Coherent states in light-front QCD

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The method of asymptotic dynamics is used to construct a set of coherent states for light-front QCD. The coherent states thus obtained are used to calculate $O(g^2)$ corrections to the quark gluon vertex using x^+-ordered perturbation theory. It has been shown that, in this order, the true infrared divergences, i.e., the divergences appearing due to vanishing energy denominators, get canceled if the Hamiltonian matrix element is calculated between the coherent states.

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I. INTRODUCTION

Recently, there has been a lot of interest in light-front field theories (LFFT's) as these theories provide a hope for solving the relativistic bound state problem [1–3]. Light front Hamiltonian methods have already been applied successfully to theories such as Yukawa theory, QED, ϕ^4 theory, and $(1+1)$-dimensional QCD to obtain the mass spectrum and wave functions [4–7]. However, there are still unresolved issues which must be addressed before one can develop nonperturbative methods for light-front quantum chromodynamics (LFQCD) [3]. One of these important issues is the problem of infrared (IR) divergences [3,8].

In LFFT's there are two kinds of IR divergences—true and spurious [9]. Spurious IR divergences are divergences arising due to $k^+ \to 0$ and we have named them spurious because they are actually a manifestation of the UV divergences of the equal time theory only. Harindranath and Zhang [8,10,11] have calculated the lowest order corrections to the quark gluon vertex in LFQCD and have shown that a consistent treatment of the boundary integrals leads to a cancellation of the spurious IR divergences. However, the true IR divergences of the theory, which correspond to the actual IR divergences of the equal time theory and which arise from the $k^+ \to 0, k_\perp \to 0$ region have been avoided by these as well as by other authors by putting a lower cutoff on both k^+ and $k_\perp$.

In a previous work [9], we suggested the use of a coherent state basis to deal with true IR divergences in LFFT's. We have obtained such a basis for light front QED by using the method of asymptotic dynamics proposed by Kulish and Faddeev [12] in the context of equal time theories. This coherent state basis was subsequently applied to a bound state calculation in QED to demonstrate the cancellation of true IR divergences [13]. The procedure of Kulish and Faddeev has been used in equal-time QCD by Nelson and Butler [14] to generate a set of states in the asymptotic region of perturbative Q.C.D. Nelson *et al.* have shown [15,16] that the asymptotic states constructed by the method of asymptotic dynamics lead to a cancellation of IR divergences in separate topological sets in the matrix element for $q+q \to q+q+g$ to lowest order in perturbative QCD.

The relevance of the coherent state method to LFFT's lies in the observation that this method is specially suited for time (x^+) ordered perturbation theory and the bound state calculations in LFFT's are based on x^+-ordered Hamiltonian perturbation theory. The cancellation of IR divergences in this approach can be shown in an almost transparent manner if one uses suitably defined coherent states instead of Fock states to calculate the Hamiltonian matrix elements.

Relevance of a coherent state representation in LFFT's has been discussed by other authors as well [17,18]. In the context of two dimensional ϕ^4 theory, it has been shown, using variational methods, that a coherent state may be a valid vacuum in LFFT's [17]. In LF Scwinger model, it has been shown that the physical vacuum is a gauge invariant superposition of coherent states of dynamical gauge field zero mode [18].

There are two approaches to bound state calculations on the light-front–light-front Tamm-Dancoff (LFTD) method [2] and the discretized light cone quantization (DLCQ) method [1]. Both of these methods are based on diagonalization of the light cone Hamiltonian in Fock basis. The basic input in such calculations are the matrix elements of H_{LC} in Fock basis. In LFTD calculations, these matrix elements are obtained by using the x^+-ordered perturbation theory. In these calculations, the true IR divergences of equal time theory appear as vanishing light cone energy denominators in the loop integrals. On the other hand, the Kulish and Faddev approach to asymptotic dynamics leads to asymptotic states in which the test functions also consist of similar energy denominators. It is the aim of this work to show, by means of an example, that these coherent states actually lead to a cancellation of IR divergences in $O(g^2)$.

