# Quark-gluon vertex in arbitrary gauge and dimension

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One-loop off-shell contributions to the quark-gluon vertex are calculated, in an arbitrary covariant gauge and in arbitrary space-time dimension, including quark-mass effects. It is shown how one can get results for all on-shell limits of interest directly from the off-shell expressions. In order to demonstrate that the Ward-Slavnov-Taylor identity for the quark-gluon vertex is satisfied, we have also calculated the corresponding one-loop contribution involving the quark-quark-ghost-ghost vertex.

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## I. INTRODUCTION

Tests of perturbative quantum chromodynamics [1,2] are steadily reaching new levels of precision [3], necessitating information on higher-loop results for a wide range of processes. Higher-order QCD effects are also frequently required for background estimates in searches for signs of new physics.

In spite of its fundamental role, the quark-gluon vertex has not been explored in detail, even at one loop. It is of course very much related to the electron-photon vertex of quantum electrodynamics, a basic and nontrivial aspect of which is the anomalous magnetic moment, or $g-2$, which provides a powerful test of the whole concept of quantum field theories. The quark-gluon vertex differs from the electron-photon vertex already at one loop, by the contributions of an additional Feynman diagram, involving the three-gluon vertex. In fact, apart from introducing additional color structure, this non-Abelian diagram introduces at the one-loop level a kinematical structure which is absent in the QED vertex.

For special kinematical configurations, and special gauges, several results are available. Already around 1980, the symmetric off-shell case was considered by Pascual and Tarrach (PT) in [4] (see also [5]) in an arbitrary covariant gauge for massless quarks.[1] The emphasis was on comparing the modified minimal subtraction ($\overline{\mathrm{MS}}$) and Weinberg's renormalization schemes. The symmetric off-shell case was also considered in [7] by Dung, Tarasov and Phuoc (DTP), for massive quarks, restricted to the scalar function multiplying $\gamma_\mu$. For massless quarks, some on-shell results are available, mainly in the Feynman gauge, presented by Nowak, Praszałowicz and Słomiński (NPS) [8].

The situation when one (gluon or quark) momentum vanishes has been studied in more detail. Technically, in this case all three-point functions effectively reduce to two-point integrals. For massless quarks, some one-loop results in an arbitrary covariant gauge have been obtained by Braaten and Leveille [9]. In the Feynman gauge, also two-loop corrections have been presented in [9]. Moreover, in a recent paper by Chetyrkin and Rétey [10], renormalized expressions for three-loop-order QCD vertices have been obtained for such zero-momentum configurations, in an arbitrary covariant gauge.

The QED contribution, proportional to the ''Abelian'' contribution to the quark-gluon vertex, has been studied more systematically. An early paper by Ball and Chiu (BC) [11] presented a systematic kinematical decomposition of the vertex, and gave off-shell results for the one-loop QED vertex in Feynman gauge. Their work was extended to arbitrary covariant gauge by Kızılersü, Reenders and Pennington (KRP) [12]. All above-mentioned papers deal with the (dimensionally regulated [13]) four-dimensional case. Results for the three-dimensional QED contribution are also available (for massless fermions), due to Bashir, Kızılersü and Pennington (BKP) [14]. A summary of all these one-loop results is given in Table I. In addition to this table, we note that another special gauge which has been investigated is the Fried-Yennie gauge [15].

Among non-covariant gauges, we would like to mention the Coulomb gauge. In some sense, it is more ''physical,'' but technically rather challenging [16]. A rather different approach to QCD vertex functions is provided by lattice calculations [17]. The quark-gluon vertex functions may also serve as a basis for modeling the photon-nucleon vertices [18] and the quark-Reggeon vertex [19].

From Table I, one can see that, even if we consider the results in (or around) four dimensions, there are still several ''white spots.'' The aim of the present paper is to cover *all*

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[1]Numerical analysis of two-loop QCD vertices in this limit is given in a recent paper [6].

 

TABLE I. Kinematics and gauges considered in other one-loop studies. None of these results is valid for arbitrary dimension *n*.

| | All momenta off-shell | | | Some momenta on-shell | |
|---|---|---|---|---|---|
| | General case | | $p_1^2=p_2^2=p_3^2$ | $p_3=0$ | $p_1^2=p_2^2=0$ or $p_2^2=p_3^2=0$ |
| | QED | QCD | | | |
| Feynman gauge | BC [11] | | special case of PT [4], $m=0$ | special case of BL [9], $m=0$ | NPS [8], $m=0$ |
| Arbitrary covariant gauge | KRP [12]; BKP [14], 3*d* | | PT [4], $m=0$ DTP [7], $\gamma_\mu$ part | BL [9], $m=0$ | |

such remaining spots. Moreover, we present results which are valid for an *arbitrary* value of the space-time dimension. Apart from the quark-gluon vertex itself, we also consider the related two-point functions, and the quark-quark-ghost-ghost vertex function, in order to be able to check that the obtained quark-gluon vertex function obeys the Ward-Slavnov-Taylor (WST) identity [20].

At the one-loop level, the simple and well-known Dirac-matrix structure of the lowest-order quark-gluon vertex gets modified. In the general case, 12 structures are needed to decompose it [11]. Thus, 12 scalar functions multiplying these tensor structures are to be calculated. These scalar functions depend on the gauge parameter, the space-time dimension, quark mass(es), and the kinematical invariants ($p_1^2$, $p_2^2$,$p_3^2$). Four of them (the ''longitudinal'' ones) are involved in the WST identity, whereas the remaining eight are unconstrained.

There are several reasons why the one-loop results calculated in arbitrary gauge and dimension *n* are of special interest:

(i) knowing the results in arbitrary gauge, one can explicitly keep track of gauge invariance for physical quantities;

(ii) if one is interested in the two-loop calculation of the quark-gluon coupling, one should know one-loop contributions in more detail;

(iii) results in arbitrary dimension make it possible to consider all on-shell limits *directly* from these expressions (see Sec. IV); this is impossible if one only has the results valid around four dimensions;

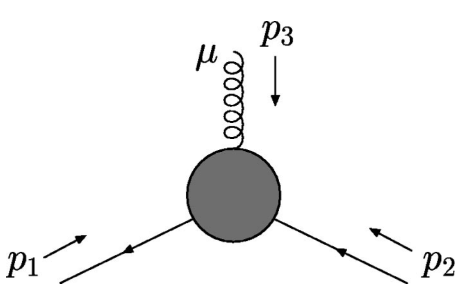

FIG. 1. Kinematics of the quark-gluon vertex.

(iv) QCD is also a theory of interest in three and two dimensions (see, e.g., [21]);

(v) as we shall see, the results for arbitrary dimension are not much more cumbersome than those considered around four dimensions (in some respects, they are even more transparent and instructive).

The paper is organized as follows. In Sec. II, we introduce the notation for the two- and three-point functions to be considered, and discuss their decomposition in terms of scalar functions as well as the corresponding Ward-Slavnov-Taylor identity. In Sec. III, we present the most general off-shell results for the quark-gluon vertex. Sec. IV contains the corresponding expressions for special limits of interest. In Sec. V, we conclude with a summary and a discussion of the results. Then, we have several appendices where some further results and technical details are presented, such as the formulas used to decompose the quark-gluon vertex (Appendix A), relevant results for the scalar integrals (Appendices B and C), results for the one-loop contribution involved in checking the WST identity (Appendix D), and general results for the transverse part of the quark-gluon vertex (Appendix E).

## II. PRELIMINARIES

We shall here establish some notation, and discuss the functions involved in the Ward-Slavnov-Taylor identity for the quark-gluon vertex.

### A. Notation

A graphical representation of the quark-gluon vertex is given in Fig. 1.[2] The momentum of the outgoing quark is denoted by $p_1$, $p_2$ is the momentum of the incoming quark, whereas $p_3$ and $\mu$ are the momentum and the Lorentz index of the gluon, respectively. All momenta are ingoing, $p_1+p_2+p_3=0$. The lowest-order quark-gluon vertex is

[2]To produce the figures, the AXODRAW package [22] was used.

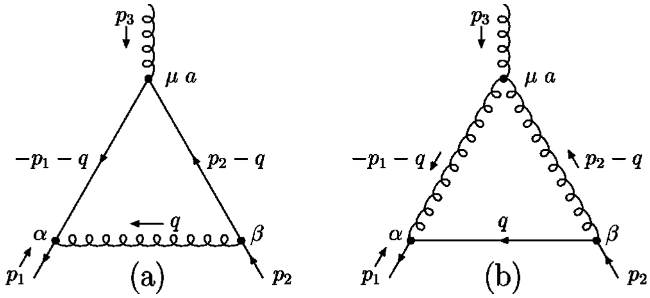


FIG. 2. The two one-loop diagrams.

$$g(T^a)_{ji}[\gamma_\mu]_{\beta\alpha}, \tag{2.1}$$

where $T^a$ are color matrices corresponding to the fundamental representation of the gauge group. As a rule, it will be implied that the SU($N$) group is considered, with $N$ being the number of colors (we can put $N=3$ in the end).

When one calculates radiative corrections to the quark-gluon vertex, other Dirac matrix structures arise, in addition to $\gamma_\mu$, Eq. (2.1). The total number of independent structures is 12 (see, e.g., in [4,11], and also in Appendix A of this paper). Extracting the over-all color structure, we can present the one-particle irreducible quark-gluon vertex as

$$\Gamma^a_\mu(p_1,p_2,p_3)=gT^a\Gamma_\mu(p_1,p_2,p_3), \tag{2.2}$$

where matrix notation in both color and Dirac matrices is understood.

At the one-loop level, we have two contributions to the quark-gluon vertex which are shown in Fig. 2. Their color factors are proportional to $(C_F-\frac{1}{2}C_A)$ and $C_A$, respectively, where $C_F$ and $C_A$ denote eigenvalues of the quadratic Casimir operator in the fundamental and adjoint representations, respectively. For the SU($N$) gauge group,

$$C_A=N, \quad C_F=\frac{N^2-1}{2N}. \tag{2.3}$$

The first, ''Abelian'' contribution is completely similar to the one-loop correction to the fermion-photon vertex in QED. The difference is only in the over-all factor. Formally, we can get the one-loop QED vertex from the considered QCD vertex by putting $C_A=0$. The second diagram in Fig. 2 is essentially non-Abelian and appears due to the self interaction of gluons.

If quarks are massive, it is clear that diagrams *a* and *b* in Fig. 2 involve ''triangle'' integrals with two and one massive lines, respectively:

$$J_2(\nu_1,\nu_2,\nu_3) \equiv \int \frac{\mathrm{d}^n q}{[(p_2-q)^2-m^2]^{\nu_1}[(p_1+q)^2-m^2]^{\nu_2}(q^2)^{\nu_3}}, \tag{2.4}$$

$$J_1(\nu_1,\nu_2,\nu_3)\equiv \int \frac{\mathrm{d}^n q}{[(p_2-q)^2]^{\nu_1}[(p_1+q)^2]^{\nu_2}[q^2-m^2]^{\nu_3}}, \tag{2.5}$$

where $n=4-2\varepsilon$ is the space-time dimension in the framework of dimensional regularization [13]. Understanding the subscript of $J$ as the number of massive propagators (cf. in [23]), we can extend this notation to the massless integrals $J(\nu_1,\nu_2,\nu_3)$ considered in [24], via

$$J(\nu_1,\nu_2,\nu_3)\leftrightarrow J_0(\nu_1,\nu_2,\nu_3). \tag{2.6}$$

Integrals with Lorentz indices can be reduced to the scalar ones using the standard techniques [25] (see also in [26,27]). Using the integration-by-parts technique [28] (see also in [29]), all integrals with higher integer powers of propagators can be algebraically reduced to integrals with the powers equal to one or zero (for details, see Appendix B).

As in [24], we shall extract from the expressions for one-loop integrals a factor

$$\eta\equiv\frac{\Gamma^2\left(\frac{n}{2}-1\right)}{\Gamma(n-3)}\Gamma\left(3-\frac{n}{2}\right)=\frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\Gamma(1+\varepsilon). \tag{2.7}$$

A natural extension of the notation used in [24] is to introduce the functions $\varphi_i$ $(i=1,2)$ such that

$$J_i(1,1,1)=\mathrm{i}\,\pi^{n/2}\eta\varphi_i(p_1^2,p_2^2,p_3^2;m). \tag{2.8}$$

In this sense, the function $\varphi$, Eq. (2.14) of [24], would correspond to $\varphi_0$, which also represents the massless limit of $\varphi_i$ $(i=1,2)$. Moreover, we can reserve the notations $J_3$ and $\varphi_3$ for the triangle integral with all three massive lines, which occurs in the three-gluon vertex (the massive quark loop contribution).

Then, for the two-point integrals we introduce functions

$$\kappa_i(p_l^2;m)\equiv\kappa_{i,l}, \tag{2.9}$$

where $p_l$ $(l=1,2,3)$ is the external momentum of the two-point function, whereas the subscript $(i=0,1,2)$ shows how many of the two internal propagators are massive. In this way, $\kappa_0$ [coinciding with the $\kappa$ defined in Eq. (2.15) of [24]] corresponds to the two-point function with massless lines,

$$J_1(1,1,0)=J_0(1,1,0)=\mathrm{i}\,\pi^{n/2}\eta\kappa_{0,3}, \tag{2.10}$$

and analogously for $J_0(0,1,1)$ and $J_0(1,0,1)$, with $\kappa_{0,1}$ and $\kappa_{0,2}$, respectively. Then, $\kappa_1$ corresponds to the two-point function with one massive and one massless line,

$$J_1(0,1,1)=J_2(0,1,1)=\mathrm{i}\,\pi^{n/2}\eta\kappa_{1,1}, \tag{2.11}$$

$$J_1(1,0,1)=J_2(1,0,1)=\mathrm{i}\,\pi^{n/2}\eta\kappa_{1,2}. \tag{2.12}$$

Finally, $\kappa_2$ corresponds to the two-point function with two massive lines,

$$J_2(1,1,0)=J_3(1,1,0)=\mathrm{i}\,\pi^{n/2}\eta\kappa_{2,3}, \tag{2.13}$$

and similarly for $J_3(1,0,1)$ and $J_3(0,1,1)$ (which would involve $\kappa_{2,2}$ and $\kappa_{2,1}$, respectively). The massless two-point functions introduced in Eq. (2.15) of [24] can be identified as $\kappa_i \leftrightarrow \kappa_{0,i}$.

The new feature, as compared to the massless case, is the appearance of the ''tadpole'' integral

$$J_1(0,0,1)=J_2(1,0,0)=J_2(0,1,0)$$
$$= \mathrm{i}\,\pi^{n/2}\frac{\Gamma(1+\varepsilon)}{\varepsilon(1-\varepsilon)}(m^2)^{1-\varepsilon}=\mathrm{i}\,\pi^{n/2}\eta m^2\tilde{\kappa}, \qquad (2.14)$$

with

$$\tilde{\kappa}\equiv\tilde{\kappa}(m^2)\equiv\frac{\Gamma(1-2\varepsilon)}{\Gamma^2(1-\varepsilon)}\frac{1}{\varepsilon(1-\varepsilon)}(m^2)^{-\varepsilon}. \qquad (2.15)$$

Let us recall that the massless tadpoles vanish in the framework of dimensional regularization [13],

$$J_1(1,0,0)=J_1(0,1,0)=J_2(0,0,1)=0. \qquad (2.16)$$

We shall also introduce some notation to keep track of the various orders in the perturbative expansion. For a quantity $X$ (e.g. any of the scalar functions contributing to the propagators or the vertices), we shall denote the zero-loop-order contribution as $X^{(0)}$, and the one-loop-order contribution as $X^{(1)}$, so that the perturbative expansion looks like

$$X=X^{(0)}+X^{(1)}+\cdots. \qquad (2.17)$$

### B. Two-point functions

The lowest-order gluon propagator is

$$-\mathrm{i}\delta^{a_1a_2}\frac{1}{p^2}\left(g_{\mu_1\mu_2}-\xi\frac{p_{\mu_1}p_{\mu_2}}{p^2}\right), \qquad (2.18)$$

where $\xi$ is the gauge parameter corresponding to a general covariant gauge, defined such that $\xi=0$ is the Feynman gauge. Here and henceforth, a causal prescription is understood, $1/p^2 \leftrightarrow 1/(p^2+\mathrm{i}0)$. For the present purposes, loop corrections to the gluon propagator [see, e.g., Eqs. (2.7) and (C.1) of [24]] are not required.

We shall denote the quark propagator as $S(p)$. The two scalar functions $\alpha(p^2)$ and $\beta(p^2)$ in the inverse quark propagator are defined via

$$\mathrm{i}S^{-1}(p)\equiv\alpha(p^2)\not{p}+\beta(p^2)I, \qquad (2.19)$$

where $\not{p}\equiv p^\mu\gamma_\mu$, whereas $I$ is the unit matrix in the space of Dirac matrices. At the lowest order, $\alpha^{(0)}=1$ and $\beta^{(0)}=-m$. For the next-to-leading order, one needs to calculate the one-loop diagram shown in Fig. 3, which yields (for $n$ near 4, see [30])

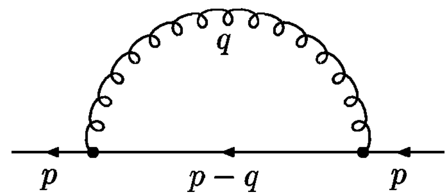


FIG. 3. Quark self-energy diagram.

$$\alpha^{(1)}(p^2)=\frac{g^2\eta}{(4\pi)^{n/2}}\frac{C_F}{2p^2}(n-2)(1-\xi)$$
$$\times[(p^2+m^2)\kappa_1(p^2;m)-m^2\tilde{\kappa}(m^2)], \qquad (2.20)$$

$$\beta^{(1)}(p^2)=-\frac{g^2\eta}{(4\pi)^{n/2}}C_F m(n-\xi)\kappa_1(p^2;m). \qquad (2.21)$$

The ghost propagator is

$$\tilde{D}^{a_1a_2}(p^2)=\mathrm{i}\,\delta^{a_1a_2}\frac{G(p^2)}{p^2}. \qquad (2.22)$$

The lowest-order result is $G^{(0)}=1$, whereas the one-loop contribution reads

$$G^{(1)}(p^2)=\frac{g^2\eta}{(4\pi)^{n/2}}\frac{C_A}{4}[2+(n-3)\xi]\kappa_0(p^2). \qquad (2.23)$$

Note that in the Fried-Yennie gauge [31] (see also in [32]), $\xi=-2$, $G^{(1)}(p^2)$ is finite as $n\to4$. Moreover, if one chooses $\xi=-2/(n-3)$ as the $n$-dimensional generalization of this gauge [33,34], then the right hand side (RHS) of Eq. (2.23) vanishes.

### C. Ward-Slavnov-Taylor identity

The WST identity [20] for the quark-gluon vertex $\Gamma_\mu(p_1,p_2,p_3)$ reads (see, e.g., in [2,35])

$$p_3^\mu\Gamma_\mu(p_1,p_2,p_3)=G(p_3^2)[S^{-1}(-p_1)H(p_1,p_2,p_3)$$
$$-\bar{H}(p_2,p_1,p_3)S^{-1}(p_2)], \qquad (2.24)$$

where $G(p^2)$ [see Eq. (2.22)] is the scalar function associated with the ghost propagator.

The function $H$ (and the ''conjugated'' function $\bar{H}$) involves the complete four-point quark-quark-ghost-ghost vertex, as shown in Fig. 4. To get the $H$ function, we need to ''join'' the out-quark and out-ghost lines in a non-standard vertex (denoted by a cross) and integrate over the resulting loop momentum. It should be noted that the complete quark-quark-ghost-ghost vertex involved in the WST identity can be decomposed into a connected and an unconnected piece, as shown in Fig. 5. Moreover, the connected part can be further split in terms of the proper (one-particle irreducible) vertices, see Fig. 6.

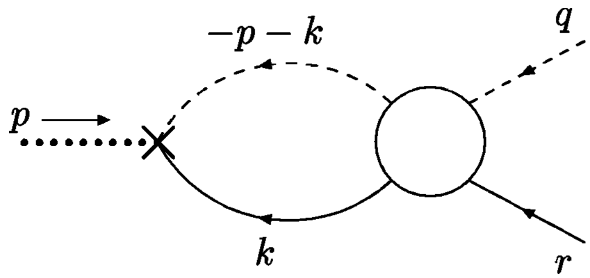


FIG. 4. Graphical representation of the $H$ function.

We note that the first diagram on the RHS of the equation shown in Fig. 6 (the diagram involving the proper four-point function) does not have a zero-loop (tree) contribution. Its perturbative expansion starts from the one-loop boxes, corresponding to the exchange by two gluons. Since the $H$ function involves an extra loop integration (see Fig. 4), this proper four-point function does not contribute to the one-loop-order $H$ function, $H^{(1)}$, which is shown in Fig. 7.

