PHYSICAL REVIEW D, VOLUME 62, 126003

Proposal for non-Bogomol'nyi-Prasad-Sommerfield D-brane action

J. Klusoň*

Department of Theoretical Physics and Astrophysics, Faculty of Science, Masaryk University, Kotlářská 2, 611 37, Brno, Czech Republic

(Received 15 May 2000; published 2 November 2000)

In this paper we would propose a form for the action of a non-BPS D-brane, which will be manifestly supersymmetric invariant and *T*-duality covariant. We also explicitly show that tachyon condensation on the world-volume of this brane leads to the Dirac-Born-Infeld action for a BPS D(p-1)-brane.

PACS number(s): 11.25.−w

I. INTRODUCTION

In Ref. [1] Sen proposed a supersymmetric invariant Dirac-Born-Infeld (DBI) action for a non-Bogomol'nyi-Prasad-Sommerfield (BPS) D-brane. Since the non-BPS branes break all supersymmetries, it seems strange to construct a supersymmetric action describing this brane. However, although there is no manifest supersymmetry of the world-volume theory, we still expect the world-volume theory to be supersymmetric, with the supersymmetry realized as a spontaneously broken symmetry. From these arguments Sen showed that the action has to contain the full number of fermionic zero modes (=32), because they are fermionic Goldstone modes of the completely broken supersymmetry, while a BPS D-brane contains 16 zero modes, because it breaks one-half of the supersymmetry. Sen showed that the DBI action for the non-BPS D-brane (without the presence of the tachyon) is the same as the supersymmetric action describing the BPS D-brane. This action is manifestly invariant under all space-time supersymmetries. Sen argued that the ordinary action for the BPS D-brane contains the DBI term and the Wess-Zumino (WZ) term, which are invariant under supersymmetry. But only when they both are present in the action for the D-brane is the action invariant under local κ symmetry, which is needed for gauging away one-half of the fermionic degrees of freedom, so that only 16 physical fermionic fields remain on the BPS D-brane, as should be the case for an object breaking 16 bulk supersymmetries. Sen showed that the DBI term for a non-BPS D-brane is exactly the same as the DBI term in the action of a BPS D-brane (when we suppose that other massive fields are integrated out, including the tachyon) that is invariant under supersymmetry transformations, but has no κ symmetry, so that the number of fermionic degrees of freedom is 32 which is the appropriate number of fermionic Goldstone modes for an object braking bulk supersymmetry completely.

Sen also showed how we could include the tachyonic field into the action. Sen proposed a form of the term expressing the interaction between the tachyon and other light fields on the world-volume of a non-BPS D-brane on the grounds of invariance under the supersymmetry and general covariance. This term has the useful property that for a constant tachyon field it is zero, so that the action for the non-BPS D-brane in that case vanishes identically.

There have been many attempts to generalize Sen's proposal for the construction of non-BPS D-branes. In a previous paper [2] we tried to construct the action for a non-BPS Dp-brane in type-IIA theory, which was manifestly supersymmetric and also had the property that tachyon condensation in the form of a kink solution leads to the BPS D-brane in type-IIA theory. Different forms of the action for a non-BPS D-brane was suggested in [3,4]. These proposals reflects the remarkable symmetry between the tachyon and other massless degrees of freedom, which was anticipated in [5]. The action presented in [3] was *T*-duality covariant, while the action presented in [2] was not *T*-duality covariant, which is the main problem of that proposal.

In the present paper we propose yet another form of the action for a single non-BPS D-brane which will combine the virtues of all previous attempts. We propose the form of the action, which will be manifestly supersymmetry invariant, *T*-duality covariant, and in the linear approximation the equation of motion for the tachyon will naturally have a smooth tachyon kink solution as a solution. As a result, the action for a non-BPS Dp-brane will reduce to the Dirac-Born-Infeld action for D(p-1)-brane, which together with the tachyon condensation in the Wess-Zumino (WZ) term for a non-BPS D-brane [6,3] will lead to the supersymmetric action for a BPS D(p-1)-brane.

