Bottom-up model for maximal ν_μ-ν_τ mixing

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We construct a model which provides maximal mixing between a pseudo-Dirac ν_μ/ν_τ pair, based on a local $U(1)_{L_\mu\text{-}L_\tau}$ symmetry. Its strengths, weaknesses and phenomenological consequences are examined. The mass gap necessitated by the pseudo-Dirac structure is most naturally associated with the LSND anomaly. The solar neutrino problem then requires a light mirror or sterile neutrino. By paying a fine-tuning price to nullify the mass gap, one can also invoke $\nu_e \to \nu_{\mu,\tau}$ for the solar problem. The model predicts a new intermediate range force mediated by the light gauge boson of $U(1)_{L_\mu\text{-}L_\tau}$. Through the mixing of μ, τ and e, this force couples to electrons and thus may be searched for in precision ''gravity'' experiments.

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I. INTRODUCTION

Mounting evidence from the SuperKamiokande [1,2] experiment suggests that muon neutrinos are mixed with neutrinos of another flavor, with a mixing angle of close to $\pi/4$, that is, maximal mixing. In this paper, we adopt the point of view that the angle $\pi/4$ is a special, unique value that ought to be explained. In other words, we will attempt to understand the origin of this maximally large mixing, as opposed to the view that it is just one possible point in parameter space which should be assigned no particular significance. The contrast between this large leptonic mixing angle and the small Cabibbo-Kobayashi-Maskawa mixing angles in the quark sector is stark, and justifies our point of view.

It is interesting that two-fold maximal mixing can be fairly easily explained if each active neutrino mixes with a *sterile* partner. There are two known ways to do this: embrace the exact parity or mirror matter model [3], or suppose a pseudo-Dirac structure [4]. The former seems especially compelling, because the exact parity model is not much more complicated than the standard model (SM) itself. The pseudo-Dirac structure, while having a degree of elegance in and of itself, suffers when one requires it to emerge from a complete extension of the SM. Both possibilities, though, provide a strong theoretical motivation for light sterile neutrino flavors. Also, the combined solar [5], atmospheric and LSND [6] data provide interesting indirect experimental support for the existence of at least one light sterile neutrino.

One of the most important problems in experimental atmospheric neutrino physics at present is to discriminate between the $\nu_\mu \to \nu_s$ and $\nu_\mu \to \nu_\tau$ possibilities. The cleanest atmospheric neutrino data (the fully and partially contained events) can be explained equally well by both oscillation modes. The modes can in principle be distinguished by processes sensitive to the matter-effect (ME) and/or the neutral current (NC). SuperKamiokande has four data sets of this type: neutrino induced π^0 production (NC), upward throughgoing muons (ME), higher energy partially contained events (ME), and multiring events (NC). The π^0 event sample is not very useful at present because the production cross-section is poorly known. A forthcoming measurement of this quantity by the K2K long baseline experiment is eagerly awaited. SuperKamiokande have recently argued that the last three data sets disfavour the $\nu_\mu \to \nu_s$ scenario [2], though this has been disputed in Ref. [7]. We await with interest a complete account of the SuperKamiokande analysis, so that independent researchers can judge the robustness of their conclusion. In any case, the future MINOS and CERN to Gran Sasso long baseline terrestrial experiments will be able to check whatever conclusions are drawn on the basis of atmospheric neutrino data.

This paper will be devoted to building a theoretical bottom-up style model for maximal ν_μ-ν_τ mixing. We do so partly to provide a foil for the mirror and pseudo-Dirac approaches to understanding two-fold maximal mixing. Can one understand active-active maximal mixing in as compelling a way as one can active-mirror or active-sterile mixing? In addition, most other proposals for understanding large angle ν_μ-ν_τ mixing [8] have invoked grand unification and/or string motivated physics [such as anomalous U(1) symmetries]. By contrast, we will use a bottom-up approach, whereby we try to keep the new physics at as low an energy scale as possible. Also, we will not attempt to connect the ν_μ-ν_τ mixing angle problem with the rest of the flavor (mass and mixing angle hierarchy) problem. We ask the question: what new physics principles are implied by the discrete hypothesis of maximal ν_μ-ν_τ mixing? Then, given these general principles, how may they be instantiated within a complete extension of the SM?

II. SYMMETRY PRINCIPLES FOR MAXIMAL ν_μ-ν_τ MIXING

Our experience with the SM strongly suggests that internal symmetry principles play a very fundamental role in nature. In this section, we will deduce some very simple symmetry principles suggested by maximal ν_μ-ν_τ mixing. Strictly speaking, the atmospheric neutrino results do not rigorously establish exact maximal mixing. It is a logical possibility that the mixing is very large but not maximal. It is reasonable to expect that exact maximal mixing would be

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correlated with a symmetry principle, because maximality arises from a special point in parameter space. For aesthetic reasons, and because of the historical precedent regarding the importance of symmetries, our fundamental supposition here is that maximal mixing is our target.