The $H$ function can be decomposed in terms of scalar functions (''form factors'') as

$$\begin{aligned}H(p_1,p_2,p_3)=&\chi_0(p_1^2,p_2^2,p_3^2)I+\chi_1(p_1^2,p_2^2,p_3^2)\not{p}_1\\&+\chi_2(p_1^2,p_2^2,p_3^2)\not{p}_2\\&+\chi_3(p_1^2,p_2^2,p_3^2)\sigma_{\mu\nu}p_1^\mu p_2^\nu,\end{aligned}\tag{2.25}$$

with

$$\sigma_{\mu\nu}=\frac{1}{2}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu).\tag{2.26}$$

The ''conjugated'' function $\overline{H}$ can be written in terms of the same scalar functions,

$$\begin{aligned}\overline{H}(p_2,p_1,p_3)=&\chi_0(p_2^2,p_1^2,p_3^2)I-\chi_2(p_2^2,p_1^2,p_3^2)\not{p}_1\\&-\chi_1(p_2^2,p_1^2,p_3^2)\not{p}_2\\&+\chi_3(p_2^2,p_1^2,p_3^2)\sigma_{\mu\nu}p_1^\mu p_2^\nu.\end{aligned}\tag{2.27}$$

At the lowest order, $\chi_0^{(0)}=1$ and $\chi_i^{(0)}=0$ $(i=1,2,3)$. The one-loop results for the $\chi_i$ functions (valid for arbitrary values of $n$ and $\xi$) are presented in Appendix D.

At the one-loop level, it is convenient to ''split'' the WST identity into two separate identities, corresponding to the contributions of the two diagrams shown in Fig. 2. To do this, we need to rewrite the one-loop contribution to the RHS of Eq. (2.24) in terms of color coefficients $(C_F-\frac{1}{2}C_A)$ and $C_A$, in analogy with the two contributions to the LHS. On

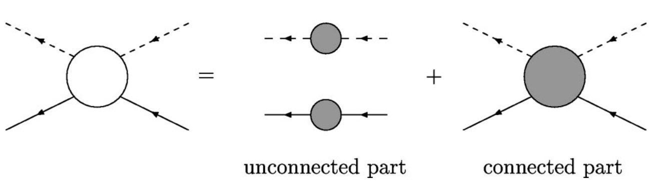


FIG. 5. Unconnected and connected parts of the quark-quark-ghost-ghost amplitude.

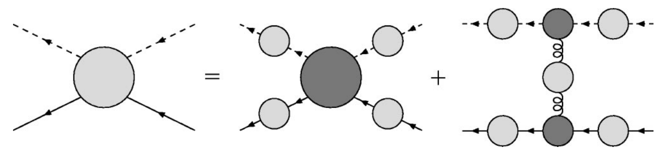

FIG. 6. Connected part of the quark-quark-ghost-ghost amplitude in terms of proper (dark) vertices.

the RHS, all one-loop contributions are proportional to $C_A$, except for the quark self-energies, which contain $C_F$. Therefore, all we need to do is to represent this $C_F$ as $(C_F-\frac{1}{2}C_A)+\frac{1}{2}C_A$. In this way, we get two separate WST identities for the contributions of diagrams $a$ and $b$ in Fig. 2,

$$\begin{aligned}p_3^\mu\Gamma_\mu^{(1a)}(p_1,p_2,p_3)=&\left(C_F-\frac{1}{2}C_A\right)C_F^{-1}[S^{-1}(-p_1)\\&-S^{-1}(p_2)]^{(1)},\end{aligned}\tag{2.28}$$

$$\begin{aligned}p_3^\mu&\Gamma_\mu^{(1b)}(p_1,p_2,p_3)\\&=[S^{-1}(-p_1)]^{(0)}H^{(1)}(p_1,p_2,p_3)\\&\quad-\overline{H}^{(1)}(p_2,p_1,p_3)[S^{-1}(p_2)]^{(0)}\\&\quad+\frac{1}{2}C_AC_F^{-1}[S^{-1}(-p_1)-S^{-1}(p_2)]^{(1)}H^{(0)}\\&\quad+2G^{(1)}(p_3^2)[S^{-1}(-p_1)-S^{-1}(p_2)]^{(0)}H^{(0)},\end{aligned}\tag{2.29}$$

where, following the convention of Eq. (2.17), the superscripts ''(0)'' and ''(1)'' correspond to the zero-loop and one-loop contributions, respectively.

The first identity, Eq. (2.28), has, up to a factor, the same form as the Abelian (QED) identity, also known as the Ward-Fradkin-Takahashi identity [36]. The second identity, Eq. (2.29), is the non-Abelian one.

### D. Decomposition of the quark-gluon vertex

Keeping in mind the WST identity (2.24), it is useful to split the quark-gluon vertex into a longitudinal part and a transverse part,

$$\Gamma_\mu(p_1,p_2,p_3)=\Gamma_\mu^{(\mathrm{L})}(p_1,p_2,p_3)+\Gamma_\mu^{(\mathrm{T})}(p_1,p_2,p_3),\tag{2.30}$$

where

$$p_3^\mu\Gamma_\mu^{(\mathrm{T})}(p_1,p_2,p_3)=0\tag{2.31}$$

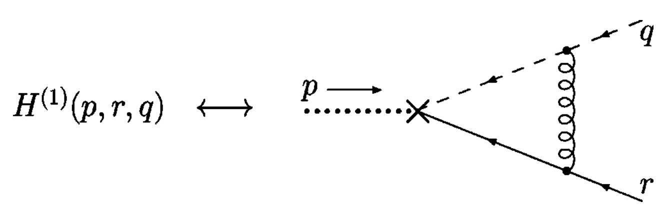


FIG. 7. The one-loop order function $H^{(1)}$.

and, therefore, $\Gamma_\mu^{(T)}$ does not contribute to the LHS of Eq. (2.24).

In general, we shall just extend the decomposition of the QED vertex suggested in [11] (see also in Ref. [12]) to the QCD case. The longitudinal part of the vertex can be represented as

$$\Gamma_\mu^{(L)}(p_1,p_2,p_3)=\sum_{i=1}^{4}\lambda_i(p_1^2,p_2^2,p_3^2)L_{i,\mu}(p_1,p_2), \tag{2.32}$$

with

$$\begin{aligned}
L_{1,\mu}&=\gamma_\mu,\\
L_{2,\mu}&=(\not{p}_1-\not{p}_2)(p_1-p_2)_\mu,\\
L_{3,\mu}&=(p_1-p_2)_\mu,\\
L_{4,\mu}&=\sigma_{\mu\nu}(p_1-p_2)^\nu.
\end{aligned} \tag{2.33}$$

Using Eq. (2.24), the functions $\lambda_i$ can be related to the functions $\alpha$, $\beta$, $G$ and $\chi_i$. For instance, in the ''Abelian'' case (i.e., when we consider only the first, QED-like diagram in Fig. 2) the functions $\lambda_i$ ($i=1,2,3,4$) would be equal to (up to a color factor)

$$\frac{1}{2}[\alpha(p_1^2)+\alpha(p_2^2)],\quad \frac{\alpha(p_1^2)-\alpha(p_2^2)}{2(p_1^2-p_2^2)},$$
$$-\frac{\beta(p_1^2)-\beta(p_2^2)}{p_1^2-p_2^2},\quad \text{and}\quad 0, \tag{2.34}$$

respectively (see in [11,12]).[3]

The transverse part of the vertex, which does not contribute to the WST identity (2.24), can be presented as [11]

$$\Gamma_\mu^{(T)}(p_1,p_2,p_3)=\sum_{i=1}^{8}\tau_i(p_1^2,p_2^2,p_3^2)T_{i,\mu}(p_1,p_2), \tag{2.35}$$

where the transverse tensors are the following:

$$\begin{aligned}
T_{1,\mu}&=p_{1\mu}(p_2p_3)-p_{2\mu}(p_1p_3), \qquad (2.36)\\
T_{2,\mu}&=-[p_{1\mu}(p_2p_3)-p_{2\mu}(p_1p_3)](\not{p}_1-\not{p}_2),\\
T_{3,\mu}&=p_3^2\gamma_\mu-p_{3\mu}\not{p}_3,\\
T_{4,\mu}&=[p_{1\mu}(p_2p_3)-p_{2\mu}(p_1p_3)]\sigma_{\nu\lambda}p_1^\nu p_2^\lambda,\\
T_{5,\mu}&=\sigma_{\mu\nu}p_3^\nu,\\
T_{6,\mu}&=\gamma_\mu(p_1^2-p_2^2)+(p_1-p_2)_\mu\not{p}_3,\\
T_{7,\mu}&=-\frac{1}{2}(p_1^2-p_2^2)[\gamma_\mu(\not{p}_1-\not{p}_2)-(p_1-p_2)_\mu]\\
&\quad-(p_1-p_2)_\mu\sigma_{\nu\lambda}p_1^\nu p_2^\lambda,\\
T_{8,\mu}&=-\gamma_\mu\sigma_{\nu\lambda}p_1^\nu p_2^\lambda+p_{1\mu}\not{p}_2-p_{2\mu}\not{p}_1.
\end{aligned}$$

The connection of $\lambda$'s and $\tau$'s with the naive decomposition basis is discussed in Appendix A.

Applying charge conjugation to the quark-gluon vertex, i.e., interchanging quark and anti-quark, the following relation is obtained (see, e.g., in Ref. [12]):

$$\mathrm{C}\,\Gamma_\mu(p_1,p_2,p_3)\mathrm{C}^{-1}=-\Gamma_\mu^{\mathrm{T}}(p_2,p_1,p_3). \tag{2.37}$$

Interchanging the quark momenta ($p_1\leftrightarrow p_2$) and using the fact that

$$\mathrm{C}\,\gamma_\mu\mathrm{C}^{-1}=-\gamma_\mu^{\mathrm{T}}, \tag{2.38}$$

one finds that all $L_\mu$ and $T_\mu$ are odd, except for $L_{4,\mu}$ and $T_{6,\mu}$:

$$\begin{aligned}
L_{i,\mu}(p_1,p_2)&=-L_{i,\mu}^{\mathrm{T}}(p_2,p_1),\quad i=1,2,3,\\
L_{4,\mu}(p_1,p_2)&=L_{4,\mu}^{\mathrm{T}}(p_2,p_1),\\
T_{i,\mu}(p_1,p_2)&=-T_{i,\mu}^{\mathrm{T}}(p_2,p_1),\quad i=1,2,3,4,5,7,8,\\
T_{6,\mu}(p_1,p_2)&=T_{6,\mu}^{\mathrm{T}}(p_2,p_1).
\end{aligned} \tag{2.39}$$

To satisfy Eq. (2.37), all $\lambda$'s and $\tau$'s must be symmetric under the interchange of $p_1^2$ and $p_2^2$, except $\lambda_4$ and $\tau_6$, which are odd:

$$\begin{aligned}
\lambda_i(p_1^2,p_2^2,p_3^2)&=\lambda_i(p_2^2,p_1^2,p_3^2),\quad i=1,2,3,\\
\lambda_4(p_1^2,p_2^2,p_3^2)&=-\lambda_4(p_2^2,p_1^2,p_3^2),\\
\tau_i(p_1^2,p_2^2,p_3^2)&=\tau_i(p_2^2,p_1^2,p_3^2),\quad i=1,2,3,4,5,7,8,\\
\tau_6(p_1^2,p_2^2,p_3^2)&=-\tau_6(p_2^2,p_1^2,p_3^2).
\end{aligned} \tag{2.40}$$

An important corollary of these relations is that in the case $p_1^2=p_2^2\equiv p^2$ the $\lambda_4$ and $\tau_6$ functions must vanish,

$$\lambda_4(p^2,p^2,p_3^2)=0,\quad \tau_6(p^2,p^2,p_3^2)=0. \tag{2.41}$$

Furthermore, in Ref. [12] a modification of the basis (2.35),(2.36) has been proposed, which has an advantage in dealing with kinematical singularities.[4] Namely, the transverse part is represented as

[3]Since in the QED case the longitudinal functions have such simple representations (2.34) in terms of $\alpha(p_{1,2}^2)$ and $\beta(p_{1,2}^2)$, there was no need in Refs. [11,12] to introduce a special notation for these functions. In the presence of the non-Abelian contribution, the situation becomes more complicated. This is why we have introduced longitudinal functions $\lambda_i$ in Eq. (2.32).

[4]In Ref. [12], the notation $\sigma_i$ and $S_i$ was used for what we call $\tilde{\tau}_i$ and $\tilde{T}_i$.

$$\Gamma_\mu^{(T)}(p_1,p_2,p_3)=\sum_{i=1}^{8}\tilde{\tau}_i(p_1^2,p_2^2,p_3^2)\tilde{T}_{i,\mu}(p_1,p_2), \tag{2.42}$$

where

$$\tilde{T}_{4,\mu}=\frac{2}{p_2^2-p_1^2}[2T_{4,\mu}-p_3^2T_{7,\mu}]$$
$$=p_3^2[p_{1\mu}-p_{2\mu}-\gamma_\mu(\not{p}_1-\not{p}_2)]-2(p_1+p_2)_\mu\sigma_{\nu\lambda}p_1^\nu p_2^\lambda,$$
$$\tilde{T}_{i,\mu}=T_{i,\mu}, \quad (i\neq 4), \tag{2.43}$$

with

$$\tilde{\tau}_4=\frac{1}{4}(p_2^2-p_1^2)\tau_4, \quad \tilde{\tau}_7=\tau_7+\frac{1}{2}p_3^2\tau_4. \tag{2.44}$$

Moreover, as we shall see below, in the on-shell limit the following modifications of $\lambda_2$ and $\lambda_3$ turn out to be useful:

$$\tilde{\lambda}_2\equiv\lambda_2+\frac{1}{2}p_3^2\tau_2, \quad \tilde{\lambda}_3\equiv\lambda_3-\frac{1}{2}p_3^2\tau_1. \tag{2.45}$$

## III. OFF-SHELL RESULTS

Before presenting results for the $\lambda_i$ and $\tau_i$ functions, let us introduce the following notation for the Gram determinants occurring in the denominators:

$$\mathcal{K}\equiv p_1^2p_2^2-(p_1p_2)^2, \tag{3.1}$$

$$\mathcal{M}_1\equiv(p_1^2-m^2)(p_2^2-m^2)+m^2p_3^2, \tag{3.2}$$

$$\mathcal{M}_2\equiv(p_1^2-m^2)(p_2^2-m^2)p_3^2+m^2(p_1^2-p_2^2)^2. \tag{3.3}$$

In fact, $\mathcal{K}$ is a symmetric function of $p_1^2$, $p_2^2$ and $p_3^2$. It can be rewritten as $-\frac{1}{4}\lambda(p_1^2,p_2^2,p_3^2)$, where $\lambda(x,y,z)$ is the Källén function [cf. Eq. (3.2) of [24]]. Note that $\mathcal{M}_1$ can also be represented as

$$\mathcal{M}_1=p_1^2p_2^2+2(p_1p_2)m^2+m^4=\mathcal{K}+[(p_1p_2)+m^2]^2.$$

To distinguish between the contributions of the two one-loop diagrams in Fig. 2, we shall use the letters *a* and *b*:

$$\lambda_i^{(1)}=\lambda_i^{(1a)}+\lambda_i^{(1b)}, \quad \tau_i^{(1)}=\tau_i^{(1a)}+\tau_i^{(1b)}. \tag{3.4}$$

For the calculation we used the algebraic programming system REDUCE [37]. Further technical details can be found in Appendices A and B.

### A. Results for the longitudinal part of the vertex

The general results for the longitudinal functions of the vertex are reasonably compact, even in a general covariant gauge and arbitrary dimension, and are given below for the two diagrams.

#### 1. Diagram a

$$\lambda_1^{(1a)}(p_1^2,p_2^2,p_3^2)=\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{(n-2)(1-\xi)}{4p_1^2p_2^2}[p_2^2(p_1^2+m^2)\kappa_{1,1}+p_1^2(p_2^2+m^2)\kappa_{1,2}-(p_1^2+p_2^2)m^2\tilde{\kappa}], \tag{3.5}$$

$$\lambda_2^{(1a)}(p_1^2,p_2^2,p_3^2)=\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{(n-2)(1-\xi)}{4p_1^2p_2^2(p_1^2-p_2^2)}[p_2^2(p_1^2+m^2)\kappa_{1,1}-p_1^2(p_2^2+m^2)\kappa_{1,2}+(p_1^2-p_2^2)m^2\tilde{\kappa}], \tag{3.6}$$

$$\lambda_3^{(1a)}(p_1^2,p_2^2,p_3^2)=\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{(n-\xi)m}{p_1^2-p_2^2}(\kappa_{1,1}-\kappa_{1,2}), \tag{3.7}$$

$$\lambda_4^{(1a)}(p_1^2,p_2^2,p_3^2)=0. \tag{3.8}$$

Note that diagram *a* does not contribute to $\lambda_4^{(1)}$.

#### 2. Diagram b

$$\lambda_1^{(1b)}(p_1^2,p_2^2,p_3^2)=-\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{1}{16\mathcal{K}}\Big\{(2-\xi)\mathcal{K}[2(p_1^2+p_2^2-2m^2)\varphi_1-n\kappa_{1,1}-n\kappa_{1,2}-4\kappa_{0,3}]+[2+(n-3)\xi]$$
$$\times(p_1^2-p_2^2)^2[(p_1p_2)\varphi_1+m^2\varphi_1+\kappa_{0,3}]+[2+(n-3)\xi](p_1^2-p_2^2)[p_2^2\kappa_{1,2}-p_1^2\kappa_{1,1}+(p_1p_2)(\kappa_{1,1}-\kappa_{1,2})]$$
$$-(n-2)(2-\xi)\mathcal{K}m^2\left[\frac{\kappa_{1,1}}{p_1^2}+\frac{\kappa_{1,2}}{p_2^2}-\frac{p_1^2+p_2^2}{p_1^2p_2^2}\tilde{\kappa}\right]\Big\}, \tag{3.9}$$

$$\lambda_2^{(1b)}(p_1^2,p_2^2,p_3^2)=\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{1}{16\mathcal{K}(p_1^2-p_2^2)}\Big\{[2+(n-3)\xi]p_3^2(p_1^2-p_2^2)[(p_1p_2)\varphi_1+m^2\varphi_1+\kappa_{0,3}]$$

$$+[2+(n-3)\xi]p_3^2[p_2^2\kappa_{1,2}-p_1^2\kappa_{1,1}+(p_1p_2)(\kappa_{1,1}-\kappa_{1,2})]+(n-2)(2-\xi)\mathcal{K}$$

$$\times\left[\frac{p_1^2+m^2}{p_1^2}\kappa_{1,1}-\frac{p_2^2+m^2}{p_2^2}\kappa_{1,2}+\frac{p_1^2-p_2^2}{p_1^2p_2^2}m^2\tilde{\kappa}\right]\Big\}, \tag{3.10}$$

$$\lambda_3^{(1b)}(p_1^2,p_2^2,p_3^2)=\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{m}{8\mathcal{M}_1}\Big\{(n-4)\xi p_3^2[(p_1p_2)+m^2]\varphi_1+(n-3)\xi[(p_2^2-p_3^2-m^2)\kappa_{1,1}+(p_1^2-p_3^2-m^2)\kappa_{1,2}+p_3^2\kappa_{0,3}]$$

$$-(n-2)\xi\frac{\mathcal{M}_1(p_1p_2)}{p_1^2-p_2^2}\left(\frac{\kappa_{1,1}}{p_1^2}-\frac{\kappa_{1,2}}{p_2^2}\right)+[4(n-1)-n\xi]\mathcal{M}_1\frac{\kappa_{1,1}-\kappa_{1,2}}{p_1^2-p_2^2}$$

$$+(n-2)\xi\left[1-\frac{2(p_1p_2)+m^2}{p_1^2p_2^2}(p_1p_2)\right]m^2\tilde{\kappa}\Big\}, \tag{3.11}$$

$$\lambda_4^{(1b)}(p_1^2,p_2^2,p_3^2)=\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{\xi m(p_1^2-p_2^2)}{16\mathcal{K}\mathcal{M}_1}\Big\{(n-3)p_3^2[(p_1p_2)+m^2]^2\varphi_1+(n-3)\mathcal{K}(\kappa_{1,1}+\kappa_{1,2})+\mathcal{K}p_3^2\varphi_1+(n-3)[(p_1p_2)$$

$$+m^2][(p_1p_3)\kappa_{1,1}+(p_2p_3)\kappa_{1,2}+p_3^2\kappa_{0,3}]+(n-2)\mathcal{K}\left[\frac{2(p_1p_2)+m^2}{p_1^2p_2^2}m^2\tilde{\kappa}+\frac{\mathcal{M}_1}{p_1^2-p_2^2}\left(\frac{\kappa_{1,1}}{p_1^2}-\frac{\kappa_{1,2}}{p_2^2}\right)\right]\Big\}. \tag{3.12}$$

We have checked that these expressions, together with the results for the two-point functions and for the $\chi_i$ functions (given in Appendix D), satisfy the WST identities (2.28), (2.29) at the one-loop level, for arbitrary values of $n$ and $\xi$.

