Contribution of inelastic rescattering to $B\rightarrow\pi\pi$, $K\bar{K}$ decays

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We discuss multichannel inelastic rescattering effects in B decays into a pair PP of pseudoscalar mesons ($PP=\pi\pi$ or $K\bar{K}$). In agreement with short-distance models it is assumed that initially B meson decays dominantly into jet-like states composed of two flying-apart low-mass resonances M_1M_2 which rescatter into PP. Since from all S-matrix elements $\langle i|S|PP\rangle$ involving PP only some ($i=M_1M_2$) contribute to the final state rescattering, the latter is treated as a correction only. The rescattering of the resonance pair M_1M_2 into the final PP state is assumed to proceed through Regge exchange. Although effects due to a single intermediate state M_1M_2 are small, it is shown that the combined effect of all such states should be large. In particular, the amplitudes of B decays into $K\bar{K}$ become significantly larger than those estimated through short-distance penguin diagrams, to the point of being comparable to the $B\rightarrow\pi\pi$ amplitudes.

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I. INTRODUCTION

Studies of CP violation in B decays must involve final state interaction (FSI) effects. Unfortunately, a reliable estimate of such effects is very hard to achieve. In the analyses of $B\rightarrow PP$ decays (P, pseudoscalar meson) only some intermediate states, believed to provide non-negligible contributions, are usually taken into account. Many authors restrict their studies to elastic rescattering $P_1P_2\rightarrow P_1P_2$ only. In Regge language this is described in terms of a Pomeron exchange. Although in $PP\rightarrow PP$ quasi-elastic rescattering at $s=m_B^2$ contributions from other nonleading Regge exchanges are much smaller, they are not completely negligible and have been included in various analyses.

The main problem, however, is posed by the sequence $B\overset{weak}{\rightarrow} i\overset{FSI}{\rightarrow} PP$ involving *inelastic* rescattering processes $i\overset{FSI}{\rightarrow} PP$. Arguments have been given that it is these inelastic processes that actually constitute the main source of soft FSI phases [1–4]. It has been also pointed out [5] that nonzero inelasticity strongly affects the extraction of FSI phases in models based on quasi-elastic rescattering. Thus, inelastic events affect model predictions even if rescattering is of quasi-elastic type only.

On the other hand, FSI phases are often attributed directly to short-distance (SD) quark-line diagrams in the hope that this will take into account all inelastic production phenomena. This belief persists despite justified skepticism about the dominance of short-distance QCD in FSI of B decays (see e.g. [3]). In fact, it is known that the resulting prescription strongly violates such tenets of strong interactions as isospin symmetry [6] (see also [7]). The origin of the problem pointed out in Ref. [6] is the lack of any correlation between the spectator quark and the products of b quark decay. By its very nature such correlation cannot be provided by SD dynamics. What must be involved here is a long-distance (LD) mechanism which ensures that quarks ''know'' about each other. Thus we are led to hadrons, hadron-level dynamics, and inelastic rescattering effects.

In this paper we perform a simplified analysis of corrections which should be introduced by inelastic rescattering into the SD-based description of some nonleptonic decays of B mesons. Of course, any such analysis must be half-qualitative in nature, because—in the presence of many decay channels—it is well beyond our ability to take them accurately into account. Since it is believed that B decays into $\pi\pi$ and $K\bar{K}$ may provide some handle on the determination of angle α of the unitarity triangle, we shall concentrate on these decays: it is important to know the effects of FSIs here. As shown in Ref. [8], inclusion of coupled-channel quasi-elastic effects ($\pi\pi\rightarrow K\bar{K}$) generates an effective long-distance penguin amplitude comparable in size to the short-distance one. One may expect that inelastic channels will also contribute to this effect.

We start with an SD-based model of nonleptonic B decays. On the basis of standard tree-dominated mechanism for these decays, enriched with the related and well-established models of semileptonic B decays, we qualitatively estimate the types and the number of states produced in the first stage of the nonleptonic decay. As in other existing models, we take these states as composed of two (flying apart) resonances M_1M_2 (Sec. II). These resonances are assumed to rescatter into PP through Regge exchange.

In order to provide the basis for an estimate of this rescattering, we recall how in a Regge picture the unitarity relation involving $M_1M_2\rightarrow PP$ and other $i\rightarrow PP$ processes looks like. This enables us to make a rough estimate as to what part of all inelastic $i\rightarrow PP$ processes is due to the $M_1M_2\rightarrow PP$ transitions and, consequently, how much the situation deviates from the case of (quasi-)elastic rescattering. Using a rough estimate for the contribution $|\langle M_1M_2|S|PP\rangle|^2$ from an average single inelastic intermediate channel M_1M_2, we estimate the number of inelastic channels involved (Sec. III).

In Sec. IV we analyze the behavior of the $B\rightarrow\pi\pi$ and $B\rightarrow K\bar{K}$ amplitudes as a function of the number of intermediate states considered. We show that although effects due to each single intermediate state are small, the combined effect

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of all intermediate states is large. In particular, amplitudes of decays into $K\overline{K}$ become significantly larger than those estimated through short-distance penguin amplitudes, to the point of being comparable to the $B\to\pi\pi$ amplitudes.

Our conclusions are given in Sec. V.

II. *B* DECAYS WITHOUT FSIs

In short-distance approaches to nonleptonic weak B decays, the relevant amplitudes are usually expressed as sums of amplitudes corresponding to different types of quark diagrams (T, tree; C, color suppressed; E, W exchange; P, penguin; A, annihilation; PA, penguin annihilation). In this paper we concentrate on $\Delta S=\Delta C=0$ decays of B mesons which are initiated by a $b\to u\bar{u}d$ transition. In these decays one expects that the dominant contribution comes from the tree diagram T, with the main corrections to it provided by the color-suppressed and penguin diagrams C and P [9]. Accordingly, neglecting contributions from other diagrams, the SD amplitudes $\langle (P_1P_2)_I|w|B^0\rangle$ for B^0 decays into a pair of octet pseudoscalar mesons P_1P_2 with total isospin I are expressed in the SU(3) symmetry case as

$$\begin{aligned}
\langle(\pi\pi)_2|w|B^0\rangle &= -\frac{1}{\sqrt{6}}(T+C)\\
\langle(K\overline{K})_1|w|B^0\rangle &= -\frac{1}{2}P\\
\langle(\pi^0\eta_8)_1|w|B^0\rangle &= -\frac{1}{\sqrt{6}}P\\
\langle(\pi\pi)_0|w|B^0\rangle &= -\frac{1}{\sqrt{3}}\left(T-\frac{1}{2}C+\frac{3}{2}P\right)\\
\langle(K\overline{K})_0|w|B^0\rangle &= \frac{1}{2}P\\
\langle(\eta_8\eta_8)_0|w|B^0\rangle &= \frac{1}{6}(C+P).
\end{aligned}\tag{1}$$

Inclusion of $\pi\eta_8$ and $\eta_8\eta_8$ into our considerations is mandatory if we want to maintain SU(3) symmetry [10].

