{f}_{L,R}$) of the corresponding fermions $f_{L,R}$, could almost completely cancel against the other two diagrams. Note that the above processes actually occur via the mass eigenstates $\tilde{\chi}^{\pm}$ and $\tilde{\chi}^0$.

SUSY diagrams, allowing $O(1)$ *CP* violating phases to be consistent with the current EDM experimental limits.

The set of phases $\{\phi_{1,3},\phi_A,\phi_\mu\}$ enter in any diagram which involves mixing between the following fields: charginos ($\tilde{\chi}^{\pm}$), ϕ_μ lies in the matrix $\mathbf{M}_{\chi^{\pm}}$, which mixes the set ($\tilde{W}^+,\tilde{H}_2^+,\tilde{W}^-,\tilde{H}_1^-$); neutralinos ($\tilde{\chi}^0$), both ϕ_μ and ϕ_1 appear in the matrix $\mathbf{M}_{\chi^0}$, which mixes the set ($\tilde{W}^0,\tilde{B},\tilde{H}_1^0,\tilde{H}_2^0$); scalars, terms in the Lagrangian such as $\mu^*\mathbf{y_u}\bar{\tilde{u}}\tilde{u}H_1^{0*}$ arising from the second term of Eq. (3), and SUSY B terms in Eq. (5), introduce ϕ_μ and ϕ_A, respectively, into the scalar mass insertions. The effect is particularly significant in $\tilde{t}$ mixing, where the Yukawa matrices are large.

The SUSY diagrams which appear in Fig. 1(b) fall into three classes, shown in Fig. 2. The charged *W*-ino–Higgsino mixing diagram, Fig. 2(a), provides the dominant contribution to d_f, with the phase of μ entering the amplitude as expected. The neutralino-mixing diagram Fig. 2(b), is numerically smaller than its charged counterpart, but it is of opposite sign and has the same dependence on ϕ_μ; therefore Figs. 2(a) and 2(b) partially cancel. The final type of diagram, shown in Fig. 2(c), has a more complicated phase dependence[2] which, in certain regions of parameter space that are not fine tuned, leads to a destructive interference with the other two diagrams consistent with current experimental bounds on both d_n and d_e [15].

If $O(1)$ phases are permissible, then, it is important to know whether or not other experimental observables can overconstrain these phases. Next we show that the *B* system alone provides enough observables to strongly constrain these phases.

IV. *B* PHYSICS CONSTRAINTS

There are many reasons to consider the *B* system in particular for measurements of *CP* violation beyond the SM, a few of which are SM contributions to the relevant observables are down by factors of small CKM matrix elements, so generic non-SM *CP* violating physics should give a very clear signal, uncertainties arising from strong interactions are small using heavy quark theory [27], and large amounts of data will be available in the near future.

The *CP* violation in question can arise in any of three ways: through B^0-$\bar{B}^0$ mixing, *B* decay, or through their combination [2,27].

A. *CP* violation in B^0-$\bar{B}^0$ mixing

At any given instant of time, the propagating meson states are linear combinations $|B_{L,H}\rangle \equiv p|B^0\rangle \pm q|\bar{B}^0\rangle$, which evolve according to the 2×2 Hamiltonian with dispersive and absorptive pieces $\mathbf{M}$ and $\mathbf{\Gamma}$, viz.

$$i\frac{d}{dt}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix} = \begin{pmatrix} M_{11}+\frac{i}{2}\Gamma_{11} & M_{12}+\frac{i}{2}\Gamma_{12} \\ M_{12}^*+\frac{i}{2}\Gamma_{12}^* & M_{11}+\frac{i}{2}\Gamma_{11} \end{pmatrix}\begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}. \quad (7)$$

If we denote the time evolution of the states as

$$|B_{L,H}(t)\rangle = |B_{L,H}(0)e^{-iM_{L,H}}\rangle e^{-1/2\Gamma_{L,H}}, \quad (8)$$

then solving Eq. (7), it follows [2] that

$$CP \text{ violation in mixing} \Leftrightarrow \mathrm{Im}(\Delta M)\neq 0, (\Delta M \equiv M_L - M_H). \quad (9)$$

