Rare radiative decay of the B_c meson

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We study the decays $B_c \to D^* \gamma$ and $B_c \to D_s^* \gamma$ in the relativistic independent quark model based on the confining potential in the scalar-vector harmonic form. Out of the two competing mechanisms contributing to these decays, we find that the weak annihilation contribution dominates the electromagnetic penguin one. Considering contributions from both mechanisms, total decay widths and branching ratios are predicted as $\Gamma(B_c \to D^* \gamma) = 5.22 \times 10^{-18}$ GeV, $\Gamma(B_c \to D_s^* \gamma) = 1.98 \times 10^{-16}$ GeV and $Br(B_c \to D^* \gamma) \simeq 3.64 \times 10^{-6}$, $Br(B_c \to D_s^* \gamma) \simeq 1.39 \times 10^{-4}$ with $\tau_{B_c} = 0.46$ ps. The decays $B_c \to D_s^* \gamma$ can well be studied at CERN LHC in the near future.

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I. INTRODUCTION

The investigation of heavy flavored hadrons is one of the most promising areas of research in high energy physics, which provides a unique opportunity for the determination of many fundamental parameters of the standard model, including the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, leptonic decay constants, and the top quark mass. The investigation of bottom mesons, particularly the B_c meson, is more interesting in that different mechanisms such as weak annihilation, the spectator, flavor changing neutral current (FCNC), as well as the charged current decays simultaneously contribute to their decay amplitudes, giving a deeper insight into the weak decay dynamics of heavy hadrons. Consisting of two heavy quarks of different flavors and masses, the B_c meson not only provides a rich source for a precise determination of CKM matrix elements but also yields reliable QCD predictions on various inclusive and exclusive decays.

There are two types of transitions at the constituent level that can contribute to rare radiative decays of heavy mesons, namely the single and two quark transitions [1]. The single quark transition comes from the so called electromagnetic penguin process induced by the flavor changing neutral current of the type $b \to s\gamma$. The two quark transition (weak annihilation process) is however described through the W-exchange annihilation diagram in two ways: one via the W-exchange annihilation diagram accompanied by the photon emission from external quarks and the other via the same W-exchange diagram but with photon emitted from the W boson. The later type two quark transition is typically suppressed by a factor of $m_q k/M_w^2$ (k being the photon energy) as compared to the former [2].

Out of the two important mechanisms contributing to the rare radiative decays of heavy mesons, the effective Lagrangian for short distance penguin process is fairly known. Extensive work has been done in this sector to study the flavor changing neutral current dynamics using several approaches such as heavy quark effective theory (HQET) [3], QCD sum rule [4], lattice QCD [5], nonrelativistic and relativistic quark models [6], heavy quark mass limit [7] and the relativistic quasipotential model in quantum field theory [8], etc. The predictions [3–8] on these decays have not only played a significant role in testing the loop effect in the standard model but also searching for physics beyond the standard model. Owing to large top quark mass involved in the penguin process, the $b \to s\gamma$ type amplitudes are neither quark-mixing nor loop suppressed. Moreover, it is largly enhanced by QCD correction [6]. Due to this, short distance penguin contributions to decay amplitudes in $\overline{B^0} \to \overline{K^0}^* \gamma$ and $B_s \to \phi\gamma$, etc. dominate over the long distance weak annihilation contributions. On the other hand, in case of the weak radiative decays of B_c meson which consists of two heavy flavored quarks, the contribution from a weak annihilation diagram is expected to be large. This is because in these cases the bound state effects are expected to seriously modify the decay amplitude.

While studying $B_c \to D^{*-}\gamma$ [9] and $B_c \to D_s^{*-}\gamma$ [10], it is argued that in $b \to d(s)\gamma$ inclusive process, the daughter quarks d(s) obtain large recoil momentum. But in order to form bound states such as $D^{*-}(D_s^{*-})$ with the much heavier spectator $\bar{c}$ quark, most of the momentum of the daughter quark must be transferred to the spectator $\bar{c}$ by a hard scattering process, as it is for the heavier constituent in the final bound state to share the major part of the momentum of $D^{*-}(D_s^{*-})$. Since hard scattering is suitable for perturbative QCD (PQCD) calculations, weak annihilation of heavy hadrons, particularly bottom meson, have been studied satisfactorily by many authors using PQCD [11]. Recently weak radiative decay of B mesons have been systematically studied by Cheng *et al.* [12] and by Grinstein *et al.* and others [13]. The generic form of the weak annihilation process considered in light cone formalism in Refs. [14,15] is $M \to M^*\gamma \to V\gamma$; where M is the massive initial 0^- state pseudoscalar meson, M^* is the virtual intermediate vector meson with the flavor quantum number of initial meson M, and V is the final state 1^- meson. In such a scheme, only the photon coupling to M is included with the assumption that photon coupling to V is suppressed by the light quark mass; which, in fact, yields to a large scale underestimation of the branching ratio especially in B_c-meson decays. A class of meson decay modes sensitive to one quark topology at leading G_F order is analyzed in Ref. [16] in the generic form: $M \to P \to V\gamma$ where P is the lighter vitual 0^- meson with flavor quantum number of the final state V. In this scheme photon coupling to M [$M \to M^*\gamma \to V\gamma$] is neglected and intermediate state P is assumed to be the lightest pseudoscalar meson. Such an assumption is thought to be reasonable partly because data exist in $PV\gamma$ coupling from the observed $V \to P\gamma$ and partly because the lightest pseudoscalar meson P among possible intermediates presumably has one of the largest coupling to $V\gamma$ due to relatively large wave function overlap. In any case the branching ratio is bound to be underestimated in this scheme due to the exclusion of $MM^*\gamma$ coupling. In the more elaborate quark model calculation based on the vector meson dominance (VMD) [17] and effective field theoretic technique [18] it is shown that $MM^*\gamma$ amplitude is about 1/4 as large as that for $PV\gamma$. In the calculation based on heavy quark spin and flavor symmetry (HQS) [19], contributions from both diagrams are systematically taken into account in analyzing B_c meson decaying to charm meson final states where $MM^*\gamma$ and $PV\gamma$ couplings are related to $DD^*\gamma$ coupling through the HQS.