From the above formulas one may find SD amplitudes $\mathbf{w}_{\mathbf{R},I}$ from state $|B^0\rangle$ into states $\langle\mathbf{R},I|$ of a given isospin I belonging to definite representations $\mathbf{R}$ of SU(3):

$$\mathbf{w}_{\mathrm{R},I}=\mathbf{O}_I\mathbf{w}_I \tag{2}$$

with

$$\begin{aligned}
\mathbf{w}_2^T &= \langle(\pi\pi)_2|w|B^0\rangle\\
\mathbf{w}_1^T &= [\langle(K\overline{K})_1|w|B^0\rangle,\langle(\pi^0\eta_8)_1|w|B^0\rangle]\\
\mathbf{w}_0^T &= [\langle(\pi\pi)_0|w|B^0\rangle,\langle(K\overline{K})_0|w|B^0\rangle,\langle(\eta_8\eta_8)_0|w|B^0\rangle]
\end{aligned}\tag{3}$$

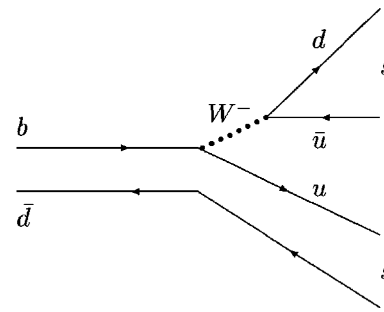

FIG. 1. Tree diagram (T) for B decay.

and the matrices $\mathbf{O}_I$ given by

$$\mathbf{O}_2=1, \tag{4}$$

$$\mathbf{O}_1=\begin{bmatrix}\sqrt{\frac{3}{5}} & \sqrt{\frac{2}{5}}\\ -\sqrt{\frac{2}{5}} & \sqrt{\frac{3}{5}}\end{bmatrix} \tag{5}$$

and

$$\mathbf{O}_0=\begin{bmatrix}-\frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{2}}\\ \frac{\sqrt{3}}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}}\\ \frac{1}{2\sqrt{10}} & \frac{\sqrt{3}}{\sqrt{10}} & -\frac{3\sqrt{3}}{2\sqrt{10}}\end{bmatrix} \tag{6}$$

where rows correspond (from top to bottom) to **27** for $\mathbf{O}_2$; **8**,**27** for $\mathbf{O}_1$; and **1**,**8**,**27** for $\mathbf{O}_0$.

One expects that SD amplitudes C and P constitute a 10–20% correction [9]. Indeed, for the factor r in the relation $C=T/(3r)$, the short-distance QCD corrections give the value of $r=(c_1+c_2/3)/(3c_2+c_1)\approx-3$ ($c_1\approx1.1$, $c_2\approx-0.25$ are Wilson coefficients), while estimates in Ref. [11] yield $|P/T|$ in the range of 0.04–0.20.

Dominance of T amplitudes is expected to hold for other decays initiated by $b\to u\bar{u}d$ as well. For larger invariant masses of $d\bar{u}$ and $u\bar{d}$ systems these quark-level states should hadronize mainly as $n\pi$ states [12,13]. It is the rescattering from these states to the final PP state that is of interest to us. Since we want to estimate rescattering at hadron level, we need to know what hadronic states are produced in the first stage of the decay. There are two groups of hadrons produced: one comes from the decay $W^-\to d\bar{u}$; the other one originates from the recombination of the u quark with spectator $\bar{d}$ (Fig. 1). The values of invariant mass squares $q^2\equiv s_1$ and s_2 (see Fig. 1 for definitions) are not large. The well-known probability distribution of b-quark decay is

$$v(q^2)=2(1-q^2/m_b^2)^2(1+2q^2/m_b^2) \tag{7}$$

which falls for increasing q^2, the average s_1 being $\overline{s_1} \approx 7$ GeV^2. The values of s_2 are smaller. Estimates obtained in various models for semileptonic and nonleptonic decays yield $\overline{s_2}$ around 1.5 GeV^2 or so, with the distribution of $\sqrt{s_2}$ extending to around 2.0 or 2.5 GeV [12,14–18]. When such a low-mass quark-antiquark state hadronizes, a resonance is produced. In the updated Isgur-Scora-Grinstein-Wise (ISGW2) model [16] for semileptonic decays, exclusive partial widths for the production of lowest-in-mass resonances have been predicted. The resonances considered were the ground-state mesons (pseudoscalars P, $1\,{}^1S_0$; vector mesons V, $1\,{}^3S_1$), the P-wave mesons (tensor mesons T, $1\,{}^3P_2$; axial mesons A, $1\,{}^3P_1$; B, $1\,{}^1P_1$; and scalar mesons S, $1\,{}^3P_0$); and the $2\,{}^1S_0$ and $2\,{}^3S_1$ states. The highest (nonstrange) resonance mass explicitly considered in [16] is below 1.5 GeV. The total partial width for the production of all these resonances is 5–6 times larger than the partial width for the production of a pseudoscalar meson. Thus, the average partial width into a resonance is smaller by a factor of 0.7 or so than that for the production of a pseudoscalar meson. The resonances explicitly considered in [16] do not saturate the inclusive decay rate which is still about 2 times larger [16]. Thus, several other resonances of masses below 2.0 or 2.5 GeV should be added to the list given above. Assuming that the average contribution from each one is similar to that just estimated, one expects that the number of types of ''average'' resonances produced should be of the order of 15.

Similar or even larger number of resonance types is expected from the hadronization of the $d\bar{u}$ created from the W boson. Thus, in nonleptonic B decays, apart from the PP state, many other resonance pairs must be produced: $M_1M_2 = VV, \ldots, PA, PB, VA, VB, VS, \ldots$, etc. Exact counting of the number of all these two-resonance states is not important for our purposes. However, it is fairly easy to give an estimate: limiting oneself to resonances of mass smaller than 2 GeV, this number will definitely be greater than 10, probably of the order of a few tens.

Clearly, there is no hope that one can reliably calculate the contribution from rescattering into PP from each of these intermediate states. However, one may try to estimate their overall contribution in an average way. For the first stage of the decay process, we will assume that the only important amplitude is the tree amplitude T and that this amplitude is approximately the same for all intermediate states considered [for the low values of s_1 that we shall be concerned with later, this is indeed the case in Eq. (7)]. Although this may seem a very rough assumption for s_2, we shall further see that our general results should be fairly independent of it as long as there is a rather large number of two-resonance states with production amplitudes scattered around the average. The next question is how to describe transitions $M_1M_2 \to PP$ (or vice versa) in an average way. This is what we shall discuss in Sec. III.