Making use of the fact that

$$\Delta\Gamma \ll \Delta M, \quad (\Delta\Gamma \equiv \Gamma_L - \Gamma_H),$$

which holds for both B_d^0 and B_s^0 [2,28], we obtain the simplifications

$$\Delta M \approx 2|M_{12}|, \quad \frac{q}{p}\approx\frac{-|M_{12}|}{M_{12}}. \quad (10)$$

The degree of *CP* violation in mixing is contained in the ratio q/p which, from Eq. (10), is directly proportional to the phase factor in the amplitude of the $\Delta S=2$ box diagram (see Fig. 3). In the SM, the box graph with internal top quarks in Fig. 3(a) dominates $\mathrm{Im}(\Delta M)$; correspondingly, the strength of SM *CP* violating mixing is CKM suppressed by a factor $\mathrm{Arg}(V_{tb}V_{ts}^*)^2 \approx \eta\lambda^2$, which in the Wolfenstein approximation [29] is ≈ 0.01. Any $\Delta S=2$ contribution from new physics, therefore, has a generically large effect if it carries any phases with it at all. The leading supersymmetric chargino

[2] We briefly outline this dependence in the Appendix.

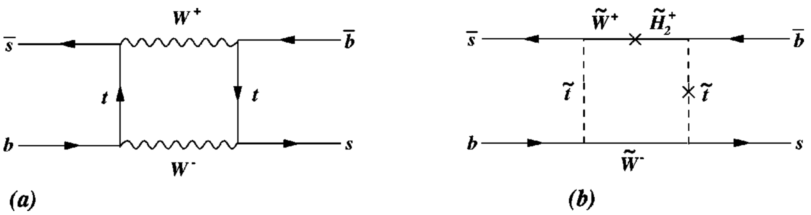

FIG. 3. Leading box diagrams in (a) the SM, and (b) the MSSM.

graph[3] in Fig. 3(b) provides the largest MSSM contribution to $\mathrm{Im}(\Delta M)$. We have computed this box (see Appendix) and find it to dominate significantly over the SM contribution. For example, at a typical point in SUSY parameter space, with $A \approx \mu$, $\tan\beta = 5$, and sparticle masses $\tilde{m}$ on the order of $O(2) \times M_W$, we obtain

$$\mathrm{Im}\left(\frac{q}{p}\right) \approx 0.1 \sin\phi_\mu \cos\phi_A, \tag{11}$$

which is an order of magnitude larger than the SM expectation, and is directly sensitive to CP violating SUSY phases. However, it is difficult to directly measure $\mathrm{Im}(q/p)$ through mixing effects alone; both time-dependent and time-integrated mixing effects are usually governed by ΔM, which is still mostly real and SM driven. We must therefore turn to other types of CP violation to constrain the MSSM phases.

B. *CP* violation from mixing combined with decay

When a particular final state f_{CP} with definite CP quantum numbers is accessible to the decays of a B^0 and $\overline{B^0}$ with amplitudes $A_{f_{CP}}$ and $\overline{A}_{f_{CP}}$, respectively, the asymmetry

$$a_{f_{CP}} \equiv \frac{\Gamma(\overline{B^0} \to f_{CP}) - \Gamma(B^0 \to f_{CP})}{\Gamma(\overline{B^0} \to f_{CP}) + \Gamma(B^0 \to f_{CP})}$$

in the limit of Eq. (10) becomes

$$a_{f_{CP}} \approx \sin(\Delta M t)\,\mathrm{Im}\left(\frac{q}{p}\overline{\rho}_{f_{CP}}\right) \tag{12}$$

where

$$\overline{\rho}_{f_{CP}} \equiv \frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}}, \quad A_{f_{CP}}(B^0 \to f_{CP}) \equiv \sum_j A_j e^{i(\delta_j + \phi_j)},$$

in general, contains a dependence on both the weak phases ϕ_j and the strong interaction phases δ_j from each diagram contributing an amplitude A_j. In Fig. 4, for example, the decay of a $\overline{B_s^0}$ to the final states J/ψ, ϕ, and $\phi\phi$ may either proceed directly ($\overline{B_s^0} \to f_{CP}$) or by oscillating first ($\overline{B_s^0} \to B_s^0 \to \overline{B_s^0} \to f_{CP}$). In the SM the asymmetries in both of these decays are tiny primarily due to the small $\Delta S = 2$ mixing effects see the discussion following Eq. (10); this effectively gives the upper bound