Results for the $\lambda_i$ functions in the Feynman gauge ($\xi=0$) can be easily obtained from the expressions presented above. We just note that

$$\lambda_3^{(1b)}(p_1^2,p_2^2,p_3^2)|_{\xi=0}=\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{(n-1)m}{2(p_1^2-p_2^2)}(\kappa_{1,1}-\kappa_{1,2}), \tag{3.13}$$

$$\lambda_4^{(1b)}(p_1^2,p_2^2,p_3^2)|_{\xi=0}=0. \tag{3.14}$$

### B. Results for the transverse part of the vertex

In an arbitrary covariant gauge, the results for most of the $\tau_i$ functions are rather cumbersome. The expressions can be made more compact by introducing certain linear combinations of the $\kappa_{i,l}$ functions, namely

$$(p_1p_3)\kappa_{1,1}+(p_2p_3)\kappa_{1,2}+p_3^2\kappa_{i,3},\quad \kappa_{1,1}+\kappa_{1,2}-2\kappa_{i,3},$$

$$\kappa_{1,1}+\kappa_{1,2}-2\tilde{\kappa},\quad \kappa_{1,1}+\kappa_{1,2},\quad \kappa_{1,1}-\kappa_{1,2}, \tag{3.15}$$

where we should take, for the subscript of $\kappa_{i,3}$, $i=2$ for diagram $a$ and $i=0$ for diagram $b$. Note that the basis (3.15) is over-complete: we have got five combinations of the four $\kappa$'s. This degree of freedom has allowed us to present the results in a more compact form.

We have collected results in an arbitrary covariant gauge in Appendix E. Here we present results for the $\tau_i$ functions in the Feynman gauge.

***1. Transverse functions in Feynman gauge: Diagram a***

$$\tau_1^{(1a)}(p_1^2,p_2^2,p_3^2)|_{\xi=0}=\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{nm}{2\mathcal{K}}\left\{2[m^2-(p_1p_2)]\varphi_2+\kappa_{1,1}+\kappa_{1,2}-2\kappa_{2,3}+(p_1-p_2)^2\frac{\kappa_{1,1}-\kappa_{1,2}}{p_1^2-p_2^2}\right\}, \tag{3.16}$$

$$\begin{aligned}\tau_2^{(1a)}(p_1^2,p_2^2,p_3^2)|_{\xi=0}=&\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{1}{8\mathcal{K}}\left\{2(4m^2-p_3^2)\varphi_2-\frac{2(n-1)}{\mathcal{K}}[(p_1p_2)-m^2]^2p_3^2\varphi_2-\frac{2(n-1)}{\mathcal{K}}[(p_1p_2)-m^2]\right.\\&\times[(p_1p_3)\kappa_{1,1}+(p_2p_3)\kappa_{1,2}+p_3^2\kappa_{2,3}]-(n-2)\frac{m^2(p_1p_2)}{p_1^2p_2^2}(\kappa_{1,1}+\kappa_{1,2}-2\tilde{\kappa})-(n-4)(\kappa_{1,1}+\kappa_{1,2})\\&\left.+(n-2)\left[\frac{m^2(p_1p_2)(p_1^2+p_2^2)}{p_1^2p_2^2}-2m^2-(p_1-p_2)^2\right]\frac{\kappa_{1,1}-\kappa_{1,2}}{p_1^2-p_2^2}\right\},\end{aligned} \tag{3.17}$$

$$\begin{aligned}\tau_3^{(1a)}(p_1^2,p_2^2,p_3^2)|_{\xi=0}=&\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{1}{16\mathcal{K}}\left\{\frac{2(n-1)}{\mathcal{K}}p_3^2(p_1-p_2)^2[(p_1p_2)-m^2]^2\varphi_2+\frac{2(n-1)}{\mathcal{K}}(p_1-p_2)^2[(p_1p_2)-m^2]\right.\\&\times[(p_1p_3)\kappa_{1,1}+(p_2p_3)\kappa_{1,2}+p_3^2\kappa_{2,3}]-8(n-2)(p_1^2-m^2)(p_2^2-m^2)\varphi_2-2(p_1-p_2)^2\\&\times[4(n-1)m^2-p_3^2]\varphi_2+4(n-2)[(p_1p_2)-m^2](\kappa_{1,1}+\kappa_{1,2}-2\kappa_{2,3})+(n-4)(p_1-p_2)^2(\kappa_{1,1}+\kappa_{1,2})\\&+(n-2)\left[1-\frac{m^2(p_1p_2)}{p_1^2p_2^2}\right](p_1^2-p_2^2)(\kappa_{1,1}-\kappa_{1,2})+(n-2)m^2\left[\frac{(p_1p_2)(p_1^2+p_2^2)}{p_1^2p_2^2}-2\right]\\&\left.\times(\kappa_{1,1}+\kappa_{1,2}-2\tilde{\kappa})\right\},\end{aligned} \tag{3.18}$$

$$\tau_4^{(1a)}(p_1^2,p_2^2,p_3^2)|_{\xi=0}=0, \tag{3.19}$$

$$\tau_5^{(1a)}(p_1^2,p_2^2,p_3^2)|_{\xi=0}=-\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}(n-4)m\varphi_2, \tag{3.20}$$

$$\begin{aligned}\tau_6^{(1a)}(p_1^2,p_2^2,p_3^2)|_{\xi=0}=&\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{p_1^2-p_2^2}{16\mathcal{K}}\left\{2(4m^2-p_3^2)\varphi_2-\frac{2(n-1)}{\mathcal{K}}p_3^2[(p_1p_2)-m^2]^2\varphi_2-\frac{2(n-1)}{\mathcal{K}}[(p_1p_2)\right.\\&-m^2][(p_1p_3)\kappa_{1,1}+(p_2p_3)\kappa_{1,2}+p_3^2\kappa_{2,3}]+(n-2)\left[\frac{m^2(p_1p_2)(p_1^2+p_2^2)}{p_1^2p_2^2}-2m^2-(p_1-p_2)^2\right]\\&\left.\times\frac{\kappa_{1,1}-\kappa_{1,2}}{p_1^2-p_2^2}-(n-2)\frac{m^2(p_1p_2)}{p_1^2p_2^2}(\kappa_{1,1}+\kappa_{1,2}-2\tilde{\kappa})-(n-4)(\kappa_{1,1}+\kappa_{1,2})\right\},\end{aligned} \tag{3.21}$$

$$\tau_7^{(1a)}(p_1^2,p_2^2,p_3^2)|_{\xi=0}=0, \tag{3.22}$$

$$\tau_8^{(1a)}(p_1^2,p_2^2,p_3^2)|_{\xi=0}=\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{(6-n)}{2\mathcal{K}}\{p_3^2[(p_1p_2)-m^2]\varphi_2+(p_1p_3)\kappa_{1,1}+(p_2p_3)\kappa_{1,2}+p_3^2\kappa_{2,3}\}. \tag{3.23}$$

The function $\tau_4^{(1a)}$ becomes non-zero when $\xi \neq 0$ (see Appendix E). In fact, the results for $\tau_1^{(1a)}$ and $\tau_8^{(1a)}$ in an arbitrary covariant gauge can be obtained from Eqs. (3.16) and (3.23) by changing the overall factors $n$ and $(6-n)$ into $(n-\xi)$ and $[6-n+(n-4)\xi]$, respectively.

### *2. Transverse functions in Feynman gauge: Diagram b*

$$\tau_1^{(1b)}(p_1^2,p_2^2,p_3^2)|_{\xi=0} = -\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{(n-1)m}{4\mathcal{K}}\left\{2[(p_1p_2)+m^2]\varphi_1-\kappa_{1,1}-\kappa_{1,2}+2\kappa_{0,3}-(p_1-p_2)^2\frac{\kappa_{1,1}-\kappa_{1,2}}{p_1^2-p_2^2}\right\}, \tag{3.24}$$

$$\begin{aligned}\tau_2^{(1b)}(p_1^2,p_2^2,p_3^2)|_{\xi=0} = &\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{1}{16\mathcal{K}}\left\{\frac{2(n-1)}{\mathcal{K}}p_3^2[(p_1p_2)+m^2]^2\varphi_1+4(n-3)[(p_1p_2)+m^2]\varphi_1+2p_3^2\varphi_1\right.\\ &+\frac{2(n-1)}{\mathcal{K}}[(p_1p_2)+m^2][(p_1p_3)\kappa_{1,1}+(p_2p_3)\kappa_{1,2}+p_3^2\kappa_{0,3}]-2(n-3)(\kappa_{1,1}+\kappa_{1,2}-2\kappa_{0,3})\\ &+(n-4)(\kappa_{1,1}+\kappa_{1,2})-(n-2)\frac{m^2(p_1p_2)}{p_1^2p_2^2}(\kappa_{1,1}+\kappa_{1,2}-2\tilde{\kappa})-(n-4)(p_1-p_2)^2\frac{\kappa_{1,1}-\kappa_{1,2}}{p_1^2-p_2^2}\\ &\left.+(n-2)m^2\left[\frac{(p_1p_2)(p_1^2+p_2^2)}{p_1^2p_2^2}-2\right]\frac{\kappa_{1,1}-\kappa_{1,2}}{p_1^2-p_2^2}\right\},\end{aligned} \tag{3.25}$$

$$\begin{aligned}\tau_3^{(1b)}(p_1^2,p_2^2,p_3^2)|_{\xi=0} = &-\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{1}{32\mathcal{K}}\left\{\frac{2(n-1)}{\mathcal{K}}p_3^2(p_1-p_2)^2[(p_1p_2)+m^2]^2\varphi_1+\frac{2(n-1)}{\mathcal{K}}(p_1-p_2)^2[(p_1p_2)+m^2]\right.\\ &\times[(p_1p_3)\kappa_{1,1}+(p_2p_3)\kappa_{1,2}+p_3^2\kappa_{0,3}]-8(n-2)[(p_1p_2)+m^2]^2\varphi_1+(n-4)(p_1-p_2)^2(\kappa_{1,1}+\kappa_{1,2})\\ &+2p_3^2(p_1-p_2)^2\varphi_1+4(n-2)[(p_1p_2)+m^2](\kappa_{1,1}+\kappa_{1,2}-2\kappa_{0,3})+(n-2)\left[\frac{m^2(p_1p_2)}{p_1^2p_2^2}+1\right](p_1^2-p_2^2)\\ &\left.\times(\kappa_{1,1}-\kappa_{1,2})-m^2(n-2)\left[\frac{(p_1p_2)(p_1^2+p_2^2)}{p_1^2p_2^2}-2\right](\kappa_{1,1}+\kappa_{1,2}-2\tilde{\kappa})\right\},\end{aligned} \tag{3.26}$$

$$\tau_4^{(1b)}(p_1^2,p_2^2,p_3^2)|_{\xi=0} = 0, \tag{3.27}$$

$$\tau_5^{(1b)}(p_1^2,p_2^2,p_3^2)|_{\xi=0} = -\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{3}{2}m\varphi_1, \tag{3.28}$$

$$\begin{aligned}\tau_6^{(1b)}(p_1^2,p_2^2,p_3^2)|_{\xi=0} = &\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{p_1^2-p_2^2}{32\mathcal{K}}\left\{\frac{2(n-1)}{\mathcal{K}}p_3^2[(p_1p_2)+m^2]^2\varphi_1+4[(p_1p_2)+m^2]\varphi_1+\frac{2(n-1)}{\mathcal{K}}[(p_1p_2)+m^2]\right.\\ &\times[(p_1p_3)\kappa_{1,1}+(p_2p_3)\kappa_{1,2}+p_3^2\kappa_{0,3}]+2p_3^2\varphi_1+(n-4)(\kappa_{1,1}+\kappa_{1,2})-2(\kappa_{1,1}+\kappa_{1,2}-2\kappa_{0,3})\\ &-(n-2)\frac{m^2(p_1p_2)}{p_1^2p_2^2}(\kappa_{1,1}+\kappa_{1,2}-2\tilde{\kappa})+(n-4)(p_1-p_2)^2\frac{\kappa_{1,1}-\kappa_{1,2}}{p_1^2-p_2^2}\\ &\left.+(n-2)m^2\left[\frac{(p_1p_2)(p_1^2+p_2^2)}{p_1^2p_2^2}-2\right]\frac{\kappa_{1,1}-\kappa_{1,2}}{p_1^2-p_2^2}\right\},\end{aligned} \tag{3.29}$$

$$\tau_7^{(1b)}(p_1^2,p_2^2,p_3^2)|_{\xi=0} = 0, \tag{3.30}$$

$$\tau_8^{(1b)}(p_1^2,p_2^2,p_3^2)|_{\xi=0}=\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{3}{4\mathcal{K}}\{p_3^2[(p_1p_2)+m^2]\varphi_1+2\mathcal{K}\varphi_1+(p_1p_3)\kappa_{1,1}+(p_2p_3)\kappa_{1,2}+p_3^2\kappa_{0,3}\}. \tag{3.31}$$

In the limit of massless quarks, $m\to 0$, the above results simplify considerably. First of all, in this limit, $\tau_1^{(1)}=\tau_4^{(1)}=\tau_5^{(1)}=\tau_7^{(1)}=0$ (this holds also in an arbitrary covariant gauge). Furthermore, $\varphi_1\to\varphi_0$, $\varphi_2\to\varphi_0$, $\kappa_{1,i}\to\kappa_{0,i}$, $\kappa_{2,i}\to\kappa_{0,i}$ and $\widetilde{\kappa}\to 0$.

### C. Comparison with other papers

As we have already mentioned, the contributions of the first diagram should coincide, up to an overall factor, with the one-loop contribution to the fermion-photon vertex in QED. Formally, to get the QED case from our expressions, we can put $C_A\Rightarrow 0$, $C_F\Rightarrow 1$, $g\Rightarrow e$ (the absolute value of the charge of the electron). Therefore, our expressions also provide a one-loop correction to the QED vertex, in an arbitrary covariant gauge and dimension.

To get the expressions for the dimensionally regulated four-dimensional case, we put $n=4-2\varepsilon$ and expand our results (including the scalar integrals, see Appendix C) in $\varepsilon$, keeping singular ($1/\varepsilon$) and finite terms. In the Feynman gauge ($\xi=0$) and in four dimensions, we reproduce the well-known results from [11]. There are a few misprints in [11] which were pointed out in [12] (p. 1252). We agree with most of these corrections given in [12], except for the following: Eqs. (3.12), (3.14) and (A19) of [11] are correct, they should not be changed.

For an arbitrary value of $\xi$ and in four dimensions, one-loop contributions to the QED vertex have been calculated by Kızılersü, Reenders and Pennington [12]. We basically reproduce their results given by Eqs. (60)–(64), (67)–(74), and (87), except for $\tau_3$ and $\tau_6$, Eqs. (69) and (72). Specifically, to get agreement with our results, we had to change the following[5]: in the sixth line of $\tau_3$,

$$+\frac{1}{4}q^2m^2[p^2(p^2-m^2)L+k^2(k^2-m^2)L']$$

$$\to-\frac{1}{4}q^2m^2[p^2(p^2+m^2)L+k^2(k^2+m^2)L'],$$

and in the eleventh line of $\tau_6$,

$$-\frac{3m^2}{8\Delta^2}p^2q^2(p^4-k^4)[(m^2+k^2)L-(m^2+p^2)L']$$

$$\to-\frac{3m^2}{8\Delta^2}q^2(p^4-k^4)[p^2(m^2+k^2)L+k^2(m^2+p^2)L'].$$

For massless three-dimensional QED, results are given by Bashir, Kızılersü and Pennington in [14]. We reproduce their results for the longitudinal vertex, Eqs. (50) and (51), as well as the transverse parts,[6] Eqs. (55), (57)–(60).

Comparison with some other papers is given in Sec. IV, where the corresponding special limits are considered.

### D. Renormalization

In the limit $n\to 4$ ($\varepsilon\to 0$) the only function in the quark-gluon vertex which has an ultraviolet (UV) singularity *at one loop* is the $\lambda_1^{(1)}$ function. In an arbitrary covariant gauge, the UV-singular part of $\lambda_1^{(1)}$ reads

$$\lambda_1^{(1,\mathrm{UV})}=\frac{g^2\eta}{(4\pi)^{2-\varepsilon}}\left[(1-\xi)\left(C_F-\frac{1}{2}C_A\right)+\frac{3}{4}(2-\xi)C_A\right]\times\left(\frac{1}{\varepsilon}+\cdots\right)$$
$$=\frac{g^2\eta}{(4\pi)^{2-\varepsilon}}\left[(1-\xi)C_F+\frac{1}{4}(4-\xi)C_A\right]\left(\frac{1}{\varepsilon}+\cdots\right). \tag{3.32}$$

In the first line, the contributions from the first and the second diagram are explicitly separated. This result coincides with Eqs. (A.55)–(A.57) of Ref. [30].[7]

The divergent parts of the two-point functions are as follows:

$$\alpha^{(1,\mathrm{UV})}=\frac{g^2\eta}{(4\pi)^{2-\varepsilon}}C_F(1-\xi)\left(\frac{1}{\varepsilon}+\cdots\right), \tag{3.33}$$

$$\beta^{(1,\mathrm{UV})}=-\frac{g^2\eta}{(4\pi)^{2-\varepsilon}}mC_F(4-\xi)\left(\frac{1}{\varepsilon}+\cdots\right), \tag{3.34}$$

[5]There is also an obvious misprint in their Eq. (A14): the denominator $[(k-2)^2-m^2]$ should read $[(k-w)^2-m^2]$. We are grateful to the authors of [12] for confirming all mentioned misprints.

[6]However, we note that some factors of $\pi$ are inconsistent. The result for $J_0$ [Appendix, Eq. (5)] should read $-2/\sqrt{-k^2p^2q^2}$ (no $\pi$). The right-hand sides of Eqs. (40) and (55) should have an extra $\pi$. Finally, $K_0$ and $K^{(0)}$ are related like $J_0$ and $J^{(0)}$. We are grateful to the authors of [14] for confirming these misprints.

[7]The contribution of the first diagram is also in agreement with Eq. (11.65) of [38], whereas for the contribution of the second diagram there seems to be a misprint in Eq. (11.70) of [38]. Namely, in *their* notation, the factor $(1-\xi)$ in Eq. (11.70) should read $(1+\xi)$. Their $\xi$ and $\varepsilon$ correspond to our $(1-\xi)$ and $2\varepsilon$, respectively.

$$G^{(1,\mathrm{UV})}=\frac{g^2\eta}{(4\pi)^{2-\varepsilon}}\frac{C_A}{4}(2+\xi)\left(\frac{1}{\varepsilon}+\cdots\right). \tag{3.35}$$

The results for $\alpha$ and $\beta$ agree with Eq. (2.5.138) of [30] (our $\alpha$ and $\beta$ correspond to his $(-B)$ and $mA$, respectively), and with Eq. (11.55) of [38]. Among the $\chi_i$ functions, only $\chi_0$ has a UV-singularity at one loop,

$$\chi_0^{(1,\mathrm{UV})}=\frac{g^2\eta}{(4\pi)^{2-\varepsilon}}\frac{C_A}{2}(1-\xi)\left(\frac{1}{\varepsilon}+\cdots\right). \tag{3.36}$$

Using our results we have checked that all other functions $\lambda_i^{(1)}$ $(i\neq 1)$, $\chi_i^{(1)}$ $(i=1,2,3)$ and $\tau_i^{(1)}$ have no UV-singularities. This was one of the important checks on self-consistency of the calculation.

To renormalize the above expressions, we need to subtract the $1/\varepsilon$ poles, (possibly) getting some constant $R$ instead, depending on the renormalization scheme:

$$\left(\frac{1}{\varepsilon}+\cdots\right)\to(R+\cdots). \tag{3.37}$$

In the $\overline{\mathrm{MS}}$ scheme $R=0$, because [see Eq. (2.7)]

$$\eta=e^{-\gamma\varepsilon}[1+\mathcal{O}(\varepsilon^2)], \tag{3.38}$$

so that $e^{-\gamma\varepsilon}$ and $(4\pi)^\varepsilon$ are absorbed by the $\overline{\mathrm{MS}}$ re-definition of the coupling constant $g^2$.

This procedure can be also re-formulated in the language of the renormalization $Z$-factors, by analogy with Sec. VIII of [39].

## IV. SOME SPECIAL CASES

A few limits are of special interest:

the symmetric case, when $p_1^2=p_2^2=p_3^2$;

the on-shell limit $p_1^2=p_2^2=m^2$ (with or without the assumption that the vertex function is being sandwiched between Dirac spinors);

the zero-momentum limit, when the gluon momentum vanishes $(p_3=0)$.

Since in all these cases we can put $p_1^2=p_2^2\equiv p^2$, we start by considering this as the first step towards all these limits.

In the case $p_1^2=p_2^2\equiv p^2$ some of the tensor structures (2.33) and (2.36) in the quark-gluon vertex become linearly dependent, namely: $L_{2,\mu}$ and $T_{2,\mu}$, $L_{3,\mu}$ and $T_{1,\mu}$, $T_{4,\mu}$ and $T_{7,\mu}$. Moreover, according to Eq. (2.41), $\lambda_4$ and $\tau_6$ vanish. Therefore, the quark-gluon vertex in this limit can be written as [cf. Eqs. (2.44) and (2.45)]

$$\Gamma_\mu|_{p_1^2=p_2^2\equiv p^2}=L_{1,\mu}\lambda_1+L_{2,\mu}\tilde{\lambda}_2+L_{3,\mu}\tilde{\lambda}_3+T_{3,\mu}\tau_3+T_{5,\mu}\tau_5 +T_{7,\mu}\tilde{\tau}_7+T_{8,\mu}\tau_8. \tag{4.1}$$

In fact, only the $L_{1,\mu}$ contribution remains "non-transverse" in this limit.

