Radiative corrections to the semileptonic Dalitz plot with angular correlation between polarized decaying hyperons and emitted charged leptons

A. Martínez,[1] J. J. Torres,[1] Rubén Flores-Mendieta,[2] and A. García[3]
[1]*Escuela Superior de Física y Matemáticas del IPN, Apartado Postal 75-702, México, Distrito Federal 07738, Mexico*
[2]*Instituto de Física, Universidad Autónoma de San Luis Potosí, Álvaro Obregón 64, Zona Centro, San Luis Potosí, S.L.P. 78000, Mexico*
[3]*Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, Apartado Postal 14-740, México, Distrito Federal 07000, Mexico*
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We obtain a model-independent expression for the Dalitz plot of semileptonic decays of polarized hyperons including radiative corrections to order α and neglecting terms of order $\alpha q/\pi M_1$, where q is the four-momentum transfer and M_1 is the mass of the decaying hyperon. We specialize our results to exhibit the correlation between the charged-lepton momentum and the spin of the decaying hyperon. We present results for the three-body region of the Dalitz plot and for the complete Dalitz plot (which includes the four-body region). From these results we also obtain the corresponding radiative corrections to the integrated lepton spin-asymmetry coefficient. Our formulas are valid for charged as well as for neutral decaying hyperons and are appropriate for model-independent experimental analyses whether the real photon is discriminated or not.

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I. INTRODUCTION

The form factors of hyperon semileptonic decays (HSD), $A \rightarrow Bl\nu_l$, contain important information about the low-energy strong interactions of spin-1/2 baryons (A and B are such baryons and l and ν_l are the accompanying charged lepton and neutrino). Their experimental determination requires the use of accurate formulas in the analysis of the measurements of several observables. An important one of these observables is the angular correlation between the spin $\hat{\mathbf{s}}_1$ of A and the direction $\hat{\mathbf{l}}$ of the momentum of l. It is the purpose of this paper to calculate the radiative corrections (RC) to the Dalitz plot (DP) with the spin correlation $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$ exhibited explicitly.

We shall obtain expressions that are suitable for a model-independent experimental analysis. The model dependence of the virtual RC is handled following the approach of Sirlin [1] to the RC of neutron beta decay, while the model dependence of the bremsstrahlung RC is controlled by the theorem of Low [2]. In previous work we have discussed the RC to the unpolarized DP [3] and to the DP with the $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ spin correlation kept explicitly [4] ($\hat{\mathbf{p}}_2$ is the direction of the momentum of the emitted baryon B). It is not possible to derive the result for the spin correlation $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$ from the final result of Ref. [4] because all kinematical integrations, except for the l and B energies E and E_2, respectively, were already performed. However, since we are going to follow the same approach of this reference, much of the work has already been advanced.

The bremsstrahlung RC is a four-body decay whose DP covers entirely the DP of the three-body decay $A \rightarrow Bl\nu_l$. We shall refer to the latter as the three-body region (TBR) and to the non-overlap of the former and the latter as the four-body region (FBR). Even when no experimental arrangement has been made to detect and discriminate real photons, it is possible to eliminate the photons that belong to the FBR by energy-momentum conservation. Therefore, in calculating the bremsstrahlung RC we shall keep a clear distinction between these two regions.

We shall also obtain the radiative corrections to the integrated lepton spin-asymmetry coefficient α_l. As we shall see, the distinction between the TBR and the FBR leads to a perceptible change in the RC to this asymmetry coefficient.

Our results will be presented in two final forms. One where the triple integration over the real photon three-momentum $\mathbf{k}$ is left indicated and ready to be performed numerically. And another one, an analytical form, where such a triple integration has been performed. Both forms can be used to numerically cross-check on one another. However, the analytical result, although tedious to feed into a Monte Carlo program, leads to a considerable savings of computer time because the triple integration does not have to be performed within the Monte Carlo calculation every time E and E_2 or the form factors are changed.

For the use of our results it is important that this paper be as self-contained as possible. In Sec. II we introduce our notation and conventions and we review the virtual RC; also the infrared divergence of this part is clearly separated. In Sec. III the real photon emission is calculated and separated into the contributions of the TBR and of the FBR. The calculation of Ref. [4] is adapted to the present case. The infrared divergence is extracted following Ref. [5] and its cancellation with the one of Sec. II is discussed in detail. Our first main result is established, allowing for the elimination (or not) of real photons from the experimental analysis. In Sec. IV we proceed to the analytical evaluation of the triple integration over the photon bremsstrahlung three-momentum. Our second main result is established, also allowing for the experimental discrimination (or not) of real photons. In Sec. V we use the analytical result to obtain the RC to the asymmetry coefficient α_l. In Sec. VI we make numerical evaluations for several HSD and also for the $\Lambda_c^+ \to \Lambda e^+ \nu$ decay. We compare with other results available in the literature. Section VII is reserved to discuss and summarize our results. To make this paper self-contained we introduce two appendices. In Appendix A we give the amplitudes for virtual and bremsstrahlung RC, emphasizing how the model dependence is kept under control. In Appendix B we give the analytical expressions of all the coefficients, both new and of Refs. [3] and [4] required to compute the RC to the $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$ correlation.

Our results have been obtained neglecting contributions of order $\alpha q/\pi M_1$ and higher (q is the momentum transfer and M_1 is the mass of A). They cover both neutral and charged A and are reliable up to 0.5% or better in HSD. Furthermore, they provide a useful result for charm decay experiments with several thousands of events. For higher statistics experiments it will be necessary to incorporate $\alpha q/\pi M_1$ contributions.

II. VIRTUAL RADIATIVE CORRECTIONS

In this section we shall discuss the virtual radiative corrections, up to order α and neglecting terms of order $\alpha q/\pi M_1$, to the DP of the HSD

$$A \to B + l + \nu_l, \tag{1}$$

with A polarized. Our results will be specialized to exhibit the angular correlation $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$.

A convenient procedure to get such results is that of Ref. [4]. So, in this paper we adopt the same approach, the same approximations and the same conventions of this reference. In this way, $p_1 = (E_1, \mathbf{p}_1)$, $p_2 = (E_2, \mathbf{p}_2)$, $l = (E, \mathbf{l})$, and $p_\nu^0 = (E_\nu^0, \mathbf{p}_\nu)$ will be the four-momenta of A, B, l, and ν_l, respectively. M_1, M_2, and m will denote the masses of the first three particles. We shall assume throughout this paper that $m_\nu = 0$. $\hat{\mathbf{p}}_2$ will denote a unit vector along $\mathbf{p}_2$, etc. We shall make our calculations in the center-of-mass (CM) frame of A. In this case, p_2, l, and p_ν will denote the magnitudes of the corresponding three-momenta. There will be no confusion because the expressions obtained will not be manifestly covariant. Because we want our results to exhibit the correlation $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$, it is convenient to choose the z-axis along $\mathbf{l}$ and not along $\mathbf{p}_2$ as done in Ref. [4].

At this point, it is convenient to mention that it is not possible to obtain the virtual RC of our present case by using the final virtual RC given by $d\Gamma_V$ of Eq. (15) of Ref. [4]. This is because in that equation the correlation $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ was singled out after the integration over the azimuthal angle ϕ_l of $\mathbf{l}$ was performed [in such Eq. (15) $d\phi_l$ is still present in the phase space. As it appears and because of the choice of z-axis its integration amounts a 2π factor]. Therefore, all the terms in $d\Gamma_V$ containing the product $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$, and which appear before the integration over $d\phi_l$ is performed, have been transformed leaving the correlation $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ only. There is no way to recover the $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$ terms from this Eq. (15) of Ref. [4]. So, for obtaining our $d\Gamma_V$, exhibiting the correlation $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$ only, we have to take a few steps back before that Eq. (15).

Our calculation now starts at the point where the scalar products $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$ and $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ appear for the first time, that is, at Eq. (13) of Ref. [4], namely,

$$\sum_{\text{spins}} |\mathsf{M}_V|^2 = \frac{1}{2} \sum_{\text{spins}} |\mathsf{M}_V'|^2 - \frac{1}{2} \sum_{\text{spins}} |\mathsf{M}_V^{(s)}|^2. \tag{2}$$

Here M_V is the sum of the order zero amplitude M_0', corrected [1] by the model-dependent part of the virtual radiative corrections through the modified form factors f_1' and g_1', and the model-independent amplitude M_v of such RC. M_0' and M_v are given explicitly in Appendix A [see Eqs. (A1) and (A3)]. Of course, Eq. (2) is now adapted to our case of a polarized hyperon A. That is, the spinor $u_A(p_1)$ now appears $\Sigma(s_1) u_A(p_1)$, with $\Sigma(s_1)$ being the spin projection operator of A given in Eq. (4) of Ref. [4].

With Eq. (2) we can express the differential decay rate $d\Gamma_V$ as

$$\begin{aligned} d\Gamma_V &= \frac{dE_2 \; dE \; d\Omega_l \; d\phi_2}{(2\pi)^5} M_2 m m_\nu \left[\frac{1}{2} \sum_{\text{spins}} |\mathsf{M}_V'|^2 \right. \\ &\quad \left. - \frac{1}{2} \sum_{\text{spins}} |\mathsf{M}_V^{(s)}|^2 \right] \\ &= d\Gamma_V' - d\Gamma_V^{(s)}. \end{aligned} \tag{3}$$

Notice the variables in the phase space of this equation, they correspond to our new choice of the z-axis along $\mathbf{l}$. $d\Omega_l$ is the differential of the solid angle of $\hat{\mathbf{l}}$ and $d\phi_2$ is the differential of the azimuthal angle of $\hat{\mathbf{p}}_2$.

In Eq. (3), $d\Gamma_V'$ corresponds to the first term within the square brackets. It can be identified with the differential decay rate with virtual radiative corrections of unpolarized A given in Eq. (10) of Ref. [3] and, therefore, there is no need to recalculate it now. $d\Gamma_V^{(s)}$ corresponds to the second term of the square brackets of Eq. (3) and it contains the scalar products $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$, $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_\nu$, and $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$. The scalar product $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_\nu$ can be expressed in terms of $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$ and $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ by three-momentum conservation. In this way, only $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$ and $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ will appear in $d\Gamma_V^{(s)}$. Now, we require that $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$ be the only scalar product present in $d\Gamma_V^{(s)}$. This can be accomplished by noting that the most general form of $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ depends on $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$, $\hat{\mathbf{l}} \cdot \hat{\mathbf{p}}_2$, and $\cos\phi_2$. The terms directly proportional to this cosine drop out after integration over ϕ_2 from 0 to 2π. This fact allows us to use the replacement (see Ref. [6] for further discussion)

$$\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2 \to (\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}})(\hat{\mathbf{l}} \cdot \hat{\mathbf{p}}_2) = \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}\, y, \tag{4}$$

in $d\Gamma_V^{(s)}$, leaving us with an expression which only contains the correlation $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$. In Eq. (4) y is the cosine of the angle between $\hat{\mathbf{p}}_2$ and $\hat{\mathbf{l}}$. Let us define, in the CM of A, the variable y_0

$$y_0 = \frac{(M_1 - E_2 - E)^2 - p_2^2 - l^2}{2 p_2 l}. \tag{5}$$

In this three body decay one can make the identification

$$E_\nu^0 = M_1 - E_2 - E, \tag{6}$$

as the neutrino energy and then one can see that $y = y_0$. Of course, this y varies within -1 and 1. When a real photon is present y_0 will not be any longer identifiable with y (see Sec. III).