$$a_{\phi\phi, J/\psi\phi}(SM) \leq \eta\lambda^2 \approx 0.01. \tag{13}$$

If SUSY exists, then both of these asymmetries can be an order of magnitude larger, as is evident from Eqs. (11) and (12). Furthermore, the value of the quantity $\overline{\rho}_{f_{CP}}$ will differ from its value in the heavy quark expansion (HQE) by powers of Λ_{QCD}/m_b only [30–32], so we set the strong phases at $\delta_j = \pi$ with an uncertainty $(\Delta\delta)/\delta < 10\%$ [33,34]. In the case of the decay to $\phi\phi$, Fig. 4(b), for example, $\overline{\rho}_{f_{CP}} \approx 1$, whereas for the decay to $J/\psi\phi$ through the dominant tree-graph, Fig. 4(a), carries no strong-phase dependence at all. From Eqs. (11) and (12), it follows as a prediction of our model that

$$a_{J/\psi\phi,\phi\phi} \approx 0.1 \sin\phi_\mu \cos\phi_A. \tag{14}$$

We now have two experimental B-physics signals of new CP violating phases which constrain a combination of ϕ_μ and ϕ_A independent from those which arise in the EDM bounds.

C. *CP* violation in decay

The charged B-mesons' decays can also serve to measure our set of phases. The asymmetry in the decay to a final state f is

$$a_f \equiv \frac{1 - |\overline{A}/A|^2}{1 + |\overline{A}/A|^2},$$

$$A \equiv \mathrm{Amp}(B^+ \to f) = \sum_j A_j e^{i(\delta_j + \phi_j)},$$

$$\overline{A} \equiv \mathrm{Amp}(B^- \to \overline{f}) = \sum_j A_j e^{i(\delta_j - \phi_j)},$$

with δ_j and ϕ_j being the strong and weak phases, respectively, for the diagram with modulus A_j. Rewritten in the form

[3]We neglect gluino boxes in the approximation that SUSY introduces negligible flavor mixing in the down sector; boxes with additional Higgsinos are likewise suppressed by small Yukawa matrices. Further, we assume that the lightest top squark dominates the loops since it is usually the lightest squark.

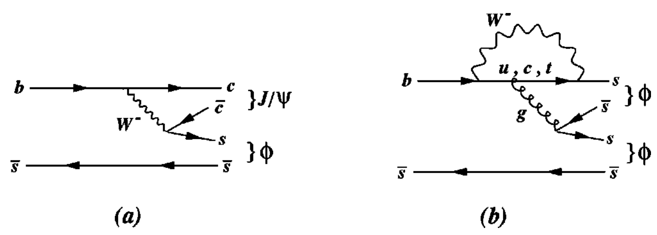

FIG. 4. Sizable *CP* violating asymmetries arise from the processes (a) $\overline{B}_s^0 \to J/\psi\phi$, and (b) $\overline{B}_s^0 \to \phi\phi$. Only in the first case is hadronic uncertainty absent.

$$a_f = \frac{\sum_{i,j} A_i A_j \sin(\phi_i - \phi_j)\sin(\delta_i - \delta_j)}{\sum_{i,j} A_i A_j \cos(\phi_i - \phi_j)\cos(\delta_i - \delta_j)}, \qquad (15)$$

it is clear that a nonzero asymmetry requires that at least two diagrams contribute with different strong and weak phases.

1. $b \to s\gamma$

One of the interesting features of this mode is that any physical model that introduces new *CP* violating phases can result in an asymmetry far greater than that which the SM predicts; yet it must not disturb the branching ratio (BR) for $b \to s\gamma$, which CLEO has measured [35] as

$$BR(b \to s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}. \qquad (16)$$

In Fig. 5 we show the dominant diagrams; as in the case of the SUSY contributions to the EDMs studied above, the most important non-SM diagrams for $b \to s\gamma$ involve the chargino loop in Figs. 5(b) [37]. Since the observed value of the branching ratio (16) agrees very well with the SM prediction, new decay channels are strongly constrained; accordingly, we follow the analysis of [36] in carefully accounting for the higher-order graphs in Figs. 5(c)–5(e).

