New fits for the non-perturbative parameters in the CSS resummation formalism

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We update the non-perturbative function of the Collins-Soper-Sterman (CSS) resummation formalism which resums the large logarithmic terms originating from multiple soft gluon emission in hadron collisions. Two functional forms in impact parameter (b) space are considered, one with a pure Gaussian form with two parameters and another with an additional linear term. The results for the two parameter fit are found to be $g_1=0.24^{+0.08}_{-0.07}$ GeV^2 and $g_2=0.34^{+0.07}_{-0.08}$ GeV^2. The results for the three parameter fit are $g_1=0.15^{+0.04}_{-0.03}$ GeV^2, $g_2=0.48^{+0.04}_{-0.05}$ GeV^2, and $g_3=-0.58^{+0.26}_{-0.20}$ GeV^{-1}. We also discuss the potential of the full Fermilab Tevatron run 1 Z boson data for further testing of the universality of the non-perturbative function within the context of Drell-Yan production.

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I. INTRODUCTION

It is a prediction of the theory of quantum chromodynamics (QCD) that at hadron colliders the production of Drell-Yan pairs or weak gauge bosons ($W^\pm$ and Z) will generally be accompanied by gluon radiation. Therefore, in order to test QCD theory or the electroweak properties of vector bosons, it is necessary to include the effects of multiple gluon emission. At the Fermilab Tevatron (a $p\bar{p}$ collider), we expect about 2×10^6 $W^\pm$ and 6×10^5 Z bosons produced at $\sqrt{S}=1.8$ TeV, per 100 pb^{-1} of luminosity. This large sample of data is useful (i) for QCD studies (with either single or multiple scales), (ii) as a tool for precision measurements of the W boson mass and width, and (iii) as a probe for new physics (e.g., Z'). Achievement of these physics goals requires accurate predictions for the distributions of the rapidity and the transverse momentum of $W^\pm/Z$ bosons and of their decay products.

Consider the production process $h_1h_2\rightarrow VX$. Denote Q_T and Q to be the transverse momentum and the invariant mass of the vector boson V, respectively. When $Q_T\sim Q$, there is only one hard scale and a fixed-order perturbation calculation is reliable. When $Q_T\ll Q$, there are two hard scales and the convergence of the conventional perturbative expansion is impaired. Hence, it is necessary to apply the technique of QCD resummation to combine the *singular* terms in each order of perturbative calculation, which yields

$$\frac{d\sigma}{dQ_T^2}\sim\frac{1}{Q_T^2}\{\alpha_S(L+1)+\alpha_S^2(L^3+L^2)+\alpha_S^3(L^5+L^4)+\alpha_S^4(L^7+L^6)+\cdots+\alpha_S^2(L+1)+\alpha_S^3(L^3+L^2)+\alpha_S^4(L^5+L^4)+\cdots+\alpha_S^3(L+1)+\alpha_S^4(L^3+L^2)+\cdots\}, \tag{1}$$

where α_s is the strong coupling constant, L denotes $\ln(Q^2/Q_T^2)$ and the explicit coefficients multiplying the logs are suppressed.

Resummation of large logarithms yields a Sudakov form factor [1,2] and cures divergences as $Q_T\rightarrow0$. This resummation was pioneered by Dokshitzer, D'yakonov and Troyan (DDT) who performed an analysis in Q_T space which led to a leading-log resummation formalism [1]. Later, Parisi-Petronzio showed [2] that for large Q the $Q_T\rightarrow0$ region can be calculated perturbatively by imposing the condition of transverse momentum conservation:

$$\delta^{(2)}\left(\sum_{i=1}^{n}\vec{k}_{T_i}-\vec{q}_T\right)=\int\frac{d^2b}{4\pi^2}e^{i\vec{Q}_T\cdot\vec{b}}\prod_{i=1}^{n}e^{i\vec{k}_{T_i}\cdot\vec{b}}, \tag{2}$$

in b-space (b is the impact parameter, which is the Fourier conjugate of Q_T). Their improved formalism also sums some subleading-logs. They showed that as $Q\rightarrow\infty$, events at $Q_T\sim0$ may be obtained asymptotically by the emission of at least two gluons whose transverse momenta are not small and add to approximately zero. The intercept at $Q_T=0$ is predicted to be [2]

$$\left.\frac{d\sigma}{dQ_T^2}\right|_{Q_T\rightarrow0}\sim\sigma_0\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)^{\eta_0}, \tag{3}$$

where $\eta_0=A\ln[1+1/A]$ with $A=12C_F/(33-2n_f)$, and $\eta_0\simeq0.6$ for $n_f=4$ and $C_F=4/3$. Collins and Soper extended [3] this work in b-space and applied the properties of the renormalization group invariance to create a formalism that resums all the large log terms to all orders in α_s.

Although various formalisms for resumming large $\ln(Q^2/Q_T^2)$ terms have been proposed in the literature [4,5], we will concentrate in this paper on the formalism given by Collins, Soper and Sterman (CSS) [6], which has been applied to studies of the production of single [7–10] and double [11] weak gauge bosons as well as Higgs bosons [12] at hadron colliders.

II. COLLINS-SOPER-STERMAN RESUMMATION FORMALISM

In the CSS resummation formalism, the cross section is written in the form

$$\frac{d\sigma(h_1h_2\to VX)}{dQ^2\,dQ_T^2dy}=\frac{1}{(2\pi)^2}\int d^2b\, e^{i\vec{Q}_T\cdot\vec{b}}\widetilde{W}(b,Q,x_1,x_2)+Y(Q_T,Q,x_1,x_2), \tag{4}$$

where y is the rapidity of the vector boson V, and the parton momentum fractions are defined as $x_1=e^yQ/\sqrt{S}$ and $x_2=e^{-y}Q/\sqrt{S}$ with $\sqrt{S}$ as the center-of-mass (c.m.) energy of the hadrons h_1 and h_2. In Eq. (4), Y is the regular piece which can be obtained by subtracting the singular terms from the exact fixed-order result. The quantity $\widetilde{W}$ satisfies a renormalization group equation with the solution of the form

$$\widetilde{W}(Q,b,x_1,x_2)=e^{-\mathcal{S}(Q,b,C_1,C_2)}\widetilde{W}\left(\frac{C_1}{C_2b},b,x_1,x_2\right). \tag{5}$$