We first make a short and apparently digressive comment. It is interesting to note that the observation of neutrino oscillations,[1] and hence the necessary introduction of neutrino masses into the SM, implies degrees of freedom beyond those in the *minimal* SM. This is true irrespective of whether the neutrino masses are of Dirac or Majorana type. In a sense, therefore, the discovery of neutrino mass is akin to the previous discoveries of new particles such as the top quark. In another sense, though, it is dissimilar: the new degrees of freedom implied by neutrino mass within the gauge theoretic rules of the SM are not uniquely specified, and not *directly* observed (as yet). It is certainly true, however, that renormalizable models of nonzero neutrino mass necessitate either an expansion of the fermion sector (right-handed neutrino states, for instance) or an expansion of the scalar sector (Higgs triplets, for instance), or both. We will begin with the second possibility, by including only the minimal left-handed neutrino degrees of freedom. In the end, however, we will find that there is a natural role within our framework for at least one light mirror or sterile neutrino state.

Consider a general mass matrix involving $\nu_{\mu L}$ and $\nu_{\tau L}$:

$$\left[\overline{(\nu_{\mu L})^c} \quad \overline{(\nu_{\tau L})^c}\right]\begin{pmatrix} \delta_\mu & m \\ m & \delta_\tau \end{pmatrix}\begin{pmatrix} \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}, \tag{1}$$

where $\delta_{\mu,\tau}$ are Majorana masses for $\nu_{\mu,\tau}$, while m is a transition mass. The mass eigenvalues are

$$m_\pm = \left|\frac{\sqrt{4m^2+(\delta_\mu-\delta_\tau)^2}\pm(\delta_\mu+\delta_\tau)}{2}\right|. \tag{2}$$

The mixing angle is given by

$$\tan 2\theta = \frac{2m}{\delta_\tau-\delta_\mu}. \tag{3}$$

Very close to maximal mixing arises in the parameter range

$$\text{Case 1:} \quad \delta_{\mu,\tau} \ll m. \tag{4}$$

The mass eigenvalues are then

$$m_\pm \simeq m \pm \frac{\delta_\mu+\delta_\tau}{2}, \tag{5}$$

with

$$\theta \simeq \frac{\pi}{4}+\frac{\delta_\mu-\delta_\tau}{4m}. \tag{6}$$

This defines a pseudo-Dirac structure [10] for the ν_μ-ν_τ system.[2] Alternatively, exact maximal mixing arises if

$$\text{Case 2:} \quad \delta_\mu = \delta_\tau. \tag{7}$$

Case 1, with its pseudo-Dirac structure, leads to a nearly degenerate pair of almost maximally mixed eigenstates with a mass gap m above zero mass. Case 2 has exact maximal mixing without the necessity of a mass gap. We will see later that this mass gap is most naturally related to the Liquid Scintillation Neutrino Detector (LSND) anomaly.

The symmetry structures underlying the two cases are very simple. Consider case 1 first. The relatively large transition mass term is invariant under any U(1) symmetry for which ν_μ and ν_τ have opposite charges. The obvious choice for this symmetry is simply $U(1)_{L_\mu - L_\tau}$ [11], where L_α is the lepton number for family $\alpha = e,\mu,\tau$. The Majorana mass terms break this symmetry. The hierarchy $\delta_{\mu,\tau} \ll m$ guarantees, however, that $U(1)_{L_\mu - L_\tau}$ is an approximate symmetry correlated with the pseudo-Dirac structure. As $\delta_{\mu,\tau} \to 0$, the symmetry becomes more exact and the mixing angle approaches complete maximality (and the masses become more degenerate). The limiting case of vanishing Majorana masses supplies a four-component massive neutrino which preserves L_μ-L_τ but breaks $L_\mu + L_\tau$ by two units. (The m and δ terms both break $L_\mu + L_\tau$.) The connection between maximal mixing and increased symmetry guarantees that the close-to-maximal mixing angle deduced at tree-level will not be spoiled by radiative corrections (unless some other sector of the theory breaks the L_μ-L_τ symmetry strongly). This is a well-known property of pseudo-Dirac states.

Is there any independent reason for considering the $U(1)_{L_\mu - L_\tau}$ symmetry to be in any way fundamental? Interestingly, it has been observed [12,13] that $U(1)_{L_\mu - L_\tau}$ is actually an anomaly free symmetry of the minimal (zero neutrino mass) standard model and may therefore be gauged. In fact the gauge group of the minimal SM may be enlarged to

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X \tag{8}$$

where X is either L_μ-L_τ or L_τ-L_e or L_e-L_μ. It is important to recognize that it is possible to gauge only one of these three alternatives, because anomalies involving two different X's do not cancel given the minimal SM fermion spectrum.