The effective operators involved are

$$O_2 \equiv \overline{s_L} \gamma_\mu q_L \overline{q_L} \gamma^\mu b_L$$

$$O_7 \equiv \frac{e m_b}{4\pi^2} \overline{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R, \qquad (17)$$

$$O_8 \equiv \frac{g_s m_b}{4\pi^2} \overline{s}_L \sigma_{\mu\nu} G^{\mu\nu} b_R.$$

We leave the evaluation of these operators and all related calculations for the Appendix.

In using these diagrams to compute observables, it is important to take into account that the photon involved in the decay $b \to s\gamma$ is monochromatic, but the photon in the observable $B \to X_s\gamma$ has a variable energy. In addition to depending on the final state X_s, the photon energy is also a function of how the b quark is bound inside the B meson; if the recoil energy of the b quark is small, nonperturbative effects arise for which no reliable models currently exist. These considerations lead us to perform all calculations using a variable outgoing photon energy, E_γ, which is bounded from below: $E_\gamma > (1-\xi)E_{max}$, where ξ is between 0 and 1 and E_{max} is a model-dependent quantity. The actual dependence on ξ and E_{max} in the computed asymmetry $a_{b\to s\gamma}$ turns out to be negligible, as we demonstrate in the Appendix. Here we present the result

$$a_{b\to s\gamma} \approx 0.01 \sin(\phi_\mu),$$

where as in the case $B_s^0 \to \phi\phi$, the strong phase dependence is included in the coefficient and imparts a 10% uncertainty to this prediction. The magnitude of the asymmetry is admittedly not very large;[4] however, it is at least twice the SM prediction [38].

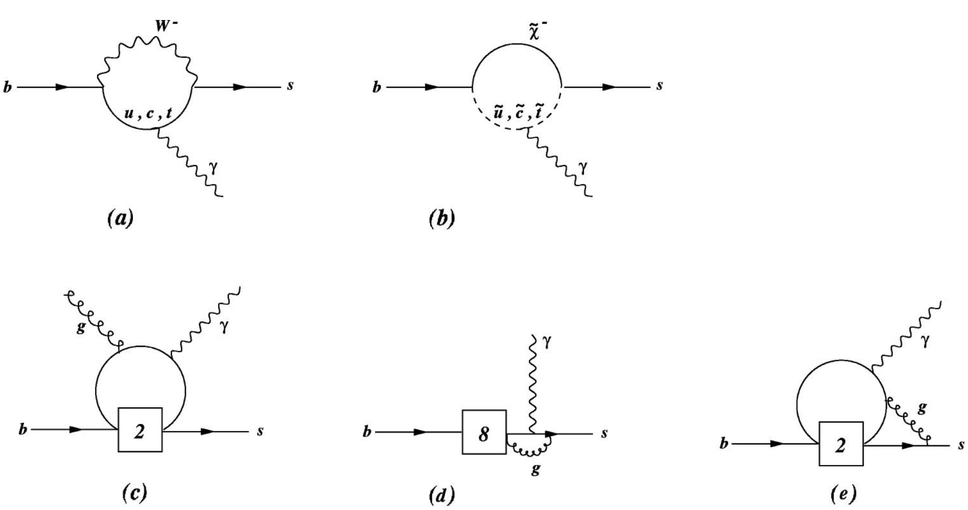

FIG. 5. Leading contributions to $b \to s\gamma$ in (a) SM and (b) MSSM. Graphs (c), (d), and (e), with the effective vertices $O_{2,8}$, play a significant role as well (mass insertions understood).

[4]Similar studies concur on this point [39].

TABLE I. Predicted asymmetries in the B system and experimental error. a_p is listed for both high luminosity (H$\mathcal{L}$) and low luminosity (L$\mathcal{L}$) experiments (see text for explanation) in one year of running. The strong phase uncertainty factors $\delta_i \approx 1 \pm 0.1$. For the explicit form of the function $f(\phi_\mu,\phi_A)$, see the Appendix.