In view of these observations it is expected that the electromagnetic penguin diagram and weak annihilation process should have comparable contributions to the decay amplitudes for $B_c \to D^{*-}\gamma$ and $B_c \to D_s^{*-}\gamma$. Therefore any theoretical treatment of these decay modes must address the question of relative importance of short and long distance contribution. With this aim we intend to study the B_c-meson weak radiative decays to charm meson final states in the relativistic independent quark model, which has provided consistently good predictions in wide ranging hadronic phenomena including the static properties of hadrons [20], elastic form factors and charge radii of mesons [21], leptonic [22], weak leptonic [23], radiative [24,25], weak radiative [26] and the semileptonic [27] decays of mesons. Recently we have studied the rare radiative decays of bottom mesons [28] using one loop electromagnetic penguin diagram in which the present model predictions are found to be in reasonable agreement with data as well as other model predictions. We have also studied successfully the weak radiative decays of charm mesons [26] in this model where we take contributions from both the diagrams corresponding to photon being emitted before and after the flavor changing weak vertex. We would like to extend the formalism developed in Ref. [28] and [26] to have a comparative study for short and long distance contributions to the decay amplitudes of $B_c \to D^{*-}\gamma$ and $B_c \to D_s^{*-}\gamma$ within the scope of the independent quark model.

The paper is organized as follows. Section II gives a brief outline of the independent relativistic quark model. Starting from the effective interaction Hamiltonians and the transition matrix element, we describe the model calculation of the decay width for B_c-meson weak radiative decays in Sec. III. Numerical results are given in Sec. IV and finally Sec. V embodies the discussion and conclusion.

II. MODEL DESCRIPTION

In this model a meson, in general, is pictured as a color-singlet assembly of a quark and an antiquark independently confined within meson by an effective average potential [20–28],

$$U(r) = \frac{1}{2}(1+\gamma^0)(ar^2+V_0), \quad a>0. \tag{1}$$

The potential $U(r)$ taken in this form represents phenomenologically the confining interaction expected to be generated by the non-perturbative multigluon mechanism. The quark-gluon interaction at short distance originating from one gluon exchange and the quark-pion interaction required in the nonstrange sector to preserve chiral symmetry are presumed to be residual interaction compared to the dominant confining interaction. Although residual interaction treated perturbatively in the model calculation are crucial in generating mass splitting [20] in hadron spectroscopy, their role in the hadronic decay process are considered less significant. Therefore, to the first approximation, it is believed that the zeroth order quark dynamics inside the meson core, generated by the confining part of interaction phenomenologically represented by the potential $U(r)$ in Eq. (1) can provide an adequate description for exclusive rare radiative decays of bottom mesons. In this picture the independent quark Lagrangian density in the zeroth order is given by

$$\mathcal{L}_q^0(x) = \bar{\psi}_q(x)\left[\frac{i}{2}\gamma^\mu \overleftrightarrow{\partial}_\mu - m_q - U(r)\right]\psi_q(x). \tag{2}$$

The ensuing Dirac equation with $E'_q = (E_q - V_0/2)$, $m'_q = (m_q + V_0/2)$, $\lambda_q = (E'_q + m'_q)$ and $r_{0q} = (a\lambda_q)^{-1/4}$ admits static solution of positive and negative energy in zeroth order, which for the ground-state meson can be obtained in the form

$$\phi^{(+)}_{q_\lambda}(\vec{r}) = \frac{1}{\sqrt{4\pi}}\begin{pmatrix} ig_q(r)/r \\ \vec{\sigma}\cdot\hat{r} f_q(r)/r \end{pmatrix}\chi_\lambda,$$

$$\phi^{(-)}_{q_\lambda}(\vec{r}) = \frac{1}{\sqrt{4\pi}}\begin{pmatrix} i(\vec{\sigma}\cdot\hat{r}) f_q(r)/r \\ g_q(r)/r \end{pmatrix}\tilde{\chi}_\lambda. \tag{3}$$

Here the two component spinors χ_λ and $\tilde{\chi}_\lambda$ stand for

$$\chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \tilde{\chi}_\uparrow = \begin{pmatrix} 0 \\ -i \end{pmatrix}, \tilde{\chi}_\downarrow = \begin{pmatrix} i \\ 0 \end{pmatrix},$$

respectively. The reduced radial parts in the upper and lower component solutions corresponding to the quark-flavor ''q'' are

$$g_q(r)=\mathcal{N}_q\left(\frac{r}{r_{0q}}\right)\exp(-r^2/2r_{0q}^2),$$
$$f_q(r)=-\frac{\mathcal{N}_q}{\lambda_q r_{0q}}\left(\frac{r}{r_{0q}}\right)^2\exp(-r^2/2r_{0q}^2), \tag{4}$$

where the normalization factor $\mathcal{N}_q$ is given by

$$\mathcal{N}_q^2=\frac{8\lambda_q}{\sqrt{\pi}r_{0q}}\frac{1}{(3E_q'+m_q')}. \tag{5}$$