We will also discuss the tachyon kink solution for the action proposed in [3,4]. We find the remarkable fact that the tachyon kink solution in the form of a piecewise function is a natural solution of the linearized equation of motion obtained from this action regardless of the form of the tachyon potential. In the conclusions, we suggest possible extensions of this work.

II. PROPOSAL FOR NON-BPS D-BRANE ACTION

We start this section with recapitulating the basic facts about non-BPS D-branes in type-IIA theory, following [1].[1] $\sigma^{\mu}, \mu=0,\ldots p$ are world-volume coordinates on a D-brane. Fields living on this D-brane arise as the lightest states from the spectrum of the open string ending on this D-brane.

*Email address: klu@physics.muni.cz

[1]For non-BPS D-brane the situation is basically the same with the difference in chirality of the Majorana-Weyl fermions. We refer to [1,8] for more details.

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These open strings have two Chan-Paton (CP) sectors [7]: The first one, with a unit 2×2 matrix, which corresponds to the states of the open string with the usual Gliozzí-Scherk-Olive (GSO) projection $(-1)^F|\psi\rangle=|\psi\rangle$, where F is the world-sheet fermion number and $|\psi\rangle$ is a state from the Hilbert space of the open string living on a Dp-brane. The second CP sector has CP matrix σ_1 and contains states having opposite GSO projection $(-1)^F|\psi\rangle=-|\psi\rangle$. The massless fields living on a Dp-brane are ten components of $X^M(\sigma), M=0,\ldots,9$; a $U(1)$ gauge field $A(\sigma)_\mu$ and a fermionic field θ with 32 real components transforming as a Majorana spinor under the Lorenz group $SO(9,1)$. We can write θ as the sum of a left-handed Majorana-Weyl spinor and a right-handed Majorana-Weyl spinor:

$$\theta=\theta_L+\theta_R, \quad \Gamma_{11}\theta_L=\theta_L, \quad \Gamma_{11}\theta_R=-\theta_R. \tag{1}$$

All fields except θ_R come from the CP sector with the identity matrix, while θ_R comes from the sector with the σ_1 matrix.[2]

As Sen [1] argued, the action for a non-BPS D-brane (without tachyon) should go to the action for BPS D-brane, when we set $\theta_R=0$ (we have the opposite convention of [1]). For this reason, the action for a non-BPS D-brane in [1] was constructed as the supersymmetric DBI action but without κ symmetry so that we are not able to gauge away half of the fermionic degrees of freedom. This action thus describes a non-BPS D-brane.

The next thing to include is the effect of the tachyon. In order to get some relation between tachyon condensation and the supersymmetric D-branes, we would like to have an effective action for the massless fields and the tachyon living on the world-volume of a non-BPS D-brane. Following [1], the effective action for a non-BPS Dp-brane with a tachyonic field on its world-volume should have the form

$$S=-C_p\int d^{p+1}\sigma \times\sqrt{-\det(\mathcal{G}_{\mu\nu}+2\pi\alpha'\mathcal{F}_{\mu\nu})}F(T,\partial T,\theta_L,\theta_R,\mathcal{G},\ldots), \tag{2}$$

$$\Pi^M_\mu=\partial_\mu X^M-\overline{\theta}\Gamma^M\partial_\mu\theta, \mathcal{G}_{\mu\nu}=\eta_{MN}\Pi^M_\mu\Pi^N_\nu \tag{3}$$

and

$$\mathcal{F}_{\mu\nu}=F_{\mu\nu}-\left[\overline{\theta}\Gamma_{11}\Gamma_M\partial_\mu\theta\left(\partial_\nu X^M-\frac{1}{2}\overline{\theta}\Gamma^M\partial_\nu\theta\right)-(\mu\leftrightarrow\nu)\right]. \tag{4}$$

The constant

$$C_p=\sqrt{2}T_p=\frac{2\pi\sqrt{2}}{g(4\pi^2\alpha')^{(p+1)/2}}$$

is a tension for a non-BPS Dp-brane, where T_p is a tension for a BPS Dp-brane and g is a string coupling constant. The function F contains the dependence of the tachyon and its derivatives and may also depend on other world-volume and background fields.