The paper is organized as follows. In Sec. II, we give the Hamiltonian of LFQCD in two component formalism and draw all possible diagrams for $O(g^2)$ corrections to the quark gluon vertex (Fig. 1). This is a summary of work done by Harindranath and Zhang [11]. In Sec. III, we use the Kulish and Faddeev procedure to obtain a set of coherent

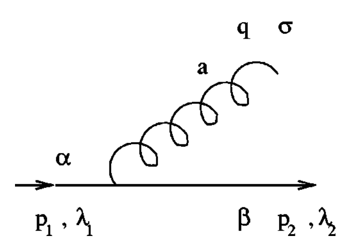

FIG. 1. Lowest order quark gluon vertex in QCD.

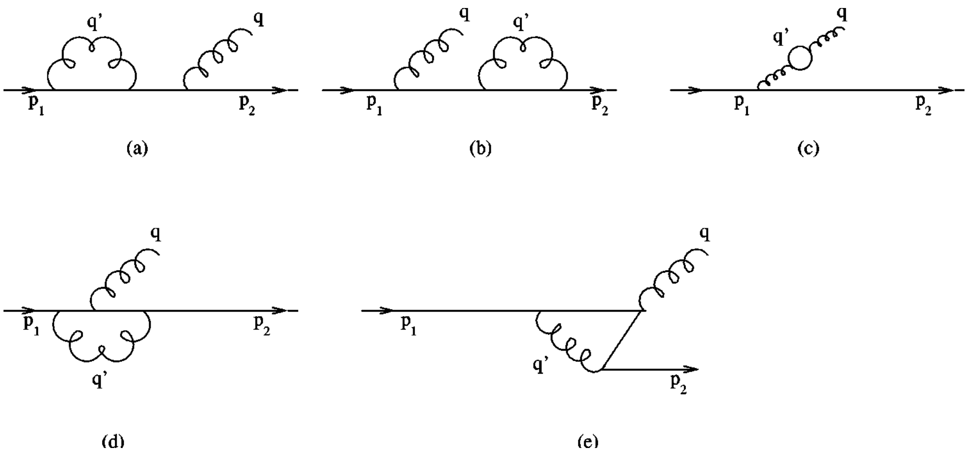

FIG. 2. O(g^2) contribution to qqg vertex represented by $\mathcal{M}_2$.

states showing explicitly the terms up-to $O(g^2)$. In Sec. IV, we calculate explicitly the matrix elements of interaction Hamiltonian between these coherent states and show the cancellation of true IR divergences. Section V contains a summary and discussion of results. In Appendixes A and B we summarize some details of calculations.

II. PRELIMINARIES

Our starting point is the LFQCD Hamiltonian in two component formalism in the light-front gauge $A_a^+ = A_a^0 + A_a^3 = 0$ [10],

$$H_{\rm LFQCD} = \int d^2x_\perp \, dx^-(H_0 + H_1), \tag{1}$$

where H_0 is the free Hamiltonian

$$H_0 = \int d^2x_\perp \, dx^- \left[(\partial^i A_a^j)(\partial^i A_a^j) + \xi^\dagger \left(\frac{-\partial_\perp^2 + m^2}{\partial^+} \right) \xi \right] \tag{2}$$

and H_I is the interaction Hamiltonian

$$H_I = H_1 + H_2 + H_3 + H_4 + H_4 + H_5, \tag{3}$$

where

$$H_1 = H_{qqg} = \int dx^- \, d^2x_\perp \, g\xi^\dagger \bigg[-2\frac{1}{\partial^+}(\partial_\perp A_\perp) + (\sigma_\perp A_\perp) \times \left(\frac{1}{\partial^+} \right)(\sigma_\perp \partial_\perp + m) + \frac{1}{\partial^+}(\sigma_\perp \partial_\perp - m)\sigma \perp A_\perp \bigg] \xi \tag{4}$$