The quark binding energy E_q of zeroth order in the meson ground state is derivable from the bound-state condition:

$$\sqrt{\frac{\lambda_q}{a}}(E_q'-m_q')=3. \tag{6}$$

From the quark-antiquark eigenmodes in Eq. (3) obtained in solving Dirac equation, it is possible to find quark-antiquark momentum probability amplitudes by taking suitable momentum space projection of the corresponding quark-antiquark orbitals. If $g_{q_1}(\vec{p}_1,\lambda_1')$ is the amplitude of the bound quark ''q'' in its lowest eigenmode to be found in the state of definite momentum $\vec{p}_1$ and spin projection λ_1', then

$$g_{q_1}(\vec{p}_1;\lambda_1')=\frac{u_{q_1}^\dagger(\vec{p}_1,\lambda_1')}{\sqrt{2E_{q_1}}}\int d\vec{r}\,\phi^{(+)}_{q_1\lambda_1}(\vec{r})\exp(-i\vec{p}_1\cdot\vec{r})$$
$$=g_{q_1}(\vec{p}_1)\,\delta_{\lambda_1\lambda_1'}, \tag{7}$$

where $E_{p_1}=\sqrt{(\vec{p}_1^{\,2}+m_{q_1}^2)}$ and $u_{q_1}(\vec{p}_1,\lambda_1')$ is the usual free Dirac spinor. Using free Dirac spinor normalization and taking $\alpha_q=1/2r_{0_q}^2$, one finds

$$g_{q_1}(\vec{p}_1)=\frac{i\pi\mathcal{N}_{q_1}}{2\alpha_{q_1}\lambda_{q_1}}\sqrt{\frac{(E_{p_1}+m_{q_1})}{E_{p_1}}}(E_{p_1}+E_{q_1})$$
$$\times\exp\left(-\frac{\vec{p}_1^{\,2}}{4\alpha_{q_1}}\right). \tag{8}$$

Thus $g_{q_1}(\vec{p}_1)$ essentially provides the momentum probability amplitude for the quark q_1 in its eigenmode $\phi^{(+)}_{q_1\lambda_1}(\vec{r})$ to have a definite momentum $\vec{p}_1$ inside the meson. Similarly the momentum probability amplitude for the antiquark $\overline{q_2}$ in its eigenmode $\phi^{(-)}_{q_2\lambda_2}(\vec{r})$ can be found in the form

$$\tilde{g}_{q_2}(\vec{p}_2)=\frac{-i\pi\mathcal{N}_{q_2}}{2\alpha_{q_2}\lambda_{q_2}}\sqrt{\frac{(E_{p_2}+m_{q_2})}{E_{p_2}}}(E_{p_2}+E_{q_2})$$
$$\times\exp\left(-\frac{\vec{p}_2^{\,2}}{4\alpha_{q_2}}\right). \tag{9}$$

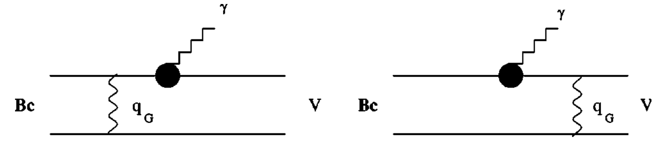

FIG. 1. The leading penguin contribution to $B_c\to V\gamma$.

Finally in the independent particle picture of the present model we take an ansatz by expressing the effective momentum distribution amplitude $G_M(\vec{p}_1,\vec{p}_2)$ for the quark and antiquark inside the meson to have momentum $\vec{p}_1$ and $\vec{p}_2$ respectively in the form

$$G_M(\vec{p}_1,\vec{p}_2)=\sqrt{g_{q_1}(\vec{p}_1)\tilde{g}_{q_2}(\vec{p}_2)}, \tag{10}$$

from which we construct meson ground state with a definite momentum $\vec{P}$ and spin projection S_V as

$$|M(\vec{P},S_V)\rangle=\frac{1}{\sqrt{N(\vec{P})}}\sum_{\lambda_1\lambda_2\in S_V}\zeta^M_{q_1q_2}(\lambda_1,\lambda_2)$$
$$\times\int d\vec{p}_1 d\vec{p}_2\,\delta^{(3)}(\vec{p}_1+\vec{p}_2-\vec{P})$$
$$\times G_M(\vec{p}_1,\vec{p}_2)\hat{b}^\dagger_{q_1}(\vec{p}_1,\lambda_1)\hat{\tilde{b}}^\dagger_{q_2}(\vec{p}_2,\lambda_2)|0\rangle. \tag{11}$$

Here $\hat{b}^\dagger_{q_1}(\vec{p}_1,\lambda_1)$ and $\hat{\tilde{b}}^\dagger_{q_2}(\vec{p}_2,\lambda_2)$ are, respectively, the quark and antiquark creation operator. $\zeta^M_{q_1q_2}(\lambda_1,\lambda_2)$ stands for the appropriate SU(6)-spin flavor coefficient for the meson $M(q_1,\overline{q_2})$. $N(\vec{P})$ represents the overall normalization factor, which is obtained in the form

$$N(\vec{P})=\int d\vec{p}_1|G_M(\vec{p}_1,\vec{P}-\vec{p}_1)|^2 \tag{12}$$

considering meson-state normalization as

$$\langle M(\vec{P})|M(\vec{P}')\rangle=\delta^{(3)}(\vec{P}-\vec{P}').$$

III. CALCULATION OF DECAY WIDTH

With the phenomenological picture showing underlying dynamics of quark and antiquark inside the meson, represented by the appropriate momentum wave packet as in Eq. (11), the hadronic matrix elements corresponding to the penguin- and weak annihilation diagram shown in Fig. 1, and Fig. 2 can be calculated using the formalism developed in the

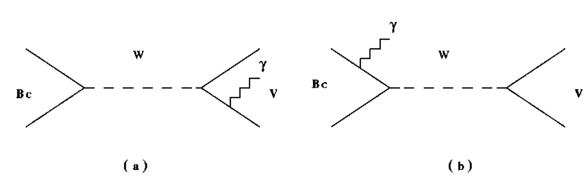

FIG. 2. The leading W-annihilation contribution to $B_c\to V\gamma$ with photon emission from external legs.

present model in our earlier works in Refs. [28] and [26], respectively. However, a brief account in the present context is provided here.