Because the pseudo-Dirac structure we want is correlated with the anomaly-free symmetry $X = L_\mu$-L_τ, we shall choose to gauge it, consistent with the common view that local symmetries are likely to be more fundamental than global symmetries [14]. Of course we cannot explain why an L_μ-L_τ symmetry should be given this status, rather than either of the two alternatives. We are simply suggesting that the maximal mixing between ν_μ and ν_τ could be associated with a gauged L_μ-L_τ which singles out ν_μ and ν_τ as special.

[1]To be precise, neutrino *oscillations* have yet to be seen—only ν_μ disappearance has been rigorously established. See, for example, Ref. [9].

[2]This type of active-active pseudo-Dirac neutrino is of course distinct from the active-sterile pseudo-Dirac neutrino states discussed in the Introduction.

Having identified L_μ-L_τ as playing a crucial role, it is tempting to speculate about further lines of development. An obvious path is to identify this quantity with the diagonal generator of a flavor SU(2) symmetry with the second and third lepton families placed in a doublet. We will not pursue this thought here, because we want to follow the simplest clues first.

Let us now turn to case 2. This obviously requires broken $U(1)_{L_\mu}$ and $U(1)_{L_\tau}$, but the central feature is an unbroken interchange symmetry $\nu_{\mu L}\leftrightarrow\nu_{\tau L}$ to enforce $\delta_\mu=\delta_\tau$. Note that, by contrast to the pseudo-Dirac case, there need be no hierarchy in the breaking scales for $L_\mu+L_\tau$ and L_μ-L_τ. The interchange symmetry can arise as a remnant of a fundamental U(2) flavor symmetry acting on the second and third family of leptons. Observe that SU(2) is not enough, because the transformation matrix within

$$\begin{pmatrix}\nu_{\mu L}\\ \nu_{\tau L}\end{pmatrix}\to\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\nu_{\mu L}\\ \nu_{\tau L}\end{pmatrix} \tag{9}$$

is an element of U(2) but not SU(2).

These symmetry principles are simple suggestions for a new physics framework that could lie behind maximal ν_μ-ν_τ mixing. We will now take the pseudo-Dirac possibility and build a complete extension of the SM around it. We will not develop case 2 further in this paper.

III. MODEL FOR A PSEUDO-DIRAC ν_μ-ν_τ SYSTEM

A. Basic framework

We shall now construct a model which realizes the pseudo-Dirac structure of case 1. Our mass matrix will have the general form of Eq. (1), with each of the mass terms arising from the vacuum expectation values (VEVs) of $SU(2)_L$ triplet Higgs fields. Note that the δ_μ and δ_τ terms require Higgs field having opposite charges under $U(1)_{L_\mu-L_\tau}$. For simplicity we shall assume that δ_μ is absent in order to limit the number of Higgs fields.

We wish for the neutrino masses to be naturally tiny, which implies a hierarchy between the VEVs of the Higgs triplets and the standard Higgs doublet, which may be achieved by invoking the VEV seesaw mechanism. This is an appealing scenario whereby the triplets acquire tiny VEVs because they have masses much greater than the electroweak scale.

The Higgs sector we shall consider consists of the following fields:

$$\phi_0\sim(1,2,1,0),\quad \chi_0\sim(1,3,2,0),$$
$$\phi_1\sim(1,2,1,1),\quad \chi_1\sim(1,3,2,1),$$
$$\chi_2\sim(1,3,2,2), \tag{10}$$

where the numbers label the $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y\otimes U(1)_{L_\mu-L_\tau}$ properties. Here ϕ_0 denotes the standard model Higgs field. The additional doublet, ϕ_1, is necessary for the implementation of the VEV seesaw mechanism for χ_1 and χ_2, and will also give rise to off-diagonal terms in the mass matrix of the charged fermions. The triplet χ_0, which has no $U(1)_{L_\mu\text{-}L_\tau}$ charge, is responsible for the m terms in Eq. (1), while the triplet χ_2, which carries an $U(1)_{L_\mu\text{-}L_\tau}$ charge of 2, produces the δ_τ mass term. We round out the model with a third triplet χ_1.