We have proposed [2] an action for a non-BPS D-brane in type-IIA theory in the form

$$S=-C_p\int d^{p+1}\sigma \times\sqrt{-\det(\mathcal{G}_{\mu\nu}+2\pi\alpha'\mathcal{F}_{\mu\nu})}F(T,\partial T,\theta_L,\theta_R,\mathcal{G},\ldots), \tag{5}$$

where the function F takes the form[3]

$$F=(\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu T\partial_\nu T+V(T)+I_{TF}), \tag{6}$$

where I_{FT} contains interaction terms between the tachyon and fermionic fields, which was determined on the base of the supersymmetric invariance. However, this term also contains the expression $f(T)\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu\overline{\theta_R}\partial_\nu\theta_L$, which, as was shown in [3], is not T-duality covariant, so that this term should not be present in the action. On the other hand, the equation of motion for the tachyon obtained from Eq. (5) does lead to the tachyon kink solution and the non-BPS D-brane reduces to the BPS D-brane of codimension one, and the presence of the term cited above leads to the elimination of one half of the fermionic degrees of freedom, which suggests that the resulting kink solution is a BPS D-brane. Then we argued that through tachyon condensation we have restored the κ symmetry on the world-volume of the resulting D-brane. It can seem that the elimination of the term $f(T)\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu\overline{\theta_R}\partial_\mu\theta_L$ on the grounds of T-duality covariance will lead to conclusion that through tachyon condensation we are not able to obtain BPS D-brane. However, as we will show, this is not completely true. We must also say that the term I_{TF} contains many interaction terms with a complicated structure, while the interaction between fermions and tachyons presented in [3] emerges in a very natural and symmetric way. This seems to tell us to follow their approach in the construction of the action for a non-BPS D-brane.

In this paper we would like to propose the Dirac-Born-Infeld action for a single non-BPS D-brane in the form

$$S_{\mathrm{DBI}}=-C_p\int d^{p+1}\sigma V(T) \times\sqrt{-\det\left(\mathcal{G}_{\mu\nu}+2\pi\alpha'\mathcal{F}_{\mu\nu}+\frac{2\pi\alpha'\partial_\mu T\partial_\nu T}{V(T)}\right)}, \tag{7}$$

[2]Our conventions are the following. Γ^M are 32×32 Dirac matrices appropriate to 10d with the relation $\{\Gamma^M,\Gamma^N\}=2\eta^{MN}$, with $\eta^{MN}=(-1,1,\ldots,1)$. For this choice of gamma matrices the massive Dirac equation is $(\Gamma^M\partial_M-M)\Psi=0$. We also introduce $\Gamma_{11}=\Gamma_0\ldots\Gamma_9,(\Gamma_{11})^2=1$.

[3]The meaning of $\tilde{\mathcal{G}}_S^{\mu\nu}$ will be explained latter.

where $V(T)$ is a tachyonic potential, which in the zeroth order approximation is equal to [9,10]

$$V(T)=-2\pi\alpha' m^2T^2/2+\lambda T^4+\frac{(2\pi\alpha' m^2)^2}{16\lambda}=\lambda(T^2-T_0^2)^2, \tag{8}$$

where $m^2=1/2\alpha', T_0^2=2\pi\alpha' m^2/4\lambda$. In the following we do not need to know the explicit value of the constant λ.

