

# Effective Lagrangian induced by the anomalous Wess-Zumino action and the exotic resonance state with $I^G(J^{PC})=1^-(1^{-+})$ in the $\rho\pi$ , $\eta\pi$ , $\eta'\pi$ , and $K^*\bar{K}+\bar{K}^*K$ channels

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A simple model for the exotic waves with  $I^G(J^{PC})=1^-(1^{-+})$  in the reactions  $VP\rightarrow VP$ ,  $VP\rightarrow PP$ , and  $PP\rightarrow PP$  is constructed beyond the scope of the quark-gluon approach. The model satisfies unitarity and analyticity and uses as a ‘‘priming’’ the ‘‘anomalous’’ nondiagonal  $VPPP$  interaction which couples together the four channels  $\rho\pi$ ,  $\eta\pi$ ,  $\eta'\pi$ , and  $K^*\bar{K}+\bar{K}^*K$ . The possibility of the resonancelike behavior of the  $I^G(J^{PC})=1^-(1^{-+})$  amplitudes belonging to the  $\{10\}-\{\bar{10}\}$  and  $\{8\}$  representations of  $SU(3)$  as well as their mixing is demonstrated explicitly in the 1.3–1.6 GeV mass range which, according to the current experimental evidence, is really rich in exotics.

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## I. INTRODUCTION

Phantoms of manifestly exotic states with  $I^G(J^{PC})=1^-(1^{-+})$  have more and more agitated the experimental and theoretical communities [1–20]. They were discovered in the 1.3–1.6 GeV mass range in the  $\eta\pi$ ,  $\eta'\pi$ ,  $\rho\pi$ ,  $b_1\pi$ , and  $f_1\pi$  systems produced in  $\pi^-p$  collisions at high energies and in  $N\bar{N}$  annihilation at rest [1–12].

Theoretical considerations concerning the mass spectra and decay properties of exotic hadrons have been based, in the main, on the MIT-bag model, constituent gluon model, flux-tube model, QCD sum rules, lattice calculations, and various selection rules. The more recent discussions of these constituent quark-gluon models and selection rules in conformity with the observed  $J^{PC}=1^{-+}$  phenomena can be found in Refs. [1–20], together with extensive analyses of the current experimental data and comprehensive references. A resonance character of the observed exotic signals and also the more popular assumption about their hybrid ( $q\bar{q}g$ ) nature are the subject of much attention and require further careful investigations [1,6,7,12,14–20].

Let us note that evidence for the possible existence of an exotic  $J^{PC}=1^{-+}$  state coupled to the  $\eta\pi$  and  $\rho\pi$  channels and belonging to the icosuplet representation of  $SU(3)$  was obtained for the first time by using the bootstrap technique of Schechter and Okubo more than 35 years ago [21] (see also Ref. [22]).

Current algebra and effective chiral Lagrangians are also important sources of theoretical information on exotic partial waves. It is sufficient to remember the prediction obtained within the framework of these approaches for the  $\pi\pi$   $s$ -wave scattering length with isospin  $I=2$  [23,24]. Constructing with the help of the effective chiral Lagrangians the series expansions of the scattering amplitudes in powers of external momenta, one can reveal explicitly exotic contributions al-

ready among the lower order terms of these series. Can at least some of these contributions found at low energies turn out to be the manifestations (‘‘the tails’’) of high-lying exotic resonances? It is well known that, for example, for the  $\pi\pi$  scattering channels involving the  $\sigma$  and  $\rho$  resonances, one can self-consistently (in the sense of agreement with experiment) sew together the resonancelike and low-energy behaviors of the scattering amplitudes by using the successfully selected unitarization scheme for the original chiral contributions, together with general analyticity requirements [25–34]. In other words, for these channels, there exist a good many of the model constructions which show that the low-energy contributions calculated within the effective chiral Lagrangians framework may in principle transform with increasing energy into resonances with the experimentally established parameters. In the present work we continue in this way and construct a model satisfying unitarity and analyticity for an exotic wave with  $J^{PC}=1^{-+}$  in the reaction  $\rho\pi\rightarrow\eta\pi$  and in the related reactions  $\rho\pi\rightarrow\eta'\pi$ ,  $\rho\pi\rightarrow\rho\pi$ ,  $\eta\pi\rightarrow\eta\pi$ ,  $\eta\pi\rightarrow(K^*\bar{K}+\bar{K}^*K)$ , and so on, using as a ‘‘priming’’ the tree exotic amplitudes generated by a simplest ‘‘anomalous’’ effective interaction of the vector ( $V$ ) and pseudoscalar ( $P$ ) mesons. The interaction is induced by the anomalous Wess-Zumino chiral Lagrangian [35] and is proportional to  $\epsilon_{\mu\nu\tau\kappa}$ .

At the tree level, the standard nonlinear chiral Lagrangian describing the low-energy dynamics of the pseudoscalar mesons belonging to the  $SU(3)$  octet generates the  $PP\rightarrow PP$  scattering amplitudes possessing only the usual quantum numbers  $J^{PC}=0^{++}$  and  $1^{--}$  in the  $s$  channel [24]. However, already in the next order of chiral perturbation theory, the  $J^{PC}=1^{-+}$  exotic contributions arise in these amplitudes at the expense of the finite parts of the one-loop diagrams. In so doing they turn out to be different from zero only owing to  $SU(3)$  symmetry breaking for pseudoscalar masses. The resonances, with which such contributions might be associated, have to possess rather odd properties. All their coupling constants to the octet of pseudoscalar mesons must vanish in the  $SU(3)$  symmetry limit. Therefore, it seems more reasonable to assume that if the exotic resonances with  $J^{PC}=1^{-+}$  exist, then they are of another origin. In such a case,

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the resources for their possible generation, which still remain within the effective chiral Lagrangian framework, seem to involve the ‘‘anomalous’’ interactions of the vector and pseudoscalar mesons [36–43]. Some indirect evidence in favor of this assumption has been given by the analysis of the  $PP \rightarrow PP$  scattering amplitudes carried out in the framework of the linear  $SU(3) \times SU(3)$   $\sigma$  model involving only scalars and pseudoscalars [28]. There operate the repulsive forces in the  $J^{PC} = 1^{-+}$  channels in this model, and any resonance states do not arise.

In Sec. II, the general properties of the  $VP \rightarrow PP$  reaction amplitudes are briefly discussed within the framework of the unitary symmetry assumption. In Sec. III, a simple model for the  $I=1$   $p$ -wave (exotic) reaction amplitudes  $VP \rightarrow PP$ ,  $VP \rightarrow VP$ , and  $PP \rightarrow PP$  is constructed. The model takes into account as a ‘‘priming’’ the nondiagonal  $VPPP$  interaction which couples together the four channels  $\rho\pi$ ,  $\eta\pi$ ,  $\eta'\pi$ , and  $K^*\bar{K} + \bar{K}^*K$ . It is essentially the summing up of all the  $s$ -channel loop diagrams with the  $VP$  and  $PP$  intermediate states. In Sec. IV, the possibility of the resonance-like behavior of the  $I^G(J^{PC}) = 1^-(1^{-+})$  amplitudes belonging to the  $\{10\} - \{\bar{10}\}$  and  $\{8\}$  representations of  $SU(3)$  as well as their mixing is demonstrated explicitly in the 1.3–1.6 GeV mass range. In quark-gluon language, the  $\{10\} - \{\bar{10}\}$  representation of  $SU(3)$  first occurs in the  $qq\bar{q}\bar{q}$  sector, whereas the states with  $J^{PC} = 1^{-+}$  belonging to the octet representation of  $SU(3)$  may in principle correspond to both  $qq\bar{q}\bar{q}$  and  $q\bar{q}g$  configurations.