Thus the Higgs-fermion couplings are given by

$$\mathcal{L}_\nu^{\rm Yuk}=\lambda_{\nu_{\mu\tau}}\overline{l^C_{\mu L}}l_{\tau L}\chi_0+\lambda_{\nu_\tau}\overline{l^C_{\tau L}}l_{\tau L}\chi_2+\lambda_{\nu_e}\overline{l^C_{eL}}l_{eL}\chi_0+\lambda_{\nu_{e\tau}}\overline{l^C_{eL}}l_{\tau L}\chi_1+\text{H.c.} \tag{11}$$

and

$$\mathcal{L}_e^{\rm Yuk}=\lambda_{\tau e}\overline{l}_{eL}\phi_1\tau_R+\lambda_{\mu e}\overline{l}_{\mu L}\phi_1 e_R+\lambda_e\overline{l}_{eL}\phi_0 e_R+\lambda_\mu\overline{l}_{\mu L}\phi_0\mu_R+\lambda_\tau\overline{l}_{\tau L}\phi_0\tau_R+\text{H.c.}, \tag{12}$$

where the l's are left-handed leptonic doublets. The Higgs potential is of the form

$$\begin{aligned}V=&\sum_i\left[m_i^2\phi_i^\dagger\phi_i+\frac{1}{2}\lambda_i^2(\phi_i^\dagger\phi_i)^2\right]+\sum_j\left[M_j^2\chi_j^\dagger\chi_j+\frac{1}{2}\Lambda_j^2(\chi_j^\dagger\chi_j)^2\right]+\alpha(\phi_0^\dagger\phi_0)(\phi_1^\dagger\phi_1)\\&+\sum_{j\neq j'}\alpha_{jj'}(\chi_j^\dagger\chi_j)(\chi_{j'}^\dagger\chi_{j'})\\&+\sum_{ij}\beta_{ij}(\phi_i^\dagger\phi_i)(\chi_j^\dagger\chi_j)+[\mu_0\chi_0^\dagger\phi_0^2+\mu_1\chi_1^\dagger\phi_0\phi_1\\&+\mu_2\chi_2^\dagger\phi_1^2+\text{H.c.}],\end{aligned} \tag{13}$$

with $i=0,1$ and $j,j'=0,1,2$. Denoting the VEVs of the Higgs fields by

$$\langle\phi_0\rangle=v_0,\quad\langle\chi_0\rangle=u_0,$$
$$\langle\phi_1\rangle=v_1,\quad\langle\chi_1\rangle=u_1,$$
$$\langle\chi_2\rangle=u_2, \tag{14}$$

it may be observed that for large M_0, M_1, and M_2, the VEV seesaw relations are given by

$$u_0\simeq\frac{\mu_0v_0^2}{M_0^2},$$

$$u_1\simeq\frac{\mu_1v_0v_1}{M_1^2}$$

and

$$u_2\simeq\frac{\mu_2v_1^2}{M_2^2}, \tag{15}$$

and thus we may obtain tiny neutrino masses by making M_0, M_1 and M_2 suitably large.

The proliferation of Higgs fields may not be as *ad hoc* as it appears at first sight. Observe that the quantum numbers of the fields are such that

$$\chi_0 \sim \phi_0^2,$$

$$\chi_1 \sim \phi_0 \phi_1,$$

$$\chi_2 \sim \phi_1^2, \tag{16}$$

hinting, speculatively, that perhaps the χ's can be reinterpreted as composite objects. This would then suggest re-expressing the model in terms of effective operator language, making the replacements,

$$\chi_0 \to \frac{1}{M}\phi_0^2,$$

$$\chi_1 \to \frac{1}{M}\phi_0\phi_1,$$

$$\chi_2 \to \frac{1}{M}\phi_1^2, \tag{17}$$

where M is a large mass scale, which of course is connected to the VEV seesaw mechanism.

While the Higgs potential (13) is undeniably ugly, it is also true that as long as the electroweak Higgs particle remains undiscovered, we cannot claim to really understand gauge symmetry breaking. One may wistfully speculate that symmetry breaking is actually achieved by a more economical mechanism that we shall eventually uncover, and all the Higgs messiness will be re-expressed in more elegant language. In the meantime though, we are forced to work with Higgs fields.

In order to realize the pseudo-Dirac form for the ν_μ-ν_τ sector, we require the hierarchy

$$u_2 \ll u_0, \tag{18}$$

which may be achieved by appropriately adjusting the values of the M_i. According to our symmetry argument, we would also tend to expect $v_1 < v_0$, that is, the VEVs which break L_μ-L_τ ought to be smaller than their L_μ-L_τ conserving counterparts. This would also help to achieve the hierarchy (18), though it is not essential. Note that we also expect $u_1 < u_0$.