We must stress that the form of the action (7) was mainly inspired with the recent proposals [3,4] where the action for non-BPS D-brane was given as

$$S=-C_p\int d^{p+1}\sigma V(T)\times\sqrt{-\det(\mathcal{G}_{\mu\nu}+2\pi\alpha'\mathcal{F}_{\mu\nu}+2\pi\alpha'\partial_\mu T\partial_\nu T)}. \tag{9}$$

We have modified the action given above to the action (7) in order to get smooth tachyon kink solution [9]. Than we will show that the tachyon condensation in the action (7) leads naturally to the DBI action for BPS D-brane and together with the tachyon condensation in the Wess-Zumino term for a non-BPS D-brane proposed in [6] and generalized to the supersymmetric invariant form in [3]

$$S_{\mathrm{WZ}}=\int C\wedge dT\wedge e^{2\pi\alpha'\mathcal{F}}, \tag{10}$$

gives a correct description of a non-BPS D-brane in the approximation of slowly varying fields. However, we will see in the next section that the equation of motion for the tachyon obtained from the linearized form of the action (9) leads to a solution which is a piecewise tachyon kink solution regardless the form of the tachyon potential.

In this paragraph we will discuss the properties of the action (7). The action is manifestly supersymmetric invariant, since it contains the supersymmetric invariant terms [1,8], together with a natural requirement that the tachyon field is invariant under supersymmetric transformations. The action is manifestly invariant under the world-volume reparametrization as well. The action is also T-duality covariant [11]. This can be easily seen from the fact that the potential $V(T)$ does not change under T-duality transformation and the T-duality covariance of the other terms in Eq. (7) was proven in [3,11].

The action (7) is equal to zero for the tachyon equal to its vacuum value T_0 which can be seen from the fact that for $T=$const its derivative is equal to zero while $V(T)\neq 0$ for $T\neq T_0$, so that the action reduces to the action anticipated by Sen [1] for the case of a constant tachyon field

$$S=-C_p\int d^{p+1}\sigma V(T)\sqrt{-\det(\mathcal{G}_{\mu\nu}+2\pi\alpha'\mathcal{F}_{\mu\nu})}\rightarrow 0, \quad \text{if } T\rightarrow T_0. \tag{11}$$

We can also see that for $T=0$, which corresponds to the $(-1)^{F_L}$ operation [12] that takes a non-BPS D-brane in type-IIA (IIB) theory into a BPS D-brane in type-IIB (IIA) theory, the derivative of the tachyon is zero, so that the term $\sim\partial T$ in the action (7) is equal to zero, while $V(0)$ is nonzero and depends on the precise form of the tachyon potential. For example, for the zeroth-order approximation of the tachyon potential, $V(T=0)$ is equal to about 0.60 of the tension of a non-BPS D-brane [9,10] so that the DBI action for a non-BPS D-brane in type-IIA (IIB) theory goes to the DBI action for a BPS D-brane in type-IIB (IIA) theory (of course, with an appropriate modification of fermionic terms, since in type-IIB theory we have spinors of the same chirality) with the tension

$$T_p=0.6T_p^c, \tag{12}$$

where

$$T_p^c=\frac{2\pi}{g(4\pi^2\alpha')^{(p+1)/2}}$$

is the correct tension for a BPS Dp-brane. In the previous equation we have used the transformation rule for the tension of the non-BPS D-brane under $(-1)^{F_L}$ operation [12]: $(-1)^{F_L}: C_p\rightarrow T_p$. We believe that with the inclusion of the higher-order corrections to the tachyon potential we get the exact result.

It is also easy to see that the action given in Eq. (7) reduces into the action (9), when we neglect the higher powers of the tachyon field in the expression $2\pi\alpha'\partial_\mu T\partial_\nu TV(T)^{-1}$, because the tachyon potential must contain the constant term ensuring that the potential is equal to zero for the tachyon equal to its vacuum value. Then we have

$$2\pi\alpha'\partial_\mu T\partial_\nu TV(T)^{-1}\approx A\cdot 2\pi\alpha'\partial_\mu T\partial_\nu T+O(T^4), \tag{13}$$

where A is some constant that depends on the precise form of the tachyon potential. As was argued in [11], the requirement of T duality does not precisely fix the numerical constant in front of the term $(\partial T)^2$, so that the presence of constant A does not affect the similarity with the term given in Eq. (9).