Process (p)	$BR(p)$	a_p(predicted)	a_p (H$\mathcal{L}$)	a_p (L$\mathcal{L}$)
$b \to s\gamma (B^0 \to X_s \gamma)$	2×10^{-4}	$0.01 \sin(\phi_\mu)\delta_1$	0.001	0.02
$B_s^0 \to \phi\phi$	$4\times 10^{-5}\times(0.50)^2$	$0.10 f(\phi_\mu,\phi_A)\delta_2$	0.003	0.09
$B_s^0 \to J/\psi\phi$	$10^{-3}\times(0.12)\times(0.50)$	$0.10 f(\phi_\mu,\phi_A)\delta_3$	0.001	0.03
$B_s^- \to K^-\phi$	$10^{-5}\times(0.50)$	$0.3 \sin(\phi_\mu)\delta_4$	0.004	0.12

2. $B^- \to K^- \phi$

One particularly striking signature of the presence of the SUSY phases is in the decay $B^- \to K^- \phi$. Here the flavor structure $b \to s\bar{s}s$ forbids a SM tree graph, so the leading SM graph is a penguin diagram (defined as ''P''), as shown in Fig. 6(a). The leading SUSY contribution in Fig. 6(b) is also a type of penguin diagram (hereafter designated a superpenguin, SP), suppressed, however, by a factor $(m_W/\tilde{m})^2$, relative to the SM due to a squark propagating in the loop instead of a W boson. So far the situation parallels the decay $B^0 \to \phi\phi$ above, but the major difference is that here the CP violation is necessarily direct. Referring back to Eq. (15), with $\{i,j\}$ running over $\{u,c,t,\tilde{u},\tilde{c},\tilde{t}\}$, we see that the asymmetry receives contributions from 36 interference terms. However, the imaginary parts of SP-SP interference terms are zero since the squarks in the loops of SP graphs are heavier than the b quark and do not give rise to absorptive phases in the amplitude [i.e., the δ's are zero in Eq. (15)]. This leaves SP-P and P-P terms. The latter, being purely SM terms, must always involve at least one u quark to get a nonzero weak phase ($V_{tb}V_{ts}^* = -V_{cb}V_{cs}^* = -A\lambda^2$ is real), whereas the former need not involve u quarks since the weak phase difference necessary for a nonzero asymmetry can come from a SUSY coupling in $\tilde{c}$ or $\tilde{t}$ graphs. Therefore, the dominant SP-P terms will [in the notation of Eq. (15)] have $i \in \{\tilde{c},\tilde{t}\}$ and $j \in \{c,t\}$, leading over the P-P terms by a factor $(V_{tb}V_{ts}^*)/\mathrm{Im}(V_{bu}V_{us}^*) \approx 1/\eta\lambda^2 \approx 100$. Ergo, the P-P terms are negligible and the asymmetry is fundamentally due to the SUSY-SM interference. Assuming as before that the lightest $\tilde{t}$ dominates the SP, the numerator of Eq. (15) only contains the term due to $\tilde{t}$-c interference:

$$a_{B^- \to K^- \phi} \approx \frac{A_c A_{\tilde{t}} \sin(\phi_c - \phi_{\tilde{t}}) \sin(\delta_c)}{A_c^2}. \tag{18}$$

We now employ the one-loop perturbative calculation of the strong phase δ_c from the c-quark loop [34,33] and, since $\phi_c \approx \eta\lambda^2$ is negligible and $\phi_{\tilde{t}}$ is essentially the same phase we calculated for $b \to s\gamma$ we obtain[5]

$$a_{B^- \to K^- \phi} \approx \left(\frac{M_W}{\tilde{m}}\right)^2 \sin(\phi_\mu) \tag{19}$$

Here we see that the absence of neutral flavor changing currents, the hierarchy of the CKM matrix, and the pattern of CP quantum numbers, all conspire in this case to give an asymmetry which is essentially zero in the SM, yet for SUSY with typical sparticle masses, can be as large as 30%.

V. DISCUSSION

We have seen that the B-system observables above provide many constraints on the phases $\{\phi_A,\phi_\mu\}$ in the model. We may classify the experiments by the size of the event samples they are expected to provide:

High Luminosity (H$\mathcal{L}$)*:* For example, experiments at the CERN Large Hadron Collider (LHC) p-p collider, producing a sample of order 10^{10} B^0-$\bar{B}^0$ pairs per year [8].