### A. Symmetric case

In Ref. [4] (see also in [5]) the "symmetric" limit of the quark-gluon vertex, $p_1^2=p_2^2=p_3^2\equiv p^2=-\mu^2$, has been examined (for the case of massless quarks, $m=0$). The decomposition of the quark-gluon vertex is given in Eq. (2.34) of [4]. It basically corresponds to the naive decomposition presented in Eq. (A1) of this paper. In Ref. [4] explicit results are given only for two scalar functions (out of twelve):

$$\Gamma_1\leftrightarrow h_1 \quad\text{and}\quad \Gamma_{12}\leftrightarrow -h_{12}. \tag{4.2}$$

Taking into account Eq. (2.41), in terms of $\lambda$'s and $\tau$'s we get (see also in Appendix A):

$$\Gamma_1=\lambda_1+p_3^2\tau_3+(p_1p_2)\tau_8, \quad \Gamma_{12}=-\tau_8. \tag{4.3}$$

Putting $p_1^2=p_2^2=p_3^2\equiv p^2$ [implying $(p_1p_2)=-\frac{1}{2}p^2$] and $m=0$, we arrive at the following one-loop results:

$$\Gamma_1^{(1)}=-\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\left\{\frac{1}{6}[4(n-4)-n\xi]p^2\varphi_0 +(3-n+\xi)\kappa_{0,3}\right\}+\frac{g^2\eta C_A}{(4\pi)^{n/2}}\left\{\frac{1}{8}[12-2(2n-5)\xi +(n-4)\xi^2]\kappa_{0,3}-\frac{1}{6}(8-n\xi)p^2\varphi_0\right\}, \tag{4.4}$$

$$\Gamma_{12}^{(1)}=-\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{1}{3}[(n-6)-(n-4)\xi]\varphi_0 -\frac{g^2\eta C_A}{(4\pi)^{n/2}}\left[1+\frac{1}{6}(n-7)\xi-\frac{1}{24}(n-6)\xi^2\right]\varphi_0. \tag{4.5}$$

Taking into account that the constant $R(1)$ used in [4,5] can be identified as

$$R(1)=p^2\varphi_0(p^2,p^2,p^2)|_{n=4}=\frac{4}{\sqrt{3}}\mathrm{Cl}_2\left(\frac{\pi}{3}\right), \tag{4.6}$$

and expanding in $\varepsilon$ (keeping the divergent and finite in $\varepsilon$ terms), we arrive at the same result as given in Eqs. (2.36) of [4].[8] In Eq. (4.6) $\mathrm{Cl}_2(\theta)=\mathrm{Im}[\mathrm{Li}_2(e^{i\theta})]$ is the Clausen function.

For the case of massive quarks $(m\neq 0)$, a similar "symmetric" limit has been considered in Ref. [7], where the renormalization factor $Z_{1F}$ was calculated at the one-loop order in the MOM scheme. For their calculation, only one of the scalar functions (namely, the one accompanying the $\gamma_\mu$

[8]There is a misprint in Eq. (2.35) of [4]: $\Gamma_{12}$ should read $p^2\Gamma_{12}=-\mu^2\Gamma_{12}$. Note that the $\varepsilon$ used in [4] has different sign, as compared to ours.

matrix) was needed, $\Gamma_1 \leftrightarrow h_1$, which is related to $\lambda$'s and $\tau$'s via Eq. (4.3). Putting $p_1^2=p_2^2=p_3^2\equiv p^2=-\mu^2$ in our expressions, we arrive at the same result[9] as given in Eqs. (13)–(14) of [7]. In particular, we have taken into account that the functions $H$ and $M$ introduced in Eq. (15) of [7] are related to our functions $\varphi_i$ through[10]

$$H=-p^2\varphi_1(p^2,p^2,p^2)|_{n=4}, \quad M=-p^2\varphi_2(p^2,p^2,p^2)|_{n=4}. \tag{4.7}$$

More details on these functions can be found in Appendix C.

### B. On-shell quarks

The tensor structure of Eq. (4.1) does not change when we put $p^2=m^2$. Let us introduce the notation

$$\varphi_{1,2}^{\rm os}\equiv\varphi_{1,2}(m^2,m^2,p_3^2) \tag{4.8}$$

for the $\varphi_i$ functions in the on-shell limit. According to Eq. (C31), in this limit the function $\varphi_2$ (corresponding to diagram $a$) reduces to a two-point function and a tadpole,

$$\varphi_2^{\rm os}=\frac{1}{(n-4)(4m^2-p_3^2)}[2(n-3)\kappa_{2,3}-(n-2)\tilde{\kappa}]. \tag{4.9}$$

This is the reason why $\varphi_2$ does not appear in the on-shell results for diagram $a$.

The results for the relevant longitudinal and transverse functions in the limit $p_1^2=p_2^2=m^2$ are presented below.

#### 1. Diagram a

$$\lambda_1^{(1a)}(m^2,m^2,p_3^2)=\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{(n-2)(1-\xi)}{2(n-3)}\tilde{\kappa}, \tag{4.10}$$

$$\tilde{\lambda}_2^{(1a)}(m^2,m^2,p_3^2)=-\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{1}{(n-3)(n-4)(4m^2-p_3^2)^2}\{(n-3)[(2-\xi)(4m^2-p_3^2)+(n-5)\xi p_3^2]\kappa_{2,3}$$
$$-(n-2)[4m^2-p_3^2+2(n-5)\xi m^2]\tilde{\kappa}\}, \tag{4.11}$$

$$\tilde{\lambda}_3^{(1a)}(m^2,m^2,p_3^2)=\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{(n-\xi)m}{(n-3)(n-4)(4m^2-p_3^2)}[(n-2)\tilde{\kappa}-2(n-3)\kappa_{2,3}], \tag{4.12}$$

$$\tau_3^{(1a)}(m^2,m^2,p_3^2)=-\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{1}{2(n-3)(n-4)p_3^2(4m^2-p_3^2)}\{[(n-2)(n-3)-(n-4)\xi]p_3^2[2(n-3)\kappa_{2,3}$$
$$-(n-2)\tilde{\kappa}]+(n-3)(n-4)(4m^2-p_3^2)[2\kappa_{2,3}-(n-2)\tilde{\kappa}]\}, \tag{4.13}$$

$$\tau_5^{(1a)}(m^2,m^2,p_3^2)=-\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{1}{4(n-3)m(4m^2-p_3^2)}\{(n-2)\xi(4m^2-p_3^2)\tilde{\kappa}+4(n-3-\xi)m^2$$
$$\times[2(n-3)\kappa_{2,3}-(n-2)\tilde{\kappa}]\}, \tag{4.14}$$

[9]Their $a$ corresponds to our $(1-\xi)$, whereas their $\lambda$ denotes the ratio $m^2/\mu^2=-m^2/p^2$. Note that there is a misprint in the *journal* version of the result for the last contribution in Eq. (14), $C_b$: in the very last term, $\ln[1/(1+\lambda)]$ should read $\ln[\lambda/(1+\lambda)]$. In the *preprint* version of [7] this equation is correct.

[10]We also note a misprint in the large-mass $(1/\lambda)$ expansion of the $M$ function presented in Appendix C of Ref. [7]: the last available term, $-2672/(11025\lambda^5)$, should read $-1523/(6300\lambda^5)$. We are grateful to O.V. Tarasov for confirming this misprint.

$$\widetilde{\tau}_7^{(1a)}(m^2,m^2,p_3^2) = -\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{\xi}{2(n-3)(n-4)m(4m^2-p_3^2)^2}\{4(n-5)m^2[2(n-3)\kappa_{2,3}-(n-2)\widetilde{\kappa}]$$
$$-(n-2)(n-4)(4m^2-p_3^2)\widetilde{\kappa}\}, \quad (4.15)$$

$$\tau_8^{(1a)}(m^2,m^2,p_3^2) = -\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{6-n+(n-4)\xi}{(n-3)(n-4)(4m^2-p_3^2)}[2(n-3)\kappa_{2,3}-(n-2)\widetilde{\kappa}]. \quad (4.16)$$

### 2. Diagram b

$$\lambda_1^{(1b)}(m^2,m^2,p_3^2) = \frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{2-\xi}{4(n-3)}[(n-3)\kappa_{0,3}+(n-2)\widetilde{\kappa}], \quad (4.17)$$

$$\widetilde{\lambda}_2^{(1b)}(m^2,m^2,p_3^2) = \frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{1}{16(n-3)(4m^2-p_3^2)^2}\{8(n-1)(n-3)(2-\xi)m^2p_3^2\varphi_1^{\text{os}}-2(n-3)^2\xi^2p_3^2[2(n-3)m^2$$
$$+p_3^2]\varphi_1^{\text{os}}-2(n-3)^2\xi^2[4(n-4)m^2+3p_3^2]\kappa_{0,3}+4(n-3)(2-\xi)[4(n-2)m^2+p_3^2]\kappa_{0,3}$$
$$+(n-1)(n-2)(n-3)\xi^2p_3^2\widetilde{\kappa}-2(n-2)(2-\xi)[8m^2+(n-3)p_3^2]\widetilde{\kappa}\}, \quad (4.18)$$

$$\widetilde{\lambda}_3^{(1b)}(m^2,m^2,p_3^2) = \frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{1}{32(n-3)m(4m^2-p_3^2)}\{2(n-3)(n-4)\xi^2p_3^2(4m^2-p_3^2)\varphi_1^{\text{os}}+[4(n-1)+2(n-4)\xi$$
$$-(n-2)(n-3)\xi^2][4(n-3)m^2p_3^2\varphi_1^{\text{os}}+8(n-3)m^2\kappa_{0,3}-(n-2)p_3^2\widetilde{\kappa}]+4(n-3)^2\xi^2(4m^2-p_3^2)\kappa_{0,3}$$
$$-4(n-1)(n-2)(4m^2-p_3^2)\widetilde{\kappa}\}, \quad (4.19)$$

$$\tau_3^{(1b)}(m^2,m^2,p_3^2) = \frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{1}{16(n-3)p_3^2(4m^2-p_3^2)}\{2(n-3)\xi(4-\xi)(4m^2-p_3^2)[p_3^2\varphi_1^{\text{os}}-(n-4)\kappa_{0,3}]$$
$$-[2(2-\xi)-(n-3)\xi^2]p_3^2[4(n-3)m^2\varphi_1^{\text{os}}+2(n-3)\kappa_{0,3}-(n-2)\widetilde{\kappa}]\}, \quad (4.20)$$

$$\tau_5^{(1b)}(m^2,m^2,p_3^2) = \frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{1}{8(n-3)m(4m^2-p_3^2)}\{(n-2)(n-4)\xi(4m^2-p_3^2)\widetilde{\kappa}+\xi[2+(n-3)\xi]m^2$$
$$\times[4(n-3)m^2\varphi_1^{\text{os}}+2(n-3)\kappa_{0,3}-(n-2)\widetilde{\kappa}]-2(n-3)[6+2(n-5)\xi+\xi^2]m^2(4m^2-p_3^2)\varphi_1^{\text{os}}\},$$
$$(4.21)$$

$$\widetilde{\tau}_7^{(1b)}(m^2,m^2,p_3^2) = -\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{\xi}{16(n-3)m(4m^2-p_3^2)^2}\{2(n-3)(4m^2-p_3^2)[4m^2+2(n-5)\xi m^2-(n-4)\xi p_3^2]\varphi_1^{\text{os}}$$
$$+(n-1)[2+(n-3)\xi][4(n-3)m^2p_3^2\varphi_1^{\text{os}}+8(n-3)m^2\kappa_{0,3}-(n-2)p_3^2\widetilde{\kappa}]$$
$$-4(n-3)^2\xi(4m^2-p_3^2)\kappa_{0,3}-(n-2)[6+(n-3)\xi](4m^2-p_3^2)\widetilde{\kappa}\}, \quad (4.22)$$

$$\tau_8^{(1b)}(m^2,m^2,p_3^2)=-\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{1}{8(n-3)(4m^2-p_3^2)}\{2(n-3)\xi(4-\xi)(4m^2-p_3^2)\varphi_1^{\rm os}-[12+2(2n-7)\xi-(n-3)\xi^2]$$
$$\times[4(n-3)m^2\varphi_1^{\rm os}+2(n-3)\kappa_{0,3}-(n-2)\widetilde{\kappa}]\}. \quad (4.23)$$

When $p_3^2\to 0$, the only problem which may arise in Eqs. (4.10)–(4.23) concerns the functions $\tau_3^{(1a)}$ and $\tau_3^{(1b)}$ containing $p_3^2$ in the denominator. However, the coefficients of $(p_3^2)^0$ in the numerators of $\tau_3^{(1a)}$ and $\tau_3^{(1b)}$ are proportional to $[2\kappa_{2,3}-(n-2)\widetilde{\kappa}]$ and $\kappa_{0,3}$, respectively. If we take into account that $\kappa_{2,3}|_{p_3^2=0}=\frac{1}{2}(n-2)\widetilde{\kappa}$ and $\kappa_{0,3}|_{p_3^2=0}=0$ (massless tadpole), we see that the limit $p_3^2\to 0$ is regular for $\tau_3^{(1)}$.

### 3. Dirac and Pauli form factors

Let us consider again the on-shell limit $p_1^2=p_2^2=m^2$, without putting any further conditions on $p_3$. If we recall that the ''physical'' quark-gluon vertex should be sandwiched between physical states obeying the Dirac equation,

$$\overline{u(}-p_1)\Gamma_\mu u(p_2),$$

$$\text{with}\quad \overline{u(}-p_1)\not{p}_1=-m\overline{u(}-p_1),\quad \not{p}_2u(p_2)=mu(p_2), \quad (4.24)$$

then (using the above Dirac conditions) we arrive at the standard decomposition

$$\begin{aligned}\overline{u(}-p_1)\Gamma_\mu u(p_2)&=F_1(p_3^2)\overline{u(}-p_1)\gamma_\mu u(p_2)\\&\quad-\frac{1}{2m}F_2(p_3^2)\overline{u(}-p_1)\sigma_{\mu\nu}p_3^\nu u(p_2)\\&=[F_1(p_3^2)+F_2(p_3^2)]\overline{u(}-p_1)\gamma_\mu u(p_2)\\&\quad+\frac{1}{2m}F_2(p_3^2)\overline{u(}-p_1)(p_1-p_2)_\mu u(p_2),\end{aligned} \quad (4.25)$$

where $F_1(p_3^2)$ and $F_2(p_3^2)$ are often associated with the Dirac and Pauli form factors, respectively. In terms of the (modified) λ and $\tau$ functions we get

$$F_1+F_2=\lambda_1+p_3^2\tau_3-2m\tau_5+\frac{1}{2}(p_3^2-4m^2)\tau_8, \quad (4.26)$$

$$\frac{1}{2m}F_2=-2m\widetilde{\lambda}_2+\widetilde{\lambda}_3-\tau_5+\frac{1}{2}p_3^2\widetilde{\tau}_7-m\tau_8. \quad (4.27)$$

Using our results we get

$$\begin{aligned}(F_1+F_2)^{(1a)}=&-\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\\&\times\frac{1}{(n-3)(n-4)(4m^2-p_3^2)}\\&\times\{(n-3)[2(n-3)p_3^2-(n^2-9n+22)\\&\times(4m^2-p_3^2)]\kappa_{2,3}-2(n-2)\\&\times[2(n-5)m^2+p_3^2]\widetilde{\kappa}\},\end{aligned} \quad (4.28)$$

$$\begin{aligned}(F_1+F_2)^{(1b)}=&-\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{1}{8(n-3)(4m^2-p_3^2)}\\&\times\{4(n-3)p_3^2(2m^2\varphi_1^{\rm os}+\kappa_{0,3})+(n-3)\\&\times[8+2(4n-13)\xi-(n-4)\xi^2]\\&\times(4m^2-p_3^2)\kappa_{0,3}-8(n-2)(5m^2-p_3^2)\widetilde{\kappa}\},\end{aligned} \quad (4.29)$$

$$\begin{aligned}\frac{1}{2m}F_2^{(1a)}=&\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{(n-5)m}{(n-3)(4m^2-p_3^2)}\\&\times[2(n-3)\kappa_{2,3}-(n-2)\widetilde{\kappa}],\end{aligned} \quad (4.30)$$

$$\begin{aligned}\frac{1}{2m}F_2^{(1b)}=&-\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{m}{2(n-3)(4m^2-p_3^2)^2}\\&\times\{(n-3)p_3^2[12m^2+(n-4)p_3^2]\varphi_1^{\rm os}+2(n\\&-3)[8m^2+(n-3)p_3^2]\kappa_{0,3}+(n-2)[(n-6)\\&\times(4m^2-p_3^2)-(n-1)p_3^2]\widetilde{\kappa}\}.\end{aligned} \quad (4.31)$$

The results for the (QED-like) diagram $a$ agree with Eqs. (5.69) and (5.70) of [40].

In the massless limit, $m\to 0$, and in Feynman gauge, $\xi=0$, our results can be compared with those of [8], where renormalized results for the quark self-energy, $\Sigma$, and the quark-gluon vertex functions, $\Lambda_1^\mu$ and $\Lambda_2^\mu$ (their notation), are collected in Tables B.I and B.II, respectively. The vertex functions are given for off-shell gluon $(p_1^2=p_2^2=0,p_3^2\neq 0)$ and the out-going quark being off-shell $(p_1^2\neq 0,p_2^2=p_3^2=0)$. We confirm these results.

### C. Zero-momentum limit

Let us consider the zero-momentum limit $p_3=0$, with $p_2=-p_1\equiv p$ being off shell. We can proceed in two steps. The first step, putting $p_1^2=p_2^2=p^2$ (without putting $p^2=m^2$), has been already done in Eq. (4.1). As the second step, we should put $p_3^2=0$ (which implies $p_3=0$). In this limit, the quark-gluon vertex becomes

$$\Gamma_\mu|_{p_2=-p_1\equiv p,p_3=0}=L_{1,\mu}\lambda_1+L_{2,\mu}\tilde{\lambda}_2+L_{3,\mu}\tilde{\lambda}_3 \\ =\gamma_\mu\lambda_1+4p_\mu \not{p}\tilde{\lambda}_2-2p_\mu\tilde{\lambda}_3. \tag{4.32}$$

One can check that the corresponding scalar functions are regular.

