Behavior of the thermal gluon self-energy in the Coulomb gauge

F. T. Brandt
Instituto de Física, Universidade de São Paulo, São Paulo, SP 05315-970, Brazil

Ashok Das
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627-0171

J. Frenkel
Instituto de Física, Universidade de São Paulo, São Paulo, SP 05315-970, Brazil
(Received 13 September 2000; published 27 November 2000)

We study, to one loop order, the behavior of the gluon self-energy in the noncovariant Coulomb gauge at finite temperature. The cancellation of the peculiar energy divergences, which arise in such a gauge, is explicitly verified in the complete two point function of the Yang-Mills theory. At high temperatures, the leading T^2 term is determined to be transverse and nonlocal, in agreement with the results obtained in covariant gauges. The coefficient of the sub-leading ln(T) contribution is nontransverse but local and coincides (up to a multiplicative constant) with that of the ultraviolet pole term of the zero temperature amplitude.

PACS number(s): 11.10.Wx

In thermal field theory, one is often interested in the contributions which arise from the region where the loop momenta are of the same order as the temperature T, with T much larger than all the masses and external momenta [1–5]. Such *hard thermal loop* contributions determine the leading gauge invariant terms of the amplitudes at high temperature, which are important in *resumming* the QCD thermal perturbation theory [6]. In general, the coefficients of these leading order terms are not directly related to the ultraviolet singular terms of the zero temperature amplitudes. In thermal QCD, for example, the n-gluon amplitudes at one-loop order behave like T^2 for high T, even though these amplitudes are ultraviolet finite, at zero temperature, for $n>4$. The hard thermal loop region is also relevant for determining the subleading, ln(T) behavior of the amplitudes. It has been argued that, in contrast with the behavior of the leading T^2 terms, the coefficients of the ln(T) terms are simply related to those of the ultraviolet pole terms of the zero temperature amplitudes [7].

These properties of the amplitudes at high temperature, which have been verified in covariant and axial gauges, were derived under the assumption that the integration over the loop energy q_0 is well defined. On the other hand, it is well known that, in the Coulomb gauge, there are spurious poles at $\vec{q}=0$, leading to divergent energy integrals simply because the denominators of some of the integrands may be independent of q_0. (For a discussion of this and other related aspects of the Coulomb gauge see, for example, [8–13].) Consequently, it is not clear, *a priori*, whether all the properties of the thermal amplitudes, derived in covariant and axial gauges, would continue to hold in the Coulomb gauge as well because of the *ill defined* energy integrals. At zero temperature, there is a proposal to regularize these singularities using a variant [14] of the conventional dimensional regularization [15]. However, it has also been pointed out that, even though individual Feynman diagrams can have divergent energy integrals in the Coulomb gauge, such divergences might cancel when all the contributions to a given amplitude are added together. This has been checked, using the conventional dimensional regularization, in the case of the one-loop self-energy for the gluon at zero temperature [16].

In this Brief Report we verify explicitly, in one-loop QCD, that the cancellation of these *ill defined* terms takes place at finite temperature as well. As a consequence of this, we show that all the properties of the hard thermal loop amplitudes alluded to earlier, continue to hold even in the Coulomb gauge. Thus, we show that the leading T^2 term in the gluon self-energy is nonlocal and is gauge invariant (namely, it is transverse and has the same value as in other gauges). The ln(T) term, on the other hand, is local but nontransverse, with the coefficient coinciding (up to a factor) with that of the ultraviolet pole term of the zero temperature amplitude in the Coulomb gauge. This latter property allows us to determine directly, from the self-energy for the Coulomb field (00 component), the ln(T) correction to the effective coupling constant at high temperature. This simple behavior arises essentially because the Coulomb field is decoupled from the ghosts [17].

To carry out the computation at finite temperature, we use the analytically continued imaginary time formalism [18], where the integration over the loop energy is replaced by a summation over the discrete values $q_0=2\pi i l T$, where l is an integer. The diagrams which contribute to the gluon self-energy, at one-loop, are shown in Fig. 1.

In the Coulomb gauge, the gluon propagator can be written as

$$D^{ab}_{\mu\nu}(q)=\frac{\delta^{ab}}{q^2}\left[\eta_{\mu\nu}+\frac{n^2 q_\mu q_\nu - q\cdot n(q_\mu n_\nu + q_\nu n_\mu)}{\vec{q}^{\,2}}\right], \quad (1)$$

where $n^\mu=(1,0,0,0)$, while the ghost propagator has the form

$$D^{ab}(q)=\frac{\delta^{ab}}{\vec{q}^{\,2}}. \quad (2)$$

This propagator is independent of energy and this is one of the reasons that divergent energy integrals arise.