We may now write down the neutrino mass matrix,

$$\left(\overline{(\nu_{e'L})^C} \quad \overline{(\nu_{\mu'L})^C} \quad \overline{(\nu_{\tau'L})^C} \right) \times \begin{pmatrix} \lambda_{\nu_e} u_0 & 0 & \lambda_{\nu_{e\tau}} u_1 \\ 0 & 0 & \lambda_{\nu_{\mu\tau}} u_0 \\ \lambda_{\nu_{e\tau}} u_1 & \lambda_{\nu_{\mu\tau}} u_0 & \lambda_{\nu_\tau} u_2 \end{pmatrix} \begin{pmatrix} \nu_{e'L} \\ \nu_{\mu'L} \\ \nu_{\tau'L} \end{pmatrix}, \tag{19}$$

where the primes signify that we are not in the basis where the charged lepton mass matrix is diagonal. To first order in u_1 and u_2 the neutrino masses are

$$m_1 = |\lambda_{\nu_e} u_0|,$$

$$m_2 = \left| -\lambda_{\nu_{\mu\tau}} u_0 + \frac{1}{2}\lambda_{\nu_\tau} u_2 \right|$$

and

$$m_3 = \left| \lambda_{\nu_{\mu\tau}} u_0 + \frac{1}{2}\lambda_{\nu_\tau} u_2 \right|. \tag{20}$$

Typical values for the combination of Yukawa coupling constants and the triplet VEV's might be

$$|\lambda_{\nu_{\mu\tau}} u_0| \sim |\lambda_{\nu_e} u_0| \sim 1 \;\; \text{eV},$$

and

$$|\lambda_{\nu_\tau} u_2| \sim 10^{-3} - 10^{-2} \;\; \text{eV}. \tag{21}$$

Nearly maximal mixing between ν_μ and ν_τ is guaranteed by the hierarchy $u_2 \ll u_0$ (provided that the relavant Yukawa coupling constants do not have a nullifying hierarchy). Mixing between ν_e and the $\nu_{\mu,\tau}$ system is controlled by $\lambda_{\nu_{e\tau}} u_1$. Its magnitude will be discussed shortly.

The mass matrix for the charged leptons has the form

$$\left(\overline{e'_L} \quad \overline{\mu'_L} \quad \overline{\tau'_L} \right) \begin{pmatrix} A & 0 & E \\ D & B & 0 \\ 0 & 0 & C \end{pmatrix} \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} \tag{22}$$

where $A,\ B,\ C \propto v_0$ and $D,\ E \propto v_1$.

In order to obtain the necessary hierarchy in the fermion masses, we must assume some hierarchy in the mass matrix parameters A, B, C, D, and E. Although the choice is by no means unique the possibility which is most natural and compelling is $D, E \ll A \ll B \ll C$. This is the case where the (off-diagonal) L_μ-L_τ violating terms D and E, are much smaller than the (diagonal) L_μ-L_τ conserving terms A, B, and C. In this limit, the charged lepton masses are given by

$$m_e^2 = A^2 + O(D^2, E^2),$$

$$m_\mu^2 = B^2 + O(D^2, E^2),$$

$$m_\tau^2 = C^2 + O(D^2, E^2). \tag{23}$$

Recall from the Introduction that our ambitions in this paper are rather limited: we want to address the $\nu_{\mu,\tau}$ maximal mixing problem without simultaneously solving the entire flavor problem. So, we just have to live with imposed hierarchies like the above.

B. Without LSND

The model building process now presents us with a choice. In this subsection, we will suppose that the LSND anomaly is not due to neutrino oscillations. Degree of freedom economy then suggests that the solar neutrino problem should be solved by $\nu_e \to \nu_{\mu,\tau}$ oscillations. Let us see what parameter range will allow this.

We are immdediately faced with a difficulty. The electron neutrino mass $\sim \lambda_{\nu_e} u_0$ is expected to be of the same order of magnitude as the average $\nu_{\mu,\tau}$ mass $\sim \lambda_{\nu_{\mu\tau}} u_0$. The combined effect of the relatively small solar neutrino δm^2 and the mass gap arising from the pseudo-Dirac structure is to demand a near degeneracy between ν_e and $\nu_{\mu,\tau}$. This entails some fine-tuning.

The mass squared difference corresponding to the atmospheric neutrino anomaly is

$$\delta m^2_{\rm atmos} \sim u_0 u_2, \tag{24}$$

whereas the mass squared difference between the electron neutrino and either of the other two mass eigenstates is naturally

$$\delta m^2_{\rm solar} \sim u_0^2. \tag{25}$$

Since $u_2 \ll u_0$, we have to adjust $|\lambda_{\nu_e}/\lambda_{\nu_{\mu\tau}}| \simeq 1$ so that

$$\delta m^2_{\rm solar} < \delta m^2_{\rm atm}. \tag{26}$$

For example, for either the small or large angle Mikheyev-Smirnov-Wolfenstein (MSW) [15] solutions we need $\delta m^2_{\rm solar} \sim 10^{-5}$ eV^2 which requires $\lambda_{\nu_e}/\lambda_{\nu_{\mu\tau}}$ to be fine-tuned to one part in 10^5 if $m_{2,3} \sim 1$ eV. This is regrettable, although perhaps not egregiously bad.

