PHYSICAL REVIEW D, VOLUME 62, 126002

Remark about a non-BPS D-brane in type-IIA theory

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(Received 11 February 2000; published 2 November 2000)

In this paper we show simple mechanisms of how, from the Dirac-Born-Infeld supersymmetric action for non-BPS Dp-brane, we can obtain the Dirac-Born-Infeld supersymmetric action describing a BPS D$(p-1)$-brane in type-IIA theory.

PACS number(s): 11.25.Mj

I. INTRODUCTION

In recent years many new results about D-branes in string theories have emerged. In a remarkable series of paper by Sen [1,2,4–6], the problem of the nonsupersymmetric configuration in string theories was studied. It is clear that non-Bogomol'nyi-Prasad-Sommerfield (BPS) D-branes in type-IIA, and -IIB theories are as important as supersymmetric ones [3]. On the other hand it is known that non-BPS D-branes are not stable, so that they can decay into a supersymmetric string vacuum. The instability of this system is a consequence of the tachyon field that lives on the world volume of the non-BPS D-brane. But the presence of a non-BPS brane in string theory is important for one reason. We can construct the kink solution of the tachyonic field on the world volume of a non-BPS D-brane, which forms a D-brane of a dimension smaller than original non-BPS D-brane. It can be shown on the basis of topological arguments that this solution is stable (for a review of this subject, see Refs. [7,8]) (also see Ref. [9]). Witten generalized this construction, and showed that all branes in type-IIB theory can be constructed as topological defects in the space-time-filling world volumes of D9-branes and D9-antibranes [10]. Hořava extended this construction to the case of type-IIA theory, and showed that all D-branes in type-IIA theory can emerge as topological solutions in space-time-filling non-BPS D9-branes. Hořava also proposed an intriguing conjecture about matrix theory and construction of D0-branes in K-theory [11,12]. For a review of the subject of D-branes and K-theory, see Ref. [13], where many references can be found.

In a recent paper, Sen [14] proposed an supersymmetric invariant action for non-BPS D-branes. Because non-BPS branes break all supersymmetries, it seems to be strange to construct supersymmetric action describing this brane. However, although there is no manifest supersymmetry of world-volume theory, we still expect world-volume theory to be supersymmetric, with the supersymmetry realized as a spontaneously broken symmetry. From these arguments Sen showed that an action has to contain a full number of fermionic zero modes (32), because they are fermionic Goldstone modes of completely broken supersymmetry, while the BPS D-brane contains 16 zero modes, because it breaks one-half of the supersymmetry. Sen showed that the Dirac-Born-Infeld (DBI) action for a non-BPS D-brane (without the presence of a tachyon) is the same as the supersymmetric action describing a BPS D-brane. This action is manifestly symmetric under all space-time supersymmetries. Sen argued that the ordinary action for a BPS D-brane contains a DBI term and a Wess-Zumino term, which are invariant under supersymmetry but only when they both are present in the action for the D-brane; the action is invariant under local symmetry on the brane κ symmetry that is needed for gauging away one-half of the fermionic degrees of freedom, so that on a BPS D-brane only 16 physical fermionic fields live, as should be the case for an object breaking 16 bulk supersymmetries. Sen showed that the DBI term for a non-BPS D-brane is exactly the same as the DBI term in the action of a BPS D-brane (when we suppose that other massive fields are integrated out, including tachyons) that is invariant under sypersymmetric transformations, but has no a κ symmetry; thus the number of fermionic degrees of freedom is 32, which is an appropriate number of fermionic Goldstone modes for an object that breaks bulk supersymmetry completely.

Sen also showed how we could include the tachyonic field in the action. Because the mass of a tachyon is of the order of a string scale, there is no systematic way to construct this effective action for a tachyon; however, on the grounds of invariance under supersymmetry and general covariance Sen proposed a form of this term expressing an interaction between a tachyon and other light fields on the world volume of a non-BPS D-brane. This term has a useful property; for a constant tachyon field it is zero, so that the action for a non-BPS D-brane vanishes identically.

In the present paper we would like to extend the analysis of Ref. [14]. We propose a form of the term containing a tachyon, and we show that the condition of invariance under a supersymmetric transformation places strong constraints on the form of this term. Then we show that tachyon condensation in the form of a kink solution leads to a DBI action for a BPS D-brane of codimension 1 with a gauged local κ symmetry. In conclusion, we will discuss other problems with non-BPS D-branes, and the relation of this construction to the K-theory.