A. Electromagnetic penguin contribution

In considering the electromagnetic penguin contribution it has been assumed that the decay process $B_c \to V+\gamma$; with $V\equiv(D^{*-},D_s^{*-})$ is goverened by the effective interaction Hamiltonian [29]

$$\mathcal{H}_{eff}=\frac{4G_F}{\sqrt{2}}\mathcal{V}_{tb}\mathcal{V}^*_{tj}C_7(m_b)\mathcal{O}_7, \tag{13}$$

where with $j\equiv(d,s)$; the electromagnetic penguin operator

$$\mathcal{O}_7=\frac{e}{32\pi^2}m_b\bar{\psi}_j\sigma^{\mu\nu}F_{\mu\nu}(1+\gamma_5)\psi_b. \tag{14}$$

This in fact describes the meson decay effectively in terms of the QCD corrected quark level decay $b\to(d,s)+\gamma$ while the other constituent quark (c) is treated as a spectator (Fig. 1). Here $F_{\mu\nu}$ denotes the field strength tensor of the photon and $C_7(m_b)$ is the Wilson-coefficient which includes the necessary QCD corrections appropriately [29,30].

Then starting with the S-matrix element for the decay process as defined in terms of the above effective interaction Hamiltonian in Eqs. (13), (14) at the constituent quark level and realizing the energy momentum conservation at the mesonic level in terms of the appropriate four momentum δ function through a loose-binding approximation at the constituent level; the invariant transition matrix element can be obtained in the form

$$\mathcal{M}=\frac{eG_Fm_b}{2\sqrt{2}\pi^2}C_7(m_b)\mathcal{V}_{tb}\mathcal{V}^*_{tj}\eta^{*\mu}(q,\delta)\sqrt{4E_{B_c}E_V}\times\langle V(k,\epsilon^*)|\bar{j}i\sigma_{\mu\nu}q^\nu b_R|B_c(P)\rangle. \tag{15}$$

Here q is the four momentum and $\eta^{*\mu}(q,\delta)$ is the polarization vector of the emitted photon. The hadronic matrix element in Eq. (15) can be expressed in a covariant decomposition as

$$\begin{aligned}\sqrt{4E_{B_c}E_V}&\langle V(k,\epsilon^*)|\bar{j}i\sigma_{\mu\nu}q^\nu b_R|B_c(P)\rangle\\ &\equiv\sqrt{4E_{B_c}E_V}\langle V(k,\epsilon^*)|V_\mu+A_\mu|B_c(P)\rangle\\ &=i\in_{\mu\nu\rho\sigma}\epsilon^{*\nu}P^\rho k^\sigma f_1(q^2)\\ &\quad+[\epsilon^*_\mu(M^2_{B_c}-M^2_V)+(\epsilon^*\cdot q)(P+k)_\mu]f_2(q^2).\end{aligned} \tag{16}$$

Here ϵ^* is the polarization vector and k is the four momentum of the final state vector meson; $q=(P-k)$ denotes the four momentum transfer and V_μ and A_μ are the vector and axial vector part of the effective current. Then replacing $|\mathcal{V}_{tb}\mathcal{V}^*_{tj}|$ by $|\mathcal{V}_{cb}\mathcal{V}^*_{cj}|$ from the unitarity relation in view of the smallness of $\mathcal{V}_{ub}$; the decay width can be expressed in the rest frame of B_c as [28]

$$\Gamma(B_c\to V\gamma)=\frac{1}{(2\pi)^2}\int\frac{d\vec{k}d\vec{q}}{2M_{B_c}2E_V2E_\gamma}\times\delta^{(4)}(P-k-q)\overline{\sum_{\delta,S_V}}|\mathcal{M}|^2,$$

which in terms of the form factors $f_1(q^2)$ and $f_2(q^2)$ becomes

$$\Gamma(B_c\to V\gamma)=\frac{\alpha G_F^2m_b^2}{8\pi^4}|\mathcal{V}_{cb}\mathcal{V}^*_{cj}|^2|C_7(m_b)|^2\bar{k}^3\times[|f_1(0)|^2+4|f_2(0)|^2]. \tag{17}$$

Here $\bar{k}=\bar{E}_\gamma=(M^2_{B_c}-M^2_V)/2M_{B_c}$ is the energy of the emitted photon at the meson level and α is the fine structure constant. Now calculation of decay width from Eq. (17) essentially boils down to an evaluation of the form factors in terms of model quantities. Using the covariant expansion in Eq. (16), which is frame independent; one can relate those form factors separately to some specific hadronic matrix elements by taking $k\equiv(E_V,0,0,|\vec{k}|)$ with $P\equiv(M_B,0,0,0)$ in the form

$$\begin{aligned}f_1(0)&=\sqrt{\frac{2E_V}{M_{B_c}}}\frac{1}{E_\gamma}[\langle V(k,\epsilon^*)|(V_1+A_1)|B_c(0)\rangle_+ +\langle V(k,\epsilon^*)|(V_1+A_1)|B_c(0)\rangle_-],\\ 2f_2(0)&=\sqrt{\frac{2E_V}{M_{B_c}}}\frac{1}{E_\gamma}[\langle V(k,\epsilon^*)|(V_1+A_1)|B_c(0)\rangle_+ -\langle V^*(k,\epsilon^*)|(V_1+A_1)|B_c(0)\rangle_-].\end{aligned} \tag{18}$$