III. ELASTIC SCATTERING AND MULTIPARTICLE PRODUCTION PROCESSES IN PP INTERACTIONS

Elastic scattering and multiparticle production processes are related to each other through unitarity of the S matrix. Since B has spin $J=0$, we shall work with the $\mathbf{S}_{J=0}$ sector only. In the following we shall suppress the subscript J and its value. In Refs. [8,10] it was shown that in the SU(3) symmetry case, with the effects of coupled channels included, one should work with states $(PP)_{\mathbf{R}}$ belonging to definite representations $\mathbf{R}=\mathbf{1},\mathbf{8},\mathbf{27}$ of SU(3). The unitarity relation for PP scattering in the $l=0$ partial wave and in SU(3) representation $\mathbf{R}$ is

$$|\langle (PP)_{\mathbf{R}}|S|(PP)_{\mathbf{R}}\rangle|^2+\sum_{k_{\mathbf{R}}}|\langle (PP)_{\mathbf{R}}|S|k_{\mathbf{R}}\rangle|^2=1 \quad (8)$$

where, for given $\mathbf{R}$, $k_{\mathbf{R}}$ labels states different from $(PP)_{\mathbf{R}}$. Matrix elements occurring in Eq. (8) may be expressed in terms of Argand amplitudes a as follows:

$$\langle (PP)_{\mathbf{R}}|S|(PP)_{\mathbf{R}}\rangle = 1+2ia((PP)_{\mathbf{R}})$$
$$\langle (PP)_{\mathbf{R}}|S|k_{\mathbf{R}}\rangle = 2ia(k_{\mathbf{R}}). \quad (9)$$

Apart from the Pomeron, there are other Regge trajectories, whose exchange in the t channel contributes to quasi-elastic scattering $(PP)_{\mathbf{R}}\to(PP)_{\mathbf{R}}$. When the leading non-Pomeron exchange-degenerate Regge trajectories ρ, f_2, ω, and a_2 and their SU(3)-symmetric partners are taken into account, one obtains the $l=0$ partial wave amplitudes [8]:

$$a((PP)_{\mathbf{27}})=\frac{1}{16\pi}\left(i\tilde{P}+\frac{2\tilde{R}f(s)}{s}\right)$$
$$a((PP)_{\mathbf{8}})=\frac{1}{16\pi}\left(i\tilde{P}+\frac{\tilde{R}[-\frac{4}{3}f(s)+\frac{5}{3}g(s)]}{s}\right) \quad (10)$$
$$a((PP)_{\mathbf{1}})=\frac{1}{16\pi}\left(i\tilde{P}+\frac{\tilde{R}[-\frac{2}{3}f(s)+\frac{16}{3}g(s)]}{s}\right)$$

with

$$f(s)=\frac{s^{\alpha(0)}}{\ln(s)}$$
$$g(s)=\frac{s^{\alpha(0)}\exp[-i\pi\alpha(0)]}{\ln(s)-i\pi} \quad (11)$$

where for the leading non-Pomeron Regge trajectory we use

$$\alpha(t)=\alpha(0)+\alpha' t\approx 0.5+t; \quad (12)$$

i.e., we have put $\alpha'=1$ GeV^{-2} and, consequently, in Eqs. (10) and further on both s and t are in GeV^2.

The amplitudes $a((PP)_{\mathbf{R}})$ are independent of isospin I as shown in [8], where their sizes at $s=m_B^2$ have also been estimated. The SU(3)-symmetric Regge residue $\tilde{R}$ is fixed from experiment as [8]

$$\tilde{R}/\alpha' = -13.1 \ \text{mb GeV}^2 = -33.6 \quad (13)$$

while for the Pomeron one has [8]

$$\tilde{P} = 3.6 \ \ \text{mb Gev}^2 = 9.25. \tag{14}$$

Using the above values in Eqs (10) one finds

$$a((PP)_{\mathbf{27}}) = -0.076 + 0.184i$$
$$a((PP)_{\mathbf{8}}) = +0.019 + 0.217i \tag{15}$$
$$a((PP)_{\mathbf{1}}) = -0.076 + 0.291i$$

while the leading non-Pomeron Regge contributions *alone* are

$$a((PP)_{\mathbf{27}}, \text{Reg}) = -0.076$$
$$a((PP)_{\mathbf{8}}, \text{Reg}) = +0.019 + 0.033i \tag{16}$$
$$a((PP)_{\mathbf{1}}, \text{Reg}) = -0.076 + 0.107i.$$

The contribution from elastic scattering (the Pomeron in Regge language) is independent of **R**:

$$a((PP)_{\mathbf{R}}, \text{Pom}) = \frac{i}{16\pi}\tilde{P} = 0.184i \tag{17}$$

(cf. $a = 0.17 \ \ i$ in Ref. [2]). Omitting the Reggeon-Pomeron interference term, the value of the contribution from the leading non-Pomeron Regge exchange to the unitarity relation of Eq. (8), after averaging over representations **R**, is equal to

$$|\langle PP|S_{Reg}|PP\rangle|^2 = 4|a(PP;\text{Reg})|^2 \approx 4|0.08|^2 = 0.025. \tag{18}$$

Neglecting the Reggeon contribution to $PP \to PP$ one obtains, from Eq. (8),

$$\left(1 - \frac{\tilde{P}}{8\pi}\right)^2 + \sum_k |\langle PP|S|k\rangle|^2 = 1. \tag{19}$$

One may conjecture that contributions to the above sum from Reggeon-exchange-induced processes $PP \to M_1M_2$ (where M_i denotes low-lying resonance) will be of a size similar or smaller than $|\langle PP|S_{Reg}|PP\rangle|^2$. Thus, if all inelastic channels k were two-resonance states M_1M_2, one would obtain from Eq. (19) the number of

$$n_{tot} = \sum_{M_1M_2} |\langle PP|S|M_1M_2\rangle|^2/0.025$$
$$= \left[1 - \left(1 - \frac{\tilde{P}}{8\pi}\right)^2\right] \Big/ \ 0.025 \approx 25 \tag{20}$$

as the number of states contributing to the unitarity relation. This should be compared with the estimate of a few tens obtained in the previous section for the number of two-resonance states produced in weak decays of B meson. Of course, the estimate of Eq. (20) is probably too low: contributions from transitions $PP \to M_1M_2$ for heavier resonances M_i are likely to be smaller and the total number of states may be larger. For example, with average value of a going down by a factor of 0.6–0.7 from 0.08 to 0.05 (Sec. II), the above estimate of the number of average quasi-two-body states increases from 25 to over 60. On the other hand, two-resonance states need not saturate Eq. (19). Thus, there are two important questions which should be answered:

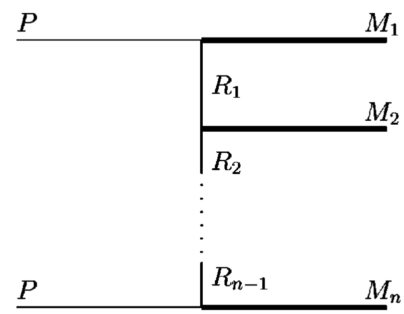

FIG. 2. Multiperipheral production of multi-resonance state $|M_1M_2 \cdots M_n\rangle$ through Regge exchanges R_k.

(1) How many of states k in the unitarity relation of Eqs. (8),(19) are indeed of the form M_1M_2, so that they can contribute to rescattering in B decays?

(2) Can the effect of all of these states be described in some average way?

At present, elastic and quasi-elastic contributions from long-distance FSIs in B decays are usually evaluated using the old language of Regge theory. This language was used in the past also for the description of resonance and multiparticle production processes that are of interest to us. In particular, the question of the buildup of elastic scattering (Pomeron exchange) as a shadow of inelastic multiparticle production processes [and thus the very content of the unitarity relation of Eqs. (8),(19)] was discussed extensively. Therefore, we must recall the essential elements of an approach which dealt with that problem. The approach, predominantly occupied with the issue of the unitarization of dual models (for reviews see [19]), was based on general properties of dual string models and on phenomenological analysis of resonance and multiparticle production data.