## II. GENERAL PROPERTIES OF THE $VP \rightarrow PP$ AMPLITUDE

The general Lorentz and  $SU(3)$  structure for the amplitude of the reaction  $V_a(k) + P_b(q_1) \rightarrow P_c(q_2) + P_d(q_3)$ , where  $V_a$  and  $P_a$  are the members of the vector and pseudoscalar octets taken in the Cartesian basis ( $a=1, \dots, 8$ ),<sup>1</sup> and  $k$ ,  $q_1$ ,  $q_2$ , and  $q_3$  are the four-momenta of the particles in the reaction, has the form

$$M_{ab;cd}^{(\lambda)} = -i\epsilon_{\mu\nu\tau\kappa} e_{(\lambda)}^{\mu} q_1^{\nu} q_2^{\tau} q_3^{\kappa} [f_{abm} d_{mcd} A(s, t, u) + d_{abm} f_{mcd} B(s, t, u) + (u_{ab})_{cd} C(s, t, u)]. \quad (1)$$

Here  $f_{abc}$  and  $d_{abc}$  are the standard structure constants of  $SU(3)$  [44],  $(u_{ab})_{cd} = f_{cam} d_{mbd} - d_{cam} f_{mbd}$  [45],  $e_{(\lambda)}^{\mu}$  is a  $\mu$  component of the  $V$  meson polarization vector with helicity  $\lambda$ ,  $s = (k + q_1)^2$ ,  $t = (k - q_2)^2$ , and  $u = (k - q_3)^2$ . From Bose symmetry it follows that the invariant amplitude  $A(s, t, u)$  is antisymmetric under the interchange of the  $t$  and  $u$  variables, whereas the invariant amplitudes  $B(s, t, u)$  and  $C(s, t, u)$  are symmetric. Note that Eq. (1) can be obtained in the usual way [45–48] by applying  $SU(3)$  symmetry, together with  $P$  and  $C$  invariance.

<sup>1</sup> $V_a = (\rho_1, \rho_2, \rho_3, K_4^*, K_5^*, K_6^*, K_7^*, \omega_8)$  and  $P_a = (\pi_1, \pi_2, \pi_3, K_4, K_5, K_6, K_7, \eta_8)$ .

The first and second terms in Eq. (1) correspond to the octet transition amplitudes  $\{8_a\} \rightarrow \{8_s\}$  and  $\{8_s\} \rightarrow \{8_a\}$  which we shall designate for short by  $A_{as}$  and  $A_{sa}$ , respectively; as usual,  $\{8_s\}$  and  $\{8_a\}$  mean the symmetric and antisymmetric octet representations of  $SU(3)$  which occur in the direct production of  $\{8\} \times \{8\}$ . The third term in Eq. (1) describes transitions via the mutually conjugate representations  $\{10\}$  and  $\{\bar{10}\}$  with the amplitudes  $A_{10}$  and  $A_{\bar{10}}$  appearing in the combination  $A_{10} - A_{\bar{10}}$ . In other words, it describes the transitions from the initial  $VP$  isocuplet  $\{10\} - \{\bar{10}\}$  to the final  $PP$  isocuplet  $\{10\} + \{\bar{10}\}$ . The transition amplitudes between the self-conjugate representations  $A_{as}$  and  $A_{sa}$  can be expanded into the partial waves with  $J^{PC} = 2^{++}, 4^{++}, \dots$  and  $J^{PC} = 1^{--}, 3^{--}, \dots$ , respectively. Hence they do not contain any explicitly exotic contributions. As for the  $\eta_8\pi$  final state, it does not occur in the  $\{8_a\}$  but can belong to the representations  $\{8_s\}$ ,  $\{10\}$ , and  $\{\bar{10}\}$  [21,22,49]. The  $SU(3)$  exotic meson amplitudes  $A_{10}$  and  $A_{\bar{10}}$  can be expanded into partial waves with  $J^P = 1^-, 3^-, \dots$ . The isotriplet amplitudes of  $A_{10} - A_{\bar{10}}$  correspond to two sets of the reactions with opposite  $G$  parity in the  $s$  channel: (a)  $\rho\eta_8 \rightarrow \pi\pi$ ,  $\rho\eta_8 \rightarrow K\bar{K}$ ,  $\omega_8\pi \rightarrow \pi\pi$ ,  $\omega_8\pi \rightarrow K\bar{K}$  and (b)  $\rho\pi \rightarrow \eta_8\pi$ ,  $K^*\bar{K} \rightarrow \eta_8\pi$ ,  $\bar{K}^*K \rightarrow \eta_8\pi$ . The partial amplitudes of the reactions belonging to set (a) possess the nonexotic quantum numbers  $I^G(J^{PC}) = 1^+(1^{--}, 3^{--}, \dots)$  and, therefore, only hidden  $SU(3)$  exotics. The reactions belonging to set (b) are purely exotic because they contain the partial waves with  $I^G(J^{PC}) = 1^-(1^{-+}, 3^{-+}, \dots)$ . In particular, it is these reactions that will be the subject of our attention in the following.

Let us now write down the amplitude for the reaction  $V_a(k) + P_b(q_1) \rightarrow P_c(q_2) + P_d(q_3)$  involving the  $V_0$  and  $P_0$   $SU(3)$  singlets:

$$N_{ab;cd}^{(\lambda)} = -i\epsilon_{\mu\nu\tau\kappa} e_{(\lambda)}^{\mu} q_1^{\nu} q_2^{\tau} q_3^{\kappa} [\delta_{a0} f_{bcd} D(s, t, u) + \delta_{b0} f_{acd} E(s, t, u) + \delta_{c0} f_{abd} F(s, t, u)], \quad (2)$$

where  $a, b, c, d$  are the flavor indices running now over  $0, 1, \dots, 8$ ,  $f_{a0} = 0$ ,  $V_0 = \omega_0$ , and  $P_0 = \eta_0$ . By the isoscalar particles with the definite masses we shall mean the pseudoscalar mesons  $\eta = \eta_8 \cos \theta_p - \eta_0 \sin \theta_p$  and  $\eta' = \eta_8 \sin \theta_p + \eta_0 \cos \theta_p$  with the mixing angle  $\theta_p \approx -20^\circ$  [50,51] and the vector mesons  $\omega = \sqrt{1/3}\omega_8 + \sqrt{2/3}\omega_0$  and  $\phi = \sqrt{2/3}\omega_8 - \sqrt{1/3}\omega_0$  with ‘‘ideal mixing.’’ Equation (2) describes the transitions via the  $SU(3)$  octet intermediate states. The first two terms in Eq. (2) do not contribute to  $\eta\pi$  and  $\eta'\pi$  production because they correspond to the transitions into the final states belonging to the  $\{8_a\}$  representation which does not contain the  $\eta_8\pi$  system. The third term in Eq. (2) describes  $\eta\pi$  and  $\eta'\pi$  production via the  $SU(3)$  singlet components of the  $\eta$  and  $\eta'$ . Under  $t \leftrightarrow u$  interchange, the invariant amplitudes  $D(s, t, u)$  and  $E(s, t, u)$  are symmetric, while the invariant amplitude  $F(s, t, u)$  does not possess a definite symmetry. Thus, the first two terms in Eq. (2) can be expanded into partial waves with  $J^{PC} = 1^{--}, 3^{--}, \dots$  and the third term into partial waves with  $J^{PC} = 1^{-+}, 2^{++}, 3^{-+}, 4^{++}, \dots$ , of which the odd waves

$1^{-+}, 3^{-+}, \dots$  are exotic. In principle, Eqs. (1) and (2) permit the  $VP \rightarrow PP$  reaction channels involving the  $\omega$ ,  $\phi$ ,  $\eta$ , and  $\eta'$  mesons to be considered in the most general form.