II. ACTION FOR A NON-BPS D-BRANE IN TYPE-IIA THEORY

We start this section by recapitulating the basic facts about non-BPS D-branes in type-IIA theory, following Ref. [14]. Let σ_μ, $\mu=0,\dots,p$, are world-volume coordinates

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on a D-brane. Fields living on this D-brane arise as the lightest states from the spectrum of an open string ending on this D-brane. These open strings have two Chan-Paton (CP) sectors [2]: the first, with a unit 2×2 matrix, corresponds to the states of an open string with a typical Gliozzí-Scherk-Olive (GSO) projection $(-1)^F|\psi\rangle=|\psi\rangle$, where F is the world-sheet fermion number and $|\psi\rangle$ is the state from the Hilbert space of an open string living on Dp-brane. The second CP sector has a CP matrix σ_1, and contains states having opposite GSO projections $(-1)^F|\psi\rangle=-|\psi\rangle$. The massless fields living on the Dp-brane are ten components of $X^M(\sigma)$, $M=0,\ldots,9$; a $U(1)$ gauge field $A(\sigma)_\mu$; and a fermionic field θ with 32 real components transforming as a Majorana spinor under a transverse Lorenz group SO(9,1). We can write θ as the sum of a left-handed Majorana-Weyl spinor and a right-handed Majorana-Weyl spinor:

$$\theta=\theta_L+\theta_R,\quad \Gamma_{11}\theta_L=\theta_L,\quad \Gamma_{11}\theta_R=-\theta_R. \tag{1}$$

All fields except θ_R come from a CP sector with an identity matrix, while θ_R comes from a sector with a σ_1 matrix.[1]

As Sen [14] argued, the action for a non-BPS D-brane (without tachyon) should lead to the action for a BPS D-brane, when we set $\theta_R=0$ (we have the opposite convention from that in Ref. [14]). For this reason, the action for the non-BPS D-brane in Ref. [14] was constructed as a supersymmetric DBI action, which is manifestly supersymmetric invariant but does not have κ symmetry, so we cannot gauge away one-half of the fermionic degrees of freedom; thus this action describes a non-BPS D-brane.

Next we include the effect of the tachyon. In order to obtain some relation between tachyon condensation and supersymmetric D-branes, we need an effective action for massless fields and a tachyon living on the world volume of a non-BPS D-brane. This effective action should appear after integrating out all massive modes of an open string ending on a Dp-brane. Because the tachyon mass is of the order of the string scale, there is no systematic way to obtain an effective action for this field, but we can still study some general properties of this action. Following Ref. [14], the effective action for a non-BPS Dp-brane with a tachyonic field on its world volume should have the forms

$$S=-C_p\int d^{p+1}\sigma\sqrt{-\det(\mathcal{G}_{\mu\nu}+(2\pi\alpha')\mathcal{F}_{\mu\nu})}\times F(T,\partial T,\theta_L,\theta_R,\mathcal{G},\ldots), \tag{2}$$

$$\Pi^M_\mu=\partial_\mu X^M-\bar\theta\Gamma^M\partial_\mu\theta,\quad \mathcal{G}_{\mu\nu}=\eta_{MN}\Pi^M_\mu\Pi^N_\nu \tag{3}$$

and

$$\mathcal{F}_{\mu\nu}=F_{\mu\nu}-[\bar\theta\Gamma_{11}\Gamma_M\partial_\mu\theta(\partial_\nu X^M-\tfrac{1}{2}\bar\theta\Gamma^M\partial_\nu\theta)-(\mu\leftrightarrow\nu)]. \tag{4}$$

The constant

$$C_p=\sqrt{2}T_p=\frac{2\pi\sqrt{2}}{g(4\pi^2\alpha')^{(p+1)/2}}$$

is the tension for a non-BPS Dp-brane, where T_p is the tension for a BPS Dp-brane and g is a string coupling constant.

We must say a few words about the function F, which expresses the presence of a tachyon on the world volume of an unstable non-BPS D-brane. We know that this function must be invariant under Poincaré symmetry and supersymmetry. We also expect that this function should express the interaction between light massless fields living on the world volume of a non-BPS D-brane and a tachyon. We will also suppose, in the construction of this function, that other massive fields were integrated out. Finally, following Ref. [14], we demand that this function is zero for a tachyon equal to its vacuum expectation value T_0, and for $T=0$ is equal to the T_p tension of a BPS D-brane. This corresponds to a $(-1)^{F_L}$ projection, which projects out the tachyon field and also the fermionic field from the σ_1 sector, so that the resulting Dp-brane is a BPS Dp-brane in type-IIB theory [8]:[2]

$$F(T=T_0)=0,\quad F(T=0)=\frac{1}{\sqrt{2}}. \tag{5}$$

Now we propose the form of this function. First, it must contain the kinetic term for a tachyon, which should be written in a manifestly supersymmetry invariant way,

$$I_{KT}=\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu T\partial_\nu T, \tag{6}$$

where $\tilde{\mathcal{G}}$ denotes the matrix inverse of $\mathcal{G}+(2\pi\alpha')\mathcal{F}$, and $\tilde{\mathcal{G}}_S$ is the symmetric part of the matrix. As argued in Ref. [14], the choice of this metric was motivated by Ref. [15], where it was argued that in a constant background field B the natural metric for open strings is $\tilde{\mathcal{G}}_S$; and an ordinary product between functions becomes a noncommutative product. On the other hand, in Refs. [16,17] it was shown that a natural noncommutative parameter is a gauge-invariant combination of $F\ 2\pi 2\ B$, where F is a constant background field strength of a gauge field living on a world-volume Dp-brane. For these