In this approach, multiparticle production processes occurring in hadron-hadron collisions at high energies are pictured as proceeding through the production of resonances or clusters in a multiperipheral model (Fig. 2) with leading non-Pomeron Reggeons exchanged in between the clusters. It is the *sum* over *all types* and *numbers* of resonances produced in this way that saturates the S-matrix unitarity relation, Eq. (8). For s above the inelastic threshold but still small enough, one may limit oneself to the production of just one pair of resonances M_1M_2. With increasing energy s, the number of resonances produced in a single collision increases on average. Although the cross section for the production of a particular number of resonances goes down at sufficiently high energy, the sum of cross sections over all possible numbers of resonances remains approximately constant. This constancy of the total inelastic cross section is ensured by a sufficiently fast increase in the number of all possible quark-line diagrams, i.e. in the number of all possible states k (and ways in which they are produced) [20].

In our case, at $s = m_B^2 = 28 \ \ \text{GeV}^2$ the model [21,22] predicts that states composed of just two resonances are produced in the fraction of $f_{2M} \approx 50\%$ cases approximately. A further 35% comes from the production of three resonances, etc. Although these numbers are obtained in [21] for resonances of any mass, the main contribution comes from the production of resonances of invariant masses squared smaller than 6 GeV^2. [The contribution from the production of objects of mass m_M is suppressed as $(m_M^2)^{2[\alpha(0)-1]}$ for larger m_M [23]]. The average mass of a resonance produced may be estimated in various ways to be around $\overline{m_M}=1.5$ (1.7 GeV in Ref. [23]), in good agreement with mass $\overline{m_M}\approx 1.3$ or so [18], expected for average $\sqrt{s_2}$ in the SD-based models of weak decays. Contributions from rescattering of states with larger values of s_1 will be suppressed because the production of such states in PP collisions is not likely. Thus, in a rescattering process $k\to PP$ one may expect that the dominant contribution will indeed come from the rescattering of states composed of two low-mass resonances M_1M_2. Translating the above expectation of a 50% share of M_1M_2 states in the unitarity relation of Eqs. (8),(19) into a number n_{2M} of contributing channels which may connect to the state originally produced by the SD dynamics (i.e. $n_{2M}=f_{2M}n_{tot}$), we conclude that this number should be around $n_{2M}=50\%\times 25\approx 12$ for average $|a(M_1M_2)|\approx 0.08$ or $n_{2M}=50\%\times 60\approx 30$ for average $|a(M_1M_2)|\approx 0.05$. The latter estimate is probably more realistic since the average size of a contribution from a single M_1M_2 channel should diminish with growing resonance masses.

IV. *B* DECAYS WITH FSIs

If one accepts that final state interactions cannot modify the probability of the original SD weak decay, it follows that the vector **W** representing the FSI-corrected amplitudes is related to the vector **w** of the original SD amplitudes through [2]

$$\mathbf{W}=\mathbf{S}^{1/2}\mathbf{w}. \tag{21}$$

Indeed, in the basis of **S**-matrix eigenstates $|\lambda\rangle$ the above equation reduces to $W_\lambda=e^{i\delta_\lambda}w_\lambda$; i.e., the condition of unchanged probability ($|W_\lambda|=|w_\lambda|$) admits Watson phases only.

The S matrix may be written in terms of the matrix **A** of amplitudes a:

$$\mathbf{S}=\mathbf{1}+2i\mathbf{A}. \tag{22}$$

We assume that we may treat the FSI-induced corrections to the SD decay amplitudes in a perturbative fashion. This is in agreement with the ideas of the dominance of SD dynamics. If this assumption is incorrect, obtaining even half-quantitative predictions will be almost impossible (cf. Ref. [2]). Although this assumption may be questioned, it has an important advantage: one may study what happens when the number of contributing two-resonance intermediate states is increased to its expected share ($f_{2M}=50\%$). In agreement with the assumption of a perturbative treatment of rescattering (i.e. small contribution from **A**), we expand the square root in $\mathbf{S}^{1/2}=(1+2i\mathbf{A})^{1/2}$ and keep only the first term. This leads to

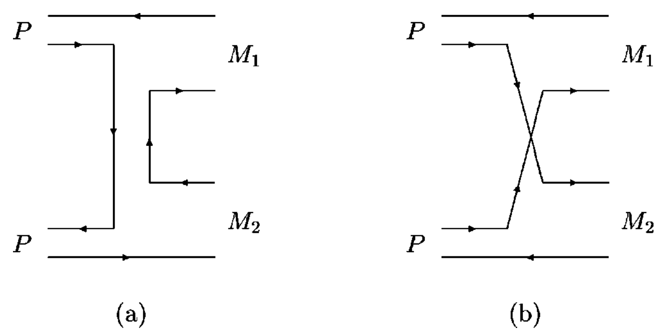

FIG. 3. Quark-line diagrams for production of two-resonance state $|M_1M_2\rangle$: (a) uncrossed Reggeon exchange, (b) crossed Reggeon exchange.

$$\mathbf{W}\approx(1+i\mathbf{A})\mathbf{w}=\frac{1+\mathbf{S}}{2}\mathbf{w}. \tag{23}$$

This is in fact the K-matrix prescription for the estimate of rescattering effects [5]. In this prescription, final state interactions modify probabilities of the SD amplitudes as can be seen explicitly from Eq. (23) written in the basis of S-matrix eigenstates. Our calculations will be based on Eq. (23).

In previous sections we considered a simplified picture of contribution from resonance pairs in which all amplitudes $a(M_1M_2)$ were equal in absolute magnitude while their phases were arbitrary. Indeed, when one row of the unitarity condition [i.e. Eq. (8)] is discussed, no knowledge of phases is needed. For the purpose of studies of CP violation the question of phases is important, however. Therefore, we have to make a very rough estimate of the FSI phases appearing in the strong rescattering amplitudes $\langle(PP)_{\mathbf{R}}|A|(M_1M_2)_{\mathbf{R}}\rangle$ in

$$\begin{aligned}\langle(PP)_{\mathbf{R}}|W|B\rangle=&\langle(PP)_{\mathbf{R}}|w|B\rangle\\&+i\sum_{M_1M_2}\langle(PP)_{\mathbf{R}}|A|(M_1M_2)_{\mathbf{R}}\rangle\\&\times\langle(M_1M_2)_{\mathbf{R}}|w|B\rangle.\end{aligned} \tag{24}$$

Thanks to CP invariance of strong interactions, the $\langle(PP)_{\mathbf{R}}|A|(M_1M_2)_{\mathbf{R}}\rangle$ amplitudes are symmetric (as is the S matrix). In order to estimate them we have to recall what are the predictions of dual string models for the production of two resonances in high energy PP collisions. In the Appendix of Ref. [22] it is shown that the dual string model predicts that the amplitude for the $PP\to M_1M_2$ production through the uncrossed diagram of Fig. 3(a) will pick up a rotating Regge phase resulting from the expression

$$[-s/(s_1s_2)]^{\alpha(t)}. \tag{25}$$

Similarly, for the crossed diagrams of Fig. 3(b) one has to remove the ''$-$'' sign in the above expression; i.e., the amplitude is real. Thus, the phase-generating factor differs from the familiar one in $PP\to PP$ scattering [i.e. $(-s)^{\alpha(t)}$] only by a different scaling factor (s_1s_2) in the denominator. Such