### III. MODEL FOR THE $I^G(J^{PC})=1^-(1^{-+})$ WAVES IN THE REACTIONS $VP \rightarrow PP$ , $PP \rightarrow PP$ , AND $VP \rightarrow VP$

Consider the  $SU(3)$  symmetric effective Lagrangian for the pointlike  $VPPP$  interaction which also possesses additional nonet symmetry with respect to the vector mesons,

$$L(VPPP) = ih \epsilon_{\mu\nu\tau\kappa} \text{Tr}(\hat{V}^\mu \partial^\nu \hat{P} \partial^\tau \hat{P} \partial^\kappa \hat{P}) + i\sqrt{1/3} h' \epsilon_{\mu\nu\tau\kappa} \text{Tr}(\hat{V}^\mu \partial^\nu \hat{P} \partial^\tau \hat{P}) \partial^\kappa \eta_0, \quad (3)$$

where  $\hat{P} = \sum_{a=1}^8 \lambda_a P_a / \sqrt{2}$ ,  $\hat{V}^\mu = \sum_{a=0}^8 \lambda_a V_a^\mu / \sqrt{2}$ , and  $\lambda_a$  are the Gell-Mann matrices [44]. The tree amplitudes for the reactions  $VP \rightarrow PP$  generated by the Lagrangian (3) are

$$T_{ij} = \begin{bmatrix} T(\rho\pi \rightarrow \rho\pi) & T(\rho\pi \rightarrow \eta\pi) & T(\rho\pi \rightarrow \eta'\pi) & T(\rho\pi \rightarrow K^*K) \\ T(\eta\pi \rightarrow \rho\pi) & T(\eta\pi \rightarrow \eta\pi) & T(\eta\pi \rightarrow \eta'\pi) & T(\eta\pi \rightarrow K^*K) \\ T(\eta'\pi \rightarrow \rho\pi) & T(\eta'\pi \rightarrow \eta\pi) & T(\eta'\pi \rightarrow \eta'\pi) & T(\eta'\pi \rightarrow K^*K) \\ T(K^*K \rightarrow \rho\pi) & T(K^*K \rightarrow \eta\pi) & T(K^*K \rightarrow \eta'\pi) & T(K^*K \rightarrow K^*K) \end{bmatrix}. \quad (4)$$

Here the subscripts  $i, j=1,2,3,4$  are the labels of the  $\rho\pi$ ,  $\eta\pi$ ,  $\eta'\pi$ , and  $K^*K$  channels, respectively, and the abbreviation  $K^*K$  implies just the  $\bar{K}^*K$  and  $K^*\bar{K}$  channels. The corresponding matrix of the  $VPPP$  coupling constants generated by the Lagrangian (3) has the form

$$h_{ij} = h \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & \gamma \\ \beta & 0 & 0 & \delta \\ 0 & \gamma & \delta & 0 \end{bmatrix}, \quad (5)$$

where

$$\begin{aligned} \alpha &= \sqrt{\frac{1}{3}} \cos \theta_P - \frac{h'}{h} \sqrt{\frac{2}{3}} \sin \theta_P, \\ \beta &= \sqrt{\frac{1}{3}} \sin \theta_P + \frac{h'}{h} \sqrt{\frac{2}{3}} \cos \theta_P, \\ \gamma &= \sqrt{\frac{2}{3}} \cos \theta_P + \frac{h'}{h} \sqrt{\frac{1}{3}} \sin \theta_P, \\ \delta &= \sqrt{\frac{2}{3}} \sin \theta_P - \frac{h'}{h} \sqrt{\frac{1}{3}} \cos \theta_P. \end{aligned} \quad (6)$$

$$\delta = \sqrt{\frac{2}{3}} \sin \theta_P - \frac{h'}{h} \sqrt{\frac{1}{3}} \cos \theta_P. \quad (7)$$

In the following we shall consider three natural limiting cases: (i)  $h'=0$ , i.e., when all exotic amplitudes belong to the  $\{10\}-\{\bar{1}0\}$  representation of  $SU(3)$ ; (ii)  $h=0$ , i.e., when

given by Eqs. (1) and (2) with the following sets of the invariant amplitudes (here we omit their arguments for short):  $(A, B, C, D) = h(0, 2, 1, \sqrt{6})$  and  $(E, F) = h'(\sqrt{2/3}, -\sqrt{2/3})$ . The presence of the amplitudes  $C$  and  $F$  such as above implies (see the discussion in Sec. II) that the Lagrangian (3) generates tree exotic amplitudes with  $I^G(J^{PC})=1^-(1^{-+})$  belonging to the  $\{10\}-\{\bar{1}0\}$  representation of  $SU(3)$  for the inelastic reactions  $\rho\pi \rightarrow \eta_8\pi$ ,  $K^*\bar{K} \rightarrow \eta_8\pi$ , and  $\bar{K}^*K \rightarrow \eta_8\pi$  as well as the amplitudes belonging to the octet representation of  $SU(3)$  for the reaction  $\rho\pi \rightarrow \eta_0\pi$ ,  $K^*\bar{K} \rightarrow \eta_0\pi$ , and  $\bar{K}^*K \rightarrow \eta_0\pi$ . In the next orders, these tree amplitudes induce as well the  $I^G(J^{PC})=1^-(1^{-+})$  exotic ones for the elastic processes  $\rho\pi \rightarrow \rho\pi$ ,  $\eta\pi \rightarrow \eta\pi$ , and so on. In this connection it is of interest to consider the following  $4 \times 4$  system of scattering amplitudes for the coupled exotic channels of the reactions  $VP \rightarrow VP$ ,  $VP \leftrightarrow PP$ , and  $PP \rightarrow PP$ :

all exotic amplitudes belong to the octet representation of  $SU(3)$ ; and (iii)  $h'=h$ , when the original pointlike  $VPPP$  interaction possesses nonet symmetry with respect to the pseudoscalar mesons.

To satisfy the unitarity condition for the coupled channel amplitudes, we sum up all the possible chains of the  $s$ -channel loop diagrams the typical examples of which are shown in Fig. 1. Such an old-fashioned field theory way of the unitarization is well known in the literature (see, for example, Refs. [52–55, 32]). Notice that in case (i) and in case (ii) the whole complex of the unitarized amplitudes, in fact, can be constructed by using only the amplitudes for the loop diagrams shown explicitly in Fig. 1. The point is that in these cases the denominator of the corresponding geometrical series for any channel turns out to be proportional to the sum of diagrams (b), (c), (d), and (e) in Fig. 1, and the loop diagrams of the type (f) and (g), or (h) and (i), play a role of a ‘‘priming’’ in the corresponding elastic channels like diagram (a) in the inelastic  $\rho\pi \rightarrow \eta\pi$  channel. However, in case (iii) the situation is considerably more complicated.

Before summing the diagrams, let us make two remarks about the model itself.