Low Luminosity (L$\mathcal{L}$)*:* Experiments run at e^+/e^- machines, such as the Cornell Electron Storage Ring (CESR, CLEO III), KEKB(Belle), and SLAC PEP-II(BaBar), as well as at hadronic machines such as Fermilab [Collider Detector at Fermilab (CDF), DØ] and DESY(HERA-B: actually e^+p), which produce similar samples of B^0-$\bar{B}^0$ pairs, of order 10^7 per year [2–6].

Any experiment which detects N B^0-$\bar{B}^0$ pairs can resolve at the one σ level an asymmetry a_p for a process p if

$$a_p > \frac{1}{\sqrt{BR(p)N}}.$$

Using this criterion, the results in Table I summarize the

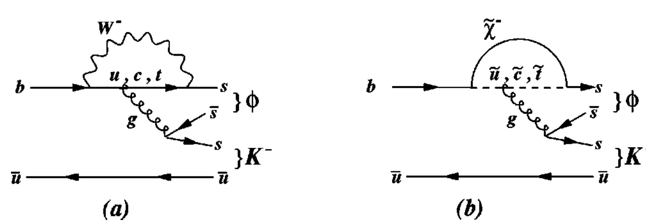

FIG. 6. $B_s^- \to K^- \phi$ in (a) the SM, where the CP violating asymmetry is dominated by the CKM-suppressed u-quark diagram, and (b) the MSSM, where the asymmetry is dominated by the relatively CKM-unsuppressed $\tilde{c}$ and $\tilde{t}$ graphs. Here mass insertions are understood.

[5]Although the helicity structure of the ingoing b quark and outgoing s quark is L-R for $b \to s\gamma$, the L-L and R-R helicity structures of $B_s^- \to K^- \phi$ give negligible contributions to the phase of the diagram.

TABLE II. Definitions of the ξ-dependent coefficients. (p) refers to the parton model and (f) includes Fermi motion.

ξ	$a_{27}^{(p)}$	$a_{87}^{(p)}$	$a_{28}^{(p)}$	$a_{27}^{(f)}$	$a_{87}^{(f)}$	$a_{28}^{(f)}$
1.00	1.06	-9.52	0.16	1.06	-9.52	0.16
0.30	1.17	-9.52	0.12	1.23	-9.52	0.10
0.15	1.31	-9.52	0.07	1.06	-9.52	0.04

TABLE III. Dependence of the asymmetry in $b \to s\gamma$, computed in the parton model and Fermi-motion model on the minimum energy of the soft photon.

ξ	$(1-\xi)E_{max}$ (GeV)	$\lvert a_{parton}\rvert$	$\lvert a_{Fermi}\rvert$
1.00	0	0.008	0.008
0.30	1.85	0.010	0.010
0.15	2.24	0.011	0.012

various modes and their asymmetries, comparing them to the experimental precision expected on each asymmetry. The reader should note that the columns designated ''H$\mathcal{L}$'' and ''L$\mathcal{L}$'' refer to the optimal choice out of the corresponding set of experiments. In some cases, experiments in the same set may have drastically different capabilities. For example, the production of B_s^0 in e^+e^- annihilation requires running on the $\Upsilon(5s)$ resonance, not currently possible at BaBar or Belle, which run at the $\Upsilon(4s)$. Correspondingly, the only observable in Table I available to these latter experiments is the inclusive decay $b \to s\gamma$ in the decay of B_d mesons. The decays of the daughter mesons necessary for detection of events is taken into account in the ''$BR(p)$'' column; for example, $BR(J/\psi \to (e^+e^-, \mu^+\mu^-)) \approx 12\%$ and $BR(\phi \to K^+K^-) \approx 50\%$. More details on the individual capabilities of each experiment may be found in [19].

Although HQE usually yields results perturbatively convergent in powers of Λ/m_b, we allow for hadronic uncertainties at the level of 10% on all observables in Table I [36,27,34]. The parameters describing this are the δ_i ($i = 1 \ldots 4$) which are, in general, completely independent for the observables in question. A valid test of the model requires a 10% measurement of the various asymmetries. Disagreement at higher precision could be ascribed to uncertainties in the strong dynamics.