[1]Our conventions are the following. Γ^M are 32×32 Dirac matrices appropriate to $10d$, with relation $\{\Gamma^M,\Gamma^N\}=2\eta^{MN}$, with $\eta^{MN}=(-1,0,\ldots,0)$. For this choice of γ matrices the massive Dirac equation is $(\Gamma^M\partial_M-M)\Psi=0$. We also introduce $\Gamma_{11}=\Gamma_0\cdots\Gamma_9,(\Gamma_{11})^2=1$.

[2]In fact, Sen argued, that in the case of a constant T, F reduces to the potential for a tachyon; as a consequence of the general form of the potential for a tachyon, this term is zero for $T=T_0$. In this paper, we slightly change the behavior of this function, because we only demand that in the world volume, where the tachyon is equal to its vacuum value T_0, we should recover a supersymmetric vacuum, so that there are no fields living on a non-BPS D-brane; thus we have Eq. (5). Sen also argued that, for $T=0$, function F should be equal to C_p; however, we think that this function should rather be equal to T_p, for reasons explained above.

reasons, we take the same metric as in Ref. [14], but, due to the vanishing of the B field and background F field strengths, we expect that noncommutative effects do not appear in our case; hence we will consider ordinary products between any functions. We will see that function F is really invariant under all symmetries presented above.

There should be a potential term for a tachyon in function F. The precise form of the tachyon potential was obtained recently in Ref. [18]. In the zeroth-order approximation, the tachyon potential has a form

$$V(T)=-m^2T^2+\lambda T^4+\frac{m^4}{4\lambda}. \tag{7}$$

This potential has a vacuum value equal to

$$\frac{dV}{dT}=0\Rightarrow T_0^2=\frac{m^2}{2\lambda}, \quad V(T_0)=0. \tag{8}$$

The precise form of parameters m and λ will be obtained later.

We also expect that some interaction terms between a tachyon and X fields and gauge fields will be presented by F. In fact, the interaction between T and X and A is presented in kinetic terms for a tachyon, which can be seen from the form of $\tilde{\mathcal{G}}_S$. We must also stress that a tachyon is not charged with respect to the gauge field, because transforms in the adjoint representation of a gauge group and $U(1)$ have no adjoint representation. For this reason there are no covariant derivatives in the action.

Finally, we will consider the interaction term between the tachyon and fermionic fields θ_L and θ_R. We propose this term in the form

$$\begin{aligned}I_{TF}=&g(T)(\theta_R^T\Gamma^0\Gamma_M\Pi_\mu^M\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\nu\theta_R+\theta_L^T\Gamma^0\Gamma_M\Pi_\mu^M\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\nu\theta_L)\\&+f(T)\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu\theta_R^T\Gamma^0\partial_\nu\theta_L\\&+\tilde{\mathcal{G}}_S^{\rho\kappa}\partial_\rho h(T^2)\Pi_\kappa^M\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu\overline{\theta_L}\Gamma_M\partial_\nu\theta_L\\&+\tilde{\mathcal{G}}_S^{\rho\kappa}\partial_\rho h(T^2)\Pi_\kappa^M\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu\overline{\theta_R}\Gamma_M\partial_\nu\theta_R,\end{aligned} \tag{9}$$

where $g(T)$ and $h(T)$ are even functions of T, and $f(T)$ is odd function of T, which comes from the fact that in perturbative diagrams in string theory, T comes with a CP factor σ_1, θ_L with a CP matrix $\mathbf{1}$, and θ_R with a CP factor σ_1. Then it is clear that $\theta_R\theta_L$ gives σ_1, so in order to have nonzero trace over CP factors, we must have $f(T)$, with odd powers of T, which gives a factor σ_1. In the same way it can be shown that $g(T)$ and $h(T)$ must be even functions of T. We have also used important properties of Majorana-Weyl spinors, which state that the expression of two Weyl spinors of the same chirality with an odd number of γ matrices is zero, and the expression of two Weyl spinors of opposite chiralities with an even number of γ matrices is zero. This can be seen from the following simple arguments

$$\begin{aligned}\theta_L^T(\Gamma_i\cdots\Gamma_{2k+1})\theta_L&=\theta_L^T\Gamma_{11}(\Gamma_i\cdots\Gamma_{2k+1})\Gamma_{11}\theta_L\\&=-\theta_L^T(\Gamma_i\cdots\Gamma_{2k+1})\theta_L,\end{aligned} \tag{10}$$

where we have used the fact that Γ_{11} commutes with an even number of γ matrices, and anticommutes with an odd numbers of γ matrices.