Here the Sudakov exponent is defined as

$$\mathcal{S}(Q,b,C_1,C_2)=\int_{C_1^2/b^2}^{C_2^2Q^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A(\alpha_s(\bar{\mu}),C_1)\ln\left(\frac{C_2^2Q^2}{\bar{\mu}^2}\right)+B(\alpha_s(\bar{\mu}),C_1,C_2)\right], \tag{6}$$

and the x_1 and x_2 dependence of $\widetilde{W}$ factorizes as

$$\widetilde{W}\left(\frac{C_1}{C_2b},b,x_1,x_2\right)=\sum_j e_j^2\mathcal{C}_{jh_1}\left(\frac{C_1}{C_2b},b,x_1\right)\mathcal{C}_{jh_2}\left(\frac{C_1}{C_2b},b,x_2\right). \tag{7}$$

Here, $\mathcal{C}_{jh}$ is a convolution of the parton distribution function ($f_{a/h}$) with calculable Wilson coefficient functions (C_{ja}), which are defined through

$$\mathcal{C}_{jh}(Q,b,x)=\sum_a\int_x^1\frac{d\xi}{\xi}C_{ja}\left(\frac{x}{\xi},b,\mu=\frac{C_3}{b},Q\right)\times f_{a/h}\left(\xi,\mu=\frac{C_3}{b}\right). \tag{8}$$

The sum on the index a is over incoming partons, j denotes the quark flavors with (electroweak) charge e_j, and the factorization scale μ is fixed to be C_3/b. A few comments about this formalism:

(a) The A, B and C functions can be calculated order-by-order in α_s.

(b) A special choice can be made for the renormalization constants C_i so that the contributions obtained from the expansion in α_s of the CSS resummed calculation agree with those from the fixed-order calculation. This is the canonical choice. It has $C_1=C_3=2e^{-\gamma_E}\equiv b_0$ and $C_2=C_1/b_0=1$, where γ_E is Euler's constant.

(c) b is integrated from 0 to ∞. For $b\gg 1/\Lambda_{QCD}$, the perturbative calculation is no longer reliable. In order to account for non-perturbative contributions from the large b region this formalism includes an additional multiplicative factor which contains measurable parameters.

We refer the readers to Ref. [9] for a more detailed discussion on how to apply the CSS resummation formalism to the phenomenology of hadron collider physics.

A. The non-perturbative function

As noted in the previous section, it is necessary to include an additional factor, usually referred to as the "non-perturbative function," in the CSS resummation formalism in order to incorporate some long distance physics not accounted for by the perturbative derivation. Collins and Soper postulated [6]

$$\widetilde{W}_{j\bar{k}}(b)=\widetilde{W}_{j\bar{k}}(b_*)\widetilde{W}_{j\bar{k}}^{NP}(b), \tag{9}$$

with

$$b_*=\frac{b}{\sqrt{1+(b/b_{max})^2}}, \tag{10}$$

so that b never exceeds b_{max} and $\widetilde{W}_{j\bar{k}}(b_*)$ can be reliably calculated in perturbation theory. (In numerical calculations, b_{max} is typically set to be of the order of 1 GeV^{-1}.) Based upon a renormalization group analysis, they found that the non-perturbative function can be generally written as

$$\widetilde{W}_{j\bar{k}}^{NP}(b,Q,Q_0,x_1,x_2)=\exp\left[-F_1(b)\ln\left(\frac{Q^2}{Q_0^2}\right)-F_{j/h_1}(x_1,b)-F_{\bar{k}/h_2}(x_2,b)\right], \tag{11}$$

where F_1, F_{j/h_1} and $F_{\bar{k}/h_2}$ must be extracted from data with the constraint that

$$\widetilde{W}_{j\bar{k}}^{NP}(b=0)=1.$$

Furthermore, the F_1 term only depends on Q, while F_{j/h_1} and $F_{\bar{k}/h_2}$ in general depend on x_1 or x_2, and their values can depend on the flavor of the initial state partons (j and $\bar{k}$ in this case). Later, in Ref. [13], it was shown that the $F_1(b)\ln(Q^2/Q_0^2)$ dependence is also suggested by infrared renormalon contributions to the Q_T distribution.

B. Testing the universality of $\widetilde{W}_{j\bar{k}}^{NP}$

The CSS resummation formalism suggests that the non-perturbative function should be universal.[1] Its role is analogous to that of the parton distribution function (PDF) in any

[1] Here we mean "universal" within the context of Drell-Yan processes, although it may apply in general to other reactions having the same initial or final state particles as in the Drell-Yan process.

fixed order perturbative calculation, as its value must be determined from data. The first attempt to determine such a universal non-perturbative function was made by Davies, Webber and Stirling (DWS) [14] in 1985 who fit data available at that time to the resummed piece (the $\widetilde{W}_{j\bar{k}}$ term) using the Duke and Owens parton distribution functions [15]. Subsequently, the DWS results were combined with a next-to-leading order (NLO) calculation [16] by Arnold and Kauffman [7] in 1991 to provide the first complete CSS prediction relevant to hadron collider Drell-Yan data. In 1994, Ladinsky and Yuan (LY) [17] observed that the prediction of the DWS set of $\widetilde{W}_{jk}^{NP}$ deviates from R209 data ($p+p\to\mu^+\mu^-+X$ at $\sqrt{S}=62$ GeV) using the CTEQ2M PDF [18]. In order to incorporate possible $\ln(\tau)$ dependence, LY postulated a model for the non-perturbative term, which was different from that of DWS, as

$$\widetilde{W}_{jk}^{NP}(b,Q,Q_0,x_1,x_2)=\exp\left[-g_1b^2-g_2b^2\ln\left(\frac{Q}{2Q_0}\right)-g_1g_3b\ln(100x_1x_2)\right], \qquad (12)$$

where $x_1x_2=\tau$. A ''two-stage fit'' of the R209, Collider Detector at Fermilab (CDF) Z ($4\,\mathrm{pb}^{-1}$ data) and E288 ($p+\mathrm{Cu}$) data gave [17]

$$g_1=0.11^{+0.04}_{-0.03}\ \mathrm{GeV}^2,\quad g_2=0.58^{+0.1}_{-0.2}\ \mathrm{GeV}^2,$$

$$g_3=-1.5^{+0.1}_{-0.1}\ \mathrm{GeV}^{-1},$$

for $Q_0=1.6$ GeV and $b_{max}=0.5\ \mathrm{GeV}^{-1}$.[2] The purpose of the project described here is twofold: First, we update fits for the non-perturbative parameters using modern, high-statistics samples of low energy Drell-Yan data and second we incorporate a fitting technique which will track the full error matrix for all fitted parameters. In a subsequent report we will update these results for the full Tevatron run I datasets [19]. Our results are given in the following sections.