The substitutions for the relevant integrals are[11]

$$J_2(1,1,1)|_{p_2=-p_1\equiv p,p_3=0}=-\frac{1}{p^2-m^2}[(n-3)J_2(0,1,1) \\ -J_2(1,1,0)], \tag{4.33}$$

$$J_2(1,1,0)|_{p_2=-p_1\equiv p,p_3=0}=\frac{n-2}{2m^2}J_2(0,1,0), \tag{4.34}$$

$$J_1(1,1,1)|_{p_2=-p_1\equiv p,p_3=0}=\frac{1}{(p^2-m^2)^2}\{(n-2)J_1(0,0,1) \\ -(n-3)(p^2+m^2)J_1(0,1,1)\}, \tag{4.35}$$

$$J_1(1,1,0)|_{p_2=-p_1\equiv p,p_3=0}=0. \tag{4.36}$$

The scalar functions from Eq. (4.32) in this limit are

$$\lambda_1^{(1a)}(p^2,p^2,0)=\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{(n-2)(1-\xi)}{2p^2} \\ \times[(p^2+m^2)\kappa_1(p^2)-m^2\tilde{\kappa}], \tag{4.37}$$

$$\tilde{\lambda}_2^{(1a)}(p^2,p^2,0)=-\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{(n-2)(1-\xi)}{8(p^2)^2(p^2-m^2)} \\ \times\{[(p^2-m^2)(p^2+3m^2)-(n-3) \\ \times(p^2+m^2)^2]\kappa_1(p^2)+[2(n-2)p^2 \\ -n(p^2-m^2)]m^2\tilde{\kappa}\}, \tag{4.38}$$

$$\tilde{\lambda}_3^{(1a)}(p^2,p^2,0)=\frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\frac{(n-\xi)m}{2p^2(p^2-m^2)} \\ \times\{[(n-4)p^2+(n-2)m^2]\kappa_1(p^2) \\ -(n-2)m^2\tilde{\kappa}\}, \tag{4.39}$$

$$\lambda_1^{(1b)}(p^2,p^2,0)=\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{2-\xi}{8p^2(p^2-m^2)} \\ \times\{(n-2)(m^2-3p^2)m^2\tilde{\kappa} \\ +[4(n-3)m^2p^2+(n-2)(p^2-m^2) \\ \times(3p^2+m^2)]\kappa_1(p^2)\}, \tag{4.40}$$

$$\tilde{\lambda}_2^{(1b)}(p^2,p^2,0)=-\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{(n-2)(2-\xi)}{32(p^2)^2(p^2-m^2)} \\ \times\{[(n-4)p^2+nm^2]m^2\tilde{\kappa} \\ -[4(n-3)m^4+(n-4)(p^2-m^2) \\ \times(p^2+3m^2)]\kappa_1(p^2)\}, \tag{4.41}$$

$$\tilde{\lambda}_3^{(1b)}(p^2,p^2,0)=\frac{g^2\eta C_A}{(4\pi)^{n/2}}\frac{m(n-1)}{4p^2(p^2-m^2)} \\ \times\{[(n-2)m^2+(n-4)p^2]\kappa_1(p^2) \\ -(n-2)m^2\tilde{\kappa}\}. \tag{4.42}$$

If we now consider the on-shell case, i.e., put $p^2=m^2$, we need to be careful, since the above expressions contain $(p^2-m^2)$ in their denominators. Using two terms of the expansion of $\kappa_1(p^2)$ in $\delta\equiv(m^2-p^2)/m^2$,

$$\kappa_1(p^2)=\frac{n-2}{2(n-3)}\tilde{\kappa}(m^2)\left[1+\frac{1}{2}\delta+\mathcal{O}(\delta^2)\right], \tag{4.43}$$

we see that the pole at $p^2=m^2(\delta=0)$ is canceled. In this way, we arrive at

$$\lambda_1^{(1)}(m^2,m^2,0)=\frac{g^2\eta}{(4\pi)^{n/2}}\frac{(n-2)\tilde{\kappa}}{2(n-3)} \\ \times\left[\left(C_F-\frac{1}{2}C_A\right)(1-\xi)+\frac{1}{2}C_A(2-\xi)\right], \tag{4.44}$$

$$\tilde{\lambda}_2^{(1)}(m^2,m^2,0)=-\frac{g^2\eta}{(4\pi)^{n/2}}\frac{(n-2)\tilde{\kappa}}{4(n-3)m^2} \\ \times\left[\left(C_F-\frac{1}{2}C_A\right)(1-\xi)+\frac{1}{4}C_A(2-\xi)\right], \tag{4.45}$$

[11] In general, one should also make sure that the next term of the expansion in $p_3^2$ does not contribute, which may happen when $p_3^2$ appears in the denominators. In this calculation, we did not need such $p_3^2$ terms.

$$\widetilde{\lambda}_3^{(1)}(m^2,m^2,0)=-\frac{g^2\eta}{(4\pi)^{n/2}}\frac{(n-2)\widetilde{\kappa}}{4(n-3)m}\times\left[\left(C_F-\frac{1}{2}C_A\right)(n-\xi)+\frac{1}{2}C_A(n-1)\right]. \tag{4.46}$$

These results coincide with those obtained by first taking the on-shell limit $p_1^2=p_2^2=m^2$ [see Eqs. (4.10)–(4.12) and (4.17)–(4.19) above] and then putting $p_3=0$. If we assume that the vertex (4.32) is sandwiched between physical states (4.24), we see that both structures containing $p_\mu$ can be transformed into $\gamma_\mu$, so that the ''effective'' vertex becomes

$$\gamma_\mu[\lambda_1(m^2,m^2,0)+4m^2\widetilde{\lambda}_2(m^2,m^2,0)-2m\widetilde{\lambda}_3(m^2,m^2,0)]. \tag{4.47}$$

Substituting our results (4.44)–(4.46) we obtain for the one-loop contribution to Eq. (4.47)

$$\gamma_\mu\frac{g^2\eta}{(4\pi)^{n/2}}\frac{(n-1)(n-2)}{2(n-3)}C_F\widetilde{\kappa}=\gamma_\mu\frac{g^2}{(4\pi)^{2-\varepsilon}}\Gamma(\varepsilon)\frac{(3-2\varepsilon)}{(1-2\varepsilon)}C_F(m^2)^{-\varepsilon}, \tag{4.48}$$

where we have taken into account the definitions of $\eta$ (2.7) and $\widetilde{\kappa}$ (2.15). We see that the result (4.48) is gauge independent, and it does not contain $C_A$. For the QED case, it coincides with Eq. (22) of [15].

In the massless ($m=0$) case, we can compare our results for the zero-momentum limit with those given in [9]. The definition of the zero-gluon-momentum vertex is given in their Eq. (A4) (the lower equation). It is proportional to (their $q\leftrightarrow$our $p$)

$$\left\{[1+\Gamma_3(q^2)]\gamma_\mu+\Gamma_4(q^2)\gamma^\nu\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\right\}. \tag{4.49}$$

If we consider our results in the off-shell zero-momentum limit, we see that on putting $m=0$ the function $\widetilde{\lambda}_3$ vanishes. The remaining two functions can be mapped into $\Gamma_{3,4}$ as

$$\lambda_1^{(\text{ren})}\leftrightarrow\Gamma_3+\Gamma_4,\quad -4p^2\widetilde{\lambda}_2\leftrightarrow\Gamma_4, \tag{4.50}$$

or, vice versa,

$$\Gamma_3\leftrightarrow\lambda_1^{(\text{ren})}+4p^2\widetilde{\lambda}_2,\quad \Gamma_4\leftrightarrow-4p^2\widetilde{\lambda}_2, \tag{4.51}$$

where the superscript ''(ren)'' means ''renormalized.''

In the massless case, taking into account that the massless tadpole function $\widetilde{\kappa}|_{m=0}$ vanishes, we get

$$\lambda_1^{(1)}=\frac{g^2\eta}{(4\pi)^{n/2}}\frac{n-2}{2}\kappa_0(p^2)\times\left[\left(C_F-\frac{1}{2}C_A\right)(1-\xi)+\frac{3}{4}C_A(2-\xi)\right], \tag{4.52}$$

$$4p^2\widetilde{\lambda}_2^{(1)}=\frac{g^2\eta}{(4\pi)^{n/2}}\frac{(n-2)(n-4)}{2}\kappa_0(p^2)\times\left[\left(C_F-\frac{1}{2}C_A\right)(1-\xi)+\frac{1}{4}C_A(2-\xi)\right]. \tag{4.53}$$

Putting $p^2=\mu^2$ (as in [9]) means that the logarithms $\ln(p^2/\mu^2)$ [in our case, $\ln(p^2)$, since we imply $\mu=1$] vanish. Therefore, we should substitute $\kappa_0\Rightarrow 1/\varepsilon+2$ (see, for example, Eq. (2.15) of [24]). Omitting the $\mathcal{O}(\varepsilon)$ terms we see that

$$\lambda_1^{(1)}\Rightarrow\frac{g^2}{16\pi^2}\left[\left(C_F-\frac{1}{2}C_A\right)(1-\xi)+\frac{3}{4}C_A(2-\xi)\right]\left(\frac{1}{\varepsilon}+1\right), \tag{4.54}$$

$$4p^2\widetilde{\lambda}_2^{(1)}\Rightarrow-\frac{g^2}{16\pi^2}\left[2\left(C_F-\frac{1}{2}C_A\right)(1-\xi)+\frac{1}{2}C_A(2-\xi)\right], \tag{4.55}$$

where we have explicitly separated the contributions of diagrams *a* and *b*. To perform the renormalization (in the $\overline{\text{MS}}$-scheme), we need just to subtract the $1/\varepsilon$ term, i.e.,

$$\lambda_1^{(1,\text{ren})}\Rightarrow\frac{g^2}{16\pi^2}\left[\left(C_F-\frac{1}{2}C_A\right)(1-\xi)+\frac{3}{4}C_A(2-\xi)\right]. \tag{4.56}$$

Now we can compare our Eqs. (4.55)–(4.56) with one-loop results for $\Gamma_{3,4}$ presented in Eq. (A12) of [9]. Taking into account that their $\xi$ corresponds to our $(1-\xi)$, and also that their $\alpha=g^2/(4\pi)$, we see that we are in agreement with [9]. Moreover, their result for the quark self-energy function $A(q^2)$ [see Eq. (A9) of [9]] is in agreement with our $\alpha(p^2)$. Note that $\beta(p^2)=0$ in the massless case.

## V. CONCLUSIONS

In this article we have given results for the one-loop quark-gluon vertex in an arbitrary covariant gauge and in an arbitrary space-time dimension. The calculation was carried out with massive quarks.

To calculate the quark-gluon vertex, we have decomposed it into longitudinal (2.32) and transverse (2.35) parts, $\Gamma_\mu^{(L)}$ and $\Gamma_\mu^{(T)}$ (like the decomposition in QED [11]). Altogether 12 scalar functions (four $\lambda$'s and eight $\tau$'s) are needed to define the quark-gluon vertex. We found that the function $\lambda_4$, the coefficient of $\sigma_{\mu\nu}(p_1-p_2)^\nu$, which is absent in QED, does not vanish in QCD [see Eq. (3.12)] and contributes to the non-Abelian part of the Ward-Slavnov-Taylor identity (2.29). The general results for the longitudinal parts ($\lambda$'s) are given in Eqs. (3.5)–(3.12) (arbitrary gauge), and results for the transverse parts ($\tau$'s) are given in Eqs. (3.16)–(3.31) (Feynman gauge) and in Appendix E (arbitrary covariant gauge). Using recurrence relations (see Appendix B), all results have been expressed in terms of integrals with powers of propagators equal to zero or one. Only two non-trivial scalar functions are required, $\varphi_i(p_1^2,p_2^2,p_3^2)$, where $i=1$ or 2 counts the number of *massive* propagators involved (see Section II A and Appendix C).

Starting from the general off-shell expressions ($p_1^2 \neq m^2, p_2^2 \neq m^2, p_3^2 \neq 0$) in an arbitrary space-time dimension, $n$, for the longitudinal and transverse parts of the vertex, we have derived results for the on-shell limit ($p_1^2=p_2^2=m^2$) which are also valid for an arbitrary $n$ (Section IV). Note that if we started from the off-shell results expanded around $n=4$ (similar to the results of [11,12] for the QED case), we would get infrared divergences from the on-shell-divergent logarithms. Keeping the arbitrary space-time dimension, we see that the corresponding infrared divergences result in extra poles in $\varepsilon=(4-n)/2$.

Various special cases of the general results were compared with those of Refs. [4,7–9,11,12,14,15] (for details, see Secs. III and IV).

At the one-loop level, the Ward-Slavnov-Taylor identity for the quark-gluon vertex can be split in an Abelian and a non-Abelian part, Eqs. (2.28) and (2.29). The Abelian part is similar to the Ward-Fradkin-Takahashi identity in QED [36], whereas the non-Abelian part has a nontrivial contribution involving the quark-quark-ghost-ghost vertex (2.25), which can be described by scalar functions $\chi_i$ ($i=0,\ldots,3$). One-loop results for these functions are presented in Appendix D. Using the results for these $\chi$ functions, and those for quark and ghost self energies, we have checked that our results for the longitudinal parts of the vertex ($\lambda_1$, $\lambda_2$, $\lambda_3$, and $\lambda_4$) satisfy the WST identity for arbitrary $n$ and $\xi$, as they should.

In principle, some techniques which can be used for the calculation of the two-loop off-shell quark-gluon vertex, at least in the $m=0$ case, are already available [41,27], although the problem of higher powers of irreducible numerators is still difficult for algorithmization. For special limits, the calculation is very similar to the three-gluon vertex, which was calculated at two loops in [39] (the zero-momentum limit) and in [42] (the on-shell case).[12]

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[12]The results of this paper are available in REDUCE format at http://wwwthep.physik.uni-mainz.de/Publications/progdata/mzth9963/

## APPENDIX A: DECOMPOSITION OF THE VERTEX

The general quark-gluon vertex can be expressed as (see [35,11,12])

$$\begin{aligned}\Gamma_\mu(p_1,p_2,p_3) = {} & \gamma_\mu h_1 + p_{2\mu} h_2 + p_{1\mu} h_3 + \gamma_\mu \not{p}_2 h_4 + \gamma_\mu \not{p}_1 h_5 \\ & + p_{2\mu}\not{p}_2 h_6 + p_{2\mu}\not{p}_1 h_7 + p_{1\mu}\not{p}_2 h_8 \\ & + p_{1\mu}\not{p}_1 h_9 + p_{2\mu}\not{p}_1\not{p}_2 h_{10} + p_{1\mu}\not{p}_1\not{p}_2 h_{11} \\ & + \gamma_\mu \not{p}_1\not{p}_2 h_{12}, \end{aligned} \tag{A1}$$

where $h_i \equiv h_i(p_1^2,p_2^2,p_3^2)$.

The longitudinal and transverse functions of Eqs. (2.32) and (2.35) are related to this naive basis as follows:

$$\begin{aligned}\lambda_1 &= h_1 - \frac{1}{2}(p_2p_3)(h_6+h_7) - \frac{1}{2}(p_1p_3)(h_8+h_9) \\ &\quad + \frac{1}{2}(p_3^2 - 2p_2^2)h_{12}, \\ \lambda_2 &= \frac{1}{2(p_1^2-p_2^2)}[(p_2p_3)(h_6-h_7) \\ &\quad + (p_1p_3)(h_8-h_9) - p_3^2 h_{12}], \\ \lambda_3 &= -\frac{1}{p_1^2-p_2^2}\{(p_2p_3)[h_2+h_4+(p_1p_2)h_{10}] \\ &\quad + (p_1p_3)[h_3+h_5+(p_1p_2)h_{11}]\}, \\ \lambda_4 &= \frac{1}{2}[-h_4+h_5+(p_2p_3)h_{10}+(p_1p_3)h_{11}], \end{aligned} \tag{A2}$$

$$\tau_1 = \frac{1}{p_1^2-p_2^2}[h_2+h_3+h_4+h_5+(p_1p_2)(h_{10}+h_{11})],$$

$$\tau_2 = \frac{1}{2(p_1^2-p_2^2)}(h_6-h_7+h_8-h_9+2h_{12}),$$

$$\tau_3 = -\frac{1}{4}(h_6+h_7+h_8+h_9),$$

$$\tau_4 = \frac{1}{p_1^2-p_2^2}(h_{10}+h_{11}),$$

$$\tau_5 = -\frac{1}{2}(h_4+h_5),$$

$$\tau_6=\frac{1}{4}(h_6+h_7-h_8-h_9-2h_{12}),$$

$$\tau_7=\frac{1}{p_1^2-p_2^2}[(p_2p_3)h_{10}+(p_1p_3)h_{11}],$$

$$\tau_8=-h_{12}, \tag{A3}$$

where $\lambda_i\equiv\lambda_i(p_1^2,p_2^2,p_3^2)$, $\tau_i\equiv\tau_i(p_1^2,p_2^2,p_3^2)$.

Inverting these relations, one finds

$$h_1=\lambda_1+\tau_3p_3^2+\tau_6(p_1^2-p_2^2)+\tau_8(p_1p_2),$$

$$h_2=-\lambda_3+\lambda_4-\tau_1(p_1p_3)+\tau_4(p_1p_2)(p_1p_3)+\tau_5+\tau_7\left(p_2^2-\frac{1}{2}p_3^2\right),$$

$$h_3=\lambda_3-\lambda_4+[\tau_1-\tau_4(p_1p_2)](p_2p_3)+\tau_5-\tau_7\left(p_2^2-\frac{1}{2}p_3^2\right),$$

$$h_4=-\lambda_4-\tau_5+\frac{1}{2}\tau_7(p_1^2-p_2^2),$$

$$h_5=\lambda_4-\tau_5-\frac{1}{2}\tau_7(p_1^2-p_2^2),$$

$$h_6=\lambda_2-\tau_2(p_1p_3)-\tau_3+\tau_6,$$

$$h_7=-\lambda_2+\tau_2(p_1p_3)-\tau_3+\tau_6-\tau_8,$$

$$h_8=-\lambda_2+\tau_2(p_2p_3)-\tau_3-\tau_6+\tau_8,$$

$$h_9=\lambda_2-\tau_2(p_2p_3)-\tau_3-\tau_6,$$

$$h_{10}=-\tau_4(p_1p_3)+\tau_7,$$

$$h_{11}=\tau_4(p_2p_3)-\tau_7,$$

$$h_{12}=-\tau_8. \tag{A4}$$

## APPENDIX B: RECURRENCE RELATIONS FOR SCALAR INTEGRALS

To calculate scalar integrals with higher (integer) powers of denominators, a recurrence procedure based on the integration-by-parts technique [28] has been used. For the three-point integrals, one can follow an approach described in [29] (see, in particular, Sec. IV of [29] where the massive case is discussed). Using the recurrence relations, we can reduce all integrals to the master integrals $J_i(1,1,1)$ ($i=1,2$) and a few two-point integrals. All basic integrals are discussed in Appendix C.

### 1. Two-point integrals

All two-point integrals occurring in this paper can be associated with certain special cases of three-point integrals $J_2$ and $J_1$ (defined in Eqs. (2.4) and (2.5), respectively), when one of the indices $\nu_i$ vanishes. The masses of internal particles $(m_1,m_2)$ can be equal to $(m,m)$, $(m,0)$, $(0,m)$, or $(0,0)$.

For two-point integrals with arbitrary masses,

$$J^{(2)}(\nu_1,\nu_2|m_1,m_2)\equiv\int \mathrm{d}^nq\frac{1}{[(p-q)^2-m_1^2]^{\nu_1}(q^2-m_2^2)^{\nu_2}}, \tag{B1}$$

such a procedure has been described in detail in Appendix A of [46]. For positive $\nu_i$, it is enough to apply

$$\begin{aligned}J^{(2)}(\nu_1+1,\nu_2|m_1,m_2)&=\frac{1}{\nu_1\Delta}\{[(n-\nu_1-2\nu_2)(p^2-m_1^2)\\&\quad+(n-3\nu_1)m_2^2]J^{(2)}(\nu_1,\nu_2|m_1,m_2)\\&\quad-\nu_1(p^2-m_1^2-m_2^2)J^{(2)}(\nu_1+1,\nu_2-1|m_1,m_2)\\&\quad-2\nu_2m_2^2J^{(2)}(\nu_1-1,\nu_2+1|m_1,m_2)\},\end{aligned} \tag{B2}$$

together with a similar equation with $(\nu_1,m_1)\leftrightarrow(\nu_2,m_2)$. In Eq. (B2),

$$\Delta\equiv\Delta(m_1^2,m_2^2,p^2)=4m_1^2m_2^2-(p^2-m_1^2-m_2^2)^2. \tag{B3}$$

In our calculation, $\Delta$ may be equal to $p^2(4m^2-p^2)$, $-(p^2-m^2)^2$ or $-(p^2)^2$.

Note that the sum of indices $\nu_i$ for any of the integrals on the RHS of Eq. (B2) is less (by one) than such a sum for the integral on the LHS. Therefore, $J^{(2)}$ with any (positive) integers $\nu_1$ and $\nu_2$ can be reduced to $J^{(2)}$ with $\nu_1=\nu_2=1$ and tadpole integrals.

The reduction of tadpole integrals is trivial:

$$J_1(0,0,N)=\frac{1}{(N-1)!}\left(1-\frac{n}{2}\right)_{N-1}(-m^2)^{1-N}J_1(0,0,1) \tag{B4}$$

[and similarly for $J_2(N,0,0)=J_2(0,N,0)$], where $(a)_j\equiv\Gamma(a+j)/\Gamma(a)$ is the Pochhammer symbol.

### 2. Three-point integrals $J_2$

For the integrals $J_2$ with positive $\nu_i$ (with at least one of them being greater than one), the following solution of recurrence relations can be used:

$$J_2(\nu_1,\nu_2,\nu_3+1)=\frac{1}{2\nu_3\mathcal{M}_2}\{[(\nu_1-\nu_3)p_1^2p_3^2+(\nu_2-\nu_3)p_2^2p_3^2+2(n-\nu_1-\nu_2-2\nu_3)p_3^2(p_1p_2)+2(\nu_2-\nu_1)m^2(p_1^2-p_2^2)$$
$$-(2n-\nu_1-\nu_2-6\nu_3)m^2p_3^2]J_2(\nu_1,\nu_2,\nu_3)-p_3^2(p_3^2-4m^2)$$
$$\times[\nu_1J_2(\nu_1+1,\nu_2,\nu_3-1)+\nu_2J_2(\nu_1,\nu_2+1,\nu_3-1)]+[(p_2^2-m^2)p_3^2+2(p_1^2-p_2^2)m^2]$$
$$\times[\nu_1J_2(\nu_1+1,\nu_2-1,\nu_3)+\nu_3J_2(\nu_1,\nu_2-1,\nu_3+1)]+[(p_1^2-m^2)p_3^2-2(p_1^2-p_2^2)m^2]$$
$$\times[\nu_2J_2(\nu_1-1,\nu_2+1,\nu_3)+\nu_3J_2(\nu_1-1,\nu_2,\nu_3+1)]\}, \tag{B5}$$

$$J_2(\nu_1+1,\nu_2,\nu_3)=\frac{1}{2\nu_1\mathcal{M}_2}\{2(n-\nu_1-\nu_2-2\nu_3)[p_1^2(p_2p_3)-m^2(p_1p_3)]J_2(\nu_1,\nu_2,\nu_3)+(p_1^2-m^2)$$
$$\times[(\nu_3-\nu_1)p_1^2+(\nu_2-\nu_3)p_2^2+(\nu_1-\nu_2)m^2]J_2(\nu_1,\nu_2,\nu_3)+[p_3^2(p_1^2-m^2)-2m^2(p_1^2-p_2^2)]$$
$$\times[\nu_1J_2(\nu_1+1,\nu_2,\nu_3-1)+\nu_2J_2(\nu_1,\nu_2+1,\nu_3-1)]+(p_1^2-m^2)(p_2^2-m^2)$$
$$\times[\nu_1J_2(\nu_1+1,\nu_2-1,\nu_3)+\nu_3J_2(\nu_1,\nu_2-1,\nu_3+1)]-(p_1^2-m^2)^2$$
$$\times[\nu_2J_2(\nu_1-1,\nu_2+1,\nu_3)+\nu_3J_2(\nu_1-1,\nu_2,\nu_3+1)]\}, \tag{B6}$$

and also an equation for $J_2(\nu_1,\nu_2+1,\nu_3)$ which can be obtained from Eq. (B6) via $(\nu_1,p_1^2)\leftrightarrow(\nu_2,p_2^2)$. The quantity $\mathcal{M}_2$ is defined in Eq. (3.3).