First, generally speaking, the pointlike exotic contributions due to the  $VPPP$  interaction might be modified by the tree diagrams involving  $V$  meson exchanges if one takes into account the ‘‘anomalous’’ Lagrangian for the  $VVP$  interaction and the ordinary one for the  $VPP$  interaction. However, such a considerable complication of the original exotic amplitudes actually does not lead to any new possibilities (or degrees of freedom) to obtain the resonancelike behavior of

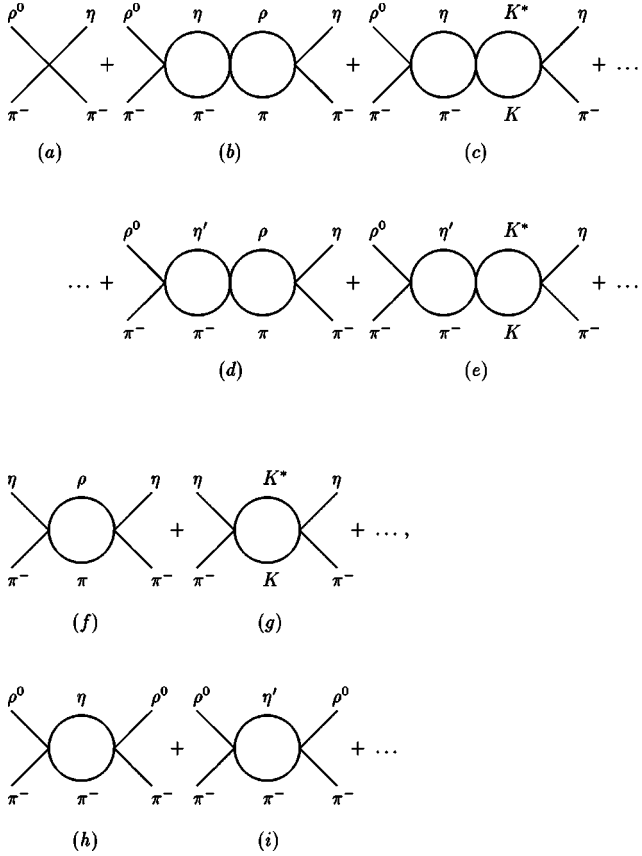


FIG. 1. The examples of the diagrams which are summed to obtain the unitarized amplitudes in coupled channels.

the complete unitarized amplitudes. This only burdens the model by additional technical difficulties and makes it much less transparent in comparison with the one based only on the Lagrangian (3). For example, after such a modification of the original exotic amplitudes, the above-mentioned obvious unitarization scheme need be changed, say, by some version of the Padé approximation [27,28] because of the impossibility of the direct calculation and summation of higher loops.

Second, the effective coupling constant  $h$  occurring in the Lagrangian (3) is not the unambiguously definite value in the theory with the “anomalous” chiral Lagrangians (comprehensive discussions of this point may be found in Refs. [36–43]). Actually, one may only claim that it is not too large in the scale defined by the combination  $2g_{\rho\pi\pi}g_{\omega\rho\pi}/m_\rho^2 \approx 284 \text{ GeV}^{-3}$  [40,43]. Therefore, we consider the coupling constants  $h$  and  $h'$  as free parameters of the model in the region of their relatively small values.

The summing up of the loop diagram chains can be easily carried out by using the matrix equation for the auxiliary amplitudes  $\tilde{T}_{ij}$ ,

$$\tilde{T}_{ij} = h_{ij} + h_{im}\Pi_{mn}\tilde{T}_{nj}, \quad (8)$$

which is shown schematically in Fig. 2. The auxiliary amplitudes  $\tilde{T}_{ij}$  pertain to the hypothetical case when all the particles in the reactions are spinless, but otherwise they are the exact analogs of the physical amplitudes  $T_{ij}$  designated in

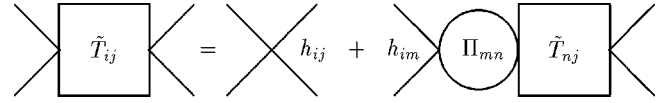


FIG. 2. The auxiliary equation for the summing up of the diagram series some examples of which are shown in Fig. 1.

Eq. (4). So in Eq. (8) the matrix  $h_{ij}$  is given by Eqs. (5), (6), and (7), and  $\Pi_{ij}$  is the diagonal matrix of the loops, i.e.,  $\Pi_{ij} = \delta_{ij}\Pi_j$ , where  $\Pi_1$ ,  $\Pi_2$ ,  $\Pi_3$ , and  $\Pi_4$  correspond to the four independent  $s$ -channel loops involving the  $\rho\pi$ ,  $\eta\pi$ ,  $\eta'\pi$ , and  $K^*K$  intermediate states, respectively (for a moment, all for the spinless case). Notice that if all the particles are spinless, then the  $h_{ij}$  and  $\Pi_{ij}$  in Eq. (8) are dimensionless, as well as the  $\tilde{T}_{ij}$  themselves. For the matrix elements  $h_{im}\Pi_{mj} = h_{ij}\Pi_j$  it is convenient to introduce the following compact notation [look at Eq. (5)]:

$$h_{im}\Pi_{mj} = h_{ij}\Pi_j = h \begin{bmatrix} 0 & \alpha_2 & \beta_3 & 0 \\ \alpha_1 & 0 & 0 & \gamma_4 \\ \beta_1 & 0 & 0 & \delta_4 \\ 0 & \gamma_2 & \delta_3 & 0 \end{bmatrix}, \quad (9)$$

where  $\alpha_1 = \alpha\Pi_1$ ,  $\beta_3 = \beta\Pi_3$ , and so on. The solution of Eq. (8) has the form

$$\tilde{T}_{ij} = [(\hat{1} - \hat{h}\hat{\Pi})^{-1}]_{im}h_{mj}, \quad (10)$$

where  $\hat{1}$  is the  $4 \times 4$  identity matrix and the matrix  $\hat{h}\hat{\Pi}$  is defined by the relations of Eq. (9).

Next, we define

$$\begin{aligned} \bar{D} &= \det(\hat{1} - \hat{h}\hat{\Pi}) \\ &= 1 - h^2(\alpha_1\alpha_2 + \beta_1\beta_3 + \gamma_2\gamma_4 + \delta_3\delta_4) \\ &\quad + h^4(\alpha_1\delta_4 - \beta_1\gamma_4)(\alpha_2\delta_3 - \beta_3\gamma_2). \end{aligned} \quad (11)$$

Let us now write down, as an example, the explicit expressions for the amplitudes of the following five reactions:

$$\begin{aligned} \tilde{T}(\rho\pi \rightarrow \rho\pi) &= h^2[\alpha\alpha_2 + \beta\beta_3 - h^2(\alpha\delta_4 - \beta\gamma_4) \\ &\quad \times (\alpha_2\delta_3 - \beta_3\gamma_2)]/\bar{D}, \end{aligned} \quad (12)$$

$$\tilde{T}(\rho\pi \rightarrow \eta\pi) = h[\alpha - h^2(\alpha\delta_3 - \beta_3\gamma)\delta_4]/\bar{D}, \quad (13)$$

$$\tilde{T}(\rho\pi \rightarrow \eta'\pi) = h[\beta + h^2(\alpha_2\delta - \beta\gamma_2)\gamma_4]/\bar{D}, \quad (14)$$

$$\tilde{T}(\rho\pi \rightarrow K^*K) = h^2[\alpha_2\gamma + \beta_3\delta]/\bar{D}. \quad (15)$$

$$\begin{aligned} \tilde{T}(\eta\pi \rightarrow \eta\pi) &= h^2[\alpha\alpha_1 + \gamma\gamma_4 - h^2(\alpha\delta_3 - \beta_3\gamma) \\ &\quad \times (\alpha_1\delta_4 - \beta_1\gamma_4)]/\bar{D}. \end{aligned} \quad (16)$$

In cases (i) and (ii), the combination  $h^2(\alpha\delta - \beta\gamma) = 0$  [see Eqs. (5), (6), and (7)], so that the contributions proportional to that vanish in Eqs. (10)–(16) and, as one can see, all the formulas are essentially simplified [for example, the numerator in Eq. (13) becomes simply equal to  $h\alpha$ , since, according to Eq. (9),  $h^2(\alpha\delta_3 - \beta_3\gamma) = h^2(\alpha\delta - \beta\gamma)\Pi_3$ ].