III. FITTING PROCEDURE

A. Choice of the parametrization form

At the present time, the non-perturbative functions in the CSS resummation formalism cannot be derived from QCD theory, so a variety of functional forms should be studied. The only necessary condition is that $\widetilde{W}_{jk}^{NP}(b=0)=1$. For simplicity, we consider only two typical functional forms for $\widetilde{W}_{jk}^{NP}(b,Q,Q_0,x_1,x_2)$ in b space: (i) 2-parameter pure Gaussian form [DWS form],

$$\exp\left[-g_1-g_2\ln\left(\frac{Q}{2Q_0}\right)\right]b^2 \qquad (13)$$

[2]A FORTRAN coding error in calculating the parton densities of the neutron led to an incorrect value for g_3.

TABLE I. Drell-Yan data used in this analysis. Here, δN is the published normalization uncertainty for each experiment. The CDF data were from Tevatron collider run 0 of 4 pb^{-1}.

Experiment	Ref.	Reaction	$\sqrt{S}$ GeV	$\langle\sqrt{\tau}\rangle$	δN
R209	[23]	$p+p\to\mu^+\mu^-+X$	62	$\sim$0.1	10%
E605	[24]	$p+\mathrm{Cu}\to\mu^+\mu^-+X$	38.8	$\sim$0.2	15%
CDF-Z	[25]	$p+\bar{p}\to Z+X$	1800	$\sim$0.05	–
E288	[26]	$p+\mathrm{Cu}\to\mu^+\mu^-+X$	27.4	$\sim$0.2	25%

and (ii) a 3-parameter form [LY form],

$$\exp\left\{\left[-g_1-g_2\ln\left(\frac{Q}{2Q_0}\right)\right]b^2-[g_1g_3\ln(100x_1x_2)]b\right\}, \qquad (14)$$

with a logarithmic x-dependent third term which is linear in b. The $\ln(100x_1x_2)$ term is equivalent to $\ln(\tau/\tau_0)$ for $\sqrt{\tau_0}=0.1$.

Both forms assume no flavor dependence for simplicity. In addition to fitting for the non-perturbative parameters, g_1, g_2, and g_3, the overall normalizations were allowed to float for some fits. One can also study another pure Gaussian form with similar x dependence such as

$$\exp\left[-g_1-g_2\ln\left(\frac{Q}{2Q_0}\right)-g_1g_3\ln(100x_1x_2)\right]b^2.$$

However, we find that current data are not yet precise enough to clearly separate the g_2 and g_3 parameters within this functional form and so it will not be discussed further in this paper. We also tested a few additional functions which did not incorporate additional parameters, but did not find any clear advantage to them when fitting the current Drell-Yan data. However, as to be shown later, the run 1 W/Z data at the Tevatron are expected to determine the g_2 coefficient with good accuracy, and these data can be combined with the low energy Drell-Yan data to further test various scenarios for x dependence and ultimately, universality.

B. Choice of the data sets

In order to determine the non-perturbative functions discussed above, we need to choose experimental data sets for which the contribution to the non-perturbative piece dominates the transverse momentum distributions. This suggests using low energy fixed target or collider data in which the transverse momentum (Q_T) of the Drell-Yan pair is much less than its invariant mass (Q). Because the CSS resummation formalism better describes data in which the Drell-Yan pairs are produced in the central rapidity region (as defined in the center-of-mass frame of the initial state hadrons) we shall concentrate on data with those properties. Based upon the above criteria we chose to consider data shown in Table I. We have also examined the E772 data [20], from the process $p+H^2\to\mu^+\mu^-+X$ at $\sqrt{S}=56.6$ GeV, and found that it was not compatible in our fits with the above data, and it is not included in this study.[3] Except where noted, all of the fits to $g_{1,2,3}$ were done using the CTEQ3M PDF [22] fits.

C. Primary fits

As to be shown later, the E288 data have the smallest errors, and would be expected to dominate the result of a global fit. That is indeed the case. When including the E288 data in a global fit, we found that the resulting fit required the NORM[4] to be too large (as compared to the experimental systematic error) for either the E288 or the E605 data. Furthermore, the shape of the R209 data cannot be well described by the theory prediction based on such a fit.

1. Fits $A_{2,3}$

As explained above, a straightforward global fit (that is, one which includes all of the available data) does not give a satisfactory χ^2 due to the large systematic uncertainties. We therefore employed a different strategy for the global fit based on the statistical quality of the data. We included the first two mass bins ($7<Q<8$ GeV and $8<Q<9$ GeV) of the E605 data, all but the highest mass bin of the R209 results, and all of the early CDF-Z boson data in an initial global fit, referred to here as Fit $A_{2,3}$ (see Figs. 1, 2, and 3). In total, 31 data points were considered. We allowed the normalization of the R209 and E605 data to float within their overall systematic normalization errors, while fixing the normalization of the CDF-Z data to unity. (The point-to-point systematic error of 10% for the E605 data has also been included in the error bars of the data points shown in Fig. 3.) In addition to the normalization factor for each experiment, the fitted parameters of our global fit include the coefficients $g_{1,2}$ and $g_{1,2,3}$ for the 2-parameter and 3-parameter fits, respectively. We found that, for $Q_0=1.6$ GeV and $b_{max}=1/(2\text{ GeV})$, both the 2-parameter and the 3-parameter forms give good fits, with χ^2 per degree of freedom about equal to 1.4.[5] The best fitted central values for Fit A_2 are: $g_1=0.24\text{ GeV}^2$, $g_2=0.34\text{ GeV}^2$. While the central values for Fit A_3 are $g_1=0.15\text{ GeV}^2$, $g_2=0.48\text{ GeV}^2$, $g_3=-0.58\text{ GeV}^{-1}$. The fitted values for the R209 NORM are 1.04 for both Fits A_2 and A_3.