Now explicit calculation of these hadronic matrix elements in the present model gives

$$\langle V(k,\epsilon^*)|(V_1+A_1)|B_c(0)\rangle_{S_V=-1}=0$$

leading to

$$f_1(0)=2f_2(0)=\frac{1}{E_\gamma}\sqrt{\frac{2E_V}{M_{B_c}}}\langle V(k,\epsilon^*)|(V_1+A_1)|B_c(0)\rangle_{S_V=+1}, \tag{19}$$

which in terms of the model quantities becomes

$$f_1(0)=2f_2(0)=\frac{1}{\sqrt{N(0)N(\vec{k})}}\int\frac{d\vec{p}_b G_{B_c}(\vec{p}_b,-\vec{p}_b)G_V(\vec{p}_b+\vec{k},-\vec{p}_b)\sqrt{E_{p_b+k}+E_{p_c}}}{\sqrt{4E_{p_b}E_{p_b+k}(E_{p_b}+E_{p_c})}}Q(\vec{p}_b,\vec{k}), \tag{20}$$

where

$$Q(\vec{p}_b,\vec{k})=\sqrt{z}\left[1+\frac{|\vec{k}|}{E_{p_b+k}+m_j}+\frac{|\vec{p}_b|^2}{3z}\right],$$

$$z=(E_{p_b}+m_b)(E_{p_b+k}+m_j). \tag{21}$$

The three momentum magnitude $|\vec{k}|(=|\vec{q}|=E_\gamma)$ of the final meson appearing inside the quark level integral in Eq. (20) is taken at the quark level as $\overline{E}_\gamma=(m_b^2-m_j^2)/2m_b$ on the consideration that the $j\equiv(d,s)$ quark in the quark level decay $b\to(d,s)+\gamma$ is recoiled with momentum $\vec{k}$ with $|\vec{k}|=\overline{E}_\gamma$.

We must point out here that realization of the S-matrix element in the standard covariant form at the composite level; starting from a picture at the constituent level has never been so straightforward. This is due to the fact that although three momentum conservation is automatically guaranteed at the mesonic level through appropriate delta function; energy conservation has not been so transparent. This however has been realized here [28] at the mesonic level by extracting out the energy delta function $\delta(E_{p_b}-E_{p_b+k}-E_\gamma)$ from within the quark level integral in the form of $\delta(E_{B_c}-E_V-E_\gamma)$ with a loose binding approximation of $(E_{p_b}+E_{p_c})\simeq E_{B_c}$ and $(E_{p_b+k}+E_{p_c})\simeq E_V$ in the δ-function argument. Keeping this in mind; in the final form of $f_1(q^2=0)$; a kinematic factor $\sqrt{E_V/M_{B_c}}$ from outside the quark level integral has been pushed back inside the integral under the same loose binding approximation in order to minimize any uncertainty due to the approximation at the first place. This procedure has been elaborated in our earlier work [28].

Finally from Eqs. (17), (20) and (21) the model expression for the decay width due to electromagnetic penguin process is obtained in the form

$$\Gamma(B_c\to V\gamma)=\frac{\alpha G_F^2 m_b^2}{32\pi^4}|\mathcal{V}_{cb}\mathcal{V}_{cj}^*|^2|C_7(m_b)|^2M_{B_c}^3 \times\left(1-\frac{M_V^2}{M_{B_c}^2}\right)^3|f_1(0)|^2. \tag{22}$$

B. Weak annihilation contribution

The weak annihilation of heavy hadrons derives its contribution from the weak and electromagnetic interaction at the constituent level. The possible Feynmann diagrams in Figs. 2(a) and 2(b) correspond to $B_c\to P\to V\gamma$ and $B_c\to B_c^*\gamma\to V\gamma$ being mediated by virtual intermediate meson states ''P'' and ''B_c^*'' respectively with photon being emitted from each of the four external legs. The S-matrix element for the process $B_c\to V\gamma$ is written as

$$S_{fi}=-2\pi i\,\delta^{(0)}(E_V+E_\gamma-M_{B_c})\mathcal{M}, \tag{23}$$

where the invariant transition matrix element $\mathcal{M}$ is

$$\mathcal{M}=\mathcal{M}^{(a)}+\mathcal{M}^{(b)},$$

such that

$$\mathcal{M}^{(a)}=\langle V\gamma|\hat{H}_{em}|P\rangle\langle P|\hat{H}_\omega|B_c\rangle/(M_{B_c}-E_V),$$

$$\mathcal{M}^{(b)}=\langle V\gamma|\hat{H}_\omega|B_c^*\rangle\langle B_c^*\gamma|\hat{H}_{em}|B\rangle/(M_{B_c}-E_{B_c^*}). \tag{24}$$

In such a representation of S-matrix element in so-called ''old perturbative theory,'' the interaction Hamiltonians taken effectively in the Schrödinger picture are written explicitly at the quark level in the following manner:

$$\hat{H}_{em}=\sum_q e_q\int d\vec{r}\,\overline{\psi}_q(\vec{r})\gamma_\mu A^\mu(\vec{r})\psi_q(\vec{r}), \tag{25}$$

where $A^\mu(\vec{r})$ is the photon field in the Coulomb gauge with $\epsilon^*(\vec{k},\delta)$ as the photon polarization having energy momentum $(E_\gamma=|\vec{k}|,\vec{k})$ and

$$\hat{H}_\omega=\frac{G_F}{\sqrt{2}}\mathcal{V}_{bc}\mathcal{V}_{cj}^*a_1[\overline{\psi}_j\Gamma^\mu\psi_c][\overline{\psi}_c\Gamma_\mu\psi_b], \tag{26}$$

where $\hat{j}=d(s)$ and $a_1=\frac{1}{3}(2C_++C_-)$ is the QCD correction factor representing the strength of weak interaction induced by and dependent on the flavor-changing charged current. C_+ and C_- are QCD coefficient [31] given in the leading log approximation by

$$C_\pm(\mu)=\left[\frac{\alpha_s(\mu^2)}{\alpha_s(M_\omega^2)}\right]^{d_\pm/2b},$$

with $d_- = 2d_+ = 8$; $b = (11 - 2/3N_f)$; N_f = number of colors; $\mu \simeq m_b$ and α_s = strong fine structure constants.