As a last check we will show that in the linear approximation the action (7) reduces to the action (5) without the interaction terms I_{TF} between the fermions and the tachyon. In fact, the interaction between tachyon and fermions is included directly in the form of the DBI action, which can be easily seen from the rewriting the determinant in Eq. (7) in the form

$$\det[(\mathcal{G}+2\pi\alpha'\mathcal{F})_{\mu\nu}]\det[\delta^\mu_\nu+2\pi\alpha'\tilde{\mathcal{G}}_S^{\mu\kappa}\partial_\kappa T\partial_\nu TV(T)^{-1}], \tag{14}$$

where $\tilde{\mathcal{G}}^{\mu\nu}$ is a inverse of $\mathcal{G}+\mathcal{F}$ and $(\dots)_S$ means the symmetric part of a given matrix. This result follows from the fact that $\partial_\mu T\partial_\nu T$ is symmetric in the world-volume indexes. When we expand the second determinant in Eq. (14) and when we restrict ourselves to the linear approximation, then we obtain, from Eq. (7),

$$S_{DBI}=-C_p\int d^{p+1}\sigma\sqrt{-\det(\mathcal{G}_{\mu\nu}+2\pi\alpha'\mathcal{F}_{\mu\nu})}\times\left(V(T)+\frac{2\pi\alpha'}{2}\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu T\partial_\nu T\right), \quad (15)$$

which is the same action as Eq. (5) without the fermionic terms.

We can show that the equation of motion for the tachyon obtained from Eq. (15) leads naturally to the solution, which has the behavior of a kink solution. This solution has been given earlier in [2] and we will review this calculation.

We get the equation of motion for the tachyon from the variation of Eq. (2), which gives (we consider dependence of the tachyon on only one of the coordinates, x say):

$$D\left[\frac{d}{dx}\left(\frac{\delta F}{\delta\partial_x T}\right)-\frac{dF}{dT}\right]+(2\pi\alpha')\partial_\mu D\tilde{\mathcal{G}}_S^{\mu x}\partial_x T=0, \quad (16)$$

where F has a form:

$$F=\left(\frac{2\pi\alpha'}{2}\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu T\partial_\nu T+V(T)\right) \quad (17)$$

and we have defined

$$D=\sqrt{-\det(\mathcal{G}_{\mu\nu}+(2\pi\alpha')\mathcal{F}_{\mu\nu})}. \quad (18)$$

First, we consider the first bracket in Eq. (16). The first expression in Eq. (16) gives

$$2\pi\alpha'\partial_\mu(\tilde{\mathcal{G}}_S^{\mu x}\partial_x T(x))=2\pi\alpha'\partial_\mu(\tilde{\mathcal{G}}_S^{\mu x})\partial_x T+2\pi\alpha'\tilde{\mathcal{G}}_S^{xx}\partial_x(\partial_x T), \quad (19)$$

where we have used the fact that the tachyon field is a function of x only. Since for the tachyon in the form of a kink solution the first derivative is nonzero, the first term in Eq. (19) leads to the result

$$\tilde{\mathcal{G}}_S^{\mu x}=\text{const}. \quad (20)$$

Since the constant in Eq. (20) has not any physical meaning we can take solution in the form

$$\tilde{\mathcal{G}}_S^{\mu x}=0, \quad x\neq\mu,$$

$$\tilde{\mathcal{G}}_S^{xx}=1\Rightarrow G_{xx}=1, \quad G_{x\mu}=0, \quad (21)$$

where $G_{\mu\nu}$ is an inverse matrix of $\tilde{\mathcal{G}}_S^{\mu\nu}$ and plays the role of the natural open string metric [13].