With these considerations in mind, we can express the $d\Gamma_V$ of Eq. (3) as

$$d\Gamma_V = \frac{G_V^2}{2}\frac{dE_2 \;\; dE \;\; d\Omega_l \;\; d\phi_2}{(2\pi)^5} 2M_1 \Bigg\{ A_0' + \frac{\alpha}{\pi}(A_1'\phi + A_1''\phi') - \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}} \left[B_0'' + \frac{\alpha}{\pi}(B_2'\phi + B_2''\phi') \right] \Bigg\}. \tag{7}$$

This $d\Gamma_V$ is the DP with virtual radiative corrections up to order α (and neglecting terms of order $\alpha q/\pi M_1$), leaving E_2 and E as the relevant variables and with only the angular correlation $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$ explicitly exhibited. The integration over ϕ_2 only amounts the factor 2π. Now, A_0', A_1', and A_1'' are given in Eqs. (B1)–(B3) of Appendix B. The new terms B_0'', B_2', and B_2'' are

$$B_0'' = Q_6 E p_2 y_0 + Q_7 E l, \tag{8}$$

$$B_2' = -D_3 E_\nu^0 l + D_4 E(p_2 y_0 + l), \tag{9}$$

$$B_2'' = D_4 E(p_2 y_0 + l). \tag{10}$$

In these equations the coefficients Q_6 and Q_7 are long quadratic functions of the form factors. They are given in Eqs. (B6) and (B7), respectively, of Ref. [4]. For completeness we repeat them too [see Eqs. (B9) and (B10)]. D_3 and D_4 depend on the leading form factors f_1' and g_1' and they are given in Eqs. (B13) and (B14), respectively. The primes on A_0', A_1', A_1'', B_0'', B_2', and B_2'' indicate that these terms contain the model-dependence of the virtual radiative corrections through the leading form factors. The model-independent functions ϕ and ϕ' are

$$\phi(E) = 2\left(\frac{1}{\beta}\operatorname{arctanh}\beta - 1\right)\ln\frac{\lambda}{m} - \frac{1}{\beta}(\operatorname{arctanh}\beta)^2 + \frac{1}{\beta}L\left(\frac{2\beta}{1+\beta}\right) + \frac{1}{\beta}\operatorname{arctanh}\beta - \frac{11}{8} + \begin{cases} \pi^2/\beta + \frac{3}{2}\ln(M_2/m) & \text{(NDH)}, \\ \frac{3}{2}\ln(M_1/m) & \text{(CDH)}, \end{cases} \tag{11}$$

$$\phi'(E) = \left(\beta - \frac{1}{\beta}\right)\operatorname{arctanh}\beta. \tag{12}$$

NDH (CDH) stands for neutral (charged) decaying hyperons. In these equations we use the definition $\beta = l/E$ and L denotes the Spence function $L(x) = \int_0^x (dt/t)\ln|1-t|$. λ represents a small photon mass which regularizes the infrared divergence in the function $\phi(E)$. Actually, Eq. (7) contains twice the infrared divergence. The first one appears in $A_1'\phi$ of the spin-independent part and the other one appears in $B_2'\phi$ of the spin-dependent part. Both divergences will be canceled by their counterparts in the bremsstrahlung contribution.

Let us close this section by comparing the $d\Gamma_V$ of Eq. (7) with the $d\Gamma_V$ of Eq. (15) of Ref. [4]. In spite of minor differences in their phase space factors, we can see that their spin-independent parts are the same. This is not the case for their spin-dependent parts. We can notice that the coefficients A_0'', A_2', and A_2'', which appear in the spin-dependent part of $d\Gamma_V$ of Ref. [4], have changed to the coefficients B_0'', B_2', and B_2'', respectively, of $d\Gamma_V$ of Eq. (7). We observe in Eqs. (8)–(10) that in the B coefficients y_0 always appears as a factor of p_2, while for the A coefficients of Ref. [4] y_0 always appears as a factor of l. This latter observation may induce us to think of the possibility of obtaining the $d\Gamma_V$ of Eq. (7) from the $d\Gamma_V$ of Ref. [4] by simply interchanging p_2 with l. Unfortunately this rule does not work because under such an interchange the A coefficients do not lead to the B coefficients and, thus, we cannot obtain Eq. (7) directly from the final $d\Gamma_V$ of Ref. [4].

III. BREMSSTRAHLUNG RADIATIVE CORRECTIONS

In addition to the virtual RC the bremsstrahlung contributions must be calculated to get the complete RC to the DP of polarized decaying hyperons. In this section, we shall obtain them, to the same order of approximation as the virtual RC, both for the TBR and for the FBR. First we shall define those regions and next we shall proceed to the calculations. As is discussed in Appendix A, these corrections are model-independent by virtue of the theorem of Low [2].

A. Kinematics, TBR, and FBR

The DP in the variables E and E_2 is the kinematically allowed region of the four-body decay

$$A \to B + l + \nu_l + \gamma, \tag{13}$$

where γ represents a real photon with four-momentum $k = (\omega, \mathbf{k})$. The DP can be seen as the union of the TBR and FBR, each one defined presently.

The TBR of the DP is the region where the three-body decay (1) and the four-body decay (13) overlap completely. The energies E and E_2 satisfy the bounds

$$E_2^- \leqslant E_2 \leqslant E_2^+, \tag{14}$$

and

$$m \leqslant E \leqslant E_m. \tag{15}$$

Here, E_2^+ (E_2^-) is the upper (lower) boundary of the TBR given by

$$E_2^{\pm}=\frac{1}{2}(M_1-E\pm l)+\frac{M_2^2}{2(M_1-E\pm l)}, \tag{16}$$

and E_m is the maximum energy of the charged lepton

$$E_m=\frac{M_1^2-M_2^2+m^2}{2M_1}. \tag{17}$$

The FBR of the DP is the region where only the four-body decay (13) can take place. The energies E and E_2 in this region satisfy the bounds

$$M_2\leqslant E_2\leqslant E_2^-, \tag{18}$$

$$m\leqslant E\leqslant E_B, \tag{19}$$

where

$$E_B=\frac{(M_1-M_2)^2+m^2}{2(M_1-M_2)}. \tag{20}$$

Both regions TBR and FBR are depicted in Fig. 1 of Ref. [3], where more details about these regions can be found.

We can now proceed to the calculation of the bremsstrahlung RC to the DP. First we shall do this for the TBR and next for the FBR. The complete bremsstrahlung RC to the DP are obtained by simply adding the results of the TBR to those of the FBR. To obtain the complete RC of each region, we must also add the virtual RC of Eq. (7).

B. TBR bremsstrahlung RC

Here we shall obtain the bremsstrahlung RC restricted to the TBR and with the angular correlation $\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}$ explicitly shown. We shall follow closely the procedure employed in Ref. [4]. In order to extract the infrared divergent terms and the finite terms that accompany them, we shall use the approach of Ref. [5], which was applied to the RC to the DP of $K_{e3}^{\pm}$ decays. However, for the same reasons discussed in Sec. II, it is not possible to use the final result of Ref. [4] for the bremsstrahlung RC to obtain the corresponding result for the RC to the $\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}$ correlation. We must start at an earlier stage of the calculations of this reference, namely, from its Eq. (31) which reads

$$\sum_{\text{spins}}|\mathsf{M}_B|^2=\frac{1}{2}\sum_{\text{spins}}|\mathsf{M}_B'|^2-\frac{1}{2}\sum_{\text{spins}}|\mathsf{M}_B^{(s)}|^2. \tag{21}$$

This equation is the square, summed over spins, of the bremsstrahlung transition amplitude M_B of the four-body process (13) with the spinor $u_A(p_1)$ replaced by $\Sigma(s_1)u_A(p_1)$. The explicit form of M_B is given in Eq. (A5) of Appendix A. Equation (21) enables us to express the bremsstrahlung differential decay rate as

$$d\Gamma_B^{\text{TBR}}=\frac{M_2 m\, m_\nu}{(2\pi)^8}\frac{d^3p_2}{E_2}\frac{d^3l}{E}\frac{d^3k}{2\omega}\frac{d^3p_\nu}{E_\nu}\sum_{\text{spins}}|\mathsf{M}_B|^2 \times\delta^4(p_1-p_2-l-p_\nu-k) \equiv d\Gamma_B'-d\Gamma_B^{(s)}, \tag{22}$$

where $d\Gamma_B'$ contains the first term of Eq. (21) and is independent of $\hat{\mathbf{s}}_1$, while $d\Gamma_B^{(s)}$ contains the second term of this equation and is spin-dependent.

There is no need to recalculate the spin-independent term $d\Gamma_B'$. This part is readily identified with the Eq. (33) of Ref. [4], namely,

$$d\Gamma_B'=d\Gamma_B^{\text{ir}}+d\Gamma_B^{a}+d\Gamma_B^{b}. \tag{23}$$

$d\Gamma_B^{\text{ir}}$ contains the infrared-divergent terms. Following the method of Ref. [5], we can use the equality (56) of Ref. [4] and write $d\Gamma_B^{\text{ir}}$ as

$$d\Gamma_B^{\text{ir}}=\frac{\alpha}{\pi}d\Omega'[A_1'I_0(E,E_2)+C_2], \tag{24}$$

instead of Eq. (34) of Ref. [4]. The phase space factor in Eq. (24) is now

$$\frac{\alpha}{\pi}d\Omega'=\frac{\alpha}{\pi}\frac{G_V^2}{2}\frac{dE_2\, dE\, d\Omega_l\, d\phi_2}{(2\pi)^5}2M_1, \tag{25}$$

instead of the $d\Omega$ of Eq. (35) of Ref. [4]. A_1' is defined in Appendix B and the result for $I_0(E,E_2)$ is given in Eq. (52) of Ref. [4], namely,

$$I_0(E,E_2)=\frac{1}{\beta}\text{arctanh}\,\beta\left[2\ln\left(\frac{2l}{\lambda}\right)+\ln\left(\frac{m\ \eta_{\max}^2}{4(E+l)r_+}\right)\right] -\frac{1}{\beta}L\left(-\frac{a^2}{4r_+}\right)+\frac{1}{\beta}L\left(-\frac{4r_-}{a^2}\right)-2\ln\left(\frac{m}{\lambda}\right) -\ln\left(\frac{\eta_{\max}^2}{2mE_\nu^0(q^2-m^2)}\right), \tag{26}$$

where

$$r_{\pm}=\frac{1}{E+l}\{[E_\nu^0 l^2(q^2-m^2)-a^2E/4] \pm\{[E_\nu^0 l^2(q^2-m^2)-a^2E/4]^2-m^2a^4/16\}^{1/2}\},$$

$$a^2=\eta_{\max}(4p_2 l-\eta_{\max}),$$

and

$$q^2=M_1^2-2M_1E_2+M_2^2.$$

$\eta_{\max}$ is defined right below Eq. (31).