The leptonic mixing matrix is given by

$$U_{\alpha i} \equiv U_e^\dagger U_\nu \simeq \begin{pmatrix} 1 & \epsilon & \epsilon' \\ \epsilon & 1 & 0 \\ \epsilon & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{N_1}} & \frac{\gamma_1}{\sqrt{N_2}} & \frac{\gamma_2}{\sqrt{N_3}} \\ \frac{\gamma_3}{\sqrt{N_1}} & \frac{1}{\sqrt{N_2}}\left(\frac{1}{\sqrt{2}}+\delta\right) & \frac{1}{\sqrt{N_3}}\left(\frac{1}{\sqrt{2}}-\delta\right) \\ \frac{\gamma_4}{\sqrt{N_1}} & -\frac{1}{\sqrt{N_2}}\left(\frac{1}{\sqrt{2}}-\delta\right) & \frac{1}{\sqrt{N_3}}\left(\frac{1}{\sqrt{2}}+\delta\right) \end{pmatrix} + O(u_1^2, u_2^2), \tag{27}$$

where

$$\epsilon = \frac{AD}{B^2} < \left(\frac{m_e}{m_\mu}\right)^2, \quad \epsilon' = \frac{E}{C} < \frac{m_e}{m_\tau}, \tag{28}$$

$$\delta = \frac{\lambda_{\nu_\tau}}{4\sqrt{2}\lambda_{\nu_{\mu\tau}}} \frac{u_2}{u_0},$$

$$\gamma_1 = \frac{\lambda_{\nu_{e\tau}}}{\sqrt{2}(\lambda_{\nu_{\mu\tau}}+\lambda_{\nu_e})} \frac{u_1}{u_0} \simeq \frac{\lambda_{\nu_{e\tau}}(\lambda_{\nu_{\mu\tau}}-\lambda_e)}{\sqrt{2}\lambda_{\nu_{\mu\tau}}\lambda_{\nu_\tau}} \frac{u_1}{u_2},$$

$$\gamma_2 = \frac{\lambda_{\nu_{e\tau}}}{\sqrt{2}(\lambda_{\nu_{\mu\tau}}-\lambda_{\nu_e})} \frac{u_1}{u_0} \simeq \frac{\lambda_{\nu_{e\tau}}(\lambda_{\nu_{\mu\tau}}+\lambda_e)}{\sqrt{2}\lambda_{\nu_{\mu\tau}}\lambda_{\nu_\tau}} \frac{u_1}{u_2},$$

$$\gamma_3 = \frac{-\lambda_{\nu_{\mu\tau}}\lambda_{\nu_{e\tau}}}{(\lambda^2_{\nu_{\mu\tau}}-\lambda^2_{\nu_e})} \frac{u_1}{u_0} \simeq \frac{-\lambda_{\nu_{e\tau}}}{\lambda_{\nu_\tau}} \frac{u_1}{u_2},$$

$$\gamma_4 = \frac{-\lambda_{\nu_e}\lambda_{\nu_{e\tau}}}{(\lambda^2_{\nu_{\mu\tau}}-\lambda^2_{\nu_e})} \frac{u_1}{u_0} \simeq -\left(\frac{\lambda_{\nu_e}}{\lambda_{\nu_{\mu\tau}}}\right) \frac{\lambda_{\nu_{e\tau}}}{\lambda_{\nu_\tau}} \frac{u_1}{u_2}, \tag{29}$$

and

$$N_1 \simeq 1+\gamma_3^2+\gamma_4^2$$

$$N_2 \simeq 1+\gamma_1^2+O(\delta^2)$$

$$N_3 \simeq 1+\gamma_2^2+O(\delta^2). \tag{30}$$

The second set of near equalities in Eq. (29) is related to the fine-tuning $|\lambda_{\nu_e}/\lambda_{\nu_{\mu\tau}}| \simeq 1$, or more specifically, $(\lambda^2_{\nu_{\mu\tau}} - \lambda^2_{\nu_e}) u_0^2 \simeq \lambda_{\nu_\tau}\lambda_{\nu_{\mu\tau}} u_0 u_2$.

The values of γ_i could be such as to provide either a small or large angle solar (MSW) solution.[3] For example, with $\lambda_{\nu_{e\tau}} u_1 \simeq 0.04 \lambda_{\nu_\tau} u_2$, we would have the small angle solution with $\sin\theta_{\rm solar} \simeq 0.05$. Alternatively, consider for example $\lambda_{\nu_e} \simeq -\lambda_{\nu_{\mu\tau}}$, and $\lambda_{\nu_{e\tau}} u_1 \simeq 0.3 \lambda_{\nu_\tau} u_2$. We then obtain a large angle solution with $\sin\theta_{\rm solar} \simeq \gamma_1 \simeq 0.4$, and $\gamma_2 \ll 1$.