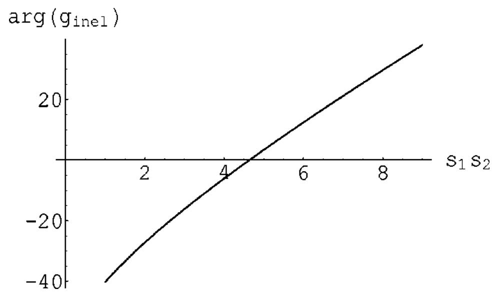

FIG. 4. Dependence of $g_{inel}(m_B^2/(s_1s_2))$ phase on s_1s_2.

a dependence of Regge amplitudes on the masses of produced resonances has been confirmed in analyses of experimental data [24,25].

At this point we have to take into account the fact that Regge amplitudes describe the scattering of two colliding resonances M_1 and M_2 in a state of definite momenta into a similar PP state. In particular, the produced PP state is a superposition of partial waves, while we are interested in the $\mathbf{S}_{J=0}$ sector of the S matrix only. Restriction to the $J=0$ sector is achieved by integrating the rescattering amplitudes $a(M_1M_2)$ with $P_{l=0}(\cos\theta)$ (with l being the angular momentum of the PP pair) over the allowed range of scattering angle θ or over the corresponding range of momentum transfer $t\in(t_{max},t_{min})$. Angular momentum conservation will then admit states M_1M_2 with total angular momentum $J=0$ only. This will have no effect on the assumed form of the SD decay amplitude since in the SD mechanism the decaying b quark does not ''know'' about the spectator quark and, consequently, about the value of J for the whole system.

One calculates that for $s_1,s_2\ll s$ the minimal value of t is $t_{min}\approx -s_1s_2/s$, while for t_{max} one may assume $t_{max}\approx -\infty$. Projecting the $M_1M_2\to PP$ Regge amplitude onto the $J=0$ sector, i.e. integrating Regge expressions over $t\in(t_{max},t_{min})$, we obtain, up to a common overall normalization factor, the following phase factors:

(1) For the uncrossed diagram [Fig. 3(a)]

$$g_{inel}(z)=\frac{z^{\alpha(0)}\exp[-(\ln z-i\pi)/z-i\pi\alpha(0)]}{(\ln z-i\pi)}. \quad (26)$$

(2) For the crossed diagram [Fig. 3(b)]

$$f_{inel}(z)=\frac{z^{\alpha(0)}\exp[-\ln(z)/z]}{\ln(z)} \quad (27)$$

where $z\equiv s/(s_1s_2)$.

The overall normalization of contributions from amplitudes $M_1M_2\to PP$ was fixed in Sec. III.

For the Regge description to be valid, the value of $s/(s_1s_2)$ must be large. With $s=m_B^2\approx 28$ GeV^2, the product s_1s_2 should not be greater than $(s_1s_2)_{max}\approx 6$ GeV^4, possibly 9 GeV^4, corresponding to the minimum value of $s/(s_1s_2)$ equal to 4 ± 0.8. For s_1 equal to s_2, this corresponds to the maximum value of resonance mass being around 1.6–1.8 GeV. These are still reasonable numbers when compared with our previous estimates of $\overline{m_M}\approx 1.5$. In Fig. 4 we show dependence of the phase of $g_{inel}(m_B^2/(s_1s_2))$ on s_1s_2 for $1<s_1s_2<9$. If one approximates s_i by their average values of around (1.5 GeV)^2, one finds that the average value of the product s_1s_2 is close to 5 GeV^4. From Fig. 4 we see that for $s_1s_2\approx 4.7$ GeV^4 the phase of g_{inel} is zero. Although the phase of g_{inel} at smaller and larger values of s_1s_2 deviates from zero quite significantly, these deviations are on average of the order of 20° and, consequently, it is still meaningful to talk about coherent superposition (i.e. with approximately similar phases) of Regge contributions from various intermediate states.

Because the quark-line structure of $PP\to PP$ and $M_1M_2\to PP$ amplitudes is the same, the eigenvalues with which the above phase factors enter into expressions for $(M_1M_2)_{\mathbf{R}}\to(PP)_{\mathbf{R}}$ in definite SU(3) representations $\mathbf{R}$ are the same as those in amplitudes $(PP)_{\mathbf{R}}\to(PP)_{\mathbf{R}}$. The latter were calculated in Ref. [8].

On the basis of Ref. [8] we have therefore

$$\begin{aligned}\delta(\mathbf{27},M_1M_2)&=\arg[+2f_{inel}(s/(s_1s_2))]\\ \delta(\mathbf{8},M_1M_2)&=\arg\left(-\frac{4}{3}f_{inel}(s/(s_1s_2))+\frac{5}{3}g_{inel}(s/(s_1s_2))\right)\\ \delta(\mathbf{1},M_1M_2)&=\arg\left(-\frac{2}{3}f_{inel}(s/(s_1s_2))+\frac{16}{3}g_{inel}(s/(s_1s_2))\right).\end{aligned} \quad (28)$$

The FSI-corrected amplitudes for B-meson decays into states in definite representations $\mathbf{R}$ of SU(3) and with isospin I are

$$\mathbf{W}_{\mathbf{R},I}=[1+ia((PP)_{\mathbf{R}})]\mathbf{w}_{\mathbf{R},I}+i\sum_{M_1M_2}e^{i\delta(\mathbf{R},M_1M_2)}|a((M_1M_2)_{\mathbf{R}})|\mathbf{w}_{\mathbf{R},I}(M_1M_2) \quad (29)$$

where $\mathbf{w}_{\mathbf{R},I}(M_1M_2)$ are SD weak decay amplitudes into M_1M_2, all assumed approximately equal to $\mathbf{w}_{\mathbf{R},I}$ [Eq. (2), Sec. II]. As in Sec. III, we assume that *all* amplitudes $|a((M_1M_2)_{\mathbf{R}})|$ are equal to some average value $\overline{a}$.

Although the phase of g_{inel} changes over the range of corresponding s_1s_2, it is very instructive first to approximate it everywhere by a constant, namely its average evaluated at, say, $\overline{s_1s_2}\approx(\overline{m_M})^2\approx 4.7$ GeV^4, where g_{inel} is real. The approximate reality of average g_{inel} is a consequence of the particular value of s (being here equal to $m_B^2\approx 28$ GeV^2) and not an s-independent feature of the approach. At $\overline{s_1s_2}=4.7$ GeV^2 we get

$$\begin{aligned}\overline{\delta(\mathbf{1})}&\approx 0^\circ\\ \overline{\delta(\mathbf{8})}&\approx \pm 180^\circ\\ \overline{\delta(\mathbf{27})}&\approx 0^\circ.\end{aligned} \quad (30)$$

The sum over all n_{2M} two-resonance states will then yield the contribution from inelastic rescattering:

TABLE I. Effects of inelastic rescattering on $\langle (PP)_I|W|B^0\rangle$ amplitudes in average phase approximation. Amplitudes are in units of input tree amplitude T.