2. Uncertainties in the fits

We have also studied the uncertainties of the fitted g parameters. For the 2-parameter fit, the 1σ error in the χ^2 plot (with an approximately elliptical contour) gives $-0.07<\delta g_1<0.08\text{ GeV}^2$ and $-0.08<\delta g_2<0.07\text{ GeV}^2$. This is shown in Fig. 4. For the 3-parameter fit, the situation is more complicated, as the fitted values of g's are highly correlated as shown in Fig. 5. In order to estimate the uncertainties of the fitted g values, we fix the value (at its best fit value) of g's, one at a time, and examine the uncertainties of the other two, in a way similar to studying the 2-parameter fit result. We found that

$$g_1\text{ fixed:}-0.05<\delta g_2<+0.04\quad\text{GeV}^2,$$

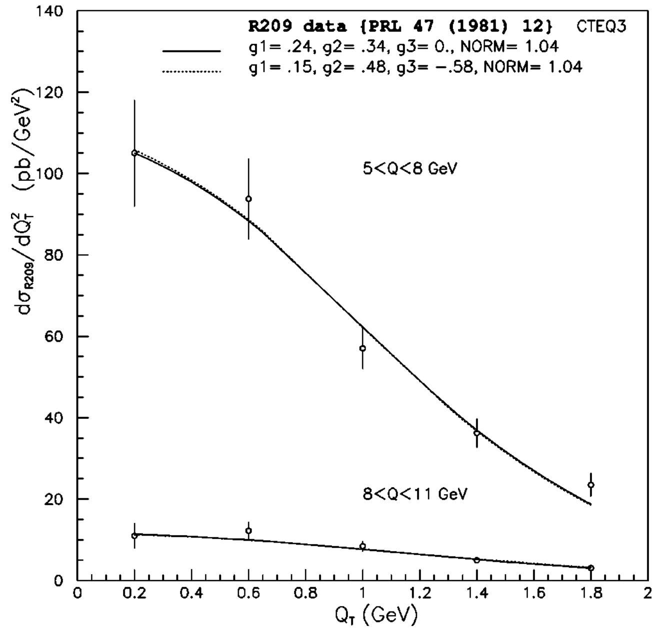

FIG. 1. R209 data, from $p+p\to\mu^+\mu^-+X$ at $\sqrt{S}=62$ GeV, with an overall systematic normalization error of 10%. The curves are the results of Fits A_2 and A_3 and are multiplied by the value of NORM, as shown in the figure and described in the text.

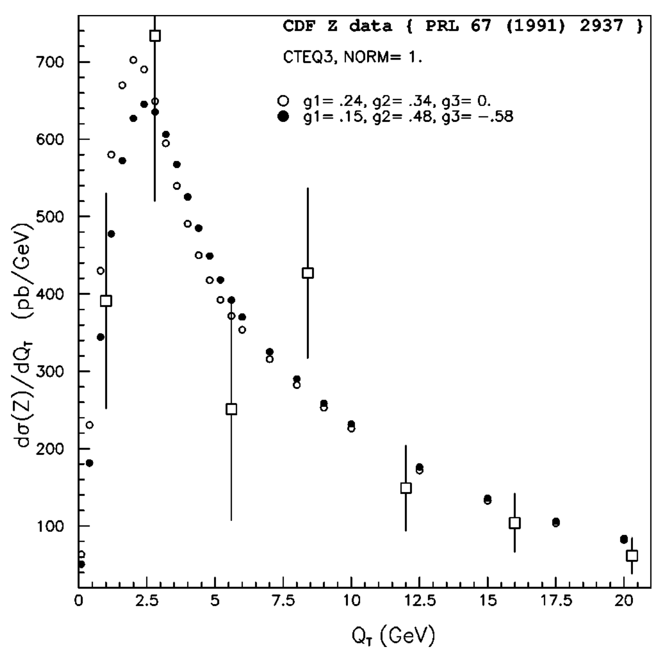

FIG. 2. Comparison of 4 pb^{-1} CDF-Z data at the Tevatron with two different theory model predictions. The dots correspond to the results of Fits A_2 and A_3.

[3]Using the fitted g values to be given below, the theory prediction for the E772 experiment is typically a factor of 2 smaller than the data. Similarly, CTEQ fitting of PDF parameters are not well fit with these data [21].

[4]Here, and in subsequent discussions, the quantity NORM is the fitted normalization factor which is applied to the prediction curves in all that follow: the data are uncorrected.

[5]We scan the values of g_1 and g_2 between 0 and 1, and g_3 between -2 and 3.

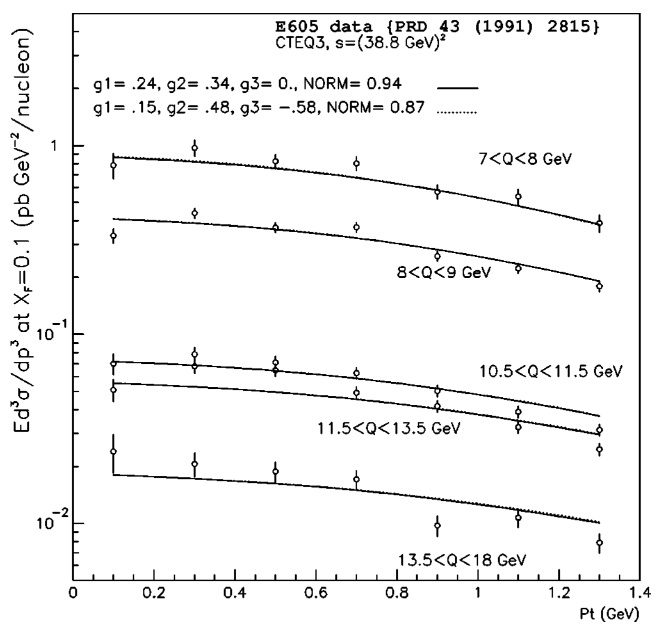

FIG. 3. E605 data, from $p+\mathrm{Cu}\rightarrow\mu^+\mu^-+X$ at $\sqrt{S}=38.8$ GeV, with an overall systematic normalization error of 15%. The curves are the results of Fits A_2 and A_3 and are multiplied by the value of NORM, as shown in the figure and described in the text.