Let us consider first the ghost loop contribution to the gluon self-energy shown in Fig. 1(c). Using the appropriate ghost-gluon-ghost vertex in the Coulomb gauge, one finds that, at finite temperature, this graph leads to the contribution

$$\Pi^{ij,ab}_{(ghost)}=\frac{\delta^{ab}Ng^2T}{2}\lim_{\eta\to 0^+}\sum_{q_0} q_0^\eta\int\frac{\mathrm{d}^d\vec{q}}{(2\pi)^d}\frac{(2q^iq^j+k^iq^j+k^jq^i)}{\vec{q}^{\,2}(\vec{q}+\vec{k})^2}$$
$$=\frac{\delta^{ab}Ng^2\pi^{\frac{d}{2}}}{2(2\pi)^d}\Gamma\left(1-\frac{d}{2}\right)\frac{\Gamma^2\left(\frac{d}{2}\right)}{\Gamma(d)}|\vec{k}|^{d-4}T\lim_{\eta\to 0^+}\sum_{q_0} q_0^\eta[\delta^{ij}\vec{k}^2+(d-2)k^ik^j]. \tag{3}$$

Here g is the gauge coupling constant, N the color factor of SU(N), d the space dimension (=3 in four space-time dimensions) and Γ denotes the gamma function [19]. The q_0 sum in Eq. (3) has been regularized in the spirit of ζ-function regularization, by introducing the factor q_0^η, with $\eta\to 0^+$ at the end [20]. A key element of this procedure is the process of analytic continuation which makes use of the relation $\Sigma_1^\infty l^\eta=\zeta(-\eta)$, where ζ denotes the Riemann zeta function [19].

It is then easy to check that all such terms cancel out when we take into consideration similar contributions which arise from the 00 components of the internal gluon propagators in the other diagrams in Fig. 1. In fact, adding all such contributions, we obtain

$$\Pi^{ij,ab}_{(00)}+\Pi^{ij,ab}_{(ghost)}=\frac{\delta^{ab}Ng^2T}{2}\lim_{\eta\to 0^+}\sum_{q_0} q_0^\eta$$
$$\times\int\frac{\mathrm{d}^d\vec{q}}{(2\pi)^d}\frac{(k^ik^j+k^iq^j+k^jq^i)}{\vec{q}^{\,2}(\vec{q}+\vec{k})^2}=0 \tag{4}$$

which follows from the anti-symmetry of the integrand in Eq. (4) under $q\to -q-k$.

This exact cancellation can also be proved using a different regularization of the energy independent contributions. An interesting alternative, in this respect, is to employ a gauge which interpolates between the Coulomb and the Feynman gauges, as described in the second paper in [16]. It is easy to verify that the only modification in Eq. (4), which this approach introduces, involves the denominators, which change as $\vec{q}^{\,2}\to\vec{q}^{\,2}-aq_0^2$ and $(\vec{q}+\vec{k})^2\to(\vec{q}+\vec{k})^2-a(q_0+k_0)^2$, where a is the parameter interpolating between the Feynman ($a=1$) and the Coulomb ($a=0$) gauges. In this case, a can be thought of as a regularization parameter which, when the limit $a\to 0$ is taken at the end, regularizes the energy singularities of the Coulomb gauge. The cancellation between the various diagrams now follows from the anti-symmetry as described above.

Since the *ill defined* terms cancel, we can now proceed with the standard method for evaluating the remaining finite temperature contributions, which is facilitated by the use of the relation [18]

$$T\sum_{l=-\infty}^{\infty}I(q_0=\pi ilT)=\frac{1}{4\pi i}\int_{-i\infty}^{+i\infty}\mathrm{d}q_0[I(q_0)+I(-q_0)]$$
$$+\frac{1}{2\pi i}\int_{-i\infty+\delta}^{+i\infty+\delta}\mathrm{d}q_0[I(q_0)$$
$$+I(-q_0)]\frac{1}{\exp(q_0/T)-1}. \tag{5}$$

Here $I(q_0)$ is given by an integral over the space components $\vec{q}$ and $\delta\to 0^+$. The first term on the right-hand side represents the zero-temperature part of the amplitude while the second term contains the thermal corrections which involve the Bose-Einstein distribution function. In the thermal part, the contour in the q_0 complex plane may now be closed in the right half-plane and the q_0 integration performed by evaluating the contributions from the poles of the gluon propagator. In this way, the second term in Eq. (5) may be expressed in terms of forward scattering amplitudes of on-shell thermal particles with four momentum $q^\mu=(|\vec{q}|,\vec{q})$, as illustrated in Fig. 2 [5,21].