In the context of radiative decays of heavy mesons, it has been shown that present model prediction based on static approximation [24] remains unaffected in a more realistic calculation beyond static approximation [25] with appropriate recoil effect being duly considered through the quark field operator and momentum wave packet for mesons. This led us to invoke static approximation in our subsequent analysis of charmed meson weak radiative decays [26] yielding satisfactory result. It is partly due to this observation and partly on the grounds of simplicity that we use first the static approximation to calculate decay amplitudes for $B_c \to D^{*-}\gamma$ and $B_c \to D_s^{*-}\gamma$. However, in the later stage of our calculation we would like to overcome the possible uncertainty inherent in the static calculation by invoking appropriate recoil effect as done in Refs. [25,26]. The static quark field operators appearing in $\hat{H}_{em}$ and $\hat{H}_\omega$ find possible expansion in terms of complete set of positive and negative energy static solutions of the independent quark model [24,26] as

$$\psi_q(\vec{r}) = \sum_\zeta [\hat{b}_{q\zeta}\phi^{(+)}_{q\zeta}(\vec{r}) + \hat{\tilde{b}}^+_{q\zeta}\phi^{(-)}_{q\zeta}(\vec{r})]. \tag{27}$$

Here "ζ" represents the set of Dirac quantum numbers specifying all possible eigenmodes. $\hat{b}_{q\zeta}$ and $\hat{\tilde{b}}^+_{q\zeta}$ are the quark annihilation and antiquark creation operators in the eigenmode "ζ." In the static approximation, the contribution to matrix element would essentially come from the lowest eigenmode in the field expansion in Eq. (27). The quark orbitals corresponding to positive and negative energy in their lowest eigenmode have been derived in the form Eq. (3). We represent the meson states by the usual spin-flavor SU(6) expressions with the quark-antiquark corresponding to the lowest eigenmodes. With these considerations, the transition moment corresponding to photon emission at electromagnetic vertex is found in the general form

$$\mu_{AB*}(k) = e_{q_1}\mu^0_{q_1}(k) + e_{q_2}\mu^0_{q_2}(k), \tag{28}$$

where

$$\mu^0_q(k) = 2\exp(-k^2 r_{0q}^2/4)/(3E'_q + m'_q),$$

with $\mu_q = e_q\mu^0_q(k=0)$ as the confined quark magnetic moment in the model.

With similar approximation, weak matrix elements corresponding to the diagrams 2(a) and 2(b) can be expressed as [26]

$$\langle P|\hat{H}_\omega|B_c\rangle = \frac{G_F}{\sqrt{2}}\mathcal{V}_{cb}\mathcal{V}^*_{cj}a_1A_1,$$

$$\langle V|\hat{H}_\omega|B^*_c\rangle = \frac{G_F}{\sqrt{2}}\mathcal{V}_{cb}\mathcal{V}^*_{cj}a_1(A_2+A_3), \tag{29}$$

where A_1, A_2 and A_3 in terms of model quantities are found to be

$$A_1 = \mathcal{N}_0\left\{1 - \frac{3}{R_0}(\Lambda_1+\Lambda_2+\Lambda_3) + \frac{15}{R_0^2}\Lambda_0\right\} \Big/ 2R_0^{3/2},$$

$$A_2 = \mathcal{N}_0\{\Lambda_3 - \Lambda_2\}/R_0^{5/2},$$

$$A_3 = -\mathcal{N}_0\left\{1 + \frac{1}{R_0}(\Lambda_1-\Lambda_2-\Lambda_3)\frac{15}{R_0^2}\Lambda_0\right\} \Big/ 2R_0^{3/2}, \tag{30}$$

with

$$\Lambda_0 = a^2\{\lambda_{q_1}\lambda_{q_2}\lambda_{q_3}\lambda_{q_4}\}^{-1/2},$$

$$\Lambda_1 = a\{(\lambda_{q_1}\lambda_{q_2})^{-1/2} + (\lambda_{q_3}\lambda_{q_4})^{-1/2}\},$$

$$\Lambda_2 = a\{(\lambda_{q_1}\lambda_{q_3})^{-1/2} + (\lambda_{q_2}\lambda_{q_4})^{-1/2}\},$$

$$\Lambda_3 = a\{(\lambda_{q_1}\lambda_{q_4})^{-1/2} + (\lambda_{q_2}\lambda_{q_3})^{-1/2}\},$$

$$\mathcal{N}_0 = \frac{1}{\sqrt{2\pi}}\{(\mathcal{N}_{q_1}\mathcal{N}_{q_2}\mathcal{N}_{q_3}\mathcal{N}_{q_4})/(r_{0q_1}r_{0q_2}r_{0q_3}r_{0q_4})\},$$

$$R_0 = \left\{\frac{1}{r^2_{0q_1}} + \frac{1}{r^2_{0q_2}} + \frac{1}{r^2_{0q_3}} + \frac{1}{r^2_{0q_4}}\right\}. \tag{31}$$

Substituing the transition moments from Eq. (28) and weak matrix element from Eq. (29) into Eq. (23) through Eq. (24), the S-matrix element for $B_c \to V\gamma$ is obtained as

$$\mathcal{S}_{fi} = -i\sqrt{\frac{\alpha}{k}}A_0\{F(k)+G(k)\}K_{s_V}\delta(k-\bar{k}), \tag{32}$$

where

$$A_0 = \frac{G_F}{\sqrt{2}}\mathcal{V}_{cb}\mathcal{V}^*_{cj}a_1,$$

$$F(k) = \left(\frac{A_1}{M_{B_c}-M_P}\right)\mu_{VP}(\bar{k}) + \left(\frac{A_2}{M_{B_c}-M_{B^*_c}}\right)\mu_{B_cB^*_c}(\bar{k}),$$

$$G(k) = \left(\frac{A_3}{M_{B_c}-M_{B^*_c}}\right)\mu_{B_cB^*_c}(\bar{k}),$$

$$\bar{k} = (M^2_{B_c} - M^2_V)/2M_{B_c}, \tag{33}$$

and

$$\vec{K} = \vec{k}\times\vec{\epsilon}^*(\vec{k},\delta).$$

Finally summing over the photon polarization index 'δ' in $\vec{\epsilon}^*(\vec{k},\delta)$ and vector meson spin projection "S_V," the decay width $\Gamma(B_c \to V\gamma)$ in the static approximation is expressed in the form