With Eqs. (36)–(38) of Ref. [4] for C_2, $d\,\Gamma_B^a$, and $d\,\Gamma_B^b$, respectively, we can express the $d\,\Gamma_B'$ of Eq. (23), with some minor rearrangements, in the compact form

$$d\,\Gamma_B'=\frac{\alpha}{\pi}d\Omega'\Bigg\{A_1'I_0(E,E_2)+\frac{p_2 l}{4\pi}\int_{-1}^{1}dx\int_{-1}^{y_0}dy\int_0^{2\pi}d\phi_k[|\mathsf{M}'|^2+|\mathsf{M}''|^2]\Bigg\}, \tag{27}$$

with

$$|\mathsf{M}'|^2=\frac{\beta^2(1-x^2)}{(1-\beta x)^2}\left[D_2-\frac{D_1E+D_2lx}{D}\right], \tag{28}$$

and

$$|\mathsf{M}''|^2=\frac{E_\nu}{ED(1-\beta x)}\left[D_1\left(\omega+E(1+\beta x)-\frac{m^2}{E(1-\beta x)}\right)+D_2\hat{\mathbf{p}}_\nu\cdot\left(\mathbf{l}+\hat{\mathbf{k}}(E+\omega)-\hat{\mathbf{k}}\frac{m^2}{E(1-\beta x)}\right)\right]. \tag{29}$$

In these last three equations, $x=\hat{\mathbf{l}}\cdot\hat{\mathbf{k}}$ and $y=\hat{\mathbf{l}}\cdot\hat{\mathbf{p}}_2$ are the cosines of the polar angles of $\mathbf{k}$ and $\mathbf{p}_2$, respectively, whereas ϕ_k is the azimuthal angle of $\mathbf{k}$. Furthermore, $E_\nu=E_\nu^0-\omega$ and $D=E_\nu^0+(\mathbf{p}_2+\mathbf{l})\cdot\hat{\mathbf{k}}$. The quantity E_ν^0 was defined in Eq. (6) and y_0 is the variable defined in Eq. (5). Notice that, even if we are still in the TBR, y_0 is no longer equal to y; however inside this region y_0 still varies within -1 and 1 (Also notice that outside, in what we called the FBR, $y_0\geqslant 1$ always.) The coefficients D_1 and D_2 depend on the leading form factors and they are given in Eqs. (B11) and (B12) of Appendix B.

Once we have $d\,\Gamma_B'$ of Eq. (22), we can turn our attention to the spin-dependent part $d\,\Gamma_B^{(s)}$ of this equation. In order to compute it we shall start from Eq. (43) of Ref. [4], namely,

$$d\,\Gamma_B^{(s)}=d\,\Gamma_B^{\mathrm{I}}+d\,\Gamma_B^{\mathrm{II}}, \tag{30}$$

where $d\,\Gamma_B^{\mathrm{I}}$ and $d\,\Gamma_B^{\mathrm{II}}$ contain $\Sigma_{\mathrm{spins}}|\mathsf{M}_a^{(s)}|^2$ and $\Sigma_{\mathrm{spins}}(|\mathsf{M}_b^{(s)}|^2+2\mathrm{Re}[\mathsf{M}_a^{(s)}][\mathsf{M}_b^{(s)}]^\dagger)$, respectively. $\mathsf{M}_a^{(s)}$ and $\mathsf{M}_b^{(s)}$ are the spin-dependent parts of the amplitudes M_a and M_b defined in Eq. (A5), after the spinor u_A is replaced by $\Sigma(s_1)u_A$. $d\Gamma_B^{\mathrm{I}}$ contains the infrared-divergent terms as well as many infrared-convergent ones. $d\,\Gamma_B^{\mathrm{II}}$ is infrared-convergent only. To compute $d\,\Gamma_B^{\mathrm{I}}$ we follow the procedure of Ref. [5] to extract the infrared divergence. According to this and using the explicit form of $\Sigma_{\mathrm{spins}}|\mathsf{M}_a^{(s)}|^2$ given in Eq. (44) of Ref. [4], we can write $d\,\Gamma_B^{\mathrm{I}}$ as

$$d\,\Gamma_B^{\mathrm{I}}=\frac{\alpha}{\pi}d\Omega'\frac{1}{8\pi}\lim_{\lambda\to 0}\int_{\lambda^2}^{\eta_{\max}}d\eta\frac{d^3k}{\omega}\frac{d^3p_\nu}{E_\nu}\delta^4(p_1-p_2-l-p_\nu-k)\times[-D_3\hat{\mathbf{s}}_1\cdot\mathbf{l}E_\nu^0+D_4\hat{\mathbf{s}}_1\cdot\mathbf{p}_2E+D_4\hat{\mathbf{s}}_1\cdot\mathbf{l}E+D_3\hat{\mathbf{s}}_1\cdot\mathbf{l}\omega+D_4\hat{\mathbf{s}}_1\cdot\mathbf{k}E]\times\sum_\epsilon\left(\frac{l\cdot\epsilon}{l\cdot k}-\frac{p_1\cdot\epsilon}{p_1\cdot k}\right)^2. \tag{31}$$

The infrared divergence is contained in the first three terms within the square brackets, the remaining two terms are infrared-convergent. $\eta=(p_\nu+k)^2$ is the invariant mass, which in the CM of A is given by $\eta=2p_2l(y_0-y)$. In the TBR, $\eta_{\max}=2p_2l(y_0+1)$ and $\eta_{\min}=\lambda^2$, with $\lambda^2\to 0$. The coefficients D_3 and D_4 depend on the leading form factors and they are given in Eqs. (B13) and (B14) of Appendix B. ϵ_μ is the polarization four-vector of the real photon.

In order to express $d\,\Gamma_B^{\mathrm{I}}$ in terms of the correlation $\hat{\mathbf{s}}_1\cdot\mathbf{l}$ we need to express the scalar products $\hat{\mathbf{s}}_1\cdot\mathbf{p}_2$ and $\hat{\mathbf{s}}_1\cdot\mathbf{k}$ of Eq. (31) in terms of $\hat{\mathbf{s}}_1\cdot\mathbf{l}$. This can be achieved by using the substitution of Eq. (4) and its analog [6],

$$\hat{\mathbf{s}}_1\cdot\mathbf{k}\quad\rightarrow\quad(\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}})\,(\hat{\mathbf{l}}\cdot\mathbf{k}). \tag{32}$$

In Eq. (31), with these substitutions, we can separate the infrared-divergent terms from the infrared-convergent ones, to get

$$d\,\Gamma_B^{\mathrm{I}}=\frac{\alpha}{\pi}d\Omega'\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}\Bigg\{B_2'\frac{1}{8\pi}\lim_{\lambda\to 0}\int_{\lambda^2}^{\eta_{\max}}d\eta\frac{d^3k}{\omega}\frac{d^3p_\nu}{E_\nu}\delta^4(p_1-p_2-l-p_\nu-k)\times\left[\frac{2p_1\cdot l}{p_1\cdot k\;\;l\cdot k}-\frac{m^2}{(l\cdot k)^2}-\frac{M_1^2}{(p_1\cdot k)^2}\right]+\frac{1}{8\pi}\lim_{\lambda\to 0}\int_{\lambda^2}^{\eta_{\max}}d\eta\frac{d^3k}{\omega}\frac{d^3p_\nu}{E_\nu}\delta^4(p_1-p_2-l-p_\nu-k)\times\left[D_3\omega l+D_4\hat{\mathbf{l}}\cdot\hat{\mathbf{k}}\,\omega E-D_4\frac{\eta}{2\beta}\right]\frac{\beta^2}{\omega^2}\frac{1-(\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})^2}{(1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})^2}\Bigg\}. \tag{33}$$

Here B_2' is given in Eq. (9). In the first integral the sum over polarizations indicated in Eq. (31) was performed in covariant form, whereas in the second one it was performed by using the Coester representation [7].

The first integral in Eq. (33) can be identified with the divergent integral I_0 of Eq. (26). The second one can be put in the convenient form of Eq. (38) of Ref. [3] by performing the integration over the δ function and leaving y as the integration variable instead of η. In this way, we can express $d\,\Gamma_B^{\mathrm{I}}$ finally as

$$d\,\Gamma_B^{\rm I}=\frac{\alpha}{\pi}d\Omega'\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}\left\{B_2'I_0(E,E_2)+\frac{p_2E}{4\pi}\int_{-1}^{1}dx\int_{-1}^{y_0}dy\int_0^{2\pi}d\phi_k\times\left[D_3\frac{\beta l}{D}-D_4\left(1-\frac{lx}{D}\right)\right]\frac{\beta^2(1-x^2)}{(1-\beta x)^2}\right\}. \tag{34}$$

The other term of Eq. (30), $d\,\Gamma_B^{\rm II}$, does not need any calculation. We can take the result of Eq. (57) of Ref. [4] and, with only minor changes in the phase space factor, we can adapt it to our case. After the application of rule (32) in such Eq. (57) is performed, we obtain

$$d\,\Gamma_B^{\rm II}=\frac{\alpha}{\pi}d\Omega'\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}\frac{p_2\beta}{4\pi}\int_{-1}^{1}\frac{dx}{1-\beta x}\int_{-1}^{y_0}dy\int_0^{2\pi}d\phi_k\frac{1}{D}\times\left\{D_3E_\nu\left[-l-(E+\omega)x+\frac{m^2}{E}\frac{x}{1-\beta x}\right]\right.$$
$$\left.+D_4\left[\omega+(1+\beta x)E-\frac{m^2}{E}\frac{1}{1-\beta x}\right](p_2y+l+\omega x)\right\}. \tag{35}$$

At this point we can collect our partial results to get our first main result, namely, the DP of polarized decaying hyperons with radiative corrections up to order α, neglecting terms of order $\alpha q/\pi M_1$, and restricted to the TBR. It can be set as

$$d\,\Gamma^{\rm TBR}=d\,\Gamma_V+d\,\Gamma_B^{\rm TBR}$$
$$=d\,\Gamma_V+[d\,\Gamma_B'-(d\,\Gamma_B^{\rm I}+d\,\Gamma_B^{\rm II})], \tag{36}$$

where $d\,\Gamma_V$ is given in Eq. (7), $d\,\Gamma_B^{\rm TBR}$ is given in Eq. (22), and $d\Gamma_B'$, $d\,\Gamma_B^{\rm I}$ and $d\,\Gamma_B^{\rm II}$ are given in Eqs. (27), (34) and (35), respectively. The integration over the three-momentum of the real photon in these last three equations is ready to be performed numerically (but it can be performed analytically too, as we shall see in the next section).

The result of Eq. (36) can be compared with the corresponding one of Ref. [4] [the sum of Eqs. (15), (33), (51), and (57) of this reference]. We observe that the spin-independent parts $d\,\Gamma_B'$ are the same, although our present $d\,\Gamma_B'$ is expressed in a more compact form, while the spin-dependent parts are different. By looking in detail at those equations of Ref. [4], it is clear that one cannot obtain from them the above Eqs. (34) and (35), as we stressed in the introduction.

C. FBR bremsstrahlung RC and complete RC

We shall now calculate the bremsstrahlung contribution of the FBR and afterwards we shall obtain the complete RC to the DP, with the addition of the TBR and virtual contributions.

The calculation of bremsstrahlung in the FBR is relatively simple because the events in this region have the same amplitude M_B of Eq. (A5) and it is infrared-convergent. We need to change the upper limit of the integrals over the variable y of Eqs. (27), (34), and (35)—this limit now becomes one—and to change the previously infrared-divergent integral I_0 of these equations into I_{0F} defined as

$$I_{0F}=\frac{\theta_{0F}}{2}\ln\left(\frac{y_0+1}{y_0-1}\right), \tag{37}$$

with

$$\theta_{0F}=4\left(\frac{1}{\beta}\operatorname{arctanh}\beta-1\right). \tag{38}$$

I_{0F} is no longer infrared-divergent because in the FBR the photon has a minimum energy which is nonzero. It can be easily calculated from Eq. (18) of Ref. [5]. The invariant mass η of this equation must be integrated now from a minimum value $\eta_{\rm min}=2p_2l(y_0-1)$ to a maximum value $\eta_{\rm max}=2p_2l(y_0+1)$. The quantity y_0 is still defined as in Eq. (5), but within this FBR it is always positive and greater than one; it can even grow indefinitely, but this causes no problem. It is also clear that I_{0F} is always real.