There is a question of whether higher-order derivatives of fermionic fields should be included in Eq. (9). In fact, we expect that there is no reason to include the higher derivatives of fermionic fields. The argument for this goes as follows. We know that the DBI action describes a D-brane in the approximation of slowly varying fields. In other words, in the DBI action only the first derivatives of massless fields are included. When we consider the higher derivatives of fermions in Eq. (9), we should then also consider higher corrections with higher derivatives of massless fields in the DBI action. Since we would like to study the DBI action only, we expect that this action gives a good description; hence we should consider only slowly varying fields where the first derivatives are good enough. For this reason we include only the terms with the first derivatives in Eq. (9).

In summary, we expect that F has a form

$$F=[\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu T\partial_\nu T+V(T)+I_{TF}]. \tag{11}$$

We know that F must be invariant under supersymmetric transformations as well as under Lorenz transformations and translations. We will see that the requirement of supersymmetric invariance places important conditions on various terms in the action.

Under space-time translation, which has forms

$$\delta_\xi X^M=\xi^M, \quad \delta_\xi\theta_{L,R}=0, \quad \delta_\xi T=0, \tag{12}$$

we have

$$\delta_\xi\Pi_\mu^M=0\Rightarrow\delta_\xi\mathcal{G}=0, \quad \delta_\xi\mathcal{F}=0, \tag{13}$$

so that F is invariant.

Under SO(1,9) Lorenz symmetry, various fields transform as

$$X'^M=\Lambda_N^MX^N, \quad \theta'=R(\Lambda)\theta, \quad \overline{\theta'}=\overline{\theta}R(\Lambda)^{-1}, \quad T'=T. \tag{14}$$

We then obtain

$$\Pi_\mu'^M=\Lambda_N^M\partial_\mu X^N-\overline{\theta}R(\Lambda)^{-1}\Gamma^MR(\Lambda)\theta=\Lambda_N^M\Pi_\mu^N, \tag{15}$$

where we have used $R(\Lambda)^{-1}\Gamma^MR(\Lambda)=\Lambda_N^M\Gamma^N$. Then

$$\mathcal{G}'_{\mu\nu}=\eta_{MN}\Lambda_K^M\Pi_\mu^K\Lambda_L^N\Pi_\nu^L=\mathcal{G}_{\mu\nu}, \tag{16}$$

using $\Lambda_K^M\eta_{MN}\Lambda_L^N=\eta_{KL}$. In a similar way we can show that $\delta_\Lambda\mathcal{F}=0$, and consequently

$$\delta_\Lambda\tilde{\mathcal{G}}_S=0. \tag{17}$$

To prove the Lorenz invariance of fermionic terms, we use

$$\theta_R=\tfrac{1}{2}(1+\Gamma_{11})\theta, \theta_L=\tfrac{1}{2}(1-\Gamma_{11})\theta, \tag{18}$$

as well as basic properties of a Γ_{11} matrix, $\Gamma_{11}^T=\Gamma_{11}$ and $R(\Lambda)^{-1}\Gamma_{11}R(\Lambda)=\Gamma_{11}$, to rewrite the expression $\partial_\mu\theta_R\Gamma^0\partial_\nu\theta_L$ as

$$\frac{1}{4}\partial_\mu\theta^T(1+\Gamma_{11})\Gamma^0(1-\Gamma_{11})\partial_\nu\theta=\frac{1}{2}\partial_\mu\overline{\theta}(1-\Gamma_{11})\partial_\nu\theta, \tag{19}$$

which transforms under Lorenz transformation as

$$\begin{aligned}\frac{1}{2}\partial_\mu\overline{\theta'}(1-\Gamma_{11})\partial_\nu\theta'&=\frac{1}{2}\partial_\mu\overline{\theta}R(\Lambda)^{-1}(1-\Gamma_{11})R(\Lambda)\partial_\nu\theta\\&=\frac{1}{2}\partial_\mu\overline{\theta}\partial_\nu\theta,\end{aligned} \tag{20}$$

which prove the Lorenz invariance of this term. In the same way we can prove the invariance of the expression

$$\theta_R^T\Gamma^0\Gamma_M\Pi_\mu^M\partial_\nu\theta_R=\frac{1}{2}\overline{\theta}\Gamma_M(1+\Gamma_{11})\Pi_\mu^M\partial_\nu\theta. \tag{21}$$