$$\Gamma(B_c \to V\gamma) = 4\alpha\bar{k}^3A_0^2\{F(\bar{k})+G(\bar{k})\}^2, \tag{34}$$

with the presupposition that the recoil effect is negligible. In the present model application to weak radiative decays of charm meson [26] it has been shown that possible recoil effect, however marginal, is more pronounced at the electromagnetic vertex than at the flavor-changing weak vertex. This is expected from the relative order of strength for the weak and electromagnetic interaction. Therefore it is trivial to say that the quantitative uncertainty, if any, due to the static calculation is mostly avoided if appropriate correction to the transition moment is obtainable within the scope of the model. To do so we replace the confined quark magnetic moment $\mu_q(k)$ appearing in the transition moment $\mu_{MM*}(\overline{k})$ and $\mu_{PV}(\overline{k})$ in Eq. (28) by the corresponding term $I_{q_i}(\overline{k})$ in an integral form found in a more realistic calculation [25] in the context of radiative decays of light and heavy mesons. In the derivation of quark magnetic moment $I_{q_i}(\overline{k})$ the recoil effect is taken care of by using the momentum wave packet for appropriate meson states instead of their SU(6) expressions and expanding the quark field operators in terms of free Dirac spinors instead of the positive and negative energy static solutions. The expression for the quark magnetic moment $I_{q_i}(\overline{k})$ so obtained [25] is

$$I_{q_i}(\overline{k})=\frac{1}{\sqrt{\overline{N}_{B_c}(0)\overline{N}_V(\overline{k})}}\int_0^\infty dpp^2\left(\frac{E_{ik}+E_j}{E_1+E_2}\right)^{1/2}X_i(\vec{p},\vec{k})\times\exp(-\beta p^2), \tag{35}$$

where

$$\beta=\frac{1}{4}\left(\frac{1}{\alpha_{q_1}}+\frac{1}{\alpha_{q_2}}\right),$$

$$\overline{N}_{B_c}=\int_0^\infty dpp^2R_A(p)\exp(-\beta p^2),$$

$$\overline{N}_V(\overline{k})=\int_0^\infty dpp^2R_{B_i}(p,k)\exp(-\beta p^2),$$

$$X_i(p,k)=\frac{R_A(p)}{2}\left[\left\{\frac{E_i+m_{q_i}}{E_{ik}+m_{q_i}}\right\}\left\{\frac{E_{ik}+E_{q_i}}{E_i+E_{q_i}}\right\}^2\left(\frac{1}{E_iE_{ik}^3}\right)\right]^{1/4}, \tag{36}$$

with

$$E_i=\sqrt{\vec{p}^2+m_{q_i}^2},\quad E_{ki}=\sqrt{(\vec{k}+\vec{p})^2+m_{q_i}^2},$$

$$R_A(p)=\prod_{j=1}^{2}\left[(E_j+E_{q_j})\left(1+\frac{m_{q_j}}{E_j}\right)^{1/2}\right],$$

$$R_{B_i}(p,k)=\left\{\frac{E_{ik}+E_{q_i}}{E_i+E_{q_i}}\right\}\left\{\frac{E_i(E_{ik}+m_{q_i})}{E_{ik}(E_i+m_{q_i})}\right\}^{1/2}R_A(p). \tag{37}$$

TABLE I. The quark mass m_q and corresponding binding energy E_q, together with λ_q, the scale factor r_{0q}, α_q and the quark normalization factor $\mathcal{N}_q$.

Quark flavor	m_q (GeV)	E_q (GeV)	λ_q (GeV)	r_{0q} $(\mathrm{GeV})^{-1}$	α_q $(\mathrm{GeV})^2$	$\mathcal{N}_q$ $(\mathrm{GeV})^{1/2}$
u	0.07875	0.47125	0.55	3.20806	0.04858	0.68901
d	0.07875	0.47125	0.55	3.20806	0.04858	0.68901
s	0.31575	0.59100	0.90675	2.83114	0.06238	0.80581
c	1.49276	1.57951	3.07227	2.08674	0.11482	1.02147
b	4.77659	4.76633	9.54292	1.57185	0.20237	1.19425

From the general expression for quark magnetic moment $I_{q_i}(\overline{k})$ in Eq. (35) the relevant transition moments at the respective electromagnetic vertices for decay modes: $B_c\rightarrow D^{*-}\gamma$ and $B_c\rightarrow D_s^{*-}\gamma$ are written as

$$\mu_{B_cB_c^*}(\overline{k})=\frac{1}{3}[2I_c(\overline{k})-I_b(\overline{k})],$$

$$\mu_{DD*}(\overline{k})=\frac{1}{3}[2I_c(\overline{k})-I_d(\overline{k})],$$

$$\mu_{D_sD_s^*}(\overline{k})=\frac{1}{3}[2I_c(\overline{k})-I_s(\overline{k})]. \tag{38}$$

These corrected forms of the transition moments when taken into account in the expressions for $F(k)$ and $G(k)$ in Eqs. (33) and (34); contribution to the decay width of $B_c\rightarrow V\gamma$ for the weak annihilation diagrams can be calculated.

IV. NUMERICAL ESTIMATION

In order to have numerical estimate, we have potential parameter (a,V_0), quark masses m_q and corresponding quark binding energy E_q. In previous applications of the model to various hadronic phenomena [20–28], the potential parameters taken are

$$(a;V_0)\equiv(0.017166\ \mathrm{GeV}^3;\ -0.1375\ \mathrm{GeV}). \tag{39}$$

We use the same quark masses m_q as used to generate the ground state hyperfine mass splitting of light $(\pi,\rho;K,K^*)$ [20] as well as the heavy $(D,D^*;B_c,B_c^*)$ [23] mesons. The quark binding energy E_q as the effective constituent quark mass and set of other model quantities such as $\lambda_q, r_{oq}, \alpha_q$ and $\mathcal{N}_q$ etc used in the calculation are those given in Table I.