With these changes we can write the differential decay rate corresponding to the FBR as

$$d\,\Gamma_B^{\rm FBR}=d\,\Gamma_B'^{\rm\ FBR}-d\,\Gamma_B^{(s)\rm\ FBR}, \tag{39}$$

with

$$d\,\Gamma_B'^{\rm\ FBR}=\frac{\alpha}{\pi}d\Omega'\left\{A_1'I_{0F}(E,E_2)+\frac{p_2l}{4\pi}\int_{-1}^{1}dx\int_{-1}^{1}dy\int_0^{2\pi}d\phi_k[|\mathsf{M}'|^2+|\mathsf{M}''|^2]\right\}, \tag{40}$$

and

$$d\,\Gamma_B^{(s)\rm\ FBR}=\frac{\alpha}{\pi}d\Omega'\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}\left\{B_2'I_{0F}(E,E_2)+\frac{p_2l}{4\pi}\int_{-1}^{1}dx\int_{-1}^{1}dy\int_0^{2\pi}d\phi_k[|\mathsf{M}'''|^2+|\mathsf{M}^{\rm IV}|^2]\right\}. \tag{41}$$

In Eq. (40), $|\mathsf{M}'|^2$ and $|\mathsf{M}''|^2$ are given in Eqs. (28) and (29), respectively. Equation (41) is the sum of Eqs. (34) and (35), after the above changes are performed. $|\mathsf{M}'''|^2$ and $|\mathsf{M}^{\rm IV}|^2$ are

$$|\mathsf{M}'''|^2=\frac{\beta^2(1-x^2)}{(1-\beta x)^2}\left[\frac{D_3l+D_4Ex}{D}-D_4\frac{1}{\beta}\right], \tag{42}$$

$$|\mathsf{M}^{\mathrm{IV}}|^2=\frac{1}{DE}\Bigg[D_3E_\nu\bigg(-l-Ex-\omega x+\frac{m^2}{E}\frac{x}{1-\beta x}\bigg)+D_4\bigg(\omega-\frac{m^2}{E}\frac{1}{1-\beta x}+(1+\beta x)E\bigg)\times(p_2y+l+\omega)\Bigg]. \tag{43}$$

Equation (39) is the FBR contribution to the RC of the DP. It can be added to the TBR contribution to obtain the complete RC to the DP of polarized hyperons within the approximations mentioned before. This completes our first main result of Eq. (36) by including the emission of all real photons allowed by energy-momentum conservation. It is displayed compactly as

$$d\,\Gamma=d\,\Gamma^{\mathrm{TBR}}+d\,\Gamma_B^{\mathrm{FBR}}, \tag{44}$$

with $d\Gamma^{\mathrm{TBR}}$ and $d\,\Gamma_B^{\mathrm{FBR}}$ given in Eqs. (36) and (39), respectively. The integrations over the photon variables are ready to be performed numerically.

Let us close this section by mentioning that all the integrals which arise in the two regions of the DP can be performed analytically. Because of this, we can obtain completely analytical results for the RC of the DP. We shall do this in the next section.

IV. ANALYTICAL INTEGRATIONS

In this section we shall perform analytically the photon three-momentum integrals contained in Eqs. (27), (34), (35), (40), and (41) to obtain an analytical expression for the RC to the DP restricted to the TBR first and for the total DP later.

A. TBR analytical form

The **k**-integrals corresponding to the TBR of the DP are those of Eqs. (27), (34), and (35). They can be performed analytically by following the procedure of Sec. V of Ref. [3], where the RC to the DP of unpolarized hyperons were obtained. Fortunately much of the work has already been advanced. The integrals that concern us now can be expressed in terms of the functions θ_i $(i=2,\ldots,9)$ given by Eq. (99) of that reference, and in terms of the θ_j $(j=10,\ldots,16)$ given by Eq. (46) of Ref. [8]. In this last reference the RC include all the terms of order $\alpha q/\pi M_1$, which are dropped here. We can express the analytical form of Eq. (27) as

$$d\Gamma'_B=\frac{\alpha}{\pi}d\Omega'[A'_1I_0+(D_1+D_2)(\theta'+\theta''')+D_2(\theta''+\theta^{\mathrm{IV}})]. \tag{45}$$

This equation is equivalent to dw_B of Eq. (92) of Ref. [3]. Similarly, the expressions for $d\Gamma_B^{\mathrm{I}}$ and $d\Gamma_B^{\mathrm{II}}$ of Eqs. (34) and (35) become

$$d\Gamma_B^{\mathrm{I}}=\frac{\alpha}{\pi}d\Omega'\,\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}[B'_2I_0+D_3\rho_1^l+D_4\rho_2^l], \tag{46}$$

$$d\Gamma_B^{\mathrm{II}}=\frac{\alpha}{\pi}d\Omega'\,\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}[D_3\rho_3^l+D_4\rho_4^l]. \tag{47}$$

In Eq. (45) the θ_i functions are contained in the functions θ', θ'', θ''' and θ^{IV} given by Eqs. (85), (86), (90), and (91) of Ref. [3], respectively. The functions ρ_i^l $(i=1,\ldots,4)$ in Eqs. (46),(47) can be expressed as

$$\rho_1^l=\frac{p_2l^2}{2}[(\beta^2-1)\theta_2+2\theta_3-\theta_4], \tag{48}$$

$$\rho_2^l=\frac{p_2E^2}{2}\bigg[-\frac{2}{E}\theta_0+(\beta^2-1)\theta_2-(\beta^2-3)\theta_3-2\theta_4-\beta\theta_5\bigg], \tag{49}$$

$$\rho_3^l=\frac{p_2}{2}\bigg\{E(E+E_\nu^0)(1-\beta^2)\theta_2-\bigg[(3-\beta^2)\frac{E^2}{2}+EE_\nu^0\bigg]\theta_3+\frac{1}{2}E^2(1+\beta^2)\theta_4-\frac{l}{2}(E+2E_\nu^0)\theta_5-\frac{m^2}{2E}\theta_6+\frac{1}{2}(2E-E_\nu^0)\theta_7-\frac{1}{2}(E-E_\nu^0)\theta_8+\frac{1}{4}\theta_9-\frac{3}{2}l^2\theta_{10}-\frac{1}{4}\theta_{15}\bigg\}, \tag{50}$$

$$\rho_4^l=\frac{p_2}{2}\bigg\{m^2\bigg[2-\beta^2+\frac{E_\nu^0}{E}\bigg]\theta_2+\bigg[-\frac{7}{2}m^2+p_2ly_0\bigg]\theta_3+\bigg[(3-\beta^2)\frac{E^2}{2}-EE_\nu^0-p_2ly_0\bigg]\theta_4-\bigg[\frac{1}{2}\beta E^2+2lE_\nu^0\bigg]\theta_5-\frac{m^2}{2E}\theta_6+\frac{1}{2}(3E+p_2\beta y_0)\theta_7-E\theta_8+\frac{1}{4}\theta_9-\frac{5}{2}l^2\theta_{10}-p_2l(1-\beta^2)\theta_{11}+2p_2l\theta_{12}-p_2l\theta_{13}-\frac{l}{2}\theta_{14}-\frac{1}{4}\theta_{15}-\frac{1}{4E}\theta_{16}\bigg\}. \tag{51}$$

With Eqs. (45)–(47) all the integrals over $\mathbf{k}$ in Eqs. (27), (34), and (35) have been expressed in an analytical form. We can obtain now the bremsstrahlung differential decay rate $d\Gamma_B^{\mathrm{TBR}}$ of decaying polarized hyperons with the photon integrals expressed analytically. Substituting in Eq. (22) the analytical forms of $d\Gamma_B'$, Eq. (45), and of $d\Gamma_B^{(s)}$, which is the sum of Eqs. (46) and (47), the analytical form of $d\Gamma_B^{\mathrm{TBR}}$ reads

$$d\Gamma_B^{\mathrm{TBR}}=\frac{\alpha}{\pi}d\Omega'\{A_1'I_0+(D_1+D_2)(\theta'+\theta''')+D_2(\theta''+\theta^{\mathrm{IV}})\\ -\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}[B_2'I_0+D_3(\rho_1^l+\rho_3^l)+D_4(\rho_2^l+\rho_4^l)]\}. \quad (52)$$

We are now in a position to obtain our second main result in this paper: the analytical RC to the DP of decaying polarized hyperons to order α and neglecting terms of order $\alpha q/\pi M_1$. This result comes from the addition of the virtual RC, $d\Gamma_V$ of Eq. (7), and of $d\Gamma_B^{\mathrm{TBR}}$ of Eq. (52). It can be put compactly as

$$d\Gamma^{\mathrm{TBR}}=\frac{G_V^2}{2}\frac{dE_2\,dE\,d\Omega_l}{(2\pi)^4}2M_1\left\{A_0'+\frac{\alpha}{\pi}\Phi_1-\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}\\ \times\left[B_0''+\frac{\alpha}{\pi}\Phi_2^l\right]\right\}. \quad (53)$$

Here A_0' and B_0'' are the same as Eqs. (B1) and (8), respectively. Φ_1 and Φ_2^l are

$$\Phi_1=A_1'(\phi+I_0)+A_1''\phi'+(D_1+D_2)(\theta'+\theta''')\\ +D_2(\theta''+\theta^{\mathrm{IV}}), \quad (54)$$

$$\Phi_2^l=B_2'(\phi+I_0)+B_2''\phi'+D_3(\rho_1^l+\rho_3^l)\\ +D_4(\rho_2^l+\rho_4^l). \quad (55)$$

The coefficients A_1', A_1'', B_2', and B_2'' are given in Eqs. (B2), (B3), (9) and (10), respectively. D_i $(i=1,\ldots,4)$ are given in Eqs. (B11)–(B14). The functions ϕ, ϕ', and I_0 appear in Eqs. (11), (12), and (26), respectively. The new model-independent functions ρ_i^l $(i=1,\ldots,4)$ were given in Eqs. (48)–(51). The sums $\theta'+\theta'''$ and $\theta''+\theta^{\mathrm{IV}}$ appear explicitly in Eqs. (93) and (94) of Ref. [3], respectively. We have corrected a misprint in that Eq. (93). Its fourth term has to be $l/2\ \theta_5$ rather than $-l/2\ \theta_5$. For completeness, we reproduce these two sums in Appendix B [see Eqs. (B39) and (B40)].

Because the infrared divergence, which appears in the virtual part $d\Gamma_V$ through the function ϕ, cancels out with its bremsstrahlung counterpart, which appears in I_0, the sum $\phi+I_0$ is no longer infrared-divergent and, therefore, $d\Gamma^{\mathrm{TBR}}$ of Eq. (53) is infrared-convergent.

$d\Gamma^{\mathrm{TBR}}$ is the result corresponding to $d\Gamma$ of Eq. (101) in Ref. [4]. Comparing both results, we can observe that the spin-independent parts are the same, but the spin-dependent parts show important differences. In Eq. (101) of Ref. [4] we have, within the square brackets that accompany the correlation $\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2$, the terms A_0'' and Φ_2, while in Eq. (53) the corresponding terms accompanying $\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}$ are B_0'' and Φ_2^l. They are different. In fact, we may notice that in Φ_2^l of Eq. (55) only the θ_i-functions $(i=2,\ldots,16)$ appear, while in Φ_2 of Eq. (103) of Ref. [4] also the η-functions appear [see Eqs. (91)-(95) of this reference]. Because of this, the RC to $\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}$ and $\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2$ correlations are quite different.