This term transforms under Lorenz transformation as

$$\begin{aligned}&\frac{1}{2}\overline{\theta}R^{-1}\Gamma_M(1+\Gamma_{11})\Lambda_K^M\Pi_\mu^K R\partial_\nu\theta\\&\quad=\frac{1}{2}\overline{\theta}\Lambda_M^L\Gamma_L(1+\Gamma_{11})\Lambda_K^M\Pi_\mu^K\partial_\nu\theta\\&\quad=\frac{1}{2}\overline{\theta}\Gamma_M(1+\Gamma_{11})\Pi_\mu^M\partial_\nu\theta.\end{aligned} \tag{22}$$

Then it is also easy to see that the term in Eq. (9) proportional to $\partial_\rho h(T^2)$ is invariant under a Lorenz transformation as well. This comes from the fact that the fermionic part of this term is the same as the term above, except for the presence of the derivative $\partial_\mu\theta_{R,L}^T$. However, this does not spoil the invariance under a Lorenz transformation, because Lorenz transformations are global transformations from the point of view of world-sheet theory, and consequently are not functions of σ^μ [19]. We see that the all integration terms between fermions and tachyons are Lorenz invariant.

Now we come to the crucial question of supersymmetry transformation, which has forms

$$\delta_\epsilon\theta=\epsilon,\qquad \delta_\epsilon X^M=\overline{\epsilon}\Gamma^M\theta.$$

It is well known that these transformations leave Π_μ^M, and consequently $\mathcal{G}$, invariant. It can be also shown [19] that $\mathcal{F}$ is invariant as well. As a result, we have

$$\delta_\epsilon\tilde{\mathcal{G}}_S=0. \tag{23}$$

Now we are ready to prove the invariance of the term

$$f(T)\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu\theta_R^T\Gamma^0\partial_\nu\theta_L=\frac{1}{2}f(T)\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu\overline{\theta}(1-\Gamma_{11})\partial_\nu\theta. \tag{24}$$

This term is clearly invariant under supersymmetry transformations, due to the presence of a partial derivative. In the same way we can prove the invariance of a term proportional to $\partial_\rho h(T^2)$. On the other hand, the term

$$\frac{1}{2}g(T)\tilde{\mathcal{G}}_S^{\mu\nu}\overline{\theta}\Gamma_M(1+\Gamma_{11})\Pi_\mu^M\partial_\nu\theta \tag{25}$$

leads, after a supersymmetric transformation, to a variation of the action

$$\begin{aligned}\delta S&=\int D\overline{\epsilon}\Gamma_M(1+\Gamma_{11})\Pi_\mu^M\partial_\nu\theta\\&=-\int\partial_\nu(D\Pi_\mu^M)\overline{\epsilon}\Gamma_M(1+\Gamma_{11})\theta,\end{aligned} \tag{26}$$

where we have used

$$D=\sqrt{-\det(\mathcal{G}+(2\pi\alpha')\mathcal{F})}\ \frac{1}{2}g(T)\tilde{\mathcal{G}}_S^{\mu\nu}. \tag{27}$$

We see that the requirement of invariance under a transformation of the supersymmetry leads to the conclusion that the term $g(T)\ldots$ should not be present in the action for a non-BPS D-brane; thus we consider the interaction term between fermions and tachyons in the form

$$\begin{aligned}I_{TF}=&f(T)\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu\theta_R^T\Gamma^0\partial_\nu\theta_L\\&+\tilde{\mathcal{G}}_S^{\rho\kappa}\partial_\rho h(T^2)\Pi_\kappa^M\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu\theta_L\Gamma^0\Gamma_M\partial_\nu\theta_L\\&+\tilde{\mathcal{G}}_S^{\rho\kappa}\partial_\rho h(T^2)\Pi_\kappa^M\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu\theta_R\Gamma^0\Gamma_M\partial_\nu\theta_R.\end{aligned} \tag{28}$$

It is important to stress that this term is consistent with the requirements that $f(T)$ should be an odd function of T and $h(T^2)$ should be an even function of T. This can be seen from the fact that $\tilde{\mathcal{G}}_S$ has a CP factor equal to the unit matrix. To prove this, we expand $\mathcal{G}$ as follows:

$$\begin{aligned}\mathcal{G}_{\mu\nu}=&\eta_{MN}\partial_\mu X^M\partial_\nu X^N-2\eta_{MN}\partial_\mu X^M\overline{\theta}\Gamma^N\partial_\nu\theta\\&+\eta_{MN}(\overline{\theta}\Gamma^M\partial_\mu\theta)(\overline{\theta}\Gamma^N\partial_\nu\theta).\end{aligned} \tag{29}$$