In fact, for our calculation we needed only the $\nu_1=\nu_2=\nu_3=1$ case of Eq. (B5),

$$J_2(1,1,2)=\frac{1}{2\mathcal{M}_2}\{2(n-4)p_3^2[(p_1p_2)-m^2]J_2(1,1,1)-2p_3^2(p_3^2-4m^2)J_2(2,1,0)+[(p_2^2-m^2)p_3^2+2(p_1^2-p_2^2)m^2]$$
$$\times[J_2(2,0,1)+J_2(1,0,2)]+[(p_1^2-m^2)p_3^2-2(p_1^2-p_2^2)m^2][J_2(0,2,1)+J_2(0,1,2)]\}, \tag{B7}$$

where we have taken into account the obvious symmetry $J_2(\nu_1,\nu_2,0)=J_2(\nu_2,\nu_1,0)$. We note a remarkable fact that the coefficient of $J_2(1,1,1)$ is proportional to $(n-4)$. Since, in the off-shell case, $J_2(1,1,1)$ is finite in four dimensions, this means that we do not get any nontrivial function in finite parts of any triangle integrals with higher $\nu_i$. In fact, this property is valid for arbitrary internal masses [43] (a special case has been discussed in [44]).

### 3. Three-point integrals $J_1$

For the integrals $J_1$ with positive $\nu_i$ (with at least one of them being greater than one), the following solution of recurrence relations can be used:

$$J_1(\nu_1,\nu_2,\nu_3+1)=\frac{1}{2\nu_3\mathcal{M}_1}\{[2(n-\nu_1-\nu_2-\nu_3)(p_1p_2)+\nu_1p_1^2+\nu_2p_2^2-\nu_3p_3^2+(2n-3\nu_1-3\nu_2-2\nu_3)m^2]J_1(\nu_1,\nu_2,\nu_3)$$
$$+(p_2^2-m^2)[\nu_1J_1(\nu_1+1,\nu_2-1,\nu_3)+\nu_3J_1(\nu_1,\nu_2-1,\nu_3+1)]+(p_1^2-m^2)[\nu_2J_1(\nu_1-1,\nu_2+1,\nu_3)$$
$$+\nu_3J_1(\nu_1-1,\nu_2,\nu_3+1)]-p_3^2[\nu_1J_1(\nu_1+1,\nu_2,\nu_3-1)+\nu_2J_1(\nu_1,\nu_2+1,\nu_3-1)]\}, \tag{B8}$$

$$J_1(\nu_1+1,\nu_2,\nu_3)=\frac{1}{2\nu_1p_3^2\mathcal{M}_1}\{2(n-\nu_1-\nu_2-\nu_3)[p_1^2(p_2p_3)+m^2(p_1p_3)]J_1(\nu_1,\nu_2,\nu_3)$$
$$-(\nu_1p_1^2-\nu_2p_2^2-\nu_3p_3^2)p_1^2J_1(\nu_1,\nu_2,\nu_3)+m^2[2\nu_1p_1^2+2\nu_2(p_1p_2)+(\nu_2-\nu_3)p_3^2-(\nu_1-\nu_2)m^2]$$
$$\times J_1(\nu_1,\nu_2,\nu_3)+(\mathcal{M}_1+p_3^2m^2)[\nu_1J_1(\nu_1+1,\nu_2-1,\nu_3)+\nu_3J_1(\nu_1,\nu_2-1,\nu_3+1)]+p_3^2(p_1^2-m^2)$$
$$\times[\nu_1J_1(\nu_1+1,\nu_2,\nu_3-1)+\nu_2J_1(\nu_1,\nu_2+1,\nu_3-1)]-(p_1^2-m^2)^2[\nu_2J_1(\nu_1-1,\nu_2+1,\nu_3)$$
$$+\nu_3J_1(\nu_1-1,\nu_2,\nu_3+1)]\}, \tag{B9}$$

and also an equation for $J_1(\nu_1,\nu_2+1,\nu_3)$ which can be obtained from Eq. (B9) via $(\nu_1,p_1^2)\leftrightarrow(\nu_2,p_2^2)$. The quantity $\mathcal{M}_1$ is defined in Eq. (3.2).

The ''highest'' integral $J_1$ which occurred in our calculation was $J_1(2,2,1)$. Using Eq. (B9) and the $(\nu_1,p_1^2)\leftrightarrow(\nu_2,p_2^2)$ symmetry, we get

$$J_1(2,2,1)=\frac{1}{2p_3^2\mathcal{M}_1}\Bigg\{(\mathcal{M}_1+m^2p_3^2)[J_1(2,1,1)+J_1(1,2,1)+J_1(1,1,2)]+(n-5)[p_2^2(p_1p_3)+m^2(p_2p_3)]J_1(2,1,1)+(n-5)$$

$$\times[p_1^2(p_2p_3)+m^2(p_1p_3)]J_1(1,2,1)-(p_1^2-m^2)^2\left[J_1(0,3,1)+\frac{1}{2}J_1(0,2,2)\right]-(p_2^2-m^2)^2\left[J_1(3,0,1)+\frac{1}{2}J_1(2,0,2)\right]$$

$$+p_3^2(p_1^2+p_2^2-2m^2)\left[J_1(3,1,0)+\frac{1}{2}J_1(2,2,0)\right]\Bigg\}. \tag{B10}$$

For the integrals $J_1(1,1,2)$, $J_1(2,1,1)$ and $J_1(1,2,1)$, direct application of Eqs. (B8), (B9) yields

$$J_1(1,1,2)=\frac{1}{2\mathcal{M}_1}\{2(n-4)[(p_1p_2)+m^2]J_1(1,1,1)-2p_3^2J_1(2,1,0)+(p_2^2-m^2)[J_1(2,0,1)+J_1(1,0,2)]$$

$$+(p_1^2-m^2)[J_1(0,2,1)+J_1(0,1,2)]\}, \tag{B11}$$

$$J_1(2,1,1)=\frac{1}{2\mathcal{M}_1p_3^2}\{2(n-4)[p_1^2(p_2p_3)+m^2(p_1p_3)]J_1(1,1,1)+2p_3^2(p_1^2-m^2)J_1(2,1,0)$$

$$+(\mathcal{M}_1+m^2p_3^2)[J_1(2,0,1)+J_1(1,0,2)]-(p_1^2-m^2)^2[J_1(0,2,1)+J_1(0,1,2)]\}, \tag{B12}$$

and similarly for $J_1(1,2,1)$.

#### 4. Integrals with numerators

We also need some integrals with negative powers of denominators (i.e., when the corresponding denominator is in the numerator). Such formulas can be obtained in a standard way, via representing the numerators in terms of the external invariants. For the cases when one of the $\nu_i$ is equal to $(-1)$, we get

$$J_i(\nu_1,\nu_2,-1)=-(p_3^2)^{-1}\{p_3^2[(p_1p_2)+\sigma_i m^2]J_i(\nu_1,\nu_2,0)+(p_1p_3)J_i(\nu_1-1,\nu_2,0)+(p_2p_3)J_i(\nu_1,\nu_2-1,0)\},$$

$$J_i(\nu_1,-1,\nu_3)=-(p_2^2)^{-1}\{[p_2^2(p_1p_3)+\sigma_i m^2(p_2p_3)]J_i(\nu_1,0,\nu_3)+(p_2p_3)J_i(\nu_1,0,\nu_3-1)+(p_1p_2)J_i(\nu_1-1,0,\nu_3)\},$$

$$J_i(-1,\nu_2,\nu_3)=-(p_1^2)^{-1}\{[p_1^2(p_2p_3)+\sigma_i m^2(p_1p_3)]J_i(0,\nu_2,\nu_3)+(p_1p_3)J_i(0,\nu_2,\nu_3-1)+(p_1p_2)J_i(0,\nu_2-1,\nu_3)\},$$

where $\sigma_1=1$ and $\sigma_2=-1$ (for the integrals $J_1$ and $J_2$, respectively). We note that similar substitutions would also be valid for the integrals $J_3$ (with three massive lines), if we put $\sigma_3=0$.

Moreover, an integral with $\nu_3=-2$ has occurred:

$$J_1(\nu_1,\nu_2,-2)=\{[(p_1p_2)+m^2]^2-(n-1)^{-1}\mathcal{K}\}J_1(\nu_1,\nu_2,0)+2(p_3^2)^{-1}\{(p_1p_3)[(p_1p_2)+m^2]+(n-1)^{-1}\mathcal{K}\}J_1(\nu_1-1,\nu_2,0)$$

$$+2(p_3^2)^{-1}\{(p_2p_3)[(p_1p_2)+m^2]+(n-1)^{-1}\mathcal{K}\}J_1(\nu_1,\nu_2-1,0)+(p_3^2)^{-2}[(p_1p_3)^2-(n-1)^{-1}\mathcal{K}]$$

$$\times J_1(\nu_1-2,\nu_2,0)+(p_3^2)^{-2}[(p_2p_3)^2-(n-1)^{-1}\mathcal{K}]J_1(\nu_1,\nu_2-2,0)+2(p_3^2)^{-2}[(p_1p_3)(p_2p_3)$$

$$+(n-1)^{-1}\mathcal{K}]J_1(\nu_1-1,\nu_2-1,0),$$

with $\mathcal{K}$ defined in Eq. (3.1).

## APPENDIX C: BASIC SCALAR INTEGRALS

As discussed in Sec. II A (see also Appendix B), all results for the scalar functions ($\lambda_i$, $\tau_i$, etc.) can be expressed in terms of three-point integrals $J_1(1,1,1)$ and $J_2(1,1,1)$, two-point integrals (2.10)–(2.13) and a tadpole integral (2.14). Remember that we are interested in results which are valid for arbitrary values of the space-time dimension $n$.

### 1. Two-point integrals

The massless two-point integral is trivial,

$$J_1(1,1,0)=J_0(1,1,0)=\mathrm{i}\,\pi^{n/2}(-p_3^2)^{n/2-2}\frac{\Gamma^2\left(\frac{n}{2}-1\right)\Gamma\left(2-\frac{n}{2}\right)}{\Gamma(n-2)}. \tag{C1}$$

Two-point integrals with one or two massive lines can be expressed in terms of the Gauss hypergeometric function (see, e.g., in [23]):

$$J_1(0,1,1)=J_2(0,1,1)=2\mathrm{i}\,\pi^{n/2}(m^2)^{n/2-2}\frac{\Gamma\left(2-\frac{n}{2}\right)}{n-2}\times{}_2F_1\left(\left.\begin{matrix}1,2-n/2\\ n/2\end{matrix}\right|\frac{p_1^2}{m^2}\right), \tag{C2}$$

$$J_2(1,1,0)=\mathrm{i}\,\pi^{n/2}(m^2)^{n/2-2}\Gamma\left(2-\frac{n}{2}\right){}_2F_1\left(\left.\begin{matrix}1,2-n/2\\ 3/2\end{matrix}\right|\frac{p_3^2}{4m^2}\right). \tag{C3}$$

The results for the integrals $J_1(1,0,1)=J_2(1,0,1)$ can be obtained from (C2) by substituting $p_1^2\to p_2^2$.

In three dimensions, Eqs. (C2) and (C3) yield (cf., e.g., in [48])

$$J_1(0,1,1)|_{n=3}=J_2(0,1,1)|_{n=3}=\frac{2\mathrm{i}\,\pi^2}{m}f\left(\frac{p_1^2}{m^2}\right),$$

$$J_2(1,1,0)|_{n=3}=\frac{\mathrm{i}\,\pi^2}{m}f\left(\frac{p_3^2}{4m^2}\right), \tag{C4}$$

with

$$f(z)=\begin{cases}\dfrac{1}{2\sqrt{z}}\ln\dfrac{1+\sqrt{z}}{1-\sqrt{z}}, & z>0,\\[2ex] \dfrac{1}{\sqrt{-z}}\arctan\sqrt{-z}, & z<0.\end{cases} \tag{C5}$$

Around four dimensions ($n=4-2\varepsilon$), they are singular (see, e.g., [45]):

$$J_1(0,1,1)=J_2(0,1,1)=\mathrm{i}\,\pi^{2-\varepsilon}m^{-2\varepsilon}\Gamma(1+\varepsilon)\times\left\{\frac{1}{\varepsilon}+2+\frac{m^2-p_1^2}{p_1^2}\ln\frac{m^2-p_1^2}{m^2}+\mathcal{O}(\varepsilon)\right\}, \tag{C6}$$

$$J_2(1,1,0)=\mathrm{i}\,\pi^{2-\varepsilon}m^{-2\varepsilon}\Gamma(1+\varepsilon)\times\left\{\frac{1}{\varepsilon}+2-2f\left(\frac{p_3^2}{p_3^2-4m^2}\right)+\mathcal{O}(\varepsilon)\right\}, \tag{C7}$$

with the same function $f$ as in Eq. (C5). Above the corresponding threshold ($p_1^2>m^2$ or $p_3^2>4m^2$), these functions acquire imaginary parts, whose sign is defined by the causal prescription $p_i^2\leftrightarrow p_i^2+\mathrm{i}0$ (for details, see Appendix A of [46]).

In the limit $p_1^2=p_2^2=p^2$ (in particular, in the on-shell case), the combinations

$$\frac{J_2(1,0,1)-J_2(0,1,1)}{p_2^2-p_1^2}$$

should be treated in the following way:

$$\left.\left\{\frac{J_2(1,0,1)-J_2(0,1,1)}{p_2^2-p_1^2}\right\}\right|_{p_1^2=p_2^2=p^2}=\frac{\mathrm{d}}{\mathrm{d}p^2}\{J_2(0,1,1)|_{p_1^2=p^2}\}. \tag{C8}$$

Using Eq. (C2), and also the fact that

$$\frac{\mathrm{d}}{\mathrm{d}z}{}_2F_1\left(\left.\begin{matrix}a,b\\ c\end{matrix}\right|z\right)=\frac{ab}{c}{}_2F_1\left(\left.\begin{matrix}a+1,b+1\\ c+1\end{matrix}\right|z\right), \tag{C9}$$

we obtain

$$\left.\left\{\frac{J_2(1,0,1)-J_2(0,1,1)}{p_2^2-p_1^2}\right\}\right|_{p_1^2=p_2^2=p^2}=-\frac{1}{2m^2}J_2(0,1,1)|_{p_1^2=p^2}. \tag{C10}$$

In the on-shell case, we get

$$J_2(0,1,1)|_{p_1^2=m^2}=\frac{n-2}{2m^2(n-3)}J_2(0,1,0). \tag{C11}$$

Moreover, for some functions we need the expansion of this integral in $\delta_1=(m^2-p_1^2)/m^2$ up to the linear term,

$$J_2(0,1,1)=\frac{n-2}{2m^2(n-3)}J_2(0,1,0)\left[1+\frac{1}{2}\delta_1+\mathcal{O}(\delta_1^2)\right]. \tag{C12}$$

### 2. Three-point integrals

For the triangle integrals $J_1$ and $J_2$, results in arbitrary dimension, and also for any powers of the propagators, can be presented in terms of multiple hypergeometric functions [23]. For our purposes, we need integer powers of propagators. Using recurrence relations [29] based on the integration-by-parts technique [28], all scalar integrals can be reduced to $J_i(1,1,1)$ and two-point functions.

#### a. General off-shell case

Transforming Feynman parametric integrals (for example, using the Cheng-Wu theorem [47]), we can present $J_1(1,1,1)$ and $J_2(1,1,1)$ as

$$J_1(1,1,1)=-\mathrm{i}\pi^{n/2}\Gamma\left(3-\frac{n}{2}\right)\int_0^\infty\int_0^\infty\frac{\mathrm{d}\xi\mathrm{d}\eta}{(1+\xi+\eta)^{n-3}[m^2\eta(1+\xi+\eta)-\eta p_1^2-\xi\eta p_2^2-\xi p_3^2]^{3-n/2}}, \tag{C13}$$

$$J_2(1,1,1)=-\mathrm{i}\pi^{n/2}\Gamma\left(3-\frac{n}{2}\right)\int_0^\infty\int_0^\infty\frac{\mathrm{d}\xi\mathrm{d}\eta}{(1+\xi+\eta)^{n-3}[m^2(1+\xi+\eta)(1+\eta)-\xi p_1^2-\xi\eta p_2^2-\eta p_3^2]^{3-n/2}}. \tag{C14}$$

In the three-dimensional case ($n=3$) the denominator $(1+\xi+\eta)$ disappears in both integrals, and one can easily integrate over $\xi$, and then over $\eta$. This yields

$$J_1(1,1,1)|_{n=3}=-\frac{\mathrm{i}\pi^2}{\sqrt{p_3^2\mathcal{M}_1}}\ln\left[\frac{m+\sqrt{\mathcal{M}_1/p_3^2}}{m-\sqrt{\mathcal{M}_1/p_3^2}}\right], \tag{C15}$$

$$J_2(1,1,1)|_{n=3}=-\frac{\mathrm{i}\pi^2}{\sqrt{\mathcal{M}_2}}\ln\left[\frac{m(2m^2-p_1^2-p_2^2)+\sqrt{\mathcal{M}_2}}{m(2m^2-p_1^2-p_2^2)-\sqrt{\mathcal{M}_2}}\right], \tag{C16}$$

where $\mathcal{M}_1$ and $\mathcal{M}_2$ are defined in Eqs. (3.2) and (3.3), respectively. If $\mathcal{M}_1/p_3^2$ or $\mathcal{M}_2$ is negative (in $J_1$ and $J_2$, respectively), the logarithms should be substituted by arctan functions [see Eq. (C5)], which correspond to the limiting cases of the result obtained in [48] (see also in [43]) for three-dimensional three-point function with arbitrary masses. Note that in the massless case we arrive at the well-known result [49] (see also in [14])

$$J_{1,2}(1,1,1)|_{n=3,m=0}=J_0(1,1,1)|_{n=3}=-\frac{\mathrm{i}\pi^3}{\sqrt{-p_1^2p_2^2p_3^2}}. \tag{C17}$$

In four dimensions ($n=4$), we can also integrate over $\xi$. Then, performing the remaining $\eta$ integral, we arrive at the known results in terms of dilogarithms [45].