Let us now take into account the spin of the particles. Consider the three different processes

$$\rho^0(k) + \pi^-(q_1) \rightarrow \rho^0(k') + \pi^-(q'_1), \quad (17)$$

$$\rho^0(k) + \pi^-(q_1) \rightarrow \eta(q_2) + \pi^-(q_3), \quad (18)$$

$$\eta(p) + \pi^-(q) \rightarrow \eta(q_2) + \pi^-(q_3). \quad (19)$$

Let  $Q = k + q_1 = k' + q'_1 = q_2 + q_3 = p + q$  and  $s = Q^2$ . Straightforward calculations with the help of the Lagrangian (3) of arbitrary terms of the relevant diagram series results in the following Lorentz structures and angular dependences for the corresponding physical amplitudes:

$$\begin{aligned} T^{(\lambda', \lambda)}(\rho^0 \pi^- \rightarrow \rho^0 \pi^-) &= \epsilon_{\mu' \nu' \tau' \sigma} e_{(\lambda')}^{\mu'} q_1^{\nu'} k'^{\tau'} \epsilon_{\mu \nu \tau}^{\sigma} e_{(\lambda)}^{\mu} q_1^{\nu} k^{\tau} \tilde{T}'(\rho \pi \rightarrow \rho \pi) \\ &= (\delta_{\lambda, +1} + \delta_{\lambda, -1})(\delta_{\lambda', +1} + \delta_{\lambda', -1})(s|\vec{q}_1|^2/2) \\ &\quad \times (\lambda \lambda' + \cos \theta) \tilde{T}'(\rho \pi \rightarrow \rho \pi), \end{aligned} \quad (20)$$

$$\begin{aligned} T^{(\lambda)}(\rho^0 \pi^- \rightarrow \eta \pi^-) &= \epsilon_{\mu \nu \tau \sigma} e_{(\lambda)}^{\mu} q_1^{\nu} q_2^{\tau} q_3^{\sigma} \tilde{T}'(\rho \pi \rightarrow \eta \pi) \\ &= -(\delta_{\lambda, +1} + \delta_{\lambda, -1}) i \sqrt{s/2} |\vec{q}_1| |\vec{q}_3| \sin \theta \tilde{T}'(\rho \pi \rightarrow \eta \pi), \end{aligned} \quad (21)$$

$$T(\eta \pi^- \rightarrow \eta \pi^-) = |\vec{q}|^2 \cos \theta \tilde{T}'(\eta \pi^- \rightarrow \eta \pi^-), \quad (22)$$

where  $\lambda$  ( $\lambda'$ ) is the initial (final)  $\rho$  meson helicity and  $\theta$  is the angle between the momenta of the initial and final pions in the reaction c.m. system. Certainly the dimensions of all physical amplitudes  $T$  in Eqs. (20)–(22) are the same: the amplitudes are dimensionless. At the same time, as is seen from Eqs. (20)–(22), The invariant amplitudes  $\tilde{T}'$  have different dimensions in the  $VP \rightarrow VP$ ,  $VP \rightarrow PP$ , and  $PP \rightarrow PP$  channels. These invariant amplitudes are obtained directly from the corresponding auxiliary amplitudes  $\tilde{T}$  [see Eqs. (10)–(16)] by substituting the physical dimensional coupling constants  $h$  and  $h'$  from the Lagrangian (3) and the following expressions for the  $p$ -wave loop integrals:

$$\Pi_i = \frac{1}{16\pi} \frac{2}{3} F_i \times \begin{cases} 4s, & i=1,4 \text{ (VP loops)}, \\ 1, & i=2,3 \text{ (PP loops)}, \end{cases} \quad (23)$$

where

$$\begin{aligned} F_i &= C_{1i} + s C_{2i} + \frac{s^2}{\pi} \int_{m_{i+}^2}^{\infty} \frac{[P_i(s')]^3 ds'}{\sqrt{s'} s'^2 (s' - s - i\epsilon)} = C_{1i} + s C_{2i} + \frac{(s - m_{i+}^2)^{3/2} (s - m_{i-}^2)^{3/2}}{8\pi s^2} \left[ \ln \left( \frac{\sqrt{s - m_{i-}^2} - \sqrt{s - m_{i+}^2}}{\sqrt{s - m_{i-}^2} + \sqrt{s - m_{i+}^2}} \right) + i\pi \right] \\ &\quad + \frac{1}{4\pi} \left\{ \frac{1}{2m_{i+}m_{i-}} \ln \left( \frac{m_{i+} - m_{i-}}{m_{i+} + m_{i-}} \right) \left[ \frac{m_{i+}^4 m_{i-}^4}{s^2} - \frac{3m_{i+}^2 m_{i-}^2}{2s} (m_{i+}^2 + m_{i-}^2) + \frac{3}{8} (m_{i+}^4 + m_{i-}^4 + 6m_{i+}^2 m_{i-}^2) \right. \right. \\ &\quad \left. \left. + \frac{s(m_{i+}^2 + m_{i-}^2)}{16m_{i+}^2 m_{i-}^2} (m_{i+}^4 - 10m_{i+}^2 m_{i-}^2 + m_{i-}^4) \right] + \frac{m_{i+}^2 m_{i-}^2}{2s} - \frac{5}{8} (m_{i+}^2 + m_{i-}^2) + \frac{s(3m_{i+}^4 + 3m_{i-}^4 + 38m_{i+}^2 m_{i-}^2)}{48m_{i+}^2 m_{i-}^2} \right\}. \end{aligned} \quad (24)$$

Here  $P_i(s) = [(s - m_{i+}^2)(s - m_{i-}^2)/(4s)]^{1/2}$ ,  $m_{i+}$  ( $m_{i-}$ ) is the sum (the difference) of the particle masses in channel  $i$ , and  $C_{1i}$  and  $C_{2i}$  are the subtraction constants. Note that the expression (24) is valid for  $s \geq m_{i+}^2$ . In the regions  $m_{i-}^2 < s < m_{i+}^2$  and  $s \leq m_{i-}^2$ , it changes according to analytic continuation [56].