We know that X^M comes from a CP sector with a unit matrix, so that only one ‘‘dangerous’’ term is

$$\begin{aligned}\overline{\theta}\Gamma^N\partial_\nu\theta&=(\theta_R+\theta_L)^T\Gamma^0\Gamma^N\partial_\nu(\theta_R+\theta_L)\\&=\theta_R^T\Gamma^0\Gamma^N\partial_\nu\theta_R+\theta_L^T\Gamma^0\Gamma^N\partial_\nu\theta_L.\end{aligned} \tag{30}$$

These two terms give CP factors of either $\mathbf{11}=\mathbf{1}$ (for θ_L) or $(\sigma_1\sigma_1)=\mathbf{1}$ for (θ_R). (In Sec. I we used the result $\theta_R^T\Gamma^0\Gamma^M\partial_\mu\theta_L=-\theta_R^T\Gamma_{11}\Gamma_0\Gamma^M\Gamma_{11}\partial_\mu\theta_L=-\theta_R^T\Gamma^0\Gamma^M\partial_\mu\theta_L$.)

In the same way we can prove that $\mathcal{F}$ comes with a unit matrix of CP factors. For example, the expression $\overline{\theta}\Gamma_{11}\Gamma_M\partial_\mu\theta$ is equal to $\theta_R^T\Gamma^0\Gamma_{11}\Gamma_M\partial_\mu\theta_R+\theta_L^T\Gamma^0\Gamma_{11}\Gamma_M\partial_\mu\theta_L$, where we have used the identity

$$\begin{aligned}\theta_L^T\Gamma^0\Gamma_{11}\Gamma_M\theta_R&=-\theta_L^T\Gamma_{11}\Gamma_0\Gamma_{11}\Gamma_M\Gamma_{11}\theta_R\\&=-\theta_L^T\Gamma_0\Gamma_{11}\Gamma_M\theta_R.\end{aligned} \tag{31}$$

It seems to us that the requirement of supersymmetry places a strong constraint on the coupling between the fermions and tachyon. In particular, we have seen that fermions must always come with a partial derivative. In Sec. III we will show that tachyon condensation in the form of a kink solution leads to the correct supersymmetric invariant DBI action of the BPS D$(p-1)$-brane.

III. TACHYON CONDENSATION ON WORLD-VOLUME OF A NON-BPS D-BRANE

In this section we will consider tachyon condensation on the world volume of a non-BPS Dp-brane in the form of a kink solution, in a way similar to that used in Ref. [18]. In that paper it was shown that tachyon condensation in the form of a kink solution gives approximately the correct value of tension of a D$(p-1)$-brane. We show, on the grounds of the action given in Eq. (2), that the tachyon condensation in the form of the kink solution really leads to the supersymmetric action for BPS D$(p-1)$-brane.

We will consider the situation where the tachyon field is a function of one single coordinate x on world-volume of a non-BPS D-brane. We obtain equation of motion for a tachyon from a variation of Eq. (2), which gives (we consider a dependence of a tachyon only on x, say a p coordinate)

$$G\left[\frac{d}{dx}\left(\frac{\delta F}{\delta\partial_x T}\right)-\frac{dF}{dT}\right]+\partial_\mu G\partial^\mu T=0, \tag{32}$$

where F has a form

$$F=[\tilde{\mathcal{G}}_S^{\mu\nu}\partial_\mu T\partial_\nu T+V(T)+I_{TF}], \tag{33}$$

and we have used

$$G=\sqrt{-\det(\mathcal{G}_{\mu\nu}+(2\pi\alpha')\mathcal{F}_{\mu\nu})}. \tag{34}$$

Using the fact that the tachyon field is a function of x only, from the last term in Eq. (32) we obtain the condition

$$\partial_x\mathcal{G}_{\mu\nu}=\partial_x\mathcal{F}_{\mu\nu}=0, \tag{35}$$

in order to obey the equation of motion for a tachyon.

Now we return to the first bracket in Eq. (32). The first equation in Eq. (32) gives

$$2\partial_\mu[\tilde{\mathcal{G}}_S^{\mu x}\partial_x T(x)]=2\partial_\mu(\tilde{\mathcal{G}}_S^{\mu x})\partial_x T+2\tilde{\mathcal{G}}_S^{xx}\partial_x\partial^x T, \tag{36}$$

where we have used the fact that the tachyon field is a function of x only. Since for a tachyon in the form of a kink solution the first derivative is nonzero, the first term in Eq. (36) leads to the result

$$\tilde{\mathcal{G}}_S^{\mu x}=\text{const}. \tag{37}$$

Since the constant in Eq. (37) does not have any physical meaning, we can take solution in the form

$$\tilde{\mathcal{G}}_S^{\mu x}=0,\ \ x\neq\mu,\quad \tilde{\mathcal{G}}_S^{xx}=1\Rightarrow(\tilde{\mathcal{G}}_S)_{xx}=1,\quad (\tilde{\mathcal{G}}_S)_{x\mu}=0. \tag{38}$$