#### b. Symmetric case

The ‘‘symmetric’’ case is of a certain interest (see e.g. in [7]), when all external invariants are equal, $p_1^2=p_2^2=p_3^2=-\mu^2$. Then we obtain

$$J_1(1,1,1)|_{n=4,p_i^2=-\mu^2}=-\frac{\mathrm{i}\pi^2}{\mu^2}\int_0^\infty\frac{\mathrm{d}\eta}{1+\eta+\eta^2}\times\ln\left[\frac{(1+\eta)(m^2+\mu^2+\mu^2\eta)}{m^2+\mu^2+m^2\eta}\right], \tag{C18}$$

$$J_2(1,1,1)|_{n=4,p_i^2=-\mu^2}=-\frac{\mathrm{i}\pi^2}{\mu^2}\int_0^\infty\frac{\mathrm{d}\eta}{1+\eta+\eta^2}\times\ln\left[\frac{(m^2+\mu^2)(1+\eta)^2}{m^2(1+\eta)^2+\mu^2\eta}\right]. \tag{C19}$$

These parametric representations are equivalent to those for the $H$ and $M$ functions given in [7]. Note that $\mu^2J_1\leftrightarrow\mathrm{i}\pi^2H$ and $\mu^2J_2\leftrightarrow\mathrm{i}\pi^2M$. These integrals can be evaluated in terms of Clausen functions as

$$J_1(1,1,1)|_{n=4,p_i^2=-\mu^2}$$
$$=-\frac{\mathrm{i}\pi^2}{\mu^2\sqrt{3}}\left\{2\,\mathrm{Cl}_2\left(\frac{\pi}{3}\right)+2\,\mathrm{Cl}_2\left(\frac{\pi}{3}+2\,\theta_{s1}\right)\right.$$
$$\left.+\mathrm{Cl}_2\left(\frac{\pi}{3}-2\,\theta_{s1}\right)+\mathrm{Cl}_2(\pi-2\,\theta_{s1})\right\}, \qquad \text{(C20)}$$
$$J_2(1,1,1)|_{n=4,p_i^2=-\mu^2}$$
$$=-\frac{2\mathrm{i}\pi^2}{\mu^2\sqrt{3}}\left\{2\,\mathrm{Cl}_2\left(\frac{2\pi}{3}\right)+\mathrm{Cl}_2\left(\frac{\pi}{3}+2\,\theta_{s2}\right)\right.$$
$$\left.+\mathrm{Cl}_2\left(\frac{\pi}{3}-2\,\theta_{s2}\right)\right\}, \qquad \text{(C21)}$$

where

$$\tan\theta_{s1}=\frac{\mu^2+2m^2}{\mu^2\sqrt{3}}, \quad \tan\theta_{s2}=\sqrt{\frac{\mu^2+4m^2}{3\mu^2}}. \qquad \text{(C22)}$$

This gives analytical results for the $H$ and $M$ functions from [7]. In the massless limit ($m\to 0$), $\theta_{s1}=\theta_{s2}=\pi/6$, and, remembering that $\mathrm{Cl}_2(2\pi/3)=\frac{2}{3}\mathrm{Cl}_2(\pi/3)$, we reproduce the well-known result [5]

$$J_0(1,1,1)|_{n=4,p_i^2=-\mu^2}=-\frac{4\mathrm{i}\pi^2}{\mu^2\sqrt{3}}\mathrm{Cl}_2\left(\frac{\pi}{3}\right). \qquad \text{(C23)}$$

### *c. On-shell limit*

Now let us consider the on-shell limit $p_1^2=p_2^2=m^2$. For $J_1(1,1,1)$, the two-fold parametric integral (C13) yields

$$J_1(1,1,1)|_{p_1^2=p_2^2=m^2}$$
$$=-\mathrm{i}\pi^{n/2}\Gamma\left(3-\frac{n}{2}\right)$$
$$\times\int_0^\infty\int_0^\infty\frac{\mathrm{d}\xi\,\mathrm{d}\eta}{(1+\xi+\eta)^{n-3}[m^2\eta^2-\xi p_3^2]^{3-n/2}}. \qquad \text{(C24)}$$

Using Mellin-Barnes contour integral for the second denominator, we find

$$J_1(1,1,1)|_{p_1^2=p_2^2=m^2}=-\mathrm{i}\pi^{n/2}(m^2)^{n/2-3}\frac{1}{\Gamma(n-3)}$$
$$\times\frac{1}{2\pi\mathrm{i}}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\mathrm{d}s\left(-\frac{p_3^2}{m^2}\right)^s\Gamma(-s)$$
$$\times\Gamma(n-5-2s)\Gamma^2(1+s)$$
$$\times\Gamma\left(3-\frac{n}{2}+s\right). \qquad \text{(C25)}$$

Closing the contour to the right, we get the result in terms of ${}_2F_1$ functions of the argument $p_3^2/(4m^2)$. Note that the integral $J_1(1,1,1)$ is not divergent in the on-shell limit, so that we can put $n=4$ in Eq. (C24). For instance, the following simple representation in terms of $\mathrm{Cl}_2$ can be mentioned

$$J_1(1,1,1)|_{n=4,p_1^2=p_2^2=m^2}=\frac{2\mathrm{i}\pi^2}{\sqrt{p_3^2(4m^2-p_3^2)}}$$
$$\times\{\mathrm{Cl}_2(\pi-2\psi_{os})-\mathrm{i}\pi\psi_{os}\}, \qquad \text{(C26)}$$

with

$$\psi_{os}=\arctan\sqrt{(4m^2-p_3^2)/p_3^2}, \quad 0<p_3^2<4m^2.$$

Let us also consider the integral $J_2(1,1,1)$ in the on-shell limit. Starting from the representation (C14) in arbitrary dimension $n$ and integrating over $\xi$, we get

$$J_2(1,1,1)|_{p_1^2=p_2^2=m^2}$$
$$=\frac{1}{2}\mathrm{i}\pi^{n/2}\Gamma\left(2-\frac{n}{2}\right)$$
$$\times\int_0^\infty\frac{\mathrm{d}\eta}{(1+\eta)^{n-4}[m^2(1+\eta)^2-\eta p_3^2]^{3-n/2}}. \qquad \text{(C27)}$$

It is easy to show that this three-point function (in the on-shell limit) reduces to a two-point function (C3) with a shifted space-time dimension $n\to n-2$,

$$J_2(1,1,1)|_{p_1^2=p_2^2=m^2}=\frac{\pi}{4-n}J_2(1,1,0)\Big|_{n\to n-2}. \qquad \text{(C28)}$$

Using Eq. (C3) we get

$$J_2(1,1,1)|_{p_1^2=p_2^2=m^2}=\frac{1}{2}\mathrm{i}\pi^{n/2}(m^2)^{n/2-3}\Gamma\left(2-\frac{n}{2}\right)$$
$$\times{}_2F_1\left(\begin{matrix}1,3-n/2\\3/2\end{matrix}\middle|\frac{p_3^2}{4m^2}\right). \qquad \text{(C29)}$$

In the limit $n\to 4$, because of the singular factor in front of the RHS of Eq. (C28), we need to expand the two-point function up to the $\varepsilon$ term. This is how dilogarithms (or Clausen functions) arise in the finite part of the three-point function.

The same result (C29) can be obtained from Eqs. (32) and (34) of [23], taking into account that the first two arguments of the $\Phi_2$ function are $z_1=z_2\equiv z=1$. Therefore, the sum over $j$ in Eq. (34) of [23] (corresponding to an ${}_2F_1$ function of unit argument) can be performed in terms of $\Gamma$ functions. As a result, we arrive at Eq. (C29).

Moreover, using Kummer relations for contiguous ${}_2F_1$ functions, we get

$$(n-3)\,{}_2F_1\left(\begin{matrix}1,2-n/2\\3/2\end{matrix}\middle|z\right)$$
$$=(n-4)(1-z)\,{}_2F_1\left(\begin{matrix}1,3-n/2\\3/2\end{matrix}\middle|z\right)+1. \qquad \text{(C30)}$$

Therefore,

$$J_2(1,1,1)|_{p_1^2=p_2^2=m^2}=\frac{2(n-3)}{(n-4)(4m^2-p_3^2)}[J_2(1,1,0)-J_2(0,1,1)|_{p_1^2=m^2}]. \quad \text{(C31)}$$

## APPENDIX D: RESULTS FOR THE $\chi$ FUNCTIONS

The one-loop results for $\chi_i$ functions from Eq. (2.25) are collected below:

$$\chi_0^{(1)}(p_1^2,p_2^2,p_3^2)=-\frac{g^2\eta}{(4\pi)^{n/2}}\frac{C_A}{8\mathcal{M}_1}\{[2+(n-3)\xi]\mathcal{M}_1(p_1^2-m^2)\varphi_1-(n-2)\xi p_1^2p_2^2p_3^2\varphi_1+(n-2)\xi\mathcal{M}_1[(p_1p_2)+m^2]\varphi_1$$
$$-\xi m^2p_3^2[n(p_1p_2)+2m^2]\varphi_1+(n-3)\xi[m^2p_3^2\kappa_{0,3}+p_1^2(p_2^2-m^2)\kappa_{1,1}+m^2(p_1^2-p_3^2-m^2)\kappa_{1,2}]$$
$$-(2-\xi)\mathcal{M}_1(\kappa_{0,3}+\kappa_{1,2})+\xi\mathcal{M}_1\kappa_{1,1}+(n-2)\xi m^2[(p_1p_2)+m^2]\tilde{\kappa}\}, \quad \text{(D1)}$$

$$\chi_1^{(1)}(p_1^2,p_2^2,p_3^2)=\frac{g^2\eta}{(4\pi)^{n/2}}\frac{mC_A}{8\mathcal{K}\mathcal{M}_1}\{(2-\xi)\mathcal{M}_1(p_2p_3)[(p_1p_2)+m^2]\varphi_1-\xi m^2\mathcal{K}p_3^2\varphi_1+[2+(n-3)\xi]\mathcal{K}\mathcal{M}_1\varphi_1$$
$$+(n-3)\xi p_3^2[(p_1p_2)+m^2][p_1^2p_2^2+m^2(p_1p_2)]\varphi_1+(2-\xi)\mathcal{M}_1[(p_1p_2)\kappa_{1,1}+p_2^2\kappa_{1,2}+(p_2p_3)\kappa_{0,3}]$$
$$+(n-3)\xi p_3^2[p_1^2p_2^2+m^2(p_1p_2)]\kappa_{0,3}+(n-3)\xi p_1^2[p_2^2(p_1p_3)+m^2(p_2p_3)]\kappa_{1,1}+(n-3)\xi p_2^2$$
$$\times[p_1^2(p_2p_3)+m^2(p_1p_3)]\kappa_{1,2}+(n-2)\xi m^2\mathcal{K}\tilde{\kappa}\}, \quad \text{(D2)}$$

$$\chi_2^{(1)}(p_1^2,p_2^2,p_3^2)=\frac{g^2\eta}{(4\pi)^{n/2}}\frac{mC_A}{8\mathcal{K}\mathcal{M}_1}\{[2+(n-3)\xi]\mathcal{K}\mathcal{M}_1\varphi_1-(2-\xi)\mathcal{M}_1(p_1p_3)[(p_1p_2)+m^2]\varphi_1$$
$$+\xi[(n-4)\mathcal{K}-(n-3)\mathcal{M}_1]p_1^2p_3^2\varphi_1-(n-3)\xi p_1^2p_3^2[(p_1p_2)+m^2]\kappa_{0,3}+(n-3)\xi p_1^2[p_3^2(p_1p_2)$$
$$+(p_1p_3)(p_2^2-m^2)]\kappa_{1,1}-(n-3)\xi p_1^2(p_2p_3)[(p_1p_2)+m^2]\kappa_{1,2}+\xi p_1^2\mathcal{K}\kappa_{1,2}-(2-\xi)\mathcal{M}_1$$
$$\times[p_1^2\kappa_{1,1}+(p_1p_2)\kappa_{1,2}+(p_1p_3)\kappa_{0,3}]+(n-2)\xi m^2\mathcal{K}(p_2^2)^{-1}[2(p_1p_2)+m^2](\kappa_{1,2}-\tilde{\kappa})\}, \quad \text{(D3)}$$

$$\chi_3^{(1)}(p_1^2,p_2^2,p_3^2)=-\frac{g^2\eta}{(4\pi)^{n/2}}\frac{C_A}{8\mathcal{K}\mathcal{M}_1}\{(n-4)\xi m^2\mathcal{K}p_3^2\varphi_1-m^2\mathcal{M}_1[2(p_1p_3)+\xi(p_2p_3)]\varphi_1-[2+(n-3)\xi]\mathcal{M}_1p_1^2(p_2p_3)\varphi_1$$
$$-(n-2)\xi\mathcal{M}_1(p_1p_2)(p_2p_3)\varphi_1-\mathcal{M}_1[2(p_1p_3)+\xi(p_2p_3)]\kappa_{0,3}-\mathcal{M}_1[2p_1^2+\xi(p_1p_2)]\kappa_{1,1}$$
$$-\mathcal{M}_1[2(p_1p_2)+\xi p_2^2]\kappa_{1,2}+(n-3)\xi p_3^2[p_1^2p_2^2+m^2(p_1p_2)]\kappa_{0,3}+(n-3)\xi p_1^2[p_2^2(p_1p_3)+m^2(p_2p_3)]\kappa_{1,1}$$
$$+(n-3)\xi p_2^2[p_1^2(p_2p_3)+m^2(p_1p_3)]\kappa_{1,2}+(n-2)\xi m^2\mathcal{K}\tilde{\kappa}\}. \quad \text{(D4)}$$

## APPENDIX E: TRANSVERSE FUNCTIONS IN AN ARBITRARY GAUGE

We here collect results for the transverse parts of the vertex, $\tau_i$, valid for arbitrary covariant gauge and dimension.

### 1. Transverse functions of diagram *a*

Using the decomposition (3.15), all $\tau$'s of diagram *a* can be presented as[13]

[13]As mentioned in Sec. III B, the combinations of $\kappa$'s in Eqs. (E1) and (E2) are linearly dependent. As a result, we can (simultaneously) shift $t_{i,1}\to t_{i,1}+2c_i$, $t_{i,2}\to t_{i,2}+p_3^2c_i$ and $t_{i,5}\to t_{i,5}+(p_1^2-p_2^2)^2c_i$ (where $c_i$ are arbitrary functions of the momenta, which can be chosen separately for each $\tau_i$), without change of the value of the corresponding $\tau_i$. This possibility allows us to write some of the $t$'s in a more compact form.

$$\tau_i^{(1a)}(p_1^2,p_2^2,p_3^2) = \frac{g^2\eta\left(C_F-\frac{1}{2}C_A\right)}{(4\pi)^{n/2}}\left\{ t_{i,0}^{(1a)}\varphi_2 + t_{i,1}^{(1a)}[(p_1p_3)\kappa_{1,1} + (p_2p_3)\kappa_{1,2} + p_3^2\kappa_{2,3}] + t_{i,2}^{(1a)}(\kappa_{1,1}+\kappa_{1,2}-2\kappa_{2,3}) + t_{i,3}^{(1a)}(\kappa_{1,1}+\kappa_{1,2}-2\widetilde{\kappa}) + t_{i,4}^{(1a)}(\kappa_{1,1}+\kappa_{1,2}) + t_{i,5}^{(1a)}\frac{\kappa_{1,1}-\kappa_{1,2}}{p_1^2-p_2^2}\right\}. \tag{E1}$$

The results for the scalar functions $t_i^{(1a)}$ (which depend on the invariants $p_1^2,p_2^2,p_3^2$) are listed below, for all eight $\tau$'s:

$$t_{1,0}^{(1a)} = -(n-\xi)m\mathcal{K}^{-1}[(p_1p_2)-m^2],$$

$$t_{1,1}^{(1a)} = t_{1,3}^{(1a)} = t_{1,4}^{(1a)} = 0,$$

$$t_{1,2}^{(1a)} = \frac{1}{2}(n-\xi)m\mathcal{K}^{-1},$$

$$t_{1,5}^{(1a)} = \frac{1}{2}(n-\xi)m\mathcal{K}^{-1}(p_1-p_2)^2,$$

$$t_{2,0}^{(1a)} = -\frac{1}{4}\mathcal{K}^{-1}\{2(n-4)\xi\mathcal{M}_2^{-1}p_3^2(p_1^2-m^2)(p_2^2-m^2)\times[(p_1p_2)-m^2]+(1+\xi)p_3^2-4m^2 +(n-1)\mathcal{K}^{-1}p_3^2[(p_1p_2)-m^2]\times[(1+\xi)(p_1p_2)-(1-\xi)m^2]+4\xi(p_1p_2)\},$$

$$t_{2,1}^{(1a)} = -\frac{1}{4}(n-1)\mathcal{K}^{-2}[(1+\xi)(p_1p_2)-(1-\xi)m^2],$$

$$t_{2,2}^{(1a)} = \frac{1}{4}\xi\mathcal{K}^{-1}[(n-2)-(n-3)m^2\mathcal{M}_2^{-1}(p_1^2-p_2^2)^2],$$

$$t_{2,3}^{(1a)} = -\frac{1}{8}(n-2)m^2\mathcal{K}^{-1}\{(1-\xi)(p_1^2p_2^2)^{-1}(p_1p_2) +2\xi\mathcal{M}_2^{-1}p_3^2[(p_1p_2)-m^2]+4\xi\mathcal{K}\mathcal{M}_2^{-1}\},$$

$$t_{2,4}^{(1a)} = -\frac{1}{8}(n-4)\mathcal{K}^{-1}\mathcal{M}_2^{-1}\{(1-\xi)\mathcal{M}_2+2\xi p_3^2[p_1^2p_2^2 -m^2(p_1p_2)]-4\xi m^2\mathcal{K}\},$$

$$t_{2,5}^{(1a)} = -\frac{1}{8}\mathcal{K}^{-1}(p_1^2p_2^2)^{-1}\{(n-2)(1-\xi)\times(p_1-p_2)^2[p_1^2p_2^2-m^2(p_1p_2)]+2(n-2)\times(1-\xi)m^2\mathcal{K}+2(n-3)\xi m^2\mathcal{M}_2^{-1}p_1^2p_2^2(p_1-p_2)^2\times(p_1^2-p_2^2)^2\},$$

$$t_{3,0}^{(1a)} = \frac{1}{8}\mathcal{K}^{-1}\{(p_1^2+p_2^2-2m^2)[(1+\xi)(p_1^2+p_2^2) -2(1-\xi)m^2]-4(n-2)\mathcal{K}+(n-1)\mathcal{K}^{-1}\times(p_1^2-p_2^2)^2[(p_1p_2)-m^2][(1+\xi)(p_1p_2) -(1-\xi)m^2]-2(n-4)\xi m^2\mathcal{M}_2^{-1}(p_1^2-p_2^2)^2\times[(p_1p_2)-m^2](p_1^2+p_2^2-2m^2)\},$$

$$t_{3,1}^{(1a)} = 0,$$

$$t_{3,2}^{(1a)} = \frac{1}{16}\mathcal{K}^{-2}\{2(n-3)\xi m^2\mathcal{K}\mathcal{M}_2^{-1}(p_1^2-p_2^2)^2\times(p_1^2+p_2^2-2m^2)-[(n-1)(p_1^2-p_2^2)^2+4\mathcal{K}]\times[(1+\xi)(p_1p_2)-(1-\xi)m^2]\},$$

$$t_{3,3}^{(1a)} = \frac{1}{16}(n-2)m^2\mathcal{K}^{-1}\{(1-\xi)(p_1^2p_2^2)^{-1}\times[(p_1-p_2)^2(p_1p_2)-2\mathcal{K}]+2\xi\mathcal{M}_2^{-1}\times(p_1^2-p_2^2)^2[(p_1p_2)-m^2]\},$$

$$t_{3,4}^{(1a)} = \frac{1}{16}(n-4)\mathcal{K}^{-1}\{(1+\xi)(p_1-p_2)^2+2\xi m^2\mathcal{M}_2^{-1}\times(p_1^2-p_2^2)^2[(p_1p_2)-m^2]\},$$

$$t_{3,5}^{(1a)} = \frac{1}{16}\mathcal{K}^{-2}(p_1^2-p_2^2)^2\{(n-1+2\xi)[2\mathcal{K}-(p_1p_2)\times(p_1-p_2)^2]-(n-n\xi+6\xi)\mathcal{K}-(1-\xi)m^2\times[(n-2)\mathcal{K}(p_1^2p_2^2)^{-1}(p_1p_2)-(n-1)(p_1-p_2)^2] +(n-3)\xi\mathcal{M}_2^{-1}(p_1-p_2)^2[2m^2p_1^2p_2^2p_3^2 -p_3^2(p_1p_2)(p_1^2p_2^2+m^4)-4m^4\mathcal{K}]\},$$

$$t_{4,0}^{(1a)} = -\frac{1}{2}\xi m\mathcal{K}^{-1}\{(n-1)\mathcal{K}^{-1}p_3^2[(p_1p_2)-m^2] -2(n-5)+2(n-4)\mathcal{K}\mathcal{M}_2^{-1}\times(p_1^2+p_2^2-2m^2)-(n-4)\mathcal{M}_2^{-1}(p_1^2-p_2^2)^2\times[(p_1p_2)-m^2]\},$$

$$t_{4,1}^{(1a)} = \frac{1}{2}\xi m\mathcal{K}^{-1}[(n-3)\mathcal{M}_2^{-1}(p_1^2+p_2^2-2m^2) -(n-1)\mathcal{K}^{-1}],$$

$$t_{4,2}^{(1a)} = 0,$$

$$t_{4,3}^{(1a)}=\frac{1}{4}(n-2)\xi m^3\mathcal{K}^{-1}\mathcal{M}_2^{-1}(p_1^2p_2^2)^{-1}\times[2\mathcal{K}(p_1^2+p_2^2)-p_1^2p_2^2p_3^2+m^2p_3^2(p_1p_2)],$$

$$t_{4,4}^{(1a)}=-\frac{1}{4}(n-4)\xi m\mathcal{K}^{-1}\mathcal{M}_2^{-1}p_3^2[(p_1p_2)-m^2],$$

$$t_{4,5}^{(1a)}=-\frac{1}{4}\xi m\{(n-2)\mathcal{K}^{-1}[(p_1^2p_2^2)^{-1}(p_1p_2)(p_1^2+p_2^2)-2]+4(n-3)\mathcal{M}_2^{-1}(p_1^2+p_2^2-2m^2)\},$$

$$t_{5,0}^{(1a)}=-\frac{1}{4}m\mathcal{K}^{-1}\{4(n-4-\xi)\mathcal{K}+(n-4)\xi\mathcal{K}\mathcal{M}_2^{-1}\times(p_1^2-p_2^2)^2(p_3^2-4m^2)-(n-3)\xi(p_1^2-p_2^2)^2-2\xi p_3^2[(p_1p_2)-m^2]\},$$

$$t_{5,1}^{(1a)}=\frac{1}{4}\xi m\mathcal{K}^{-1}\{(n-3)\mathcal{M}_2^{-1}(p_1^2-p_2^2)^2\times[(p_1p_2)-m^2]+2\},$$

$$t_{5,2}^{(1a)}=0,$$

$$t_{5,3}^{(1a)}=\frac{1}{8}(n-2)\xi m^3\mathcal{M}_2^{-1}(p_1^2p_2^2)^{-1}[2(p_1^2-p_2^2)^2(p_1p_2)+2p_1^2p_2^2p_3^2-m^2p_3^2(p_1^2+p_2^2)],$$

$$t_{5,4}^{(1a)}=\frac{1}{8}(n-4)\xi m\mathcal{M}_2^{-1}p_3^2(p_1^2+p_2^2-2m^2),$$

$$t_{5,5}^{(1a)}=\frac{1}{8}\xi m(p_1^2-p_2^2)^2\{(n-2)(p_1^2p_2^2)^{-1}-4(n-3)\mathcal{M}_2^{-1}[(p_1p_2)-m^2]\},$$