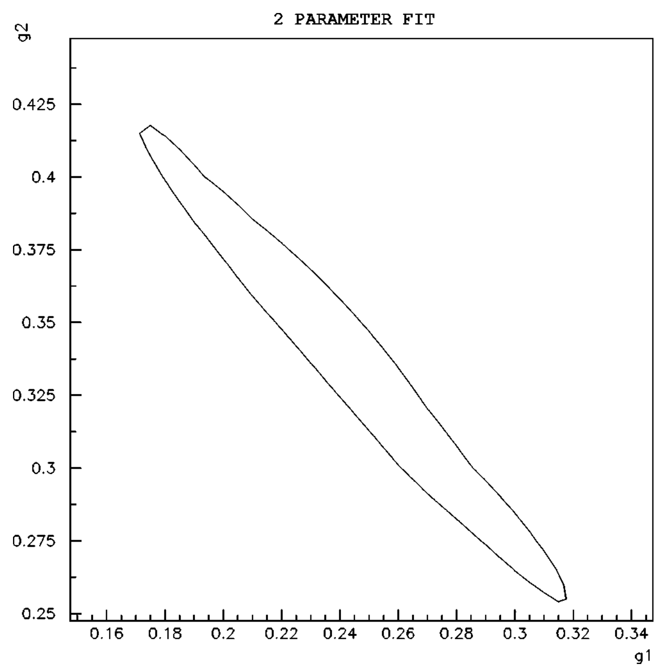

FIG. 4. The error ellipse on the g_1 and g_2 plane from which the errors of the 2-parameter fit A_2 were interpreted.

$$-0.20<\delta g_3<+0.26\ \ \mathrm{GeV}^{-1};$$

$$g_2\ \text{fixed:}-0.03<\delta g_1<+0.03\ \ \mathrm{GeV}^2,$$

$$-0.11<\delta g_3<+0.11\ \ \mathrm{GeV}^{-1};$$

$$g_3\ \text{fixed:}-0.02<\delta g_1<+0.04\ \ \mathrm{GeV}^2,$$

$$-0.03<\delta g_2<+0.02\ \ \mathrm{GeV}^2;$$

constitute a conservative set of uncertainty ranges. With an appreciation for the complexity discussed above, we characterize the uncertainties of the fitted g's in the 3-parameter form conservatively by their maximal deviations among all of the fixed parameter choices just described. Therefore, the best fitted g values are:

$$\text{Fit } A_2: \ g_1=0.24^{+0.08}_{-0.07}\ \ \mathrm{GeV}^2, \quad g_2=0.34^{+0.07}_{-0.08}\ \ \mathrm{GeV}^2;$$

$$\text{Fit } A_3: \ g_1=0.15^{+0.04}_{-0.03}\ \ \mathrm{GeV}^2, \quad g_2=0.48^{+0.04}_{-0.05}\ \ \mathrm{GeV}^2,$$

$$g_3=-0.58^{+0.26}_{-0.20}\ \ \mathrm{GeV}^{-1};$$

for the 2 and 3 parameter fits, respectively. In summary, fits A_2 and A_3 constitute the main results of this paper. The g_1 and g_2 parameter spaces of the above two fits do not strongly overlap because the g parameters are found to be highly correlated.

3. Cross checks

Given these values of g's and the fitted normalization factors for the E605 data, we can calculate the two different predictions for the other three high mass bins not used in Fits $A_{2,3}$, and the results are also shown in Fig. 3. In order to compare with the E288 data, we created Fits $N_{2,3}$ in which we fix the g's to those obtained from Fits $A_{2,3}$ and fit for NORM from the E288 data alone. Figure 6 shows the resulting fits are acceptable, with values of NORM close to the quoted 25%, namely, NORM$=0.92$ and 0.79 for Fits $N_{2,3}$, respectively. It is encouraging that the quality of the fit for the E288 results is very similar to that for the E605 data and that the normalizations are now acceptably within the range quoted by the experiment. Hence, we conclude that the fitted values of g's reasonably describe the wide-ranging, complete set of data, as discussed above.

We note that although the CDF-Z data, as shown in Fig. 2, contain only 7 data points with large statistical uncertainties, they prove to be very useful in determining the value of g_2. To test this observation, we performed an additional fit, L_2. Following the method suggested in [17], we set g_3 to be zero and fit the g_1 and g_2 parameters using the R209 and CDF-Z data alone. (Note that for the R209 data, the typical value of $\sqrt{\tau}$ is of the order 0.1, which motivates the choice of $\sqrt{\tau_0}=0.1$ in the LY form. Effectively, the g_3 contribution to the R209 data can be ignored.) We found that the best fit[6] gives $g_2=0.47$ GeV^2, which is in good agreement with the result of the global Fit A_3 discussed above. Hence, we conclude that the CDF-Z data already play an important role in constraining the g_2 parameter, which can be further improved with large Z data samples from run 1 of the Tevatron collider experiments. We shall defer its discussion to the next section.

[6] Also, $g_1=0.18$ GeV^2.