From the table, we see that L$\mathcal{L}$ experiments can only contribute in the asymmetries in $B_s^- \to K^-\phi$ and $B_s^0 \to J/\psi\phi$. Combined with the H$\mathcal{L}$ measurements of all of the decays studied, a determination of ϕ_μ and ϕ_A will be possible. Combined with the linear combination of phases which electron and neutron EDM's constrain (see Appendix), this provides a complete determination of the set of the phases $\{\phi_{1,3}, \phi_A, \phi_\mu\}$ studied in this model of CP violation.

In summary, the possibility that the phase structure of the MSSM extends beyond the trivial one where CP violation is confined to the CKM matrix leads not only to the requirement that SUSY phases respect the present EDM bounds, but also that the range of phases consistent with these bounds agrees with the values which can be extracted from the various B-system asymmetries considered above. Collectively, measurements of these asymmetries at present and future B-physics experiments will either determine the phases or rule out this particular SUSY model.

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APPENDIX

1. Cancellation of the EDMs

That the contributions to the EDM in Figs. 2(a) and 2(b) tend to cancel in a way dependent only on $\sin(\phi_\mu)$ is evident from the form of the chargino and neutralino mixing matrices:

$$\mathcal{L} \supset \tilde{\chi}^{\pm T}\mathbf{M}_{\tilde{\chi}^\pm}\tilde{\chi}^\pm + \tilde{\chi}^{0T}\mathbf{M}_{\tilde{\chi}^0}\tilde{\chi}^0,$$

where

$$\mathbf{M}_{\tilde{\chi}^\pm} \approx \begin{pmatrix} 0 & 0 & M_2 & 0 \\ 0 & 0 & 0 & \mu \\ M_2 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 \end{pmatrix},$$

$$\mathbf{M}_{\tilde{\chi}^0} \approx \begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & 0 & -\mu \\ 0 & 0 & -\mu & 0 \end{pmatrix},$$

assuming $M_W \ll M_2, \mu$.

The graph in Fig. 2(c) (where f is, say, an electron) receives phases from three sources: the $U(1)$ propagator carries a factor $e^{i\phi_1}$; the scalar mass insertion has a SUSY piece given from the first term of Eq. (3), which has the form $\mathbf{y_e}\mu^*\overline{\tilde{e}}\tilde{e}H_2^{0*}$ which, after EW breaking, becomes $\mathbf{y_e}\mu^*\overline{\tilde{e}}\tilde{e}v\sin\beta$; the scalar mass insertion also has a SUSY B piece [see Eq. (5)] of the form $A\mathbf{y_e}\overline{\tilde{e}}\tilde{e}H_1^0$, which becomes $A\mathbf{y_e}\overline{\tilde{e}}\tilde{e}v\cos\beta$ after EW breaking.

Putting the above pieces together, the imaginary piece of the neutralino graph in Fig. 2(c) carries a phase dependent factor $(|A|\sin(\phi_A+\phi_1)+|\mu|\tan\beta\sin(\phi_1-\phi_\mu))$. Likewise, the corresponding graph for the neutron in the $SU(6)$ model,[6] where $d_n=1/3(4d_d-d_u)$ obtains a factor similar to the electron case, with the replacement $\phi_1\rightarrow\phi_3$; the numerical demonstration that these diagrams can nearly cancel is given in [15].

2. Calculating the B_s^0-$\bar{B}_s^0$ box

We follow the notation of [37] in calculating the contribution to M_{12} from the chargino box in Fig. 3(b):

$$\Delta M_{\tilde{\chi}^\pm}=\frac{\alpha_w^2}{16}\sum_{h,k=1}^{6}\sum_{i,j=1}^{2}\frac{1}{m^2_{\tilde{\chi}^\pm}}(G_{UL}^{jkb}-H_{UR}^{jkb})(G_{UL}^{*iks}-H_{UR}^{*iks})(G_{UL}^{ihb}-H_{UR}^{ihb})(G_{UL}^{*jhs}-H_{UR}^{*jhs})G'_{ijkh}$$

where G and H are gauge and Higgs vertices, and the form factor G' depends on the masses of the particles involved. We make the assumption that the lightest chargino and lightest top squark dominate the loop, and that $M_W \ll M_2,\mu$. The final result is