$(PP)_I$	Amplitude modulus/phase	No FSIs	Quasi-elastic FSIs	Inelastic FSIs $n_{2M}=12$	Inelastic FSIs $n_{2M}=30$
$(\pi\pi)_2$	$\lvert W\rvert$	0.41	0.33	0.49	0.67
	$\arg(W/w)$	0°	−5.3°	47°	60°
$(\pi\pi)_0$	$\lvert W\rvert$	0.58	0.44	0.44	0.61
	$\arg(W/w)$	0°	−1.4°	−16°	−17°
$(K\bar{K})_0$	$\lvert W\rvert$	0	0.028	0.36	0.57
	$\arg(W)$		223°	93°	93°

$$\mathbf{W}_{\mathbf{R},I}(inel)=i\, n_{2M}e^{i\overline{\delta(\mathbf{R})}}\,\bar{a}\,\mathbf{w}_{\mathbf{R},I}. \tag{31}$$

Note that for fixed f_{2M} the average amplitude

$$\bar{a}=\sqrt{\frac{\left[1-\left(1-\frac{\tilde{P}}{8\pi}\right)^2\right]f_{2M}}{4n_{2M}}} \tag{32}$$

is inversely proportional to $\sqrt{n_{2M}}$. Thus, contribution from inelastic events in Eq. (31) is proportional to $\sqrt{n_{2M}}$: the smaller the value of $\bar{a}$, the larger the summed contribution from all two-resonance states, provided their contribution in the unitarity relation [Eq. (8)] is fixed by the same value of f_{2M}. Obviously, this is a general feature of any perturbative treatment of rescattering contribution from several channels, the reason being the *linear* nature of perturbatively treated FSIs as compared to the *quadratic* nature of the unitarity relation. Thus, estimates of FSI effects given here are most likely estimates from below: although amplitudes $a(M_1M_2)$ corresponding to rescattering from states composed of resonances of larger masses are expected to be smaller, their combined rescattering effect should be relatively larger.

The amplitudes for B decays into $(\pi\pi)_I,(K\bar{K})_I$ are calculated from the inverse of Eq. (2):

$$\mathbf{W}_I=\mathbf{O}_I^T\mathbf{W}_{\mathbf{R},I}. \tag{33}$$

In Table I we give the values of $\langle (PP)_I|W|B^0\rangle$ calculated from the above equation with the approximation of average phase [Eq. (31)] for $PP=\pi\pi,K\bar{K}$ and $n_{2M}=12,30$ ($\bar{a}=0.08,0.05$ respectively). Predictions of the model without inelastic rescattering, i.e. with quasi-elastic FSIs (PP intermediate states) only, are also given for comparison. Amplitudes are given in units of input tree amplitude T.

From Table I one can see that quasi-elastic FSIs do not affect the decays $B\to(\pi\pi)_I$ strongly. The amplitude moduli are somewhat smaller than those without the FSIs. This is due mainly to the $1+i[i\tilde{P}/(8\pi)]$ factor originating from Pomeron exchange. Phase changes are small. For the $B\to(K\bar{K})_0$ decays, quasi-elastic FSIs affect the amplitude significantly: the amplitude driven in SD dynamics by the penguin diagram (here vanishing) receives a contribution from the coupled-channel-generated long-distance penguin diagram (hereafter denoted P_{LD}) [8]. Using $|\langle (K\bar{K})_0|W|B^0\rangle/\langle(\pi\pi)_0|W|B^0\rangle|\approx\sqrt{3}P_{LD}/(2T)$ to estimate the effective LD penguin diagram, the size of P_{LD} is (as in Ref. [8]) of the order of 5% of the tree amplitude T, thus permitting significant interference effects with the short-distance penguin amplitude when the latter is taken from standard SD estimates.

The main results of this work are given in the last two columns of Table I.

(i) One can see that the $B\to(\pi\pi)_I$ amplitudes, when compared with their estimates taking into account quasi-elastic FSIs only, increase in absolute magnitude by a factor between 1 and 2. This is due to the additional contribution coming from rescattering chain $B\to M_1M_2\to\pi\pi$.

(ii) An important change can be seen in phase sizes: they are now one order of magnitude larger than in the quasi-elastic case. The origin of this effect is as follows. The W amplitude is composed of two parts: one (approximately real) is the SD amplitude w weakly suppressed by elastic (quasi-elastic) rescattering, while the other contains the contribution from the inelastic $M_1M_2\to\pi\pi$ rescattering. The latter part, being proportional to $ia(M_1M_2)w$, is mainly imaginary (for approximately real a). With a large contribution from inelastic rescattering, the resulting phase must therefore be large.

(iii) Finally, we see that the $B\to(K\bar{K})_0$ amplitude becomes much larger than in the case of quasi-elastic rescattering only, and is dominantly imaginary. Since the $B\to(K\bar{K})_0$ amplitude is fed from no-hidden-strangeness states $(M_1M_2)_{I=0}$, one might be tempted to compare this with Ref. [1]. Namely, it was shown there for the case of a simple two-channel S matrix that the phase of amplitude in channel 1 is large if the particle originally decays to channel 2. In our case, although a similar result holds, it is not general—it depends on the value of s at which amplitudes $a(M_1M_2)$ are evaluated. Note that inelastic rescattering renders the $B\to(K\bar{K})_0$ amplitude larger than the SD penguin estimate: in fact, it is comparable to the $B\to(\pi\pi)_0$ amplitude.

In our approach the full (FSI-corrected) amplitudes for $B\to(K\bar{K})_1$ decays vanish since

(1) we have assumed that only the tree amplitude T is nonzero and

(2) the rescattering from the isospin $|I=1,I_3=0\rangle$ state of $(M_1M_2)_{I=1}$, which in principle might feed the final $(K\bar{K})_1$ channel, is zero.[1]

The dependence of the LD rescattering-induced effective penguin diagram on the isospin channel—with vanishing (large) effects in the $I=1$ (0) channel—should be compared with the SD mechanism which assigns the same size and phase to penguin amplitude P in both the $(K\bar{K})_0$ and $(K\bar{K})_1$ decay channels. This is a general feature of long-distance dynamics: the size and phase of a quark-level diagram depend on what isospin [SU(3)] amplitude it contributes to [10].