Using the definition [15]

$$\tilde{\mathcal{G}}_S=\mathcal{G}-(2\pi\alpha')^2\mathcal{F}\mathcal{G}^{-1}\mathcal{F}, \tag{39}$$

we obtain

$$1=\mathcal{G}_{xx}-(2\pi\alpha')^2\mathcal{F}_{x\alpha}\mathcal{G}^{\alpha\beta}\mathcal{F}_{\beta x}, \tag{40}$$

where $\alpha,\beta=0,\ldots,p-1, x=x^p$. Since $\mathcal{G}^{\alpha\beta}\neq 0$, we obtain the natural solution of the previous equation in the form

$$\mathcal{G}_{xx}=1,\quad \mathcal{G}_{x\alpha}=\mathcal{F}_{\alpha\beta}=0. \tag{41}$$

Then we obtain

$$\det[\mathcal{G}_{\mu\nu}+(2\pi\alpha')\mathcal{F}_{\mu\nu}]=\det\begin{pmatrix}\mathcal{G}_{\alpha\beta}+(2\pi\alpha')\mathcal{F}_{\alpha\beta} & 0\\ 0 & 1\end{pmatrix}=\det[\mathcal{G}_{\alpha\beta}+(2\pi\alpha')\mathcal{F}_{\alpha\beta}]. \tag{42}$$

The second term in the first bracket in Eq. (32) gives

$$-\frac{dV}{dT}-\frac{dI_{TF}}{dT}. \tag{43}$$

In order to obtain the kink solution for a tachyon, the second term in the previous equation must vanish separately. We return to this term in a moment. When we combine the first term in the previous equation with the second term in Eq. (36), we obtain the equation

$$2\partial_x\partial^x T-\frac{dV}{dT}=0, \tag{44}$$

which is a precisely the equation for the tachyon kink solution. The solution of the previous equation can be found in many textbooks about topological configurations in field theory (see, for example, Refs. [20,21]):

$$2T''T'=\frac{dV}{dT}T'\Rightarrow(T')^2=V(T). \tag{45}$$

In order to find an exact solution of the previous equation, we take the zeroth-order form of the potential for a tachyon [18,22],

$$V(T)=-m^2T^2+\lambda T^4+\frac{m^4}{4\lambda}, \tag{46}$$

with the properties $V(T_0)=0$ and $T_0=m^2/2\lambda$.

Using Eq. (46), we can easily solve Eq. (45) as

$$T(x)=T_0\tanh\left(\frac{mx}{\sqrt{2}}\right). \tag{47}$$

Due to the fact that $m=1/\sqrt{2\alpha'}$, we see that the tachyon field is in its vacuum value almost on the whole axis x, except for the small region of size of string length $l_s\sim\sqrt{\alpha'}$. In other words, through tachyon condensation in the form of kink solution we obtain the object, which is approximately localized around the point $x=0$ on the world volume of a non-BPS Dp-brane. In the zero slope limit $\alpha'\to 0\Rightarrow m\to\infty$, the tachyon field $T(x)$ will be equal to its vacuum value almost on the whole x axis, except the small region around the point $x=0$, and the derivative of the tachyon field, which is equal to $dT/dx=(m/\sqrt{2})[1-\tanh^2(mx/\sqrt{2})]$ will be zero almost on the whole x axis except the small region around the point $x=0$.

Now we come to the second term in Eq. (32). In order to obtain the tachyon solution in the form of a kink solution, we must demand

$$\frac{dI_{TF}}{dT}=0, \tag{48}$$

where I_{TF} is given in Eq. (28). It is easy to see that the term proportional to $\partial_\rho h(T^2)$ does not contribute to the equation of motion. This follows from the fact that all massless fields are independent of x, so we can symbolically write

$$\int D\cdots\partial_x h(T^2)\cdots=\int \partial_x[D\cdots h(T^2)\cdots], \tag{49}$$

where the dots mean terms which are present in the second and third terms in Eq. (28). We see that this term contributes to the action as a total derivative, and so do not contribute to the equation of motion for a tachyon. Then the only nontrivial term in Eq. (28) is the term proportional to $f(T)$. Since $f(T)$ is an odd function of T, we can expect that its derivative $df(T)/dT$ is nonzero. The only possibility to obey the equation of motion for a tachyon is to pose the condition that $\partial_\mu\theta_R$ or $\partial_\mu\theta_L$ should be equal to zero. We choose the condition

$$\partial_\mu\theta_R=0 \tag{50}$$

for all σ^μ, $\mu=0,\ldots,p$. In other words, through tachyon condensation we have eliminated one-half of the fermionic degrees of freedom with a direct parallel to gauge the κ symmetry on the world volume of a BPS D-brane.