C. With LSND

As we have just seen, the pseudo-Dirac mass gap causes some problems with solving the solar neutrino problem. A more elegant and natural alternative is to exploit the mass gap, rather than trying to fight it. The ν_e and $\nu_{\mu,\tau}$ mass

[3] We note that recent SuperKamiokande data disfavor the small-angle MSW solution [16]. The statistical significance is, however, not yet severe enough to make this type of solution completely uninteresting.

eigenvalues are naturally of the same order, with the scale set by u_0. The ν_e-$\nu_{\mu,\tau}$ δm^2 scale is thus of order u_0^2 if we do not fine tune a near degeneracy.

Indeed, the guideline values of Eq. (21) put the ν_e-ν_μ δm^2 scale in the LSND range. It is certainly noteworthy that the LSND scale is a few orders of magnitude larger than the solar and atmospheric scales, and it is quite attractive to associate this higher scale with the mass gap *which was constructed for another reason*.

The LSND mixing angle must come out of the first equalities in Eq. (29) (the near equalities do not hold in the absence of the previous fine tuning). Using Eq. (21) as a guide, we see that $\lambda_{\nu_{e\tau}} u_1$ should be of order 0.1 eV to put γ_1 or γ_2 in the LSND mixing angle range. Since this is intermediate between the u_0 and u_2 scales, it fits in nicely with our fundamental $u_2 < u_1 < u_0$ symmetry breaking pattern, dictated by approximate $U(1)_{L_\mu\text{-}L_\tau}$ symmetry.

The solar neutrino problem then requires the introduction of a light sterile neutrino ν_s which mixes with ν_e. The most attractive possibility is to have this mixing also being maximal. This scenario fits all of the data, except for Homestake, extremely well [3,17]. The proper incorporation of a light sterile neutrino into the model is really beyond the scope of this paper. However, it is pretty obvious that the mirror matter or exact parity idea [3] is quite relevant, not only for providing a reason for the sterile state to be light, but also to explain the ν_e/ν_s maximal mixing. One could imagine that the mirror matter solution to the neutrino anomalies summarized in Ref. [3] is half correct: that the solar oscillations are into mirror partners, but the atmospheric oscillations are $\nu_\mu \leftrightarrow \nu_\tau$. Intriguingly, Ref. [18] has also recently canvassed this possibility.

IV. OTHER PHENOMENOLOGY

Let us now consider constraints on the model. There will be a new gauge boson Z', corresponding to the $U(1)_{L_\mu\text{-}L_\tau}$ gauge symmetry. Due to the VEV of ϕ_1, there will be a mass mixing term involving the Z' and the ordinary Z gauge boson. In order not to significantly modify the properties of the standard Z boson, the Z' boson must have a mass that is much smaller.

The constraints on the model, however, are not terribly stringent, since the family lepton number violating processes involve essentially only the neutrinos. We suppress the processes involving the charged leptons simply by making the off-diagonal (L_μ-L_τ violating) terms D and E in the mass matrix suitably small, which will also avoid flavor changing neutral current type processes due to exchange of ϕ bosons. Note that this is not an ad hoc requirement, but is in perfect accord with our symmetry argument.

There is also the issue of kinetic mixing of the neutral gauge bosons to consider, which arises whenever you have a theory with two local $U(1)$ symmetries. The kinetic part of the Lagrangian may be written as

$$\mathcal{L}^{\text{kinetic}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} - \frac{2}{4}\kappa F^{\mu\nu}F'_{\mu\nu}, \tag{31}$$

where, in general, the kinetic mixing term, κ, will arise directly in the Lagrangian [19], though it may also be generated radiatively [20]. The physical neutral gauge boson states are then found by diagonalizing the Z and Z' kinetic and mass terms [19]. This alters the coupling of the L_μ-L_τ gauge boson by adding a term proportional to the hypercharge—hence Z' will couple not only to the leptons but also to the quarks, though in a generation independent fashion. We have to assume that κ is small.

An interesting point to consider, since the Z' gauge boson is light, is whether there will be any ''5th force'' effects. In other words, an effective violation of the equivalence principle through a tiny Z' boson mediated repulsion of matter [21,22]. In fact, for suitable parameters, a signature of the model would be a new intermediate range force of nature. The Z,Z' mass matrix is

$$\begin{pmatrix} \dfrac{g^2}{8\cos^2\theta_W}(v_0^2+v_1^2) & \dfrac{gg'}{8\cos\theta_W}v_1^2 \\ \dfrac{gg'}{8\cos\theta_W}v_1^2 & \dfrac{g'^2}{8}v_1^2 \end{pmatrix}, \tag{32}$$

where g' denotes the gauge coupling constant of the $U(1)_{L_\mu\text{-}L_\tau}$ symmetry, and for simplicity, kinetic mixing and the tiny triplet VEVs have been neglected. The light eigenstate, which will be predominately the Z', will have a mass given by

$$M^2_{Z_{\text{light}}} \simeq \frac{g'^2}{8}\frac{v_1^2 v_0^2}{v_1^2+v_0^2}. \tag{33}$$