#### IV. ANALYSIS OF THE POSSIBLE RESONANCE PHENOMENA

First of all let us note that a number of free parameters in the present model can be reduced essentially, leaving its potentialities almost unchanged. So we shall assume that  $C_{11} = C_{14}$ ,  $C_{21} = C_{24}$  for the  $VP$  loops and  $C_{12} = C_{13}$ ,  $C_{22} = C_{23}$  for  $PP$  loops. Moreover, near a feasible resonance, the smooth  $s$  dependence of the combinations  $C_{1i} + s C_{2i}$  is not of crucial importance. Thus, as the essential free parameters

we can leave only the  $C_{11}$  and  $C_{12}$  ones, setting  $C_{21} = C_{22} = 0$ . Just this will be done in most variants considered below. A simplest way to discover ‘‘by hand’’ a possible resonance situation is that to find zero of  $\text{Re}(\bar{D})$  at fixed values of  $h$ ,  $h'$ , and  $\sqrt{s}$ , for example, at  $\sqrt{s} = 1.43$  GeV [see Eqs. (11)–(16)]. In so doing the left free subtraction constants  $C_{11}$  and  $C_{12}$  are not uniquely determined. For example, in cases (i) and (ii), the condition  $\text{Re}(\bar{D}) = 0$  gives only a relation of the type  $C_{12} = (\xi_1 + \xi_2 C_{11}) / (\xi_3 + \xi_4 C_{11})$ , where  $\xi_i$  are the known numbers. However, this is not the weak point of the model; on the contrary, this allows the shapes of the resonance curves and the relations between the absolute cross section values in the different channels to be easily changed by changing  $C_{11}$ .

According to the detailed analysis performed in Refs. [36–40,43], the acceptable tentative values of the parameter  $\tilde{h} \equiv F_{\pi}^3 h$  (where  $F_{\pi} \approx 130$  MeV) lie within the range  $|\tilde{h}|$

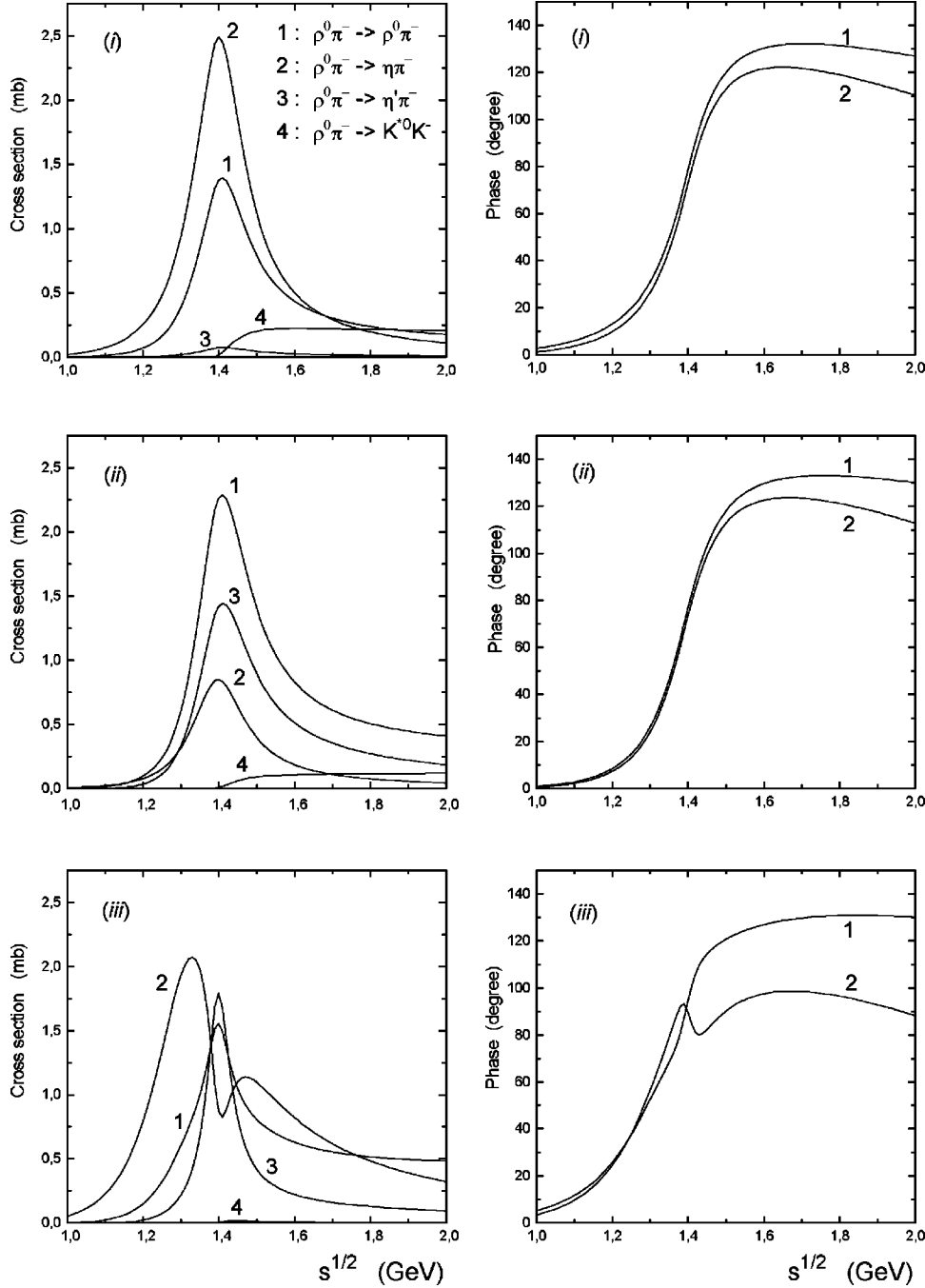


FIG. 3. The cross sections of the reactions  $\rho^0\pi^- \rightarrow \rho^0\pi^-$ ,  $\rho^0\pi^- \rightarrow \eta\pi^-$ ,  $\rho^0\pi^- \rightarrow \eta'\pi^-$ , and  $\rho^0\pi^- \rightarrow K^{*0}K^-$  and the phases of the  $\rho\pi \rightarrow \rho\pi$  and  $\rho\pi \rightarrow \eta\pi$  amplitudes for cases (i), (ii), and (iii). The correspondence between the curve numbers and the reaction channels is shown just in the figure. The values used of the parameters are listed in Table I.

$\leq 0.4$ . To illustrate the existence of the resonance phenomena in our toy model we are guided by the values of  $\tilde{h}$  (and  $\tilde{h}' \equiv F_\pi^3 h'$ ) near 0.1. We would like particularly to emphasize that, in fact, the resonance phenomena are possible in the present model for any  $|\tilde{h}| \leq 0.4$ . However, as  $|\tilde{h}|$  (and/or  $|\tilde{h}'|$ ) increases from 0.1 to 0.4, the distinct resonancelike enhancements in the reaction cross sections move into the region  $\sqrt{s} \approx 1 - 1.3$  GeV. Note that the unitarized amplitudes essentially depend on the second and fourth powers of coupling constants and therefore are very sensitive to changes of  $|\tilde{h}|$  and  $|\tilde{h}'|$ .