$$\mathrm{Im}(M_{12})\approx\left(\frac{2M_W}{\tilde{m}}\right)^3\left(\frac{\sin\phi_\mu(|A|\sin\beta\cos\beta\cos\phi_A-|\mu|\cos^2\beta y_t\cos\phi_\mu)}{\sqrt{|A|^2\sin^2\beta+|\mu|^2\cos^2\beta-|A||\mu|\sin\beta\cos\beta y_t\cos(\phi_A+\phi_\mu)}}\right),$$

which assumes a simpler form for typical points in parameter space, as noted above in Eq. (11). In keeping with prior notation, this defines

$$f(\phi_\mu,\phi_A)\equiv\frac{\sin\phi_\mu(|A|\sin\beta\cos\beta\cos\phi_A-|\mu|\cos^2\beta y_t\cos\phi_\mu)}{\sqrt{|A|^2\sin^2\beta+|\mu|^2\cos^2\beta-|A||\mu|\sin\beta\cos\beta y_t\cos(\phi_A+\phi_\mu)}}.$$

3. The operators $O_{2,7,8}$

We again follow [37] in calculating the coefficients $C_{7,8}$ of the operators $O_{7,8}$ from chargino loops:

$$\begin{aligned}C_{7,8\tilde{\chi}^\pm}=&\frac{\alpha_w\sqrt{\alpha}}{2\sqrt{\pi}}\sum_{k=1}^{6}\sum_{j=1}^{2}\frac{1}{m^2_{\tilde{u}_k}}(G_{UL}^{jkb}-H_{UR}^{jkb})(G_{UL}^{*jks}-H_{UR}^{*jks})\\&\times(F_{1,jk}+e_UF_{2,jk})-H_{UL}^{jkb}(G_{UL}^{*jks}-H_{UR}^{*jks})\\&\times\frac{m_{\tilde{\chi}_j^\pm}}{m_b}(F_{3,jk}+e_UF_{4,jk}),\end{aligned}$$

where $F_{1,2,3,4}$ are form factors and C_7 is obtained from the above by setting $\alpha=e^2/(4\pi)$ and $e_U=2/3$; for C_8, $\alpha=g^2/(4\pi)$ and $e_U=0$. Including the SM contributions given in [36], we obtain

$$C_2\approx1.11,$$

$$C_7\approx-0.31-0.19e^{i\phi_\mu},$$

$$C_8\approx-0.15-0.14e^{i\phi_\mu}.$$

Note that C_2, the real part of M_{12}, is not significantly affected by SUSY.

[6]For alternatives, see [40].

4. Soft photons in $b\rightarrow s\gamma$

In Sec. IV C 1 we noted that the energy dependence of the outgoing photon could have a significant effect on the BR and asymmetry. Here we quote the expression in [36] for the BR and asymmetry as functions of $C_{2,7,8}$ and ξ $[E_\gamma>(1-\xi)E_{max}]$:

$$\begin{aligned}BR(B\rightarrow X_s\gamma)\approx&\,2.57\times10^{-3}K_{NLO}(\xi)\\&\times BR(B\rightarrow X_ce\nu)/10.5\%,\end{aligned}$$

where

$$\begin{aligned}K_{NLO}(\xi)\equiv&\sum_{i\leqslant j=2,7,8}(k_{ij}(\xi)\mathrm{Re}(C_iC_j^*)\\&+k_{77}^{(1)}(\xi)\mathrm{Re}(C_7^{(1)}C_7^*)),\end{aligned}$$

$$\begin{aligned}a_{b\rightarrow s\gamma}=&\frac{1}{|C_7|^2}(a_{27}(\xi)\mathrm{Im}(C_2C_7^*)\\&+a_{87}(\xi)\mathrm{Im}(C_8C_7^*)\\&+a_{28}(\xi)\mathrm{Im}(C_2C_8^*)).\end{aligned}$$

The ξ-dependent quantities are listed in Table II, which we paraphrase from [36]; using this, it is straightforward to explicitly calculate the BR and asymmetry for the values of $C_{2,7,8}$ given above. The dependence is insignificant for a wide range of ξ in two different models (see Table III).

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