B. FBR analytical form

The $\mathbf{k}$-integrals of the FBR are those contained in Eqs. (40) and (41) and they can be performed analytically, too. Because $d\Gamma_B'^{\mathrm{FBR}}$ of Eq. (40) has the same form as the corresponding $d\Gamma_B'$ of Eq. (27) of the TBR, we can follow the same procedure of Sec. V of Ref. [3] to calculate the analytical form of this $d\Gamma_B'^{\mathrm{FBR}}$. The result has the same structure as $d\Gamma_B'$ of Eq. (45),

$$d\Gamma_B'^{\mathrm{FBR}}=\frac{\alpha}{\pi}d\Omega'[A_1'I_{0F}+(D_1+D_2)(\theta_F'+\theta_F''')\\ +D_2(\theta_F''+\theta_F^{\mathrm{IV}})], \quad (56)$$

with

$$\theta_F'+\theta_F'''=\frac{p_2l}{2}\Bigg[-E_\nu^0(1-\beta^2)\theta_{2F}+\left(E_\nu^0-\frac{1+\beta^2}{2}E\right)\theta_{3F}\\ +\frac{E}{2}\theta_{4F}+\frac{l}{2}\theta_{5F}+\frac{1-\beta^2}{2}\theta_{6F}-\frac{2E-E_\nu^0}{2E}\theta_{7F}\\ +\frac{1}{2}\theta_{8F}-\frac{1}{4E}\theta_{9F}\Bigg], \quad (57)$$

and

$$\theta_F''+\theta_F^{\mathrm{IV}}=\frac{p_2l}{2}[\theta_{0F}-(E+E_\nu^0+\beta p_2y_0)\theta_{3F}\\ +(E_\nu^0+E)\theta_{4F}+l\theta_{5F}]. \quad (58)$$

These last two equations have the same structure as $\theta'+\theta'''$ and $\theta''+\theta^{\mathrm{IV}}$ of the TBR analytical form of $d\Gamma_B'$ of Eq. (45). Thus, I_0 and θ_i in the latter are changed into I_{0F} and θ_{iF}, respectively, in the analytical form of $d\Gamma_B'$ in the FBR, Eq. (56).

The change of θ_i into θ_{iF} occurs because in the former y is integrated between -1 and y_0 and in the latter y is integrated between -1 and 1. The new set $\{\theta_{iF}\}$ $(i=2,\ldots,16)$ is explicitly given in Appendix B and θ_{0F} is given in Eq. (38). We can compare with the θ_i^T of Ref. [9] where the RC of the DP for the FBR were calculated up to order $\alpha q/\pi M_1$. We cannot take readily the result of this reference because, according to our approximations, we would have to neglect all the terms of order $\alpha q/\pi M_1$ in that result to obtain ours. We find the procedure of Ref. [3] more adequate and straightforward for our purposes. However, in order to check our results we have reproduced the Table I of

Ref. [9]. Our numerical evaluations coincide very well within the approximation of neglecting terms of order $\alpha q/\pi M_1$; we shall not display here this numerical evaluation.

In a similar way, we can see that the spin-dependent part of $d\Gamma_B^{\rm FBR}$ has the same structure as the corresponding spin-dependent part of $d\Gamma_B^{\rm TBR}$. Thus, we get

$$d\Gamma_B^{(s)\,\rm FBR}=\frac{\alpha}{\pi}d\Omega'\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}[B_2'I_{0F}+D_3(\rho_{1F}^l+\rho_{3F}^l)+D_4(\rho_{2F}^l+\rho_{4F}^l)], \tag{59}$$

with the functions ρ_{iF}^l $(i=1,\ldots,4)$ having the same structure as the previous ρ_i^l of Eqs. (48)–(51). Explicitly, they are

$$\rho_{1F}=\frac{p_2l^2}{2}[(\beta^2-1)\theta_{2F}+2\theta_{3F}-\theta_{4F}], \tag{60}$$

$$\rho_{2F}=\frac{p_2E^2}{2}\left[-\frac{2}{E}\theta_{0F}+(\beta^2-1)\theta_{2F}-(\beta^2-3)\theta_{3F}-2\theta_{4F}-\beta\theta_{5F}\right], \tag{61}$$

$$\rho_{3F}=\frac{p_2}{2}\Bigg\{E(E+E_\nu^0)(1-\beta^2)\theta_{2F}-\left[(3-\beta^2)\frac{E^2}{2}+EE_\nu^0\right]\theta_{3F}+\frac{1}{2}E^2(1+\beta^2)\theta_{4F}-\frac{l}{2}(E+2E_\nu^0)\theta_{5F}-\frac{m^2}{2E}\theta_{6F}$$
$$+\frac{1}{2}(2E-E_\nu^0)\theta_{7F}-\frac{1}{2}(E-E_\nu^0)\theta_{8F}+\frac{1}{4}\theta_{9F}-\frac{3}{2}l^2\theta_{10F}-\frac{1}{4}\theta_{15F}\Bigg\}, \tag{62}$$

$$\rho_{4F}=\frac{p_2}{2}\Bigg\{m^2\left[2-\beta^2+\frac{E_\nu^0}{E}\right]\theta_{2F}+\left[-\frac{7}{2}m^2+p_2ly_0\right]\theta_{3F}+\left[(3-\beta^2)\frac{E^2}{2}-EE_\nu^0-p_2ly_0\right]\theta_{4F}-\left[\frac{1}{2}\beta E^2+2lE_\nu^0\right]\theta_{5F}-\frac{m^2}{2E}\theta_{6F}$$
$$+\frac{1}{2}(3E+p_2\beta y_0)\theta_{7F}-E\theta_{8F}+\frac{1}{4}\theta_{9F}-\frac{5}{2}l^2\theta_{10F}-p_2l(1-\beta^2)\theta_{11F}+2p_2l\theta_{12F}-p_2l\theta_{13F}$$
$$-\frac{l}{2}\theta_{14F}-\frac{1}{4}\theta_{15F}-\frac{1}{4E}\theta_{16F}\Bigg\}. \tag{63}$$

From Eqs. (56) and (59) we obtain the analytical bremsstrahlung differential decay rate $d\Gamma_B^{\rm FBR}$ of decaying polarized hyperons corresponding to Eq. (39),

$$d\Gamma_B^{\rm FBR}=\frac{\alpha}{\pi}d\Omega'[\Phi_{1F}-\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}\ \ \Phi_{2F}^l], \tag{64}$$

with

$$\Phi_{1F}=A_1'I_{0F}+(D_1+D_2)(\theta_F'+\theta_F''')+D_2(\theta_F''+\theta_F^{\rm IV}), \tag{65}$$

$$\Phi_{2F}^l=B_2'I_{0F}+D_3(\rho_{1F}^l+\rho_{3F}^l)+D_4(\rho_{2F}^l+\rho_{4F}^l). \tag{66}$$

At this point we complete our second main result. The addition of $d\Gamma_B^{\rm FBR}$ of Eq. (64) and $d\Gamma^{\rm TBR}$ of Eq. (53) gives us the complete analytical RC to the DP of decaying polarized hyperons to order α and neglecting terms of order $\alpha q/\pi M_1$. This complete result can be expressed compactly as

$$d\Gamma_{\rm TOT}=\frac{G_V^2}{2}\frac{dE_2\,dE\,d\Omega_l}{(2\pi)^4}2M_1\Bigg\{A_0'+\frac{\alpha}{\pi}(\Phi_1+\Phi_{1F})-\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}\times\left[B_0''+\frac{\alpha}{\pi}(\Phi_2^l+\Phi_{2F}^l)\right]\Bigg\}. \tag{67}$$

Here A_0', B_0'', Φ_1, Φ_{1F}, Φ_2^l, and Φ_{2F}^l are given in Eqs. (B1), (8), (54), (65), (55), and (66), respectively.

From Eq. (67) we can obtain easily Eq. (53) to the RC of the DP with the TBR only by dropping Φ_{1F} and Φ_{2F}^l. It is this Eq. (67) which must be used to obtain, in HSD, totally integrated observables, such as the spin-asymmetry coefficient of the charged lepton. We shall calculate this asymmetry coefficient in the next section, allowing for the possibility that real photon emission be discriminated experimentally via energy-momentum conservation or via detection.

V. SPIN ASYMMETRY COEFFICIENT α_l

In this section we shall obtain the RC to the spin-asymmetry coefficient of the charged lepton α_l. We shall consider the two cases discussed all along, namely, that bremsstrahlung photons not be discriminated at all or that directly or indirectly the photons belonging to the FBR be eliminated from the experimental analysis. We will discuss the former case first and afterwards we will discuss the latter case. As we shall see in the numerical evaluation of the next section, an appreciable difference can be observed between these two cases.

α_l can be calculated from the total DP of Eq. (67). This equation can be used to get the quantities $N^{\pm}$ which appear in the definition of α_l,

$$\alpha_l = 2\frac{N^+ - N^-}{N^+ + N^-}. \tag{68}$$

Here $N^+(N^-)$ denotes the number of the emitted charged leptons with momenta in the forward (backward) hemisphere with respect to the polarization of the decaying hyperon. With those numbers calculated, we may express α_l as

$$\alpha_l^{\mathrm{T}} = -\frac{B_2^l + (\alpha/\pi)(a_2^l + a_{2F}^l)}{B_1 + (\alpha/\pi)(a_1 + a_{1F})}. \tag{69}$$

Here

$$B_2^l = \int_m^{E_m}\int_{E_2^-}^{E_2^+} B_0'' dE_2 dE, \tag{70}$$

$$a_2^l = \int_m^{E_m}\int_{E_2^-}^{E_2^+} \Phi_2^l dE_2\, dE, \tag{71}$$

$$a_{2F}^l = \int_m^{E_B}\int_{M_2}^{E_2^-} \Phi_{2F}^l dE_2\, dE, \tag{72}$$

$$B_1 = \int_m^{E_m}\int_{E_2^-}^{E_2^+} A_0' dE_2\, dE, \tag{73}$$

$$a_1 = \int_m^{E_m}\int_{E_2^-}^{E_2^+} \Phi_1 dE_2\, dE, \tag{74}$$

$$a_{1F} = \int_m^{E_B}\int_{M_2}^{E_2^-} \Phi_{1F}\, dE_2\, dE. \tag{75}$$

In these integrals, B_0'', Φ_2^l, Φ_{2F}^l, A_0', Φ_1, and Φ_{1F} are given in Eqs. (8), (55), (66), (B1), (54), and (65), respectively.

In Eq. (69) we have attached an upper index T to denote that the asymmetry coefficient includes the total DP of the real photons. The contributions of the TBR to the RC of α_l are given by the terms a_2^l and a_1, while the contributions of the FBR are given by the terms a_{2F}^l and a_{1F}. We can now rewrite α_l^{T} to comply with our approximations, i.e., in such a way that only the terms of order α, neglecting terms of order $\alpha q/\pi M_1$, appear. The corresponding expression is

$$\alpha_l^{\mathrm{T}} = \alpha_0^l\left[1 + \frac{\alpha}{\pi}\left(\frac{a_2^l + a_{2F}^l}{B_2^l(0)} - \frac{a_1 + a_{1F}}{B_1(0)}\right)\right], \tag{76}$$

where α_0^l is the spin-asymmetry coefficient of the charged lepton without RC. It is obtained from Eq. (69) by dropping the terms proportional to α, namely,

$$\alpha_0^l = -\frac{B_2^l}{B_1}. \tag{77}$$

$B_2^l(0)$ and $B_1(0)$ in the denominators of Eq. (76) are the zero-order q/M_1 approximations of the B_2^l of Eq. (70) and of the B_1 of Eq. (73), respectively. Explicitly, they are

$$B_2^l(0) = \int_m^{E_m}\int_{E_2^-}^{E_2^+} B_2'\, dE_2\, dE, \tag{78}$$

$$B_1(0) = \int_m^{E_m}\int_{E_2^-}^{E_2^+} A_1'\, dE_2\, dE, \tag{79}$$

with B_2' and A_1' given in Eqs. (9) and (B2), respectively.