We now come to the final result. When we use Eq. (45), we obtain

$$F(T=\text{kink})=2V(T). \tag{51}$$

In the previous equation we used Eq. (50), and the fact that the term proportional to $\partial_x h(T^2)$ gives a zero contribution to the action, since (we again use the independence of all massless fields on x)

$$\int D\cdots\partial_x h(T)^2=h[T(\infty)^2]-h[T^2(-\infty)]D\cdots=0. \tag{52}$$

Previous result comes from the fact that $h(T^2)$ is an even function.

As a last step, we put Eq. (51) into Eq. (2); then we will integrate over x and we obtain the final result, which is a DBI action for a D$(p-1)$-brane:

$$S=-T_{p-1}\int d^p\sigma\sqrt{-\det[\mathcal{G}_{\alpha\beta}+(2\pi\alpha')\mathcal{F}_{\alpha\beta}]}, \tag{53}$$

where

$$T_{p-1}=2C_p\int_{-\infty}^{\infty}dx V[T(x)]. \tag{54}$$

For the tachyon kink solution [Eq. (47)], which corresponds to zeroth order approximation of the tachyon potential [18,22], we obtain

$$V(T_{kink})=\frac{m^4}{4\lambda}\left[1-\tanh^2\left(\frac{mx}{\sqrt{2}}\right)\right]^2, \tag{55}$$

and Eq. (54) gives

$$T_{p-1}=\frac{8\sqrt{2}}{3m}C_pV_k, \tag{56}$$

where we have denoted $V_k=m^4/4\lambda$. Using $m=1/\sqrt{2\alpha'}$ and $C_p=\sqrt{2}2\pi/(4\pi^2\alpha')^{(p+1)/2}g$, we obtain

$$T_{p-1}=\left(\frac{8\sqrt{2}V_k}{3\pi}\right)\left(\frac{2\pi}{(4\pi^2\alpha')^{p/2}g}\right). \tag{57}$$

In Ref. [18] it was shown that for the zeroth-order approximation of potential $-m^2T^2+\lambda T^4$ the vacuum value of the potential V_k cancels about 0.60 of the tension of the non-BPS D-brane. Then $V_k=0.60$, and we obtain the final result

$$T_{p-1}=0.72T^c_{p-1}, \tag{58}$$

where T^c_{p-1} is the correct value of the tension for a D$(p-1)$-brane. This result is similar to the result given in Ref. [18] in the zeroth-order approximation.[3]

As a final point we must also discuss the situation when T is equal to its vacuum value everywhere. Naively we could expect from the form of Eq. (33) that for this value of a tachyon we would not obtain a supersymmetric vacuum due to the presence of the interaction term between the tachyon and fermions. However, the tachyon vacuum value must be a solution of the equation of motion, and, as we have seen, this leads to the requirement of a constant spinor field θ_R. Then the interaction term between the fermions and tachyon is equal to zero, and from this definition $V(T=T_0)=0$, $\partial_xT=0$; thus we will obtain the result that the second bracket is

[3] We must stress two important points. First, we do not claim that the solution given above is the most general one. Rather, we wanted to show that there is a one particular solution which leads to the emergence of a BPS D-brane. It would certainly be illuminating to study other possible solutions. Second, from the previous analysis it is not completely clear why the world-volume field should be confined near $x=0$. This can be seen from the form of the kink solution, which is nonzero only in the small region of the size $\sim l_s$ around the core of the kink solution. Moreover, we have shown that the dimension of the world volume of the resulting D-brane is effectively reduced by 1, thanks to the fact that all fields are independent of x, which is a nontrivial requirement arising from the form of the kink solution. It is also important to note that we are not working in a static gauge, in which we would find that the resulting D-brane sits in the point $x=0$.

equal to zero, and consequently that the whole action disappears, in agreement with Ref. [14].

IV. CONCLUSION

In previous sections we proposed a possible form of supersymmetric DBI action for a non-BPS D-brane in type-IIA theory. (For type-IIB theory the situation will be basically the same, with the difference that both spinors have the same chirality.) We have seen that the requirement of invariance under supersymmetric transformations places strong constraints on the possible form of this action. Then we studied the kink solution of a tachyon on the world volume of a non-BPS Dp-brane in type-IIA theory, and showed that this solution truly describes the BPS D$(p-1)$-brane in type-IIA theory. This is in agreement with the results of Refs. [1,10,11], and in some sense can serve as a further support of their results. We have also seen a striking similarity with the results in Ref. [18].

We would like to outline the other possible extension of this work. It would certainly be illuminating to study a situation when we have N non-BPS D-branes, and a tachyon condenses in a more general configuration. It would also be interesting to study tachyon condensation on a system of D9-branes and antibranes in type-IIB theory, following Ref. [10]. We hope to return to these important questions in the future.

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