Denoting by $A^{\mu\nu,ab}(q,k)$ the total forward scattering amplitude, where the sum over the polarizations and the color states of the thermal gluon is to be understood, we can write the thermal contributions in terms of a momentum integral of $A^{\mu\nu,ab}$ as follows:

$$\Pi^{\mu\nu,ab}_{thermal}=\int\frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\frac{1}{2|\vec{q}|}\frac{1}{\exp(|\vec{q}|/T)-1}$$
$$\times[A^{\mu\nu,ab}(q,k)+A^{\mu\nu,ab}(-q,k)]_{q_0=|\vec{q}|}, \tag{6}$$

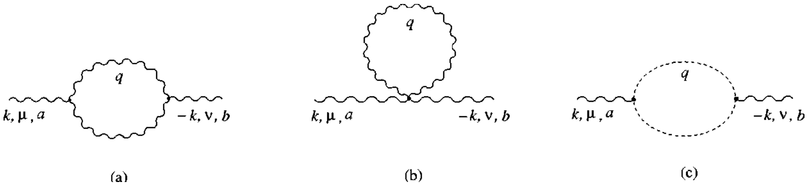

FIG. 1. One-loop diagrams which contribute to the gluon-self energy. Wavy and dashed lines denote, respectively, gluons and ghosts.

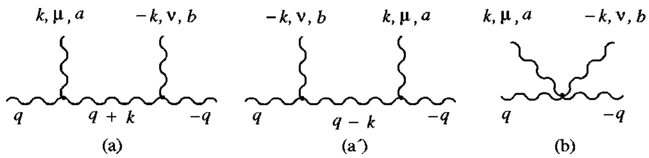

FIG. 2. Forward scattering graphs associated with diagrams (1a) and (1b).

where we have set the space dimension to $d=3$, which is the case of interest to us. Note that the temperature provides a natural ultraviolet cut-off for the thermal corrections. We may now extract from Eq. (6) a series of high-temperature contributions which arise from the region of large q. In the hard thermal region, we can use the expansion

$$\left.\frac{1}{(q+k)^2}\right|_{q_0=|\vec{q}|}=\frac{1}{2q\cdot k}-\frac{k^2}{(2q\cdot k)^2}+\frac{k^4}{(2q\cdot k)^3}+\cdots \quad (7)$$

in the denominators of the forward scattering amplitude, and expand its numerator similarly in powers of k/q. One thus gets for $A^{\mu\nu,ab}$ in Eq. (6) terms which are homogeneous in q of degree $1,0,-1,-2$ and so on. The first term has a denominator of the form $1/(q\cdot k)$ and a numerator which is quadratic in q and independent of k. Such odd terms cancel out in Eq. (6) by symmetry under $q\rightarrow -q$. The next contributions are down by a power of k/q and arise from the terms in $A^{\mu\nu,ab}$ which are of zero degree in q. Such terms yield the leading T^2 contributions. The next non-vanishing contributions come from those terms in $A^{\mu\nu,ab}$ which are of degree -2 in q. By power counting, these give rise to the $\ln(T)$ contributions. Performing the integration over q, we determine the leading T^2 contribution to be

$$\Pi^{\mu\nu,ab}_{(T^2)}=\frac{g^2N\delta^{ab}}{48\pi}T^2\int d\Omega\left(\frac{\hat{q}^\mu k^\nu+\hat{q}^\nu k^\mu}{\hat{q}\cdot k}-\frac{\hat{q}^\mu\hat{q}^\nu k^2}{(\hat{q}\cdot k)^2}-\eta^{\mu\nu}\right), \quad (8)$$

where $\int d\Omega$ denotes the integration over the directions of $\hat{q}=\vec{q}/|\vec{q}|$ and $\hat{q}^\mu=(1,\hat{q})$. This contribution, which is clearly nonlocal and manifestly transverse, agrees with the well known hard thermal loop result obtained in covariant and axial gauges [1–7]. Such a gauge independent contribution has a physical interpretation in terms of plasma frequencies and screening lengths [18].

Furthermore, we have determined the $\ln(T)$ terms in the thermal part of the gluon self-energy to be

$$\Pi^{\mu\nu,ab}_{(\ln T)}=\frac{g^2N\delta^{ab}}{8\pi^2}\left[k^\mu k^\nu-k^2\eta^{\mu\nu}+\frac{4}{3}\frac{k\cdot n}{n^2}(k^\mu n^\nu+k^\nu n^\mu)-\frac{8}{3}\frac{k^2}{n^2}n^\mu n^\nu\right]\ln\left(\frac{T}{\kappa}\right), \quad (9)$$

where $n^\mu=(1,0,0,0)$ and κ is a typical external momentum scale. This expression which is local, but nontransverse, coincides with the coefficient (up to a factor) of the ultraviolet pole term of the zero-temperature self-energy in the Coulomb gauge [14], as expected [7].