The coefficient α_l when only the TBR of the DP is allowed can be easily obtained now. All that has to be done is to drop a_{2F}^l and a_{1F} from Eq. (76) so that

$$\alpha_l^{\mathrm{R}} = \alpha_0^l\left[1 + \frac{\alpha}{\pi}\left(\frac{a_2^l}{B_2^l(0)} - \frac{a_1}{B_1(0)}\right)\right]. \tag{80}$$

We attached an upper index R to denote that the bremsstrahlung correction is restricted to the TBR.

In Ref. [4] we only calculated the emitted baryon asymmetry-coefficient α_B corresponding to α_l^{R}. In this reference it was assumed that the FBR photons were always discriminated. The contribution of these photons should be calculated and added to the results of this reference in order to get an α_B corresponding to the above α_l^{T}.

In the next section we shall display numerical evaluations that will allow us to compare our results with others available in the literature and, also, to appreciate the relevance of discriminating or not FBR photons.

VI. NUMERICAL RESULT

In order to compare the coefficients α_l^{R} of the TBR of the DP and α_l^{T} of the total DP, we shall make numerical evaluations of them for several decays. These results will enable us to establish the relevance of the difference between α_l^{R} and α_l^{T} in the study of HSD. We shall also compare them with other results reported in the literature. In Table I we give the values of the form factors used in the numerical evaluation of the coefficients α_l^{R} and α_l^{T} for the decays $n\to pe\overline{\nu}$, $\Lambda\to pe\overline{\nu}$, $\Sigma^-\to ne\overline{\nu}$, $\Sigma^-\to\Lambda e\overline{\nu}$, $\Sigma^+\to\Lambda e^+\nu$, $\Xi^-\to\Lambda e\overline{\nu}$, $\Xi^-\to\Sigma^0 e\overline{\nu}$, $\Xi^0\to\Sigma^+ e\overline{\nu}$, and $\Lambda_c^+\to\Lambda e^+\nu$. For this last decay we take the form factors of Ref. [10]. The sign of the form factor g_1 must be changed when the charged lepton is positive [11]. In the radiatively uncorrected amplitudes the q^2-dependence of the form factor was neglected along with the contributions arising from f_3, g_2, and g_3 as was done in other calculations in the literature.

To evaluate α_l^{R} we use Eq. (80) and for α_l^{T} we use Eq.

TABLE I. Values of the form factors used in our numerical calculations. For the first three decays we take the form factor ratios of Ref. [6] while for the other decays we use Ref. [13]. For $\Lambda_c^+\to\Lambda$ we take the values of Ref. [10]. For convenience we include in the last column the uncorrected α_l^0 of Eq. (77) corresponding to this choice of form factors.

Decay	f_1	f_2	g_1	α_0^l
$n\to p$	1.000	1.970	1.261	−0.0850
$\Lambda\to p$	1.236	1.199	0.890	0.0200
$\Sigma^-\to n$	1.000	−0.970	−0.340	−0.6319
$\Sigma^-\to\Lambda$	0.000	1.172	0.601	−0.7030
$\Sigma^+\to\Lambda$	0.000	1.172	0.601	−0.6474
$\Xi^-\to\Lambda$	1.225	−0.074	0.354	0.2579
$\Xi^-\to\Sigma^0$	0.707	1.310	0.899	−0.1989
$\Xi^0\to\Sigma^+$	1.000	1.853	1.267	−0.1913
$\Lambda_c^+\to\Lambda$	0.350	0.090	0.610	−0.9513

(76). These equations involve the double integration over the energies E and E_2. At this point is convenient to mention a technical aspect that we have to deal with when calculating the integrals

$$\int_{E_2^-}^{E_2^+} E_2^n \ln(y_0+1)\,dE_2 \tag{81}$$

and

$$\int_{M_2}^{E_2^-} E_2^n \ln\left(\frac{y_0+1}{y_0-1}\right) dE_2, \tag{82}$$

with $n=0,1$. Integral (81) is contained in Eqs. (71) and (74) of the TBR. In this region y_0 may become -1 and accordingly the logarithm in its integrand diverges in this limit. Integral (82) is contained in Eqs. (72) and (75) of the FBR. In this region y_0 cannot reach -1 but it can reach asymptotically $+1$ so that the logarithm in its integrand diverges in this last limit. The first limit occurs when $E_2\to E_2^+$ and also when $E_2\to E_2^-$ provided $E>E_B$. The second limit occurs when $E_2\to E_2^-$ provided $E<E_B$. However, even if the integrands in Eqs. (81) and (82) diverge in these limits, the integrals themselves have a finite result. A detailed analysis of this technicality can be found in Ref. [12]. To numerically perform these integrals we can either follow the approach of this reference and implement it in the program to evaluate the α_l's or we may neglect the points where $y_0\to\pm1$. This last approach is equivalent to leaving out the boundaries of the TBR and the FBR of the DP. The numerical difference between these two alternatives is negligible. Here we follow the second alternative.

In Table II we display our numerical results for the radiative corrections to the asymmetry coefficients $\alpha_l^{\rm R}$ and $\alpha_l^{\rm T}$. We compute them by taking the percentage differences (that is, we multiply by 100)

$$\delta\alpha_l^{\rm R,T}=\alpha_l^{\rm R,T}-\alpha_0^l, \tag{83}$$

where α_0^l is the uncorrected spin-asymmetry coefficient of the charged lepton, Eq. (77).

In the second column of Table II we display the $\delta\alpha^{\rm R}$ corresponding to the TBR of the DP, in the third column the $\delta\alpha^{\rm T}$ corresponding to the complete DP are given, in the fourth column we give the results for $\delta\alpha$ obtained from Eq. (23) and Table I of p. 58 of Ref. [13] and, finally, in the fifth column we give the two values reported in Ref. [6].

From Table II we see that there is a very good agreement between our $\delta\alpha^{\rm T}$ and the $\delta\alpha$ of Refs. [13] and [6]. In both references the FBR was included. The inclusion or exclusion of the FBR is appreciable, as can be seen by comparing the second and third columns, except for the decays $n\to pe\bar\nu$ and $\Sigma^-\to ne\bar\nu$. In several instances the inclusion of the FBR contribution reduces the total radiative corrections, even to the point of making them negligibly small. It may even be that the values in the second column are one order of magnitude larger than the corresponding ones in the third column. Therefore, in general, there is an important difference between $\alpha_l^{\rm R}$ and $\alpha_l^{\rm T}$.

VII. CONCLUSIONS

In this paper we have obtained the radiative corrections to order α to the Dalitz plot of the semileptonic decays of po-

TABLE II. Percentage radiative corrections [that is, Eq. (83) multiplied by 100] of the spin-asymmetry coefficient of the charged lepton in hyperon semileptonic decays. The prediction in the fourth column for $\Lambda_c^+\to\Lambda$ uses the approach of Ref. [13], but it was not actually given there.

Decay	$\delta\alpha^{\rm R}=\alpha_l^{\rm R}-\alpha_0^l$	$\delta\alpha^{\rm T}=\alpha_l^{\rm T}-\alpha_0^l$	$\delta\alpha$ Ref. [13]	$\delta\alpha$ Ref. [6]
$n\to p$	0.0119	0.0095	0.0101	
$\Lambda\to p$	0.0813	0.0014	−0.0023	−0.0
$\Sigma^-\to n$	0.0832	0.0815	0.0758	0.1
$\Sigma^-\to\Lambda$	0.1432	0.0836	0.0770	
$\Sigma^+\to\Lambda$	0.1287	0.0755	0.0911	
$\Xi^-\to\Lambda$	0.1024	−0.0246	−0.0310	
$\Xi^-\to\Sigma^0$	0.3327	0.0371	0.0212	
$\Xi^0\to\Sigma^+$	0.3312	0.0350	0.0208	
$\Lambda_c^+\to\Lambda$	0.0757	0.1294	0.1098	

larized spin-1/2 baryons, neglecting terms of order $\alpha q/\pi M_1$ and higher. Our main result has two forms. One in which the triple **k**-integration is ready to be performed numerically, given in Eq. (44) and which is the sum of Eqs. (36) and (39). And another one in which such integration has been performed analytically, given in Eq. (67) and which is the sum of Eqs. (53) and (64).

Since real photons may be discriminated either directly (by detection) or indirectly (by energy-momentum conservation) we have split our main result to cover this possibility. If photon discrimination indeed takes place, instead of Eq. (44) one should use only Eq. (36) and instead of Eq. (67) one should use only Eq. (53)

An important integrated observable is the charged-lepton spin-asymmetry coefficient α_l. Using the analytical forms of Eqs. (67) and (53) we obtained the radiative corrections to this observable. The integrations over E and E_2 were performed numerically and the results are displayed in Table II. A systematic behavior of the RC to α_l is observed. The contribution of the FBR bremsstrahlung may be as important as the RC from the TBR and even of opposite sign, in such a way that when no photon discrimination takes place the complete RC to α_l may become almost negligible. In this table we also compared with results reported in other references [6,13]. This comparison is satisfactory. For completeness, we evaluated also the RC to the α_l of the process $\Lambda_c^+ \to \Lambda e^+ \nu$.

Our results are model-independent and are not compromised to any particular value of the form factors. All the model dependence of radiative corrections has been absorbed into the f_1 and g_1 form factors in our approximation of neglecting contributions of order $\alpha q/\pi M_1$. This is indicated by putting a prime on them. For hyperons our results are reliable up to a precision of around 0.5%. This precision is useful for experiments involving several thousands of events. For high statistics experiments involving several hundreds of thousands of events or for decays involving charm such as $\Lambda_c^+ \to \Lambda e^+ \nu$ or even heavier quarks our equations provide a good first approximation. To improve the precision of our formulas it becomes necessary to include $\alpha q/\pi M_1$ contributions. Our results are valid for both neutral or charged decaying hyperons and whether the emitted positively or negatively charged lepton is either electron-type or muon-type. To conclude let us remark that in a Monte Carlo analysis the advantage of the analytical form is that the triple **k**-integration does not have to be repeated every time the values of f_1 and g_1, or of E and E_2, are changed. This leads to a considerable saving of computer time.

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APPENDIX A

In this appendix we give for completeness the amplitudes for the RC of the decay (1). All of them are also given in Ref. [4]. The uncorrected transition amplitude M_0 for process (1) is

$$\mathsf{M}_0 = \frac{G_V}{\sqrt{2}}[\overline{u}_B(p_2) W_\mu(p_1,p_2) u_A(p_1)][\overline{u}_l(l) O_\mu v_\nu(p_\nu)], \tag{A1}$$

where

$$W_\mu(p_1,p_2) = f_1(q^2)\gamma_\mu + \frac{f_2(q^2)}{M_1}\sigma_{\mu\nu}q_\nu + \frac{f_3(q^2)}{M_1}q_\mu + \left[g_1(q^2)\gamma_\mu + \frac{g_2(q^2)}{M_1}\sigma_{\mu\nu}q_\nu + \frac{g_3(q^2)}{M_1}q_\mu\right]\gamma_5. \tag{A2}$$

Here $O_\mu = \gamma_\mu(1+\gamma_5)$ and q is the four-momentum transfer.