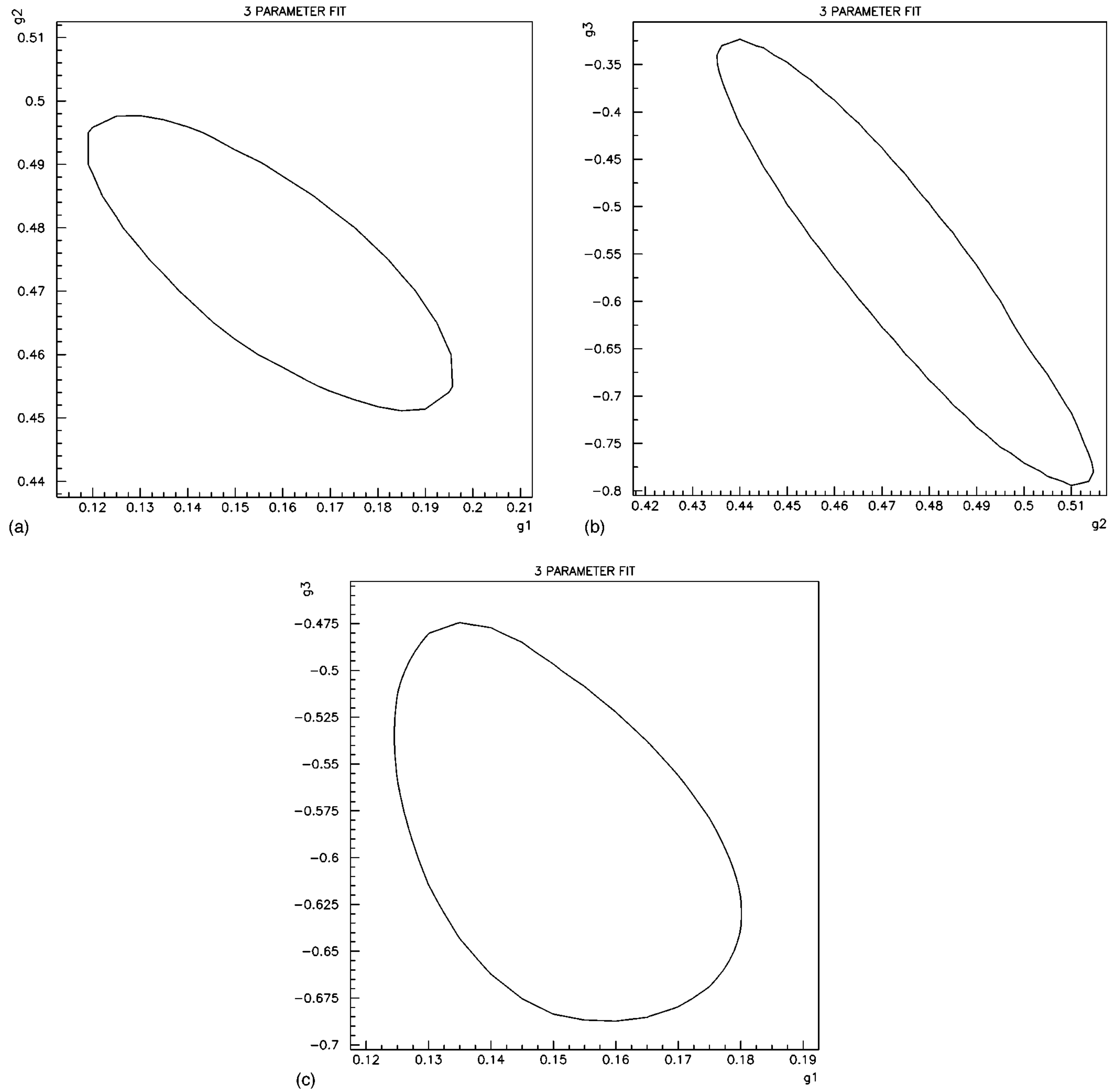

FIG. 5. The error ellipse projections from which the errors of the 3-parameter fit A_3 were interpreted. (a) g_1 and g_2 plane, (b) g_2 and g_3 plane, and (c) g_1 and g_3 plane.

IV. RUN 1 *W* AND *Z* BOSON DATA AT THE TEVATRON

The run 1 *W* and *Z* boson data at the Tevatron can be useful as a test of universality and the *x* dependence of the non-perturbative function $\widetilde{W}_{jk}^{NP}(b,Q,Q_0,x_1,x_2)$. This is clearly demonstrated in Fig. 2, where we give the predictions for the two different global fits (2-parameter and 3-parameter fits) obtained in the previous section using the CTEQ3M PDF parametrizations. (The CTEQ4M PDF [27] gives similar results.) With the large *Z* boson data sets anticipated from Tevatron run 1 (1a and 1b), it should be possible to distinguish these two example models.

As shown in Ref. [9], for $Q_T>10$ GeV the non-perturbative function has little effect on the Q_T distribution, although in principle it affects the whole Q_T range (up to $Q_T \sim Q$). In order to study the resolving power of the full Tevatron run 1 *Z* boson data in determining the non-perturbative function, we have performed a ''toy global fit,'' Fit F_3 as follows. First, we generate a set of fake run 1 *Z* boson data (assuming 5,500 reconstructed *Z* bosons in 24 Q_T bins between $Q_T=0$ and 20 GeV/c) using the original LY fit results ($g_1=0.11$ GeV^2, $g_2=0.58$ GeV^2 and $g_3=-1.5$ GeV^{-1}). Then, we combine these fake-*Z* boson data with the R209 and E605 Drell-Yan data as discussed above to per-

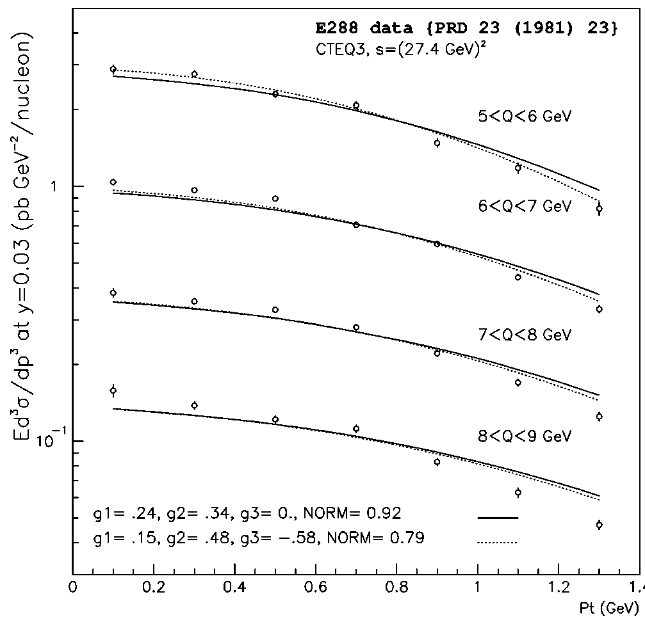

FIG. 6. E288 data, from $p+\mathrm{Cu}\rightarrow\mu^{+}\mu^{-}+X$ at $\sqrt{S}=27.4\,\mathrm{GeV}$, with an overall systematic normalization error of 25%. The curves are the results of Fits N_2 and N_3 and are multiplied by the value of NORM, as shown in the figure and described in the text.

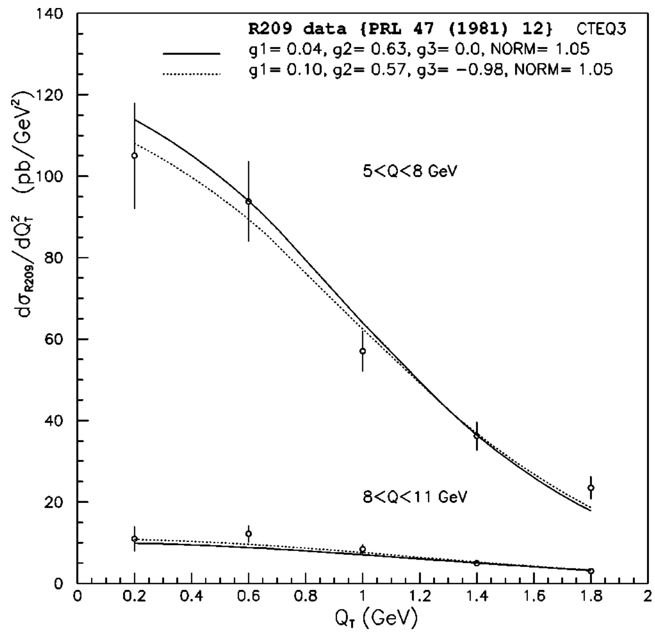

FIG. 7. Comparison of R209 data with two different theory model predictions obtained from the ''toy global fit.'' The curves are the results of Fits F_2 and F_3 and are multiplied by the value of NORM, as shown in the figure and described in the text.

form a global fit. The 3-parameter form results in

$$\text{Fit } F_3\text{:}\ \ g_1=0.10^{+0.02}_{-0.02}\ \mathrm{GeV}^2,\quad g_2=0.57^{+0.01}_{-0.02}\ \mathrm{GeV}^2,$$

$$g_3=-0.98^{+0.15}_{-0.17}\ \mathrm{GeV}^{-1},$$

with a χ^2 per degree of freedom of approximately 1.3.[7] These fitted values for the g's agree perfectly with those used to generate the fake-Z data except for the value of g_3, which is smaller by a factor of 2. It is interesting to note that this result agrees within 2σ with that of Fit A_3, using only the current low energy data, although the uncertainties on g's are smaller by a factor of 2.