TABLE II. Predictions for the ratios of the penguin, weak annihilation and total contribution of decay modes.

Decay mode	$\frac{\Gamma_P}{\Gamma_T}$	$\frac{\Gamma_A}{\Gamma_T}$	$\frac{\Gamma_A}{\Gamma_P}$
$B_c\rightarrow D^{*-}\gamma$	0.16	0.37	2.3
$B_c\rightarrow D_s^{*-}\gamma$	0.17	0.33	1.9

The CKM parameters, renormalization Wilson coefficient $C_7(m_b)$ and strength of weak interaction in the mass scale of b-quark a_1 are taken as

$$(\mathcal{V}_{cb},\mathcal{V}^*_{cs},\mathcal{V}^*_{cd})\equiv(0.041,1.04,0.224),$$
$$C_7(m_b)=0.311477\ [17]$$
$$a_1=1.01\ [32]. \tag{40}$$

With these inputs, the penguin and weak annihilation contributions to the decay widths are estimated from Eq. (21) and Eq. (34), respectively. It may be noted that since penguin and annihilation amplitudes interfere with each other, their summation should be taken at the level of matrix element of the amplitude to determine the total decay width. Our results calculated separately from the penguin diagram and weak annihilation diagrams as well as when all these are taken together can yield the decay width for $B_c\to V\gamma$ as follows:

$$\begin{aligned}\Gamma(B_c\to D^{*-}\gamma)&=0.817\times10^{-18}\ \text{GeV};\quad \text{only penguin}\ (\Gamma_P)\\&=1.904\times10^{-18}\ \text{GeV};\quad \text{only annihilation}\ (\Gamma_A)\\&=5.215\times10^{-18}\ \text{GeV};\quad \text{penguin + annihilation}\ (\Gamma_T),\end{aligned}\tag{41}$$

$$\begin{aligned}\Gamma(B_c\to D_s^{*-}\gamma)&=3.434\times10^{-17}\ \text{GeV};\quad \text{only penguin}\ (\Gamma_P)\\&=6.443\times10^{-17}\ \text{GeV};\quad \text{only annihilation}\ (\Gamma_A)\\&=1.982\times10^{-16}\ \text{GeV};\quad \text{penguin + annihilation}\ (\Gamma_T).\end{aligned}\tag{42}$$

The uncertainty in model predictions is sometimes reduced if expressed in ratio forms. We give in Table II the ratios of the penguin, weak annihilation and total (penguin + annihilation) contributions for a more realistic assessment of relative importance of two possible mechanism in the decay modes $B_c\to D^{*-}\gamma$ and $B_c\to D_s^{*-}\gamma$.

We find that weak annihilation contribution to decay width $\Gamma[B_c\to D^{*-}(D_s^{*-})\gamma]$ is 2.3 (1.9) times larger than the penguin contribution and 0.37 (0.33) times the total decay width. Our prediction on $\Gamma(B_c\to D^{*-}\gamma)$ has order of magnitude agreement with that of Ref. [19]. On the other hand, the present prediction on $\Gamma(B_c\to D_s^{*-}\gamma)$ is found to be one order of magnitude higher than that obtained in Ref. [10].

V. DISCUSSION AND CONCLUSION

We have studied two mechanisms contributing to the decay modes: $B\to D^{*-}\gamma$ and $B\to D_s^{*-}\gamma$: the short distance one induced by electromagnetic penguin and other competing one namely the long distance weak annihilation shown in Fig. 1 and Fig. 2, respectively within the framework of relativistic independent quark model based on an average confining potential in the scalar-vector harmonic form. We present a comparative study of contributions coming from the two competing sets of diagrams representing the B_c meson weak radiative decays into charm meson final states; which shows that both the penguin and weak annihilation mechanisms bear equal significance and in absolute terms the latter even dominates the former in such decay modes. In contrast, in the case of $\bar{B}^0\to\bar{K}^{0*}\gamma$ and $B_s\to\phi\gamma$ involving the initial meson $(\bar{B}^0,B_s)$ consisting of one heavy and one light quark, the penguin contribution overwhelmingly dominates weak annihilation contribution. The reason for it may be (i) a more compact size of B_c meson compared to that of $\bar{B}^0$ or B_s which can enhance the importance of annihilation decay and (ii) the relative magnitude of the relevant CKM factors. The CKM factors for contributions for the decays $B_c\to D^{*-}\gamma$ and $B_c\to D_s^{*-}\gamma$ are $|\mathcal{V}_{cb}\mathcal{V}^*_{cs}|$ and $|\mathcal{V}_{cb}\mathcal{V}^*_{cd}|$ respectively and that for $B^{\pm}\to K^{\pm*}\gamma$ is $|\mathcal{V}_{ub}\mathcal{V}^*_{us}|$ which is much smaller than the former. This factor being altogether absent, it is only the penguin contribution which is effective in decay modes such as $\bar{B}^0\to\bar{K}^{0*}\gamma$ and $B_s\to\phi\gamma$, etc.

For possible observation of decay modes involving B_c meson, one may have to wait for the future experiments either at the CERN Large Hadron Collider (LHC) or Fermilab Tevatron. However, based on the numbers of B_c currently produced at Tevatron and LHC and the present model prediction of the branching ratios such as $Br^{total}(B_c\to D^{*-}\gamma)\simeq3.64\times10^{-6}$ and $Br^{total}(B_c\to D_s^{*-}\gamma)\simeq1.39\times10^{-4}$ taking $\tau_{B_c}=0.46^{+0.18}_{-0.16}\pm0.03$ ps [33], one may believe that although these decay channels in general might remain unobservable at Tevatron, at least the $B_c\to D_s^{*-}\gamma$ channel can be expected to be observable at LHC in the near future.

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