The model-independent part of the virtual radiative corrections has the amplitude

$$\mathsf{M}_v = \frac{\alpha}{2\pi}[\mathsf{M}_0 \phi(E) + \mathsf{M}_{p_1}\phi'(E)], \tag{A3}$$

where $\phi(E)$ and $\phi'(E)$ are given in Eqs. (11) and (12), respectively. M_{p_1} is

$$\mathsf{M}_{p_1} = \left(\frac{E}{mM_1}\right)\frac{G_V}{\sqrt{2}}[\overline{u}_B W_\lambda u_A][\overline{u}_l \not{p}_1 O_\lambda v_\nu]. \tag{A4}$$

The model-dependent part of the virtual radiative corrections is absorbed into M_0 through the definition of effective form factors f_1' and g_1'. This fact is denoted by putting a prime on M_0.

The bremsstrahlung transition amplitude M_B is obtained following the Low theorem [2],

$$\begin{aligned}\mathsf{M}_B &= \frac{eG_V}{\sqrt{2}}[\overline{u}_B W_\lambda u_A][\overline{u}_l O_\lambda v_\nu]\left[\frac{l\cdot\epsilon}{l\cdot k} - \frac{p_1\cdot\epsilon}{p_1\cdot k}\right] \\ &\quad + \frac{eG_V}{\sqrt{2}}[\overline{u}_B W_\lambda u_A]\left[\frac{\overline{u}_l \not\epsilon \not k O_\lambda v_\nu}{2l\cdot k}\right] \\ &= \mathsf{M}_a + \mathsf{M}_b.\end{aligned} \tag{A5}$$

In this equation, the amplitudes M_a and M_b correspond to the first and second summands, respectively. Within our approximations the Low theorem guarantees that no model-dependence appears here.

APPENDIX B

In order to make this paper self-contained, we reproduce here all the coefficients which appear in our final results. For Eq. (7) they come from Ref. [4],

$$A_0' = Q_1 E E_\nu^0 - Q_2 E p_2(p_2 + l y_0) - Q_3 l(p_2 y_0 + l) + Q_4 E_\nu^0 p_2 l y_0 - Q_5 p_2^2 l y_0 (p_2 + l y_0), \tag{B1}$$

$$A_1' = D_1 E E_\nu^0 - D_2 l(p_2 y_0 + l), \tag{B2}$$

$$A_1'' = D_1 E E_\nu^0. \tag{B3}$$

The coefficients Q_i $(i=1, \ldots, 7)$ are given in Eqs. (B1)–(B7) of Ref. [4], respectively. For completeness we reproduce them here:

$$Q_1 = F_1^2\left[\frac{2E_2 - M_2}{M_1}\right] + \frac{1}{2}F_2^2\left[\frac{M_2+E_2}{M_1}\right] + F_1F_2\left[\frac{M_2+E_2}{M_1}\right] + F_1F_3\left[1+\frac{M_2}{M_1}\right]\times\left[1-\frac{E_2}{M_1}\right] + F_2F_3\left[\frac{M_2+E_2}{M_1}\right]\left[1-\frac{E_2}{M_1}\right]$$
$$+ G_1^2\left[\frac{2E_2+M_2}{M_1}\right] - \frac{1}{2}G_2^2\left[\frac{M_2-E_2}{M_1}\right] + G_1G_2\left[\frac{M_2-E_2}{M_1}\right] + G_1G_3\left[\frac{M_2}{M_1}-1\right]\left[1-\frac{E_2}{M_1}\right] - G_2G_3\left[\frac{M_2-E_2}{M_1}\right]\left[1-\frac{E_2}{M_1}\right]$$
$$+ M_1^2 Q_5\left\{\left[\frac{M_1-E_2}{M_1}\right]^2 - \frac{1}{2}\frac{q^2}{M_1^2}\right\}, \tag{B4}$$

$$Q_2 = -\frac{F_1^2}{M_1} - \frac{G_1^2}{M_1} - \frac{F_1F_2}{M_1} + \frac{G_1G_2}{M_1} + \frac{F_1F_3}{M_1}\left[1+\frac{M_2}{M_1}\right] + \frac{F_2F_3}{M_1}\left[\frac{M_2+E_2}{M_1}\right] + \frac{G_1G_3}{M_1}\left[\frac{M_2}{M_1}-1\right] - \frac{G_2G_3}{M_1}\left[\frac{M_2-E_2}{M_1}\right]$$
$$+ 2\frac{F_1G_1}{M_1} + M_1Q_5\left[\frac{M_1-E_2}{M_1}\right], \tag{B5}$$

$$Q_3 = Q_1 - 2F_1^2\left[\frac{E_2-M_2}{M_1}\right] - 2G_1^2\left[\frac{E_2+M_2}{M_1}\right] - M_1^2Q_5\left\{\left[1-\frac{E_2}{M_1}\right]^2 - \frac{q^2}{M_1^2}\right\}, \tag{B6}$$

$$Q_4 = Q_2 - 4\frac{F_1G_1}{M_1}, \tag{B7}$$

$$Q_5 = \frac{F_3^2}{M_1^2}\left[\frac{M_2+E_2}{M_1}\right] - \frac{G_3^2}{M_1^2}\left[\frac{M_2-E_2}{M_1}\right] - 2\frac{F_1F_3}{M_1^2} + 2\frac{G_1G_3}{M_1^2}, \tag{B8}$$

$$Q_6 = F_1^2\left[\frac{E_2-M_2}{M_1} - \frac{p_2\beta y_0}{M_1}\right] + G_1^2\left[\frac{E_2+M_2}{M_1} - \frac{p_2\beta y_0}{M_1}\right] + 2F_1G_1\left[\frac{E_2 - p_2\beta y_0}{M_1}\right] + (G_1G_2 - F_1F_2)\left[\frac{p_2\beta y_0}{M_1}\right]$$
$$+ F_2G_2\left[-1 + (1+\beta^2)\frac{E}{M_1} + \frac{E_2}{M_1} + \frac{p_2\beta y_0}{M_1}\right] + F_1G_2\left[-1 + \frac{M_2}{M_1} + (1+\beta^2)\frac{E}{M_1} + \frac{p_2\beta y_0}{M_1}\right]$$
$$- G_1F_2\left[-1 - \frac{M_2}{M_1} + (1+\beta^2)\frac{E}{M_1} + \frac{p_2\beta y_0}{M_1}\right] - F_3G_3\left[\frac{m^2}{M_1^2}\left(1 - \frac{E_2}{M_1} - (1-\beta^2)\frac{E}{M_1} + \frac{p_2\beta y_0}{M_1}\right)\right]$$
$$+ F_1G_3\left[\frac{m^2}{M_1E}\left(-1 + \frac{M_2}{M_1} + \frac{E}{M_1}\right)\right] - F_3G_1\left[\frac{m^2}{M_1E}\left(-1 - \frac{M_2}{M_1} + \frac{E}{M_1}\right)\right] - (F_2G_3 + F_3G_2)\left[\frac{m^2}{M_1E}\left(\frac{M_1 - E_2 - E}{M_1}\right)\right], \tag{B9}$$

$$Q_7 = F_1^2\left[\frac{(M_1+M_2)(E_2-M_2)}{M_1E}\right] + G_1^2\left[\frac{(M_1-M_2)(E_2+M_2)}{M_1E}\right] + 2F_1G_1\left[\frac{M_1(-M_1+E_2+2E) - m^2}{M_1E}\right] + F_1G_2\left(\frac{E_2-M_2}{M_1}\right)$$
$$\times\left(\frac{M_1-2E-E_2}{E}\right) - G_1F_2\left(\frac{E_2+M_2}{M_1}\right)\left(\frac{M_1-2E-E_2}{E}\right) + F_3G_1\left(\frac{E_2+M_2}{M_1}\right)\left(\frac{m^2}{M_1E}\right) - G_3F_1\left(\frac{E_2-M_2}{M_1}\right)\left(\frac{m^2}{M_1E}\right)$$
$$+ (F_1F_2 - G_1G_2)\left(\frac{E_2^2 - M_2^2}{M_1E}\right). \tag{B10}$$

The coefficients D_j $(j=1, \ldots, 4)$ read

$$D_1 = f_1'^2 + 3g_1'^2, \tag{B11}$$

$$D_2 = f_1'^2 - g_1'^2, \tag{B12}$$

$$D_3 = 2(f_1'g_1' - g_1'^2), \tag{B13}$$

$$D_4 = 2(f_1'g_1' + g_1'^2). \tag{B14}$$

y_0 and E_ν^0 were defined in Eqs. (5) and (6), respectively.

The functions θ_i which appear in Eqs. (47)–(50) corresponding to the TBR of the DP are given by

$$\theta_i = \frac{1}{p_2}(T_i^+ + T_i^-), \tag{B15}$$

where $i=2,\ldots,16$, and

$$T_2^\pm = \pm\frac{1\mp a^\pm}{(1\pm\beta)(1+\beta a^\pm)}\ln\left[\frac{1\mp\beta}{1-\beta x_0}\right] \pm \frac{(1\pm x_0)\ln(1\pm x_0)}{(1\pm\beta)(1-\beta x_0)} \pm \frac{1\pm a^\pm}{(1\mp\beta)(1+\beta a^\pm)}\ln(1\pm a^\pm) - \frac{(x_0+a^\pm)\ln(\pm x_0 \pm a^\pm)}{(1+\beta a^\pm)(1-\beta x_0)}, \tag{B16}$$

$$T_3^+ = T_3^- = \frac{1}{2\beta}\Bigg\{L\left[\frac{1-\beta}{1-\beta x_0}\right] - L\left[\frac{1-\beta x_0}{1+\beta}\right] - L\left[\frac{1+\beta a^-}{1-\beta x_0}\right] + L\left[\frac{1+\beta a^-}{1+\beta}\right] + L\left[\frac{1-\beta x_0}{1+\beta a^+}\right] - L\left[\frac{1-\beta}{1+\beta a^+}\right]$$
$$+ \ln\left[\frac{1-\beta x_0}{1-\beta}\right]\ln\left[\frac{1+\beta a^+}{1+\beta}\right]\Bigg\}, \tag{B17}$$

$$T_4^\pm = (x_0\pm 1)\ln(1\pm x_0) \pm (1\pm a^\pm)\ln(1\pm a^\pm) - (x_0+a^\pm)\ln(\pm x_0 \pm a^\pm), \tag{B18}$$

$$T_5^\pm = -\frac{1}{2}\{(1-x_0^2)\ln(1\pm x_0) + (x_0\mp 1)a^\pm + 1 - (1-a^{\pm 2})\ln(1\pm a^\pm) + (x_0^2 - a^{\pm 2})\ln[\pm(x_0+a^\pm)]\}, \tag{B19}$$

$$T_6^\mp = \left[-l + p_2 \pm \frac{\beta E_\nu^0(x_0+a^\mp)}{1+\beta a^\mp}\right] I_4 \pm \frac{\beta E_\nu^0(x_0+a^\mp)}{(1+\beta a^\mp)^2} I_1 + \left[E_\nu^0 - \frac{\beta E_\nu^0(x_0+a^\mp)}{1+\beta a^\mp}\right] J_4 - \frac{\beta E_\nu^0(x_0+a^\mp)}{(1+\beta a^\mp)^2} J_1$$
$$\pm \frac{E_\nu^0(x_0+a^\mp)}{(1+\beta a^\mp)^2} I_2^\mp - \frac{E_\nu^0(x_0+a^\mp)}{(1+\beta a^\mp)^2} J_2^\mp, \tag{B20}$$