With using Eq. (21), the second expression in Eq. (16) gives the condition

$$\partial_x D=0\Rightarrow\partial_x\mathcal{G}_{\mu\nu}=\partial_x\mathcal{F}_{\mu\nu}=0. \quad (22)$$

When we return to Eq. (21) and use

$$G_{\mu\nu}=\mathcal{G}_{\mu\nu}-(2\pi\alpha')^2\mathcal{F}_{\mu\kappa}\mathcal{G}^{\kappa\delta}\mathcal{F}_{\delta\nu}, \quad (23)$$

we get

$$1=\mathcal{G}_{xx}-(2\pi\alpha')^2\mathcal{F}_{x\alpha}\mathcal{G}^{\alpha\beta}\mathcal{F}_{\beta x}, \quad (24)$$

where $\alpha,\beta=0,\dots,p-1,x=x^p$. Since $\mathcal{G}^{\alpha\beta}\neq0$, we obtain the natural solution of the previous equation in the form:

$$\mathcal{G}_{xx}=1,\mathcal{G}_{x\alpha}=\mathcal{F}_{\alpha\beta}=0. \quad (25)$$

Then we get

$$\det(\mathcal{G}_{\mu\nu}+(2\pi\alpha')\mathcal{F}_{\mu\nu})=\det\begin{pmatrix}\mathcal{G}_{\alpha\beta}+(2\pi\alpha')\mathcal{F}_{\alpha\beta} & 0\\ 0 & 1\end{pmatrix}=\det(\mathcal{G}_{\alpha\beta}+(2\pi\alpha')\mathcal{F}_{\alpha\beta}). \quad (26)$$

When we combine dV/dT with the second term in Eq. (19), we get the equation

$$T''=\frac{1}{2\pi\alpha'}\frac{dV}{dT}, \quad (27)$$

where $T'=dT/dx$. This equation has been solved in many textbooks (see, for example, [14,15]) and we will follow their approach. The integration of the previous equation leads to

$$\frac{dT}{\sqrt{V}}=\sqrt{\frac{2}{2\pi\alpha'}}dx. \quad (28)$$

This equation can be easily integrated for the potential given in Eq. (8) and we get

$$T=T_0\tanh\left(\frac{m}{\sqrt{2}}x\right). \quad (29)$$

Using Eqs. (26), (28) the action (15) has the form

$$S=-C_p\int d^p\sigma dx\sqrt{-\det(\mathcal{G}_{\alpha\beta}+2\pi\alpha'\mathcal{F}_{\alpha\beta})}2V(T(x)), \quad (30)$$

where we have used $V(T)+(2\pi\alpha'/2)T'^2=2V(T)$. We can easily integrate over the x coordinate using Eq. (22) and we get the final result

$$S=-T_{(p-1)}\int d^p\sigma\sqrt{-\det(\mathcal{G}_{\alpha\beta}+2\pi\alpha'\mathcal{F}_{\alpha\beta})}, \quad (31)$$

where the tension for the D$(p-1)$-brane has the form

$$T_{(p-1)}=C_p\int_{-\infty}^{\infty}dx2V(T(x))=C_p\cdot2V_k\int_{-\infty}^{\infty}dx\left[1-\tanh^2\left(\frac{m}{\sqrt{2}}x\right)\right]^2=\frac{2\pi}{g(4\pi^2\alpha')^{(p+1)/2}}\left(\frac{8\sqrt{2}V_k}{3\pi}\right)(4\pi^2\alpha')^{1/2}, \quad (32)$$

where $V_k=(2\pi\alpha' m^2)^2/16\lambda$. As was shown in [9], the vacuum value of the tachyon potential in the zeroth-order approximation cancels about 0.60 of the tension of the non-BPS D-brane, so we have the value of V_k equal to $V_k=0.60$ and the previous equation gives the result

$$T_{(p-1)}=0.72\frac{2\pi}{g(4\pi^2\alpha')^{p/2}}, \tag{33}$$

which is in agreement with the result [9]. We believe that the higher-order correction to the potential as well as using the direct form of the action (without restriction to the linear approximation) (7) could give a correct value of the tension of a D$(p-1)$-brane.