We have also performed the same fit for the 2-parameter form and obtained an equally good fit with

$$\text{Fit } F_2\text{:}\ \ g_1=0.04^{+0.03}_{-0.03}\ \mathrm{GeV}^2,\quad g_2=0.63^{+0.04}_{-0.03}\ \mathrm{GeV}^2.$$

While the new g_2 value (0.63 GeV^2) obtained in Fit F_2 is very different from that of A_2 (0.34 GeV^2) given in the previous section, it is actually in good agreement with the g_2 value (0.57 GeV^2) obtained from Fit A_3. This implies that the run 1 Z boson data, when combined with the low energy Drell-Yan data, can be extremely useful in determining the parameter g_2. In Figs. 7–10, we compare the two theory predictions (derived from $F_{2,3}$) with the R209, fake-Z, E605, and E288 data. As shown, they both agree well with all of the data.

Before closing this section, we would like to comment on the result of a single parameter study of the fake Z data, Fit S_1. Given the large sample of run 1 Z data, one can consider fitting the non-perturbative function with only one, Q-independent, non-perturbative parameter. With this in mind, we fitted the fake Z data with the non-perturbative function

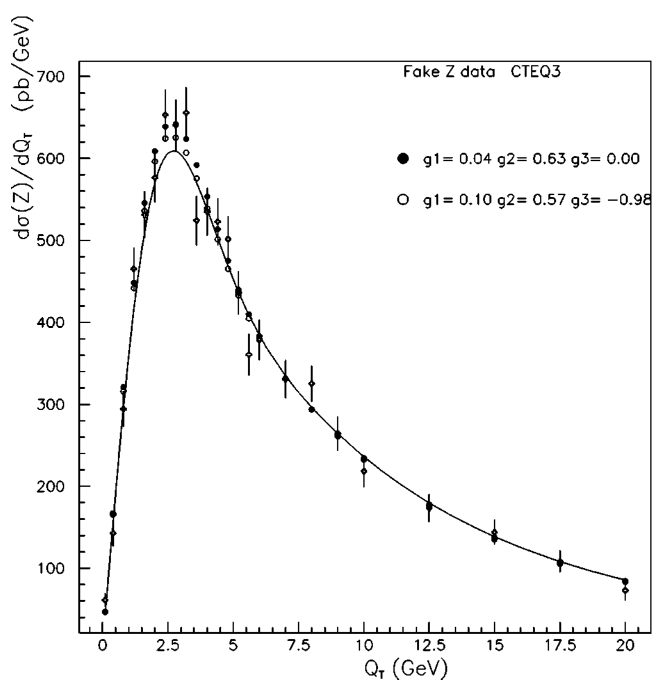

FIG. 8. Comparison of ''fake Z'' data (solid curve) with two different theory model predictions (dots) obtained from the ''toy global fits'' F_2 and F_3.

[7]This amounts to a shift in the prediction for the mass and the width of the W boson by about 5 MeV and 10 MeV, respectively [28].

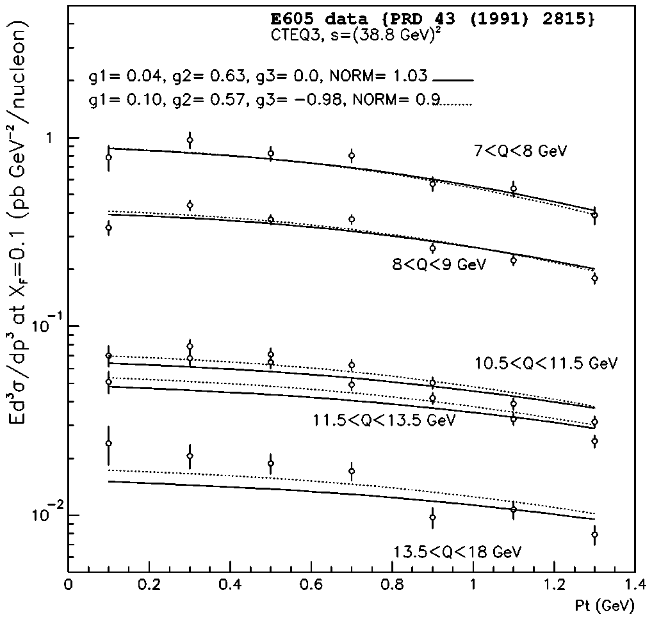

FIG. 9. Comparison of E605 data with two different theory model predictions obtained from the ''toy global fit.'' The curves are the results of Fits F_2 and F_3 and are multiplied by the value of NORM, as shown in the figure and described in the text.