$$T_7^\pm = \left[p_2 - l \mp \frac{\beta E_\nu^0(x_0+a^\pm)}{1+\beta a^\pm}\right] I_1 \mp \frac{E_\nu^0(x_0+a^\pm)}{1+\beta a^\pm} I_2^\pm + \left[E_\nu^0 - \frac{\beta E_\nu^0(x_0+a^\pm)}{1+\beta a^\pm}\right] J_1 - \frac{E_\nu^0(x_0+a^\pm)}{1+\beta a^\pm} J_2^\pm, \tag{B21}$$

$$T_8^\pm = -2(l - p_2 + E_\nu^0 x_0) \mp E_\nu^0(x_0+a^\pm) I_2^\pm - E_\nu^0(x_0+a^\pm) J_2^\pm, \tag{B22}$$

$$\frac{T_9^\pm}{4l^2} = -\frac{3E}{2l^2}(l - p_2 + E_\nu^0 x_0) + \left[\frac{3(l-p_2)}{4\beta l} + \frac{3E_\nu^0 p_2}{4l^2} + \beta G^\pm\right] I_1 \mp \frac{(E_\nu^0)^2(x_0+a^\pm)^2}{4l^2(1+\beta a^\pm)} I_3^\pm - \frac{(E_\nu^0)^2(x_0+a^\pm)^2}{4l^2(1+\beta a^\pm)} J_3^\pm + G^\pm I_2^\pm$$
$$+ \left[-\frac{3E_\nu^0}{4\beta l} + \frac{3E_\nu^0(E_\nu^0 + l x_0)}{4l^2} \pm \beta G^\pm\right] J_1 \pm G^\pm J_2^\pm, \tag{B23}$$

$$T_{10}^\mp = \frac{1}{3}(x_0^3 \mp 1)\ln(1\mp x_0) + \frac{1}{3}[(a^\mp)^3 \mp 1]\ln(1\mp a^\mp) - \frac{1}{3}[x_0^3 + (a^\mp)^3]\ln[\mp(x_0+a^\mp)] + \frac{1}{6}(1-x_0^2)(a^\mp \pm 1) - \frac{1}{3}(x_0 \pm 1)$$
$$\times[1-(a^\mp)^2], \tag{B24}$$

$$T_{11}^+ = T_{11}^- = \frac{1}{2p_2\beta}\{E_\nu^0[(1-\beta x_0)J_4 - J_1] - (\beta E_\nu^0 + l - p_2)I_4 + (l-p_2)I_1\}, \tag{B25}$$

$$T_{12}^{+}=T_{12}^{-}=\frac{1}{2p_2\beta}[E_\nu^0(1-\beta x_0)J_1+2E_\nu^0 x_0+2(l-p_2)-(\beta E_\nu^0+l-p_2)I_1], \tag{B26}$$

$$T_{13}^{+}=T_{13}^{-}=-\frac{1}{2p_2}E_\nu^0(1-x_0^2), \tag{B27}$$

$$T_{14}^{\pm}=E_\nu^0[1+x_0^2+2a^{\pm}(x_0\mp 1)\pm a^{\pm}(x_0+a^{\pm})(I_2^{\pm}\pm J_2^{\pm})], \tag{B28}$$

$$T_{15}^{\pm}=3E_\nu^0[2p_2(1+y_0)+l(1-x_0^2)]-(E_\nu^0)^2(x_0+a^{\pm})^2(J_3^{\pm}\pm I_3^{\pm})-2lE_\nu^0(x_0+a^{\pm})a^{\pm}(J_2^{\pm}\pm I_2^{\pm}), \tag{B29}$$

$$T_{16}^{\pm}=4l^2\Bigg[\frac{3}{2\beta^2}[2(l-p_2+E_\nu^0 x_0)+\beta E_\nu^0(1-x_0^2)]+\left(-\frac{3(l-p_2+\beta E_\nu^0)}{2\beta^2}-p_2(1+y_0)+\frac{p_2(E_\nu^0)^2}{2l^2}\right)I_1$$
$$-\frac{(E_\nu^0)^2(x_0+a^{\pm})^2}{2l(1+\beta a^{\pm})}(\beta J_1+J_2^{\pm}\pm\beta I_1\pm I_2^{\pm})+\left(\frac{3E_\nu^0(1-\beta x_0)}{2\beta^2}+\frac{(E_\nu^0)^2(E_\nu^0+lx_0)}{2l^2}\right)J_1\Bigg]. \tag{B30}$$

The following definitions are used in the above expressions:

$$x_0=-\frac{p_2 y_0+l}{E_\nu^0}, \quad a^{\pm}=\frac{E_\nu^0\pm p_2}{l}, \tag{B31}$$

$$I_1=\frac{2}{\beta}\operatorname{arctanh}\beta, \quad I_2^{\pm}=\ln\left|\frac{a^{\pm}+1}{a^{\pm}-1}\right|, \tag{B32}$$

$$I_3^{\pm}=\frac{2}{a^{\pm 2}-1}, \quad I_4=\frac{2}{1-\beta^2}, \tag{B33}$$

$$J_1=-\frac{1}{\beta}\left\{\ln\left[\frac{1+\beta}{1-\beta x_0}\right]+\ln\left[\frac{1-\beta}{1-\beta x_0}\right]\right\}, \tag{B34}$$

$$J_2^{\pm}=\ln\left|\frac{a^{\pm}-1}{a^{\pm}+x_0}\right|+\ln\left|\frac{a^{\pm}+1}{a^{\pm}+x_0}\right|, \tag{B35}$$

$$J_3^{\pm}=-2\left[\frac{a^{\pm}}{a^{\pm 2}-1}-\frac{1}{a^{\pm}+x_0}\right], \tag{B36}$$

$$J_4=\frac{2}{\beta}\left[\frac{1}{1-\beta^2}-\frac{1}{1-\beta x_0}\right], \tag{B37}$$

$$G^{\pm}=\mp\frac{\beta(E_\nu^0)^2(x_0+a^{\pm})^2}{4l^2(1+\beta a^{\pm})^2}\mp\frac{a^{\pm}(a^{\pm 2}-1)}{4(1+\beta a^{\pm})}. \tag{B38}$$

The sums $\theta'+\theta'''$ and $\theta''+\theta^{\mathrm{IV}}$ which appear in Eq. (53) are

$$\theta'+\theta'''=\frac{p_2 l}{2}\left[-E_\nu^0(1-\beta^2)\theta_2+\left(E_\nu^0-\frac{1+\beta^2}{2}E\right)\theta_3+\frac{E}{2}\theta_4+\frac{l}{2}\theta_5+\frac{1-\beta^2}{2}\theta_6-\frac{2E-E_\nu^0}{2E}\theta_7+\frac{1}{2}\theta_8-\frac{1}{4E}\theta_9\right], \tag{B39}$$

$$\theta''+\theta^{\mathrm{IV}}=\frac{p_2 l}{2}[\theta_0-(E+E_\nu^0+\beta p_2 y_0)\theta_3+(E_\nu^0+E)\theta_4+l\theta_5]. \tag{B40}$$

Here $\theta_0=(1+y_0)(I_1-2)$.

The explicit forms of the photon integrals corresponding to the FBR of the DP are

$$\theta_{2F}=\frac{1}{\beta p_2}\left[\frac{I_2{}^-}{b^-}-\frac{I_2{}^+}{b^+}+\frac{E^2}{m^2}\left(I_2{}^+-I_2{}^-+\beta\ln\left|\frac{I_3{}^-}{I_3{}^+}\right|\right)\right]+\frac{2I_1}{Eb^-b^+},\tag{B41}$$

$$\theta_{3F}=\frac{I_1}{p_2}\ln\left|\frac{b^+}{b^-}\right|+\frac{1}{\beta p_2}\left[L\left(\frac{1-\beta}{b^-}\right)-L\left(\frac{1-\beta}{b^+}\right)+L\left(\frac{1+\beta}{b^+}\right)-L\left(\frac{1+\beta}{b^-}\right)\right],\tag{B42}$$

$$\theta_{4F}=\frac{1}{p_2}\left[a^+I_2{}^+-a^-I_2{}^-+\ln\left|\frac{I_3{}^-}{I_3{}^+}\right|\right],\tag{B43}$$

$$\theta_{5F}=\frac{1}{2p_2}\left[(1-a^{+2})I_2{}^+-(1-a^{-2})I_2{}^-+\frac{4p_2}{l}\right],\tag{B44}$$

$$\theta_{6F}=2\frac{y_0^-}{(b^-)^2}(I_2{}^-+\beta I_1)-2\frac{y_0^+}{(b^+)^2}(I_2{}^++\beta I_1)+2\left[2+\beta\left(\frac{y_0^-}{b^-}-\frac{y_0^+}{b^+}\right)\right]I_4,\tag{B45}$$

$$\theta_{7F}=2\left[2I_1+\frac{y_0^-}{b^-}(\beta I_1+I_2{}^-)-\frac{y_0^+}{b^+}(\beta I_1+I_2{}^+)\right],\tag{B46}$$

$$\theta_{8F}=2[4+(y_0^-)I_2{}^--(y_0^+)I_2{}^+],\tag{B47}$$

$$\theta_{9F}=24E+2[6(E_\nu^0-E)+\beta(G_F^-+G_F^+)]I_1+2(G_F^-I_2^-+G_F^+I_2^+)+2p_2\left[\frac{(y_0^-)^2}{b^-}I_3{}^--\frac{(y_0^+)^2}{b^+}I_3{}^+\right],\tag{B48}$$

$$\theta_{10F}=\frac{1}{3p_2}\left[2(a^{-2}-a^{+2})-a^{-3}I_2{}^-+a^{+3}I_2{}^++\ln\left|\frac{I_3{}^-}{I_3{}^+}\right|\right],\tag{B49}$$

$$\theta_{11F}=\frac{2(I_4-I_1)}{\beta p_2},\tag{B50}$$

$$\theta_{12F}=\frac{2(I_1-2)}{\beta p_2},\tag{B51}$$

$$\theta_{13F}=0,\tag{B52}$$

$$\theta_{14F}=2[(2-a^-I_2{}^-)(y_0^-)-(2-a^+I_2{}^+)(y_0^+)],\tag{B53}$$

$$\theta_{15F}=24E_\nu^0+4l[a^-y_0^-I_2{}^--a^+y_0^+I_2{}^+]+2p_2[(y_0^-)^2I_3{}^--(y_0^+)^2I_3{}^+],\tag{B54}$$

$$\theta_{16F}=24E^2(I_1-2)+8[(E_\nu^0)^2-2E^2\beta^2]I_1+4lp_2\left[\frac{(y_0^-)^2}{b^-}(\beta I_1+I_2{}^-)-\frac{(y_0^+)^2}{b^+}(\beta I_1+I_2{}^+)\right],\tag{B55}$$

where $a^\pm$, I_1, $I_2^\pm$, $I_3^\pm$, I_4 are given in Eqs. (B31)–(B33) and

$$b^\pm=1+\beta a^\pm,\quad y_0^\pm=y_0\pm a^\pm,$$

$$G_F^\pm=\mp\beta\left(2Ea^\pm+p_2\frac{y_0^\pm}{b^\pm}\right)\frac{y_0^\pm}{b^\pm}.$$

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