Although the above expectation of FSI effect larger than that naively expected is fairly general, we have to analyze in some detail our assumption of replacing the s_1s_2-dependent phase of g_{inel} with an average (and vanishing) phase. Such an assumption would be well justified if a large fraction of M_1M_2 states led to phases close to the average. Since for a given value of the product s_1s_2 the phase is fixed, we need to know the density of two-resonance states as a function of s_1s_2. Regge models [23] predict that the dependence of $|a(M_1M_2)|^2$ on $m^2_{M_i}$ is proportional to $1/m^2_{M_i}$ for larger values of m_{M_i}. This fall of the distribution for larger mass values should be clearly visible already at the beginning of the region where using the Regge description becomes sensible, i.e. at $m^2_{M_i}\approx 4$ GeV^2 or so. The distribution of s_2 in SD models vanishes even faster [it is fairly negligible above $s_2\approx(2.0–2.5\ \text{GeV})^2$]. Since we want to estimate corrections to SD models, direct use of the mass distributions generated in SD models might seem to be the simplest and most natural choice. Furthermore, as the rescattering of states with larger values of s_1 is suppressed by the small size of the contribution from transitions $M_1M_2\to PP$, one might use the SD distribution of s_2 as that of s_1 as well. The problem is, however, that a significant part of the SD distribution of s_2 (or s_1) corresponds to s_2 (or s_1)<1 GeV^2. On the other hand, Regge amplitudes assume the form of $[s/(s_1s_2)]^{\alpha(t)}$ only for $s_1,s_2>1$ GeV^2. For $s_{1(2)}<1$ GeV^2, one should replace $s_{1(2)}$ with $(\alpha')^{-1}=1$ GeV^2. Thus, if Regge phases are to be reasonably evaluated, we must use an appropriately modified distribution of s_2. We model this situation in the simplest possible way: by assuming that the distribution of $m_{M_k}=\sqrt{s_k}$ vanishes below 1 GeV and above 2.25 GeV, while in between these values it is given by

$$\rho(m_M)=2.88-1.28m_M. \tag{34}$$

[1]The vanishing of the rescattering contribution can be seen as follows. The $I=1$ state of M_1M_2 is antisymmetric [i.e. of the form $(M^+M^- - M^-M^+)/\sqrt{2}$], while the rescattering contribution due to the uncrossed diagram of Fig. 3(a) is zero when evaluated in between the antisymmetric state $|(M_1M_2)_{I=1}\rangle$ and the symmetric state $|(K\bar{K})_1\rangle$ [for the definitions of states, see Eq. (1) of Ref. [8]]. Rescattering through diagram of Fig. 3(b) cannot change the type of quarks and, consequently, cannot induce a transition from the no-hidden-strangeness state $|M_1M_2\rangle$ into $|K\bar{K}\rangle$.

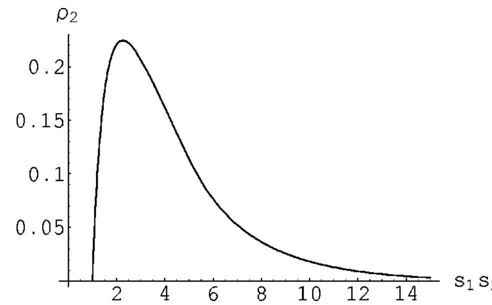

FIG. 5. Distribution of s_1s_2 in model defined in text.

This yields the average value of ''effective'' m_M equal to 1.42, which is also the value obtained from the ACCMM distribution of $\sqrt{s_2}$ (see e.g. Ref. [18]) if contributions from $m_M<1$ GeV are replaced with contributions at $m_M=1$ GeV. Using $\rho(m_M)$ as in Eq. (34), it is straightforward to evaluate the distribution ρ_2 of s_1s_2. This distribution (Fig. 5) is peaked at $s_1s_2\approx 2.2$ GeV^4, its median being at 3.6 GeV^4 and the average $\overline{s_1s_2}$ at 4.4 GeV^4. The tail above 9 GeV^4 contributes a few percent only. Thus, the assumption of $\overline{s_1s_2}\approx 4.7$ GeV^4 used in our previous discussion appears quite reasonable.

With two-resonance states k spread in s_1s_2 according to distribution ρ_2 (larger values of $k=1,\dots,n_{2M}$ correspond to states with appropriately larger values of s_1s_2), it is simple to analyze the predictions of the model numerically. We are interested in the question how the FSI effects change when heavier and heavier intermediate states k are included. For decays into states in definite representations of SU(3), the amplitudes of interest to us are therefore

$$\mathbf{W}_{\mathbf{R},I}(n)=[1+ia((PP)_{\mathbf{R}})]\mathbf{w}_{\mathbf{R},I}+i\sum_{k=1}^{n} e^{i\delta(\mathbf{R},s_1s_2)}\bar{a}\,\mathbf{w}_{\mathbf{R},I} \tag{35}$$

where n is the number of intermediate inelastic states considered $(0\leqslant n\leqslant n_{2M})$ and $\delta(\mathbf{R},s_1s_2)$ are given in Eq. (28).

In Fig. 6 we present predictions of the n-dependent version of Eq. (33) for the absolute values and phases of the amplitudes of B decays into $(\pi\pi)_0$ and $(K\bar{K})_0$ states. These predictions are given as a function of the number n of intermediate states considered. We show the case with $n_{2M}=12$ (for $n_{2M}=30$ one obtains very similar plots with features discussed below being even more pronounced). The point most to the right in each plot (i.e. at $n=n_{2M}$) corresponds to a large value of s_1s_2, where Regge approximation breaks down. Consequently, this point should be discarded. One can see that for $B\to(\pi\pi)_0$ the size of the amplitude does not depend very strongly on n and is close to the value of the input FSI-free amplitude $|\langle(\pi\pi)_0|w|B^0\rangle|$, which is 0.58. Furthermore, the FSI-induced phase is still relatively small (of the order of $-5°$). This cannot be said of the $B\to(K\bar{K})_0$ process. Here, the absolute value of the amplitude grows fast with the increasing number of intermediate states. In addition, already at $n=4$ or so, the phase becomes close

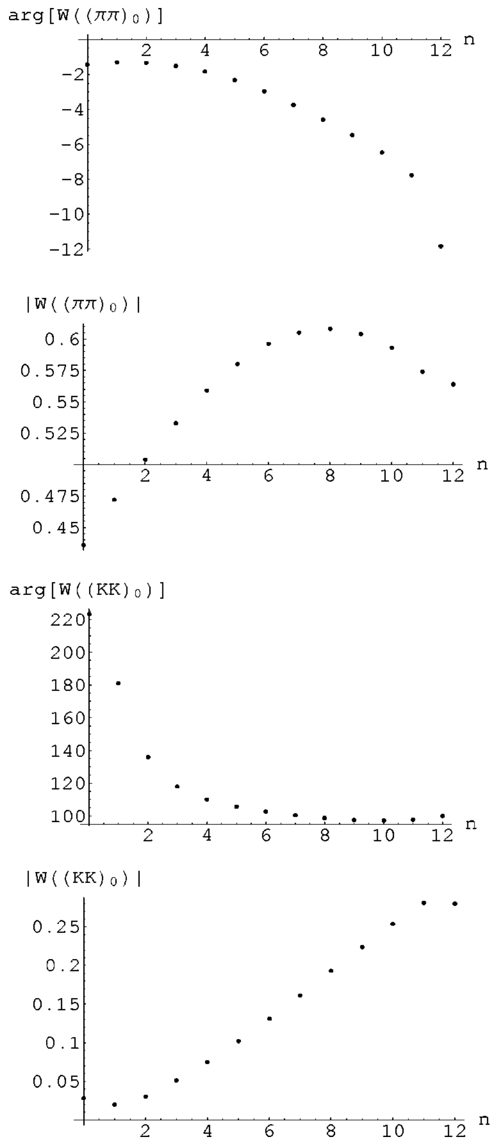

FIG. 6. Dependence of FSI-induced effects on the number of intermediate channels included.

to 100°, not far from the previous result of 93° (Table I) obtained for constant phases. One can also see that when the number of intermediate states approaches its maximum allowed value, the size of the $B \to (K\bar{K})_0$ amplitude becomes significant when compared to the tree $B \to (\pi\pi)_0$ amplitude. As the number of intermediate states increases, the long-distance-induced penguin amplitude starts to dominate over the SD one (which was estimated at 0.04–0.20 of the tree amplitude [11]). Furthermore, assuming applicability of Eq. (1), one estimates from Fig. 6 that at large n the effective long-distance penguin amplitude P_{LD} is around 0.6–0.8. This should be compared with the input or effective T amplitudes which are around 1.0. Thus, the long-distance effective penguin amplitudes become really large.