$$\widetilde{W}_{jk}^{NP}(b,Q,Q_0,x_1,x_2)=\exp[-\overline{g_1}b^2], \tag{15}$$

and found that

$$\text{Fit } S_1: \overline{g_1}=2.1^{+0.09}_{-0.08}\ \text{GeV}^2,$$

which gives a good description of the Q_T distribution of the ''fake-Z'' data. It is easy to see that this fitted value agrees with that of the 2-parameter fit just by considering the coefficient of b^2 in first two terms of Eq. (14). For the results of F_2 with the value of the Z boson mass, $M_Z=91.187\ \text{GeV}/c^2$, we obtain $0.04+0.63\ln(M_Z/2Q_0)=2.14$, which is essentially the same as the coeficient of b^2 in S_1 . One interesting question is whether the result of this one-parameter fit alone can be used to also describe the Q_T distribution of the $W^{\pm}$ boson produced at the Tevatron (at the same energy). A quantitative estimate can be easily obtained by noting again that the difference between $0.04+0.63\ln(M_Z/2Q_0)=2.14$ and $0.04+0.63\ln(M_W/2Q_0)=2.06$ with the W-boson mass $M_W=80.3\ \text{GeV}/c^2$, is 0.08, which are essentially the same, given the uncertainty of 0.09 GeV^2 from S_1. We conclude that it is indeed a good approximation to use the one-parameter fit result from fitting Z boson data in order to predict the Q_T distribution of the $W^{\pm}$ boson using the CSS resummation formalism. On the other hand, a single parameter without Q dependence (i.e. the parameter $\overline{g_1}$ alone) does not give a reasonable global fit to all of the Drell-Yan data discussed above. For instance, for the R209 data, the 2-parameter fit gives $0.04+0.63\ln(8/2Q_0)=0.6$ for the coefficient of Eq. (13), which is not consistent with the value of $\overline{g_1}$ from S_1. Hence, we conclude that in order to test the universality of the non-perturbative function of the CSS formalism, one must consider its functional form with Q (and x) dependence.

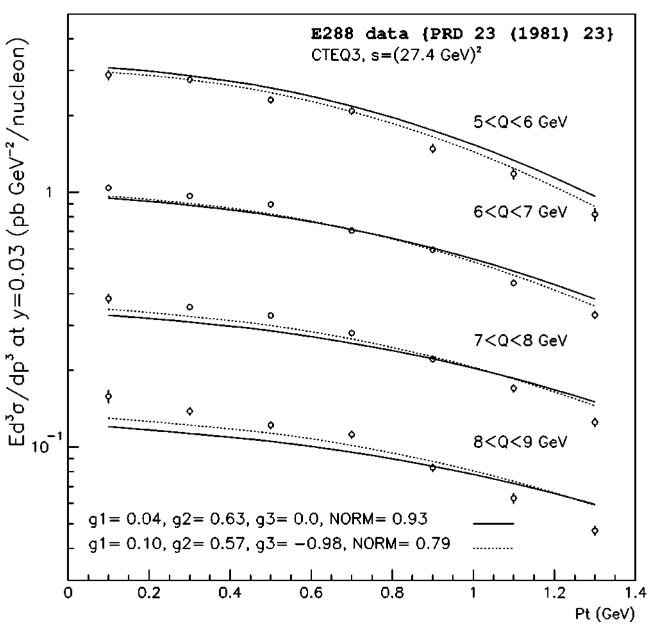

FIG. 10. Comparison of E288 data with two different theory model predictions obtained from the ''toy global fit.'' The curves are the results of Fits F_2 and F_3 and are multiplied by the value of NORM, as shown in the figure and described in the text.

V. CONCLUSIONS

The effects of QCD gluon resummation are important in many precision measurements. In order to make predictions using the CSS resummation formalism for the Q_T distributions of vector bosons at hadron colliders, it is necessary to include contributions from the phenomenological non-perturbative functions inherent to the formalism. In this paper, we have extended previous results by making use of 2-parameter and the 3-parameter fits to modern, low energy Drell-Yan data. We found that both parametrizations result in good fits. In particular our results are

$$\text{Fit } A_2: \ g_1=0.24^{+0.08}_{-0.07}\ \text{GeV}^2, \quad g_2=0.34^{+0.07}_{-0.08}\ \text{GeV}^2.$$

$$\text{Fit } A_3: \ g_1=0.15^{+0.04}_{-0.03}\ \text{GeV}^2, \quad g_2=0.48^{+0.04}_{-0.05}\ \text{GeV}^2,$$

$$g_3=-0.58^{+0.26}_{-0.20}\ \text{GeV}^{-1}.$$

Each functional form predicts measurably different Q_T distributions for Z bosons produced at the Fermilab Tevatron. We showed that the full Tevatron run 1 Z boson data can potentially distinguish these two different models. Table II summarizes all of the fits described in this paper.

In particular, using the results from a toy global fit, we concluded that the large sample of the run 1 Z data can help to determine the value of g_2, which is the coefficient of the $\ln(Q/2Q_0)$ term. Work to incorporate the full run I data is in progress [19]. Given these new data, one can hope to study the x dependence of the non-perturbative function in more detail. We also confirmed that it is reasonable to use a single non-perturbative parameter $\overline{g_1}$ to fit Z boson data, and use

TABLE II. Summary of the results of all fits described in this paper. The last column denotes the figure numbers in which the results are displayed.

Fit	Contents	Label	Results	Figure
$A_{2,3}$	full 2 and	A_2	$g_1=0.24^{+0.08}_{-0.07}$ GeV^2	1,2,3,4,5
	3 parameter		$g_2=0.34^{+0.07}_{-0.08}$ GeV^2	
	global fit	A_3	$g_1=0.15^{+0.04}_{-0.03}$ GeV^2	
			$g_2=0.48^{+0.04}_{-0.05}$ GeV^2	
			$g_3=-0.58^{+0.26}_{-0.20}$ GeV^{-1}	
L_2	$g_3=0$	L_2	$g_1=0.18$ GeV^2	
	R209 plus CDF		$g_2=0.47$ GeV^2	
$N_{2,3}$	fix g's to $A_{2,3}$	N_2	NORM$=0.92$	6
	fit NORM from E288	N_3	NORM$=0.79$	
$F_{2,3}$	g's from LY	F_2	$g_1=0.04^{+0.03}_{-0.03}$ GeV^2	7,8,9,10
	to generate fake Z data		$g_2=0.63^{+0.04}_{-0.03}$ GeV^2	
	plus R209 and E605	F_3	$g_1=0.10^{+0.02}_{-0.02}$ GeV^2	
			$g_2=0.57^{+0.01}_{-0.02}$ GeV^2	
			$g_3=-0.98^{+0.15}_{-0.17}$ GeV^{-1}	
S_1	fake Z data	S_1	$\overline{g}_1=2.1^{+0.09}_{-0.08}$ GeV^2	

that result to study the Q_T distribution of the $W^{\pm}$ boson for $Q_T<10$ GeV. Recently this point has been made in the context of a momentum-space fit [5] using a single parameter. Such an approach might indeed alleviate the computational overhead required in order to generate complete $y-Q_T$ grids of simulated W bosons necessary for M_W analyses. However, if one is interested in testing the universality property of the CSS resummation formalism or making predictions about W and Z boson production at future colliders, such as the CERN Large Hadron Collider, then one must include the Q (and, possibly, x) dependent term in the non-perturbative function.

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