In Figs. 3 and 4, we show the typical energy dependences, which occur in our model for cases (i), (ii), and (iii), for the

four reaction cross sections  $\sigma(\rho^0\pi^- \rightarrow \rho^0\pi^-)$ ,  $\sigma(\rho^0\pi^- \rightarrow \eta\pi^-)$ ,  $\sigma(\rho^0\pi^- \rightarrow \eta'\pi^-)$ , and  $\sigma(\rho^0\pi^- \rightarrow K^{*0}K^-)$  and for the phases of the  $\rho\pi \rightarrow \rho\pi$  and  $\rho\pi \rightarrow \eta\pi$  amplitudes (note that the inelastic amplitude  $\rho\pi \rightarrow \eta\pi$  is defined only up to the sign). Figure 3, together with Table I, and Fig. 4, together with Table II, illustrate the resonance effects when they concentrate mainly in the regions  $\sqrt{s} \approx 1.3 - 1.4$  GeV and  $\sqrt{s} \approx 1.5 - 1.6$  GeV, respectively. As a rule, the channel  $\rho\pi \rightarrow \eta\pi$  is dominant in case (i), when all considered amplitudes belong to the  $\{10\} - \{\overline{10}\}$  representation of  $SU(3)$ . In case (ii), when the amplitudes belong to the octet representation of  $SU(3)$ , the main channels are the  $\rho\pi \rightarrow \rho\pi$  and  $\rho\pi \rightarrow \eta'\pi$  ones. In case (iii), when  $h' = h$  and the Lagrang-

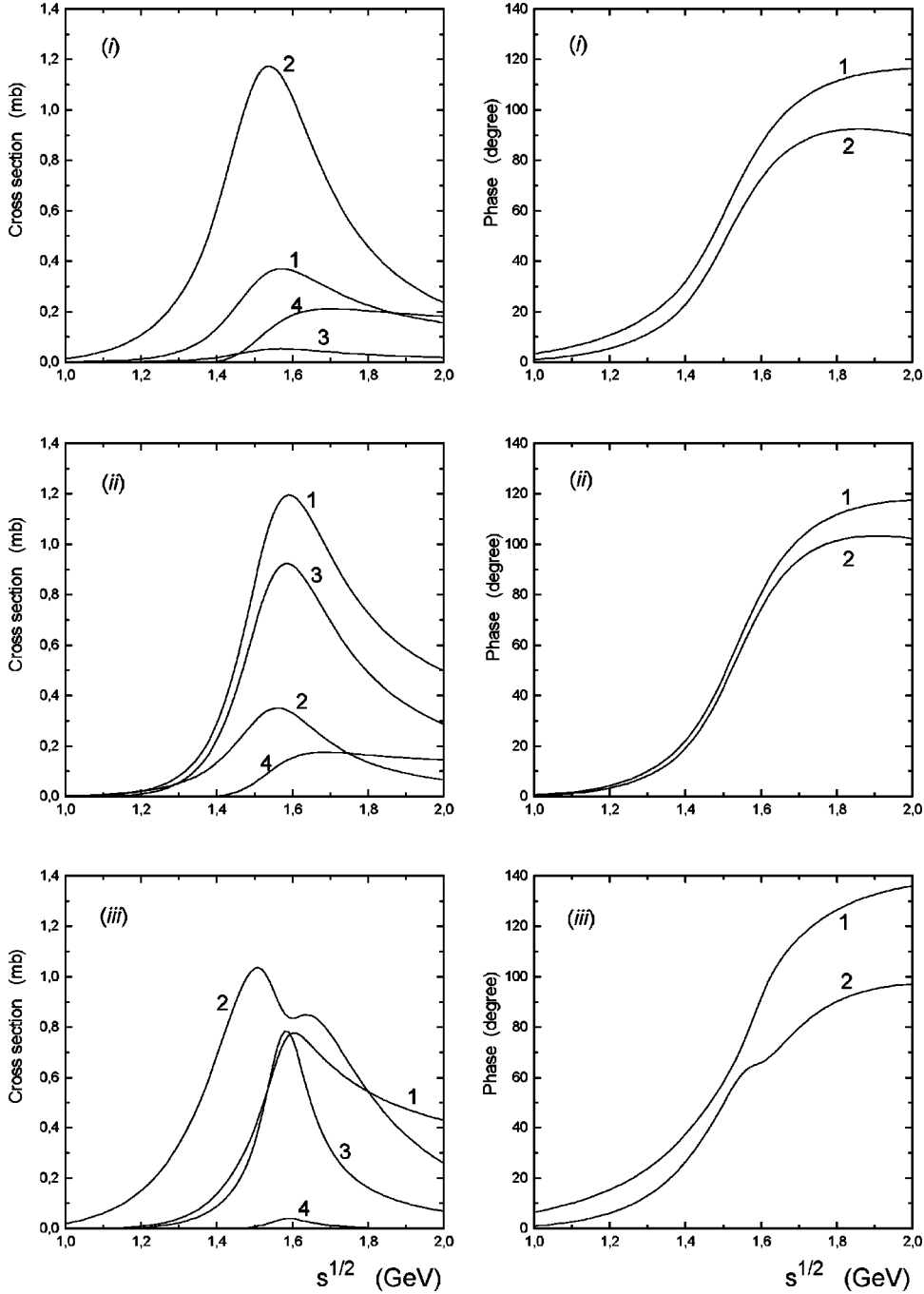


FIG. 4. The cross sections of the reactions  $\rho^0\pi^- \rightarrow \rho^0\pi^-$ ,  $\rho^0\pi^- \rightarrow \eta\pi^-$ ,  $\rho^0\pi^- \rightarrow \eta'\pi^-$ , and  $\rho^0\pi^- \rightarrow K^{*0}K^-$  and the phases of the  $\rho\pi \rightarrow \rho\pi$  and  $\rho\pi \rightarrow \eta\pi$  amplitudes for cases (i), (ii), and (iii). The correspondence between the curve numbers and the reaction channels is the same as in Fig. 3. The values used of the parameters are listed in Table II.

ian (3) possesses additional nonet symmetry, the cross sections for all channels except the  $K^*K$  one turn out to be comparable, and the general situation is rather complicated. The branching ratios of the presented resonancelike enhancements to the  $\rho^0\pi^- \rightarrow (\rho^0\pi^-, \eta\pi^-, \eta'\pi^-, K^{*0}K^-)$  channels are listed in Tables I and II. Such characteristics for the complicated broad resonance structure can be defined as follows. For example,  $B(\rho^0\pi^-) = \bar{\sigma}(\rho^0\pi^- \rightarrow \rho^0\pi^-) / \Sigma$ ,  $B(\eta\pi^-) = \bar{\sigma}(\rho^0\pi^- \rightarrow \eta\pi^-) / \Sigma$ , etc., where

$$\begin{aligned} \Sigma &= 2\bar{\sigma}(\rho^0\pi^- \rightarrow \rho^0\pi^-) + \bar{\sigma}(\rho^0\pi^- \rightarrow \eta\pi^-) \\ &+ \bar{\sigma}(\rho^0\pi^- \rightarrow \eta'\pi^-) + 2\bar{\sigma}(\rho^0\pi^- \rightarrow K^{*0}K^-) \end{aligned}$$

and every  $\bar{\sigma}$  is the integral of the corresponding cross section over the  $\sqrt{s}$  range from 1.2 to 1.8 GeV, where an enhancement concentrates.

Let us now compare the cross section values shown in Figs. 3 and 4 with those of  $a_2(1320)$  resonance production. Using the tabular branching ratios [1] we get  $\sigma(\rho^0\pi^- \rightarrow a_2 \rightarrow \rho^0\pi^-) \approx 5.7$  mb and  $\sigma(\rho^0\pi^- \rightarrow a_2 \rightarrow \eta\pi^-) \approx 2.36$  mb at  $\sqrt{s} = m_{a_2} = 1.32$  GeV. Taking also into account the ratio of the factors  $(2J+1)/|\vec{k}|^2$  for the  $a_2(1320)$  resonance and for the  $J=1$  enhancement found at  $\sqrt{s} \approx 1.3-1.4$  GeV, or at  $\sqrt{s} \approx 1.5-1.6$  GeV, we can conclude that we are certainly dealing with the resonancelike behavior of the  $I^G(J^{PC}) = 1^-(1^{-+})$  exotic waves in the re-

TABLE I. The parameter values of the model for the curves in Fig. 3.  $C_{11}$  and  $C_{12}$  are in  $\text{GeV}^2$ ; the other parameters are dimensionless. Also, in the four right columns, the branching ratios of the resonancelike enhancement obtained to the partial channels are presented.