Of course, in order for the Z' to be detectable through violation of the equivalence principle its mass must be incredibly tiny. For example, if the mass of the Z' was larger than say 10^{-5} eV, the corresponding range of the force would be less than of order 1 cm. The electron couples to the light gauge boson, both through mixing between the e,μ and τ, and mixing of Z and Z'. The coupling is given by

$$\begin{aligned} &\frac{1}{2}g'\cos\theta_Z(\epsilon^2-\epsilon'^2)\overline{e_L}\not{Z}_{\text{light}}e_L \\ &\quad+\frac{1}{2}g'\cos\theta_Z(\zeta^2-\zeta'^2)\overline{e_R}\not{Z}_{\text{light}}e_R \\ &\quad+\sin\theta_Z\frac{g}{4\cos\theta_W}\overline{e}\not{Z}_{\text{light}}(1-4\sin\theta_W-\gamma_5)e \end{aligned} \tag{34}$$

where θ_Z is the Z,Z' mixing angle such that

$$\sin\theta_Z \simeq \tan\theta_Z = \frac{g'\cos\theta_W}{g}\frac{v_1^2}{v_0^2+v_1^2}, \tag{35}$$

and

$$\zeta = \frac{D}{B}, \quad \zeta' = \frac{AE}{C^2}. \tag{36}$$

Note that the strength of the force is diminished both by g' and the small parameters ϵ^2, ϵ'^2, ζ^2, ζ'^2, and $(v_1/v_0)^2$. If one demands consistency with standard big bang Nucleosynthesis, then the very rough bound $g' \lesssim 10^{-10}$ is indicated to prevent the Z' boson being in thermal equilibrium during the relevant epoch of the early universe.

Tests of the equivalence principle on short distance scales are the subject of a proposed experiment [21] which aims to explore the range from about 10 μm to 1 cm, for forces of strength relative to gravity of about 10^{-2} upward. See Fig. 1 of Ref. [21]. The predicted Z' gauge boson may be observable in this experiment, for a certain parameter region.

The Yukawa potential associated with the Z' gauge boson will be of the form

$$V_{\text{Yukawa}} = f^2 \frac{e^{-r/\lambda}}{r}, \tag{37}$$

where $\lambda \simeq 1/M_{Z_{\text{light}}}$ is the range of the force. For the coupling to electrons given by Eq. (34) we have

$$f^2/\hbar c \sim g'^2 \left(\frac{v_1}{v_0}\right)^4, \tag{38}$$

and the strength of the force, relative to gravity, will be

$$\alpha = \frac{f^2}{G_N u^2}, \tag{39}$$

where u is the atomic mass, and G_N is the Newtonian gravitational constant. The range and strength of the force have the approximate values

$$\lambda \sim 10^{-8} \left(\frac{v_0}{v_1}\right)\left(\frac{10^{-10}}{g'}\right) \text{ m},$$

$$\alpha \sim 10^{18} \left(\frac{v_1}{v_0}\right)^4 \left(\frac{g'}{10^{-10}}\right)^2. \tag{40}$$

If we assume for example $g' \sim 10^{-10}$ and $v_1/v_0 \sim 0.1$, the range λ will be too short for the force to be detected, while for smaller values of v_1/v_0 it should be observable in the proposed experiment [21]. If we assume smaller values of g', the range of the force increases and would violate current experimental constraints.

V. CONCLUSION

We have taken a bottom-up approach to constructing a model with maximal mixing between two standard neutrinos. The simple $U(1)_{L_\mu - L_\tau}$ symmetry naturally provides a pseudo-Dirac form for ν_μ and ν_τ with close to maximal mixing, with a mass gap. The model can incorporate mixing with the ν_e, consistent with either a small or large angle MSW solution of the solar neutrino anomaly. However, a mild fine-tuning price must be paid to achieve $\delta m^2_{\text{solar}} < \delta m^2_{\text{atmos}}$. If this price is taken to be too high, then one needs to introduce a light mirror or sterile neutrino to solve the solar deficit problem by $\nu_e \to \nu_s$. It is certainly suggestive that the natural scale for the ν_e-ν_μ mass squared difference could be in the LSND range, due to the mass gap required by the pseudo-Dirac structure. The breaking of the $U(1)_{L_\mu - L_\tau}$ symmetry occurs at a low scale, and we predict a new intermediate range force which may be detectable as an apparent violation of the equivalence principle.

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