$$t_{6,0}^{(1a)}=-\frac{1}{8}\mathcal{K}^{-1}\mathcal{M}_2^{-1}(p_1^2-p_2^2)\times\{\mathcal{M}_2[(1+\xi)p_3^2-4m^2]+6\xi m^2p_3^2[(p_1p_2)-m^2]\times(p_1^2+p_2^2-2m^2)+(n-1)\mathcal{K}^{-1}p_3^2[(p_1p_2)-m^2]^2\times[\mathcal{M}_2+\xi p_3^2(p_1^2p_2^2-m^4)]\},$$

$$t_{6,1}^{(1a)}=\frac{1}{8}\mathcal{K}^{-1}(p_1^2-p_2^2)\{2(n-3)\xi m^2\mathcal{M}_2^{-1}(p_1^2+p_2^2-2m^2)-(n-1)\mathcal{K}^{-1}[(1+\xi)(p_1p_2)-(1-\xi)m^2]\},$$

$$t_{6,2}^{(1a)}=0,$$

$$t_{6,3}^{(1a)}=-\frac{1}{16}(n-2)m^2\mathcal{K}^{-1}(p_1^2-p_2^2)\times\{(1-\xi)(p_1^2p_2^2)^{-1}(p_1p_2)+2\xi\mathcal{M}_2^{-1}p_3^2[(p_1p_2)-m^2]\},$$

$$t_{6,4}^{(1a)}=-\frac{1}{16}(n-4)\mathcal{K}^{-1}\mathcal{M}_2^{-1}(p_1^2-p_2^2)\{(1-\xi)\mathcal{M}_2+2\xi p_1^2p_2^2p_3^2-2\xi m^2[4\mathcal{K}+p_3^2(p_1p_2)]\},$$

$$t_{6,5}^{(1a)}=\frac{1}{16}\mathcal{K}^{-1}(p_1^2-p_2^2)\{(n-2)(1-\xi)\times m^2[(p_1^2p_2^2)^{-1}(p_1p_2)(p_1^2+p_2^2)-2]-(n-2)(1+\xi)(p_1-p_2)^2-8(n-3)\xi m^2\mathcal{K}\mathcal{M}_2^{-1}(p_1^2+p_2^2-2m^2)\},$$

$$t_{7,0}^{(1a)}=\frac{1}{2}\xi mp_3^2[(n-4)\mathcal{M}_2^{-1}(p_1^2+p_2^2-2m^2)-(n-3)\mathcal{K}^{-1}],$$

$$t_{7,1}^{(1a)}=-\frac{1}{2}(n-3)\xi m\mathcal{M}_2^{-1}\{\mathcal{K}^{-1}p_3^2[(p_1p_2)-m^2]+2\},$$

$$t_{7,2}^{(1a)}=0,$$

$$t_{7,3}^{(1a)}=-\frac{1}{4}(n-2)\xi m^3\mathcal{M}_2^{-1}(p_1^2p_2^2)^{-1}\times\{p_3^2[2(p_1p_2)-m^2]+4\mathcal{K}\},$$

$$t_{7,4}^{(1a)}=-\frac{1}{4}(n-4)\xi m\mathcal{M}_2^{-1}p_3^2,$$

$$t_{7,5}^{(1a)}=\frac{1}{4}\xi m\{4(n-3)\mathcal{M}_2^{-1}[2\mathcal{K}+p_3^2(p_1p_2)-m^2p_3^2]-(n-2)(p_1^2p_2^2)^{-1}(p_1^2+p_2^2)\},$$

$$t_{8,0}^{(1a)}=-\frac{1}{2}[n-6-(n-4)\xi]\mathcal{K}^{-1}p_3^2[(p_1p_2)-m^2],$$

$$t_{8,1}^{(1a)}=-\frac{1}{2}[n-6-(n-4)\xi]\mathcal{K}^{-1},$$

$$t_{8,2}^{(1a)}=t_{8,3}^{(1a)}=t_{8,4}^{(1a)}=t_{8,5}^{(1a)}=0.$$

#### 2. Transverse functions of diagram *b*

By analogy with diagram *a*, (E1), all $\tau$'s of diagram *b* can be presented as

$$\tau_i^{(1b)}(p_1^2,p_2^2,p_3^2)=\frac{g^2\eta C_A}{(4\pi)^{n/2}}\Bigg\{t_{i,0}^{(1b)}\varphi_1+t_{i,1}^{(1b)}[(p_1p_3)\kappa_{1,1}+(p_2p_3)\kappa_{1,2}+p_3^2\kappa_{0,3}]+t_{i,2}^{(1b)}(\kappa_{1,1}+\kappa_{1,2}-2\kappa_{0,3})+t_{i,3}^{(1b)}(\kappa_{1,1}+\kappa_{1,2}-2\tilde{\kappa})+t_{i,4}^{(1b)}(\kappa_{1,1}+\kappa_{1,2})+t_{i,5}^{(1b)}\frac{\kappa_{1,1}-\kappa_{1,2}}{p_1^2-p_2^2}\Bigg\}. \tag{E2}$$

The results for the scalar functions $t_i^{(1b)}$ (which depend on the invariants $p_1^2,p_2^2,p_3^2$) are listed below, for all eight $\tau$'s:

$$t_{1,0}^{(1b)}=\frac{1}{8}m\mathcal{K}^{-1}\mathcal{M}_1^{-1}\{(n-4)(n-6)\xi^2m^2\mathcal{K}\mathcal{M}_1^{-1}p_3^2\times[(p_1p_2)+m^2]+(n-4)\xi\mathcal{K}p_3^2+(n-4)\xi^2\times[(p_1p_2)+m^2][(n-2)\mathcal{K}+(n-3)p_3^2(p_1p_2)]-(n-3)\xi(1-\xi)\mathcal{M}_1p_3^2-2(2n-2-\xi)\mathcal{M}_1\times[(p_1p_2)+m^2]\},$$

$$t_{1,1}^{(1b)}=-\frac{1}{8}(n-3)\xi m\mathcal{K}^{-1}\mathcal{M}_1^{-1}\{[1-(n-3)\xi](p_1p_2)+[1-\xi-(n-6)\xi\mathcal{K}\mathcal{M}_1^{-1}]m^2\},$$

$$t_{1,2}^{(1b)}=\frac{1}{16}m[2(2n-2-\xi)\mathcal{K}^{-1}-(n-2)(n-3)\xi^2\mathcal{M}_1^{-1}],$$

$$t_{1,3}^{(1b)}=\frac{1}{16}(n-2)\xi m\{(p_1^2p_2^2)^{-1}-\mathcal{M}_1^{-1}-(n-6)\xi m^2\mathcal{M}_1^{-2}[(p_1p_2)+m^2]\},$$

$$t_{1,4}^{(1b)}=\frac{1}{16}(n-4)\xi m\mathcal{M}_1^{-1}\{3\xi-1+(n-6)\xi\mathcal{M}_1^{-1}\times[p_1^2p_2^2+m^2(p_1p_2)]\},$$

$$t_{1,5}^{(1b)}=\frac{1}{16}m\mathcal{K}^{-1}[2(2n-2-\xi)(p_1-p_2)^2-(n-2)\xi\mathcal{K}(p_1^2p_2^2)^{-1}(p_1^2+p_2^2)],$$

$$t_{2,0}^{(1b)}=\frac{1}{16}\mathcal{K}^{-1}\{[2-\xi-(n-3)\xi^2]p_3^2-(n-3)\times(n-4)\xi^2m^2\mathcal{M}_1^{-1}p_3^2[(p_1p_2)+m^2]+(n-1)\times(2-\xi)\mathcal{K}^{-1}p_3^2[(p_1p_2)+m^2]^2-(n-1)\times(n-3)\xi^2\mathcal{K}^{-1}p_3^2(p_1p_2)[(p_1p_2)+m^2]+[4(n-3)-2(2n-5)\xi-(n-2)(n-3)\xi^2]\times[(p_1p_2)+m^2]\},$$

$$t_{2,1}^{(1b)}=\frac{1}{16}\mathcal{K}^{-2}\{(n-1)(2-\xi)[(p_1p_2)+m^2]-(n-3)\xi^2\times[(n-1)(p_1p_2)+(n-3)m^2\mathcal{K}\mathcal{M}_1^{-1}]\},$$

$$t_{2,2}^{(1b)}=-\frac{1}{32}\mathcal{K}^{-1}[4(n-3)-2(2n-5)\xi-(n-2)\times(n-3)\xi^2],$$

$$t_{2,3}^{(1b)}=\frac{1}{32}(n-2)m^2\mathcal{K}^{-1}\{(n-3)\xi^2\mathcal{M}_1^{-1}[(p_1p_2)+m^2]-(2-\xi)(p_1^2p_2^2)^{-1}(p_1p_2)\},$$

$$t_{2,4}^{(1b)}=\frac{1}{32}(n-4)\mathcal{K}^{-1}\mathcal{M}_1^{-1}\{(2-\xi)\mathcal{M}_1-(n-3)\xi^2\times[p_1^2p_2^2+m^2(p_1p_2)]\},$$

$$t_{2,5}^{(1b)}=\frac{1}{32}\mathcal{K}^{-1}\{(n-2)(2-\xi)m^2[(p_1^2p_2^2)^{-1}(p_1p_2)\times(p_1^2+p_2^2)-2]-[2(n-4)-(3n-8)\xi]\times(p_1-p_2)^2\},$$

$$t_{3,0}^{(1b)}=\frac{1}{32}\mathcal{K}^{-1}\{(n-4)\xi m^2\mathcal{M}_1^{-1}[(p_1p_2)+m^2]\times[(n-3)\xi(p_1^2-p_2^2)^2-4(4-\xi)\mathcal{K}]+\xi[4+(n-4)\xi][(p_1p_2)+m^2][(n-3)(p_1-p_2)^2-4(n-4)\times\mathcal{K}(p_3^2)^{-1}]+(2-\xi)[(n-2)(p_1^2-p_2^2)^2\times(1-\mathcal{K}^{-1}\mathcal{M}_1)-\mathcal{K}^{-1}\mathcal{M}_1p_3^2(p_1-p_2)^2]+8(n-3)\xi\mathcal{K}+(n-3)\xi^2(p_1^2-p_2^2)^2+(n-3)\times\xi^2(p_1p_2)[(p_1p_2)+m^2][4+(n-1)\mathcal{K}^{-1}\times(p_1^2-p_2^2)^2]\},$$

$$t_{3,1}^{(1b)}=\frac{1}{32}\mathcal{K}^{-1}(p_3^2)^{-1}\{(n-3)\xi m^2\mathcal{M}_1^{-1}\times[(n-3)\xi(p_1^2-p_2^2)^2-4(4-\xi)\mathcal{K}]+[(n-1)\mathcal{K}^{-1}(p_1^2-p_2^2)^2+4][(n-3)\xi^2(p_1p_2)-(2-\xi)(p_1p_2)-(2-\xi)m^2]\},$$

$$t_{3,2}^{(1b)}=-\frac{1}{64}\xi\mathcal{K}^{-1}(p_3^2)^{-1}[4+(n-4)\xi][4\mathcal{K}+(n-3)\times(p_1^2-p_2^2)^2],$$

$$t_{3,3}^{(1b)}=\frac{1}{64}(n-2)m^2\mathcal{K}^{-1}(p_3^2)^{-1}\{\xi\mathcal{M}_1^{-1}[(p_1p_2)+m^2]\times[4(4-\xi)\mathcal{K}-(n-3)\xi(p_1^2-p_2^2)^2]+(2-\xi)\times(p_1^2p_2^2)^{-1}p_3^2[(p_1p_2)(p_1-p_2)^2-2\mathcal{K}]\},$$

$$t_{3,4}^{(1b)}=\frac{1}{64}(n-4)\mathcal{K}^{-1}(p_3^2)^{-1}\{(n-3)\xi^2\mathcal{M}_1^{-1}\times(p_1^2-p_2^2)^2[p_1^2p_2^2+m^2(p_1p_2)]-(2-\xi)p_3^2(p_1-p_2)^2-4\xi(4-\xi)\mathcal{K}\mathcal{M}_1^{-1}[p_1^2p_2^2+m^2(p_1p_2)]\},$$

$$t_{3,5}^{(1b)}=\frac{1}{64}\mathcal{K}^{-1}(p_3^2)^{-1}(2-\xi)(p_1^2-p_2^2)^2\{4(n-2)\times m^2\mathcal{K}(p_1^2p_2^2)^{-1}-2(n-3)\xi(p_1-p_2)^2-(n-2)(p_1-p_2)^2[m^2(p_1^2p_2^2)^{-1}(p_1p_2)+1]\},$$

$$t_{4,0}^{(1b)}=\frac{1}{8}\xi m\mathcal{K}^{-1}\mathcal{M}_1^{-1}\{(n-4)(n-6)\xi m^2\mathcal{K}\mathcal{M}_1^{-1}p_3^2+(n-2)(n-4)\xi\mathcal{K}+[1+(n-3)\xi]p_3^2(p_1p_2)\times[n-4-(n-1)\mathcal{K}^{-1}\mathcal{M}_1]-(n-1)m^2\mathcal{K}^{-1}\mathcal{M}_1p_3^2+(n-4)[1-(n-3)\xi]m^2p_3^2-(n-2)\times(n-3)\xi\mathcal{M}_1\},$$

$$t_{4,1}^{(1b)}=\frac{1}{8}\xi m\mathcal{K}^{-1}\mathcal{M}_1^{-1}\{(n-3)\xi[p_1^2p_2^2+m^2(p_1p_2)]\times[(n-6)\mathcal{M}_1^{-1}-(n-1)\mathcal{K}^{-1}]-(n-1)\mathcal{K}^{-1}\mathcal{M}_1+(n-3)(1+3\xi)\},$$

$$t_{4,2}^{(1b)}=\frac{1}{16}(n-2)(n-3)\xi^2m\mathcal{K}^{-1}\mathcal{M}_1^{-1}[(p_1p_2)+m^2],$$

$$t_{4,3}^{(1b)}=\frac{1}{16}(n-2)\xi m\mathcal{K}^{-1}\mathcal{M}_1^{-1}[\mathcal{M}_1(p_1p_2)(p_1^2p_2^2)^{-1}-(p_1p_2)-m^2+(n-3)\xi m^2-(n-6)\xi m^2\mathcal{K}\mathcal{M}_1^{-1}],$$

$$t_{4,4}^{(1b)}=-\frac{1}{16}(n-4)\xi m\mathcal{K}^{-1}\mathcal{M}_1^{-1}\{(p_1p_2)+m^2+(n-3)\xi(p_1p_2)+(n-6)\xi m^2\mathcal{K}\mathcal{M}_1^{-1}\},$$

$$t_{4,5}^{(1b)}=-\frac{1}{16}(n-2)\xi m\mathcal{K}^{-1}[(p_1^2p_2^2)^{-1}(p_1p_2)\times(p_1^2+p_2^2)-2],$$

$$t_{5,0}^{(1b)}=-\frac{1}{8}m\mathcal{K}^{-1}\{\xi p_3^2[p_1^2+p_2^2+(2-\xi)m^2]\times[(n-4)\mathcal{K}\mathcal{M}_1^{-1}-(n-3)]+[(n-3)(2-\xi)-1]\xi p_3^2[(p_1p_2)+m^2]+[12-2\xi+(n-4)\times\xi(4-\xi)]\mathcal{K}\},$$

$$t_{5,1}^{(1b)}=\frac{1}{8}\xi m\mathcal{K}^{-1},$$

$$t_{5,2}^{(1b)}=\frac{1}{16}(n-3)\xi m\mathcal{K}^{-1}\mathcal{M}_1^{-1}\{2\mathcal{K}(p_1^2+p_2^2-2m^2)-(p_1^2-p_2^2)^2[(p_1p_2)+m^2]-\xi p_3^2[p_1^2p_2^2+m^2(p_1p_2)]\},$$

$$t_{5,3}^{(1b)}=\frac{1}{16}(n-2)\xi m\mathcal{M}_1^{-1}\{(2-\xi)m^2-[\mathcal{M}_1(p_1^2p_2^2)^{-1}-1](p_1^2+p_2^2)\},$$

$$t_{5,4}^{(1b)}=\frac{1}{16}(n-4)\xi m\mathcal{M}_1^{-1}[p_1^2+p_2^2+(2-\xi)m^2],$$

$$t_{5,5}^{(1b)}=\frac{1}{16}\xi m\mathcal{K}^{-1}\mathcal{M}_1^{-1}(p_1^2-p_2^2)^2\{(n-3)(2-\xi)[p_1^2p_2^2+m^2(p_1p_2)]+(n-2)\mathcal{K}\mathcal{M}_1(p_1^2p_2^2)^{-1}-(n-3)(p_1^2+p_2^2)[(p_1p_2)+m^2]\},$$

$$t_{6,0}^{(1b)}=-\frac{1}{32}\mathcal{K}^{-1}(p_1^2-p_2^2)\{(n-3)(n-4)\xi^2(m^2\mathcal{M}_1^{-1}p_3^2+1)[(p_1p_2)+m^2]+(n-1)(n-3)\xi^2\mathcal{K}^{-1}p_3^2\times(p_1p_2)[(p_1p_2)+m^2]+[(n-2)\times(2-\xi)+(n-3)\xi^2]p_3^2-2[2-(n-3)\xi][(p_1p_2)+m^2]-(n-1)(2-\xi)\mathcal{K}^{-1}\mathcal{M}_1p_3^2\},$$

$$t_{6,1}^{(1b)}=\frac{1}{32}\mathcal{K}^{-2}(p_1^2-p_2^2)\{(n-1)(2-\xi)[(p_1p_2)+m^2]-(n-1)(n-3)\xi^2(p_1p_2)-(n-3)^2\xi^2m^2\mathcal{K}\mathcal{M}_1^{-1}\},$$

$$t_{6,2}^{(1b)}=-\frac{1}{64}\mathcal{K}^{-1}(p_1^2-p_2^2)[4-2(n-3)\xi-(n-3)\times(n-4)\xi^2],$$

$$t_{6,3}^{(1b)}=\frac{1}{64}(n-2)\mathcal{K}^{-1}(p_1^2-p_2^2)\{(n-3)\xi^2m^2\mathcal{M}_1^{-1}\times[(p_1p_2)+m^2]-(2-\xi)m^2(p_1^2p_2^2)^{-1}(p_1p_2)\},$$

$$t_{6,4}^{(1b)}=\frac{1}{64}(n-4)\mathcal{K}^{-1}\mathcal{M}_1^{-1}(p_1^2-p_2^2)\{(2-\xi)\mathcal{M}_1 -(n-3)\xi^2[p_1^2p_2^2+m^2(p_1p_2)]\},$$

$$t_{6,5}^{(1b)}=\frac{1}{64}\mathcal{K}^{-1}(p_1^2-p_2^2)\{(n-2)(2-\xi)m^2 \times[(p_1^2p_2^2)^{-1}(p_1p_2)(p_1^2+p_2^2)-2] +(p_1-p_2)^2[(n-4)(2+\xi) -2(n-3)\xi^2]\},$$

$$t_{7,0}^{(1b)}=\frac{1}{8}\xi m p_3^2[(n-4)\mathcal{M}_1^{-1}-(n-3)\mathcal{K}^{-1}],$$

$$t_{7,1}^{(1b)}=-\frac{1}{8}(n-3)\xi m\mathcal{K}^{-1}\mathcal{M}_1^{-1}[(p_1p_2)+m^2],$$

$$t_{7,2}^{(1b)}=0,$$

$$t_{7,3}^{(1b)}=\frac{1}{16}(n-2)\xi m[(p_1^2p_2^2)^{-1}-\mathcal{M}_1^{-1}],$$

$$t_{7,4}^{(1b)}=-\frac{1}{16}(n-4)\xi m\mathcal{M}_1^{-1},$$

$$t_{7,5}^{(1b)}=-\frac{1}{16}(n-2)\xi m(p_1^2p_2^2)^{-1}(p_1^2+p_2^2),$$

$$t_{8,0}^{(1b)}=\frac{1}{8}\mathcal{K}^{-1}\{(n-4)\xi(4-\xi)\mathcal{K}(m^2\mathcal{M}_1^{-1}p_3^2+1) +(n-3)\xi(4-\xi)p_3^2(p_1p_2)+(6-\xi)[p_3^2(p_1p_2) +m^2p_3^2+2\mathcal{K}]\},$$

$$t_{8,1}^{(1b)}=\frac{1}{8}\mathcal{K}^{-1}\mathcal{M}_1^{-1}\{(n-3)\xi(4-\xi)[p_1^2p_2^2+m^2(p_1p_2)] +(6-\xi)\mathcal{M}_1\},$$

$$t_{8,2}^{(1b)}=t_{8,5}^{(1b)}=0,$$

$$t_{8,3}^{(1b)}=-\frac{1}{16}(n-2)\xi(4-\xi)m^2\mathcal{M}_1^{-1},$$

$$t_{8,4}^{(1b)}=-\frac{1}{16}(n-4)\xi(4-\xi)m^2\mathcal{M}_1^{-1}.$$

In the limit $m\to 0$, $\tau_1^{(1)}$, $\tau_4^{(1)}$, $\tau_5^{(1)}$, and $\tau_7^{(1)}$ vanish. We also note that all $t_{i,4}^{(1a)}$ and $t_{i,4}^{(1b)}$ are proportional to $(n-4)$, as they should, since the transverse part cannot contain UV-poles in $\varepsilon$ at one loop.


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