Cases	$F_{\pi}^3 h$	$F_{\pi}^3 h'$	$C_{11}$	$C_{21}$	$C_{12}$	$C_{22}$	$B(\rho^0 \pi^-)$	$B(\eta \pi^-)$	$B(\eta' \pi^-)$	$B(K^{*0} K^-)$
(i)	0.10746	0	0.17	0	1.25	0	0.2377	0.3614	0.0125	0.0754
(ii)	0	0.10746	0.34	0	0.67	0	0.3259	0.0890	0.1924	0.0289
(iii)	0.10746	0.10746	0.49	0	0.50	0	0.2534	0.3619	0.1276	0.0019

gion  $1.3 \leq \sqrt{s} \leq 1.6$  GeV, at least, in the  $\rho\pi$ ,  $\eta\pi$ , and  $\eta'\pi$  channels.

The most appreciable manifestation of an  $I^G(J^{PC}) = 1^-(1^{-+})$  exotic state in the mass region 1.3–1.4 GeV has been observed in the  $\eta\pi^0$  channel in the charge exchange reaction  $\pi^- p \rightarrow \eta\pi^0 n$  at 32, 38, and 100 GeV/c [11] (currently the exotic states of such a type are denoted most commonly as  $\pi_1$ ). It was found that the intensity of the  $\pi_1$  signal at its maximum in this reaction is only 3.5 times smaller than the corresponding intensity of the  $a_2(1320)$  signal. It is very essential that  $a_2(1320)$  production and  $\pi_1$  production both proceed in this case via a single mechanism, namely, via the Reggeized  $\rho$  exchange. If the  $\pi_1$  really represents a complicated structure of the  $qq\bar{q}\bar{q}$  or  $q\bar{q}g$  type, then the fact that the  $\pi_1$  production cross section in the charge exchange reaction has been found fully comparable with that of the conventional  $q\bar{q}$  resonance  $a_2(1320)$  is certainly strong evidence for the resonance nature of the observed exotic signal.

On the other hand, the intensity of  $\pi_1$  production in the  $\eta\pi^-$  channel in the reactions  $\pi^- p \rightarrow \eta\pi^- p$  at 37 GeV/c [3] and 18 GeV/c [12] has been found to be about 15 and, respectively, 30 times smaller than the  $a_2(1320)$  production intensity. This is evidently due to the more complicated mechanism of the reaction  $\pi^- p \rightarrow \eta\pi^- p$  than that of the charge exchange reaction. In fact, there are three competing Regge exchanges with natural parity in this reaction: the  $\rho$  exchange, the  $f_2$  exchange, and the Pomeron one. Also, as is known, the last two are dominant in the case of  $a_2(1320)$  production [57]. Note that  $\pi_1$  production can proceed via Pomeron mechanism only owing to the octet component of the  $\pi_1$ . However, if this component is small, that is, if the  $\pi_1$  belongs mainly to the  $\{10\} - \{\bar{10}\}$  representation of  $SU(3)$ , then  $\pi_1$  production via Pomeron exchange has to be suppressed.

Another opportunity to observed  $\pi_1$  and  $a_2(1320)$  formation with comparable cross sections appears by using

photoproduction (and electroproduction) processes, for example,  $\gamma p \rightarrow \rho^0 \pi^- \Delta^{++} \rightarrow \pi^+ \pi^- \pi^- \Delta^{++}$ ,  $\gamma p \rightarrow \rho^0 \pi^+ n \rightarrow \pi^+ \pi^- \pi^+ n$ ,  $\gamma p \rightarrow \eta \pi^+ n$ , and so on, which go at low momentum transfer mainly via the Reggeized one-pion exchange mechanism. Indeed, the existing data on the reactions  $\gamma p \rightarrow \rho^0 \pi^- \Delta^{++} \rightarrow \pi^+ \pi^- \pi^- \Delta^{++}$  and  $\gamma p \rightarrow \rho^0 \pi^+ n \rightarrow \pi^+ \pi^- \pi^+ n$  show a clear signature of the  $a_2(1320)$  resonance and the appreciable enhancements in the  $3\pi$  mass spectra in the range 1.5–2 GeV [58]. However, they do not yet allow certain conclusions to be made concerning the presence of the exotic wave in the  $\rho\pi$  system and further investigations of the above reactions are needed.

At the present time an extensive program of the search for the exotic  $\pi_1$  states in photoproduction experiments with high statistics and precision is planned for the Jefferson Laboratory [6,14,15,18–20]. A careful study of the  $\pi_1 \rightarrow \gamma\pi$  radiative decays in hadroproduction from nuclei via the Primakoff one-photon exchange mechanism is also planned with the CERN COMPASS spectrometer [59]. A collection of the data on  $\pi_1$  photoproduction, electroproduction, and hadroproduction and on the decays of the  $\pi_1$  into  $\rho\pi$  will also allow for the first time to verify the vector meson dominance model for states with exotic quantum numbers.

Summarizing we conclude that our calculation gives a further new reason in favor of the plausibility of the existence of an explicitly exotic resonance with  $I^G(J^{PC}) = 1^-(1^{-+})$  in the mass range 1.3–1.6 GeV. Currently two exotic states at 1.4 and 1.6 GeV are extensively discussed in the literature [7–20]. In our scheme one does not succeed in simultaneously generating both the 1.4 and the 1.6 resonances, although the variants with a ‘‘fine structure’’ exist [see, for example, the cross sections for case (iii) in Figs. 3 and 4]. Such a ‘‘fine structure’’ will be smoothed by the experimental resolution and we cannot certainly say about

TABLE II. The parameter values of the model for the curves in Fig. 4.  $C_{11}$  and  $C_{12}$  are in  $\text{GeV}^2$ ; the other parameters are dimensionless. Also, in the four right columns, the branching ratios of the resonancelike enhancement obtained to the partial channels are presented.

Cases	$F_{\pi}^3 h$	$F_{\pi}^3 h'$	$C_{11}$	$C_{21}$	$C_{12}$	$C_{22}$	$B(\rho^0 \pi^-)$	$B(\eta \pi^-)$	$B(\eta' \pi^-)$	$B(K^{*0} K^-)$
(i)	0.10746	0	0.18	0	0.76	0	0.1616	0.4634	0.0217	0.0959
(ii)	0	0.08417	0.33	0	0.78	0	0.3032	0.0804	0.2184	0.0474
(iii)	0.10746	0.10746	0.11	0.11	0.11	0.11	0.2429	0.3686	0.1356	0.0050



two resonances. The question may be raised as to whether this is a crucial result. It is not improbable that the inclusion of the  $b_1\pi$  and  $f_1\pi$  channels, where the exotic signals have also been found, can change the situation. However, the issue of the additional  $b_1\pi$  and  $f_1\pi$  channels remains open in the effective chiral Lagrangian approach. Notice also that at

present the situation with the two exotic resonances at 1.4 and 1.6 GeV is not yet finally arranged.

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