We must also stress that we do not obtain any constraints on the fermionic degrees of freedom. This follows from the fact that there are no interaction terms relating left-handed and right-handed spinors with the tachyon field, since these terms are not allowed through principles of the T-duality covariance. However, this does not contradict the claim that tachyon condensation on the world-volume of a non-BPS Dp-brane leads to the action for the BPS D$(p-1)$-brane, because we must also consider the tachyon condensation in expression (10). It was shown in [6] that the tachyon condensation in this term leads to the correct term for a BPS D$(p-1)$-brane. Using this result and Eq. (31) the whole action after tachyon condensation on the world-volume of a non-BPS Dp-brane has the form

$$S=-T_{p-1}\int d^p\sigma\sqrt{-\det(\mathcal{G}_{\alpha\beta}+2\pi\alpha'\mathcal{F}_{\alpha\beta})}+\mu_{p-1}\int C\wedge e^{2\pi\alpha'\mathcal{F}}. \tag{34}$$

When we assume that tachyon condensation leads to the correct values of D-brane tension T_{p-1} and D-brane charge μ_{p-1}, then Eq. (34) is the supersymmetric action for the D(p-1)-brane with κ symmetry restored.

III. ANOTHER PROPOSAL FOR NON-BPS D-BRANE ACTION

In the recent papers [3,4], the action for a non-BPS Dp-brane was proposed in the explicit form

$$S=-C_p\int d^{p+1}\sigma V(T)\times\sqrt{-\det(\mathcal{G}_{\mu\nu}+2\pi\alpha'\mathcal{F}_{\mu\nu}+2\pi\alpha'\partial_\mu T\partial_\nu T)}. \tag{35}$$

This action is manifestly supersymmetric invariant and also T-duality covariant [3,16,11]. This action obeys the property proposed in [1] that for the tachyon equal to its vacuum value [this is the value of the tachyon that minimizes the potential $V(T)$] $T=T_0, V(T_0)=0$ the action is equal to zero. This action incorporates in a very nice way the tachyon field and the interaction between the tachyon and the massless fields and suggests the deep symmetry between the tachyon and the other fields which was anticipated in [5].

We would like to discuss the equation of motion for the tachyon obtained from the linearized form of action (35). The linearized form of the action is

$$S=-C_p\int d^{p+1}\sigma\sqrt{-\det(\mathcal{G}_{\mu\nu}+2\pi\alpha'\mathcal{F}_{\mu\nu})}\times\left(\frac{2\pi\alpha'}{2}\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu T\partial_\nu T V(T)+V(T)\right), \tag{36}$$

from which we obtain the same equation of motion for the tachyon as in Eq. (16) with function F now given as

$$F=\frac{2\pi\alpha'}{2}V(T)\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu T\partial_\nu T+V(T). \tag{37}$$

The analysis of the this equation is the same as in the previous section and we obtain the same form of the constrains on the massless fields as before. The analysis of the resulting equation for the tachyon is very interesting:

$$\frac{dV}{dT}+\frac{2\pi\alpha'}{2}\frac{dV}{dT}(T')^2-2\pi\alpha'(VT')'=\frac{dV}{dT}+\frac{2\pi\alpha'}{2}\frac{dV}{dT}(T')^2-2\pi\alpha'(V'T'+VT'')=0, \tag{38}$$

where $(\ldots)'=d/dx$. We can immediately see that the solution $T=T_0$ is the solution of equation of motion. This result comes from the fact that $V(T_0)=0, dV/dT|_{T=T_0}=0$ and from the trivial fact that the derivative of a constant function is equal to zero. This confirms the results presented in [3]. To obtain the other solution, we multiply the previous equation with T' and we get

$$V'+\frac{2\pi\alpha'}{2}V'(T')^2-2\pi\alpha'V'(T')^2-\frac{2\pi\alpha'}{2}V((T')^2)'=V'-\frac{2\pi\alpha'}{2}(V(T')^2)'=0, \tag{39}$$