The above correspondence implies that the coefficient of $\ln(T/\kappa)$ in Eq. (9) must be the same as the coefficient of $\ln(\kappa/\mu)$ in the renormalized amplitude at zero temperature, where μ is the renormalization scale. This property allows us to determine, in a simple way, the $\ln(T)$ corrections in the running coupling constant at high temperature. To this end, we use the fact that the logarithmic contribution to the running coupling constant, $\overline{g}(\kappa/\mu)$ at $T=0$, can be determined directly from the renormalized Coulomb field amplitude [17]

$$\Pi^{00,ab}_{(\ln\mu)}=\delta^{ab}\vec{k}^2\left[\frac{11Ng^2}{24\pi^2}\ln\left(\frac{\kappa}{\mu}\right)\right]. \quad (10)$$

From the temperature dependent part of the 00 amplitude in Eq. (9), we see that the κ dependence cancels in the total amplitude so that the complete Coulomb thermal amplitude contains only a logarithmic factor $\ln(T/\mu)$. This term will then determine the logarithmic contribution to the running coupling constant $\overline{g}(T/\mu)$ at high temperature.

We would like to thank Professor J. C. Taylor for a helpful correspondence. This work was supported in part by U.S. Dept. of Energy Grant DE-FG 02-91ER40685, NSF-INT-9602559 as well as by CNPq, Brazil.

[1] H. A. Weldon, Phys. Rev. D **26**, 1394 (1982).
[2] K. Kajantie and J. Kapusta, Ann. Phys. (N.Y.) **160**, 477 (1985).
[3] R. D. Pisarski, Nucl. Phys. **B309**, 476 (1988).
[4] E. Braaten and R. D. Pisarski, Phys. Rev. D **45**, 1827 (1992).
[5] J. Frenkel and J. C. Taylor, Nucl. Phys. **B334**, 199 (1990); **B374**, 156 (1992).
[6] E. Braaten and R. D. Pisarski, Nucl. Phys. **B337**, 569 (1990); **B339**, 310 (1990).
[7] F. T. Brandt and J. Frenkel, Phys. Rev. Lett. **74**, 1705 (1995); Phys. Rev. D **55**, 7808 (1997); F. T. Brandt, J. Frenkel, and F. R. Machado, *ibid.* **61**, 125014 (2000).
[8] R. N. Mohapatra, Phys. Rev. D **4**, 378 (1971).
[9] R. Jackiw, I. Muzinich, and C. Rebbi, Phys. Rev. D **17**, 1576 (1978).
[10] N. H. Christ and T. D. Lee, Phys. Rev. D **22**, 939 (1980).
[11] H. Cheng and E.-C. Tsai, Phys. Rev. D **34**, 3858 (1986); **36**, 3196 (1987).
[12] P. J. Doust and J. C. Taylor, Phys. Lett. B **197**, 232 (1987); J. C. Taylor, in *Physical and Nonstandard Gauges*, Lecture Notes in Physics Vol. 361, edited by P. Gaigg, W. Kummer, and M. Schweda (Springer-Verlag, Berlin, 1990), p. 137.
[13] H. A. Weldon, Ann. Phys. (N.Y.) **271**, 141 (1999).
[14] G. Leibbrandt and J. Williams, Nucl. Phys. **B475**, 469 (1996); G. Heinrich and G. Leibbrandt, *ibid.* **B575**, 359 (2000).
[15] G. 't Hooft and M. Veltman, Nucl. Phys. **B44**, 189 (1972).

[16] D. Zwanziger, Nucl. Phys. **B518**, 237 (1998); L. Baulieu and D. Zwanziger, *ibid.* **B548**, 527 (1999).
[17] J. Frenkel and J. C. Taylor, Nucl. Phys. **B109**, 439 (1976).
[18] J. I. Kapusta, *Finite Temperature Field Theory* (Cambridge University Press, Cambridge, England, 1989); M. L. Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, England, 1996); A. Das, *Finite Temperature Field Theory* (World Scientific, New York, 1997).
[19] I. S. Gradshteyn and M. Ryzhik, *Tables of Integral Series and Products* (Academic, New York, 1980).
[20] F. T. Brandt, J. Frenkel, and J. C. Taylor, Phys. Rev. D **44**, 1801 (1991).
[21] F. T. Brandt and J. Frenkel, Phys. Rev. D **56**, 2453 (1997); F. T. Brandt, A. Das, J. Frenkel, and A. J. da Silva, *ibid.* **59**, 065004 (1999).