Large rescattering effects obtained above stem from an *approximately coherent superposition of contributions from several intermediate channels*. One might argue that in the calculation of this paper sizes of individual rescattering amplitudes a (i.e. contributions from individual intermediate channels) are overestimated. However, the trend of the results expected for the case of smaller amplitudes a may be seen from Table I and Eqs. (31),(32). The column of $n_{2M} = 30$ corresponds to a smaller value of average amplitude $\bar{a} = 0.05$. The connection existing between the values of n_{2M} to $\bar{a}$ stems from the assumption that at energy $s = m_B^2$ a two-resonance state is produced in approximately half of all inelastic PP collisions. As long as this fraction (f_{2M}) is kept constant and the perturbative treatment of FSIs is valid, smaller values of amplitudes a—after summing over all intermediate channels—result in rescattering corrections to weak decays larger than naively expected. One may also try to estimate roughly the rescattering contribution in a direct way (i.e. without using the value of f_{2M}) by just adding rescattering contributions with amplitudes $a(M_1M_2)$ assumed to be of the order of $a(PP)$ or so (in agreement with experiment). Then, for the number of average intermediate states taken as equal to the number of final states in standard models of SD decays, i.e. of the order of 10 or more, one is bound to obtain a large rescattering effect. Note that our whole approach starts with the generally accepted features of the SD decay (and resonance production) mechanism and assumes that subsequent FSIs may be treated perturbatively. We have shown that even in this case the corrections tend to become large. Of course, if they are too large, the whole perturbative scheme of their estimation (as well as the SD mechanism for the description of B decays) ceases to be viable.

V. CONCLUSIONS

We have discussed multichannel inelastic rescattering effects in B decays into $\pi\pi$ and $K\bar{K}$. Generally accepted features of short-distance decay mechanism have been assumed as part of our input. These assumptions included estimates of the types and number of resonances produced in two-resonance states initiated by the $b \to u\bar{u}d$ transition. Rescattering of these two-resonance states into the final state consisting of one pair PP of pseudoscalar mesons was evaluated under the assumption that such FSIs may be treated perturbatively. The basis for this evaluation was provided by existing knowledge about how the inelastic multiparticle (resonance) production in PP collisions is correlated with elastic PP (Pomeron exchange) scattering. This knowledge permitted us to estimate that at $s = m_B^2$ PP scatter into a two-resonance M_1M_2 state in approximately 50% of cases. Thus, the total size of rescattering from M_1M_2 to PP was fixed.

Using the Regge model for the description of the $M_1M_2 \to PP$ processes, we have shown that the rescattering contributions from the individual intermediate channels add approximately coherently. As a result, the combined effect of rescattering through many two-resonance intermediate states was shown to be quite large. This was demonstrated under the assumption that FSIs may be treated perturbatively. If that assumption is overoptimistic, reliable estimate of (presumably even larger) FSI effects will almost certainly be much more difficult.

In our calculations the amplitude for the decays $B \rightarrow (K\bar{K})_0$ was induced by LD rescattering from no-hidden-strangeness isospin 0 states produced via a short-distance tree diagram. This FSI-induced amplitude was shown to be larger than its short-distance penguin counterpart. The phase of the LD amplitude was estimated to be around 100°.

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[1] J. F. Donoghue *et al.*, Phys. Rev. Lett. **77**, 2178 (1996).
[2] M. Suzuki and L. Wolfenstein, Phys. Rev. D **60**, 074019 (1999).
[3] M. Suzuki, talk given at the 3rd International Conference on B Physics and CP Violation, Taipei, Taiwan, 1999, hep-ph/0001170.
[4] A. N. Kamal and C. W. Luo, Phys. Rev. D **57**, 4275 (1998).
[5] A. N. Kamal, Phys. Rev. D **60**, 094018 (1999).
[6] J.-M. Gérard and J. Weyers, Eur. Phys. J. C **7**, 1 (1999).
[7] L. Wolfenstein, Phys. Rev. D **52**, 537 (1995).
[8] P. Żenczykowski, Phys. Lett. B **460**, 390 (1999).
[9] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Phys. Rev. D **52**, 6356 (1995).
[10] P. Żenczykowski, Acta Phys. Pol. B **28**, 1605 (1997).
[11] M. Gronau, Phys. Lett. B **300**, 163 (1993).
[12] A. V. Dobrovolskaya, A. B. Kaidalov, K. A. Ter-Martirosyan, and V. R. Zoller, Phys. Lett. B **229**, 293 (1989).
[13] L. Wolfenstein, Phys. Rev. D **43**, 151 (1991).
[14] G. Altarelli *et al.*, Nucl. Phys. **B208**, 365 (1982).
[15] V. Barger, C. S. Kim, and R. J. N. Phillips, Phys. Lett. B **251**, 629 (1990).
[16] D. Scora and N. Isgur, Phys. Rev. D **52**, 2783 (1995).
[17] I. Bigi, R. D. Dikeman, and N. Uraltsev, Eur. Phys. J. C **4**, 453 (1998).
[18] DELPHI Collaboration, M. Battaglia, ''Study of $b \rightarrow u l \bar{\nu}$ Decays with an Inclusive Generator,'' hep-ex/0002040.
[19] Hong-Mo Chan and Sheung Tsun Tsou, Report No. RL-76-080, 1976, Bielefeld Sum. Inst. 1976, p. 83; G. F. Chew and C. Rosenzweig, Phys. Rep. **41**, 263 (1978).
[20] Huan Lee, Phys. Rev. Lett. **30**, 719 (1973); G. Veneziano, Phys. Lett. **43B**, 413 (1973).
[21] Hong-Mo Chan, J. E. Paton, Sheung Tsun Tsou, and Sing Wai Ng, Nucl. Phys. **B92**, 13 (1975).
[22] Hong-Mo Chan, J. E. Paton, and Sheung Tsun Tsou, Nucl. Phys. **B86**, 479 (1975).
[23] N. Sakai, Nucl. Phys. **B99**, 167 (1975).
[24] P. Hoyer, R. G. Roberts, and D. P. Roy, Nucl. Phys. **B56**, 173 (1973).
[25] A. C. Irving and R. P. Worden, Phys. Rep. **34**, 117 (1977).