which can be easily integrated with the result

$$V=\frac{2\pi\alpha'}{2}VT'^2+k, \tag{40}$$

where k is an integration constant. We determine this constant from the fact that in order to get the solution with the finite energy, the solution must approache the vacuum value at spatial infinity, where we have $V(T_0)=0, T'\to 0$. We immediately see that $k=0$. Then the next integration gives

$$T=\sqrt{\frac{1}{\pi\alpha'}}x, \tag{41}$$

where this solution does not depend on the precise form of the tachyon potential. We can show that the solution of the equation of the motion is given in terms of the function

$$T=\begin{cases} -T_0, & x<L \\ \sqrt{\frac{1}{\pi\alpha'}}x, & -L<x<L, \\ T_0, & x>L, \end{cases} \tag{42}$$

where the parameter L is determined from the condition that for $x=L$ the tachyon field given in Eq. (41) is equal to its vacuum value

$$T_0=\sqrt{\frac{1}{\pi\alpha'}}L\Rightarrow L=T_0\sqrt{\pi\alpha'}. \tag{43}$$

We see that this solution in the zero slope limit $\alpha'\to 0$ reduces to the piecewise kink solution discussed in [3] that depends only on the vacuum value of the tachyon field.

The next calculation is the same as in the previous section. The tension of the resulting D-brane is given as

$$T_{p-1}=C_p\int_{-\infty}^{\infty}dxF(T)=2C_p\int_{-\infty}^{\infty}dxV(T), \tag{44}$$

where we have used Eq. (40). Using Eq. (42) we obtain from the equation given above

$$T_{p-1}=2C_p\int_{-L}^{L}dxV(T)=2C_p\sqrt{\pi\alpha'}\int_{-T_0}^{T_0}dTV(T), \tag{45}$$

For the zeroth-order approximation to the potential (8) we obtain the result

$$T_{(p-1)}=(\pi\alpha')^{1/2}C_p\frac{32}{15}\lambda T_0^5=0.25\frac{(4\pi^2\alpha')^{1/2}}{\sqrt{2}}C_p$$
$$=0.25T^c_{(p-1)}. \tag{46}$$

We must emphasize again that in the linear approximation this solution does not depend on the exact form of the tachyon potential, it depends only on the vacuum value of the tachyon field. However, we must stress that from these simple calculations we cannot determine the exact form of the action for a non-BPS D-brane. Perhaps more detailed calculations in the string theory could answer the question of what a DBI action for a non-BPS D-brane looks like.

IV. CONCLUSION

In this paper we have proposed the form of the action for a non-BPS Dp-brane, which is manifestly supersymmetric invariant, T-duality covariant, and in the linear approximation we have obtained through the tachyon condensation the supersymmetric action for a D(p-1)-brane with κ symmetry restored. We have also discussed the tachyon kink solution obtained as a solution of the equation of motion which arises from the variation of the linearized action proposed in [3]. We have seen the remarkable fact that we can get a solution which does not depend on the form of the tachyon potential explicitly, it is a function of the tachyon vacuum value only. At present we cannot determine whether our proposal is the correct one only on the grounds of supersymmetry invariance and T-duality covariance. It seems to us that more detailed calculations in string theory could determine the correct form of the DBI action for a non-BPS D-brane.[4]

It would be interesting to extend the action (7) to the non-Abelian case, following [4]. This result could have a direct relation to the classification of D-branes in K theory [17,5,18,19]. We have made some progress in this direction in [20,21], where we have tried to extend the action (5) to the non-Abelian case. It would be nice to see whether the action presented in [20,21] could be modified in order to be related to the non-Abelian extension of the action (7). We hope to return to this question in the future.

ACKNOWLEDGMENTS

I would like to thank Zdeněk Kopecký and especially Rikard von Unge for conversations and critical comments.

[4]We thank A. A. Tseytlin for discussing this point.

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