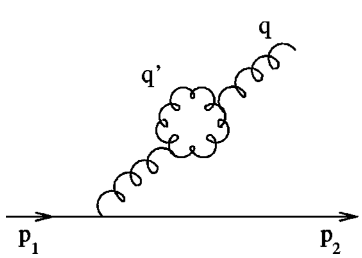

FIG. 3. $O(g^2)$ contribution to qqg vertex represented by $\mathcal{M}_3$.

is the qqg interaction,

$$H_2 = H_{ggg} = \int dx^- \, dx_\perp \bigg[g f^{abc} \bigg\{ \partial^i A_a^j A_b^j A_c^j + (\partial^i A_a^i) \left(\frac{1}{\partial^+} \right) (A_b^j \partial^+ A_c^j) \bigg\} \bigg] \tag{5}$$

represents the 3-gluon interaction

$$H_3 = H_{gggg}^{(1)} = \int dx^- \, d^2x_\perp \left[\frac{g^2}{4} f^{abc} f^{ade} A_b^i A_c^j A_d^i A_e^j \right] \tag{6}$$

is the usual quartic interaction,

$$H_4 = H_{gggg}^{(2)} = \int dx^- \, dx_\perp \bigg[\frac{g^2}{4} f^{abc} f^{ade} 2 \left(\frac{1}{\partial^+} \right) \times (A_b^i \partial^+ A_c^i) \left(\frac{1}{\partial^+} \right) (A_d^j \partial^+ A_e^j) \bigg] \tag{7}$$

is the instantaneous 4-gluon interaction and H_5 and H_6 also represent instantaneous interactions

$$H_5 = H_{qqgg} = \int dx^- \, d^2x_\perp \bigg[\xi^\dagger \sigma_\perp A_\perp \left(\frac{1}{i\partial^+} \right) (\sigma_\perp A_\perp) \xi + 2 \left(\frac{1}{\partial^+} \right) (f^{abc} A_b^i \partial^+ A_c^i) \left(\frac{1}{\partial^+} \right) (\xi^\dagger T^a \xi) \bigg], \tag{8}$$

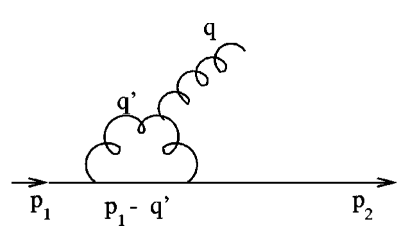

FIG. 4. $O(g^2)$ contribution to qqg vertex represented by $\mathcal{M}_4$.

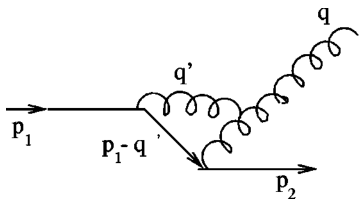

FIG. 5. $O(g^2)$ contribution to qqg vertex represented by $\mathcal{M}_5$.

$$H_6 = H_{qqqq} = \int dx^- \, d^2x_\perp \left[\left(\frac{1}{\partial^+} \right) (\xi^\dagger T^a \xi) \left(\frac{1}{\partial^+} \right) (\xi^\dagger T^a \xi) \right]. \tag{9}$$

Substituting the plane wave expansions

$$A^-(x) = \sum_\lambda \int \frac{dq^+ \, d^2q_\perp}{2(2\pi)^3 2q^+} [\epsilon^i_\lambda a(q,\lambda) e^{-iqx} + \text{H.c.}], \tag{10}$$

$$\xi(x) = \sum_\lambda \chi_\lambda \int \frac{dp^+ \, d^2p_\perp}{2(2\pi)^3} [b(p,\lambda) e^{-ipx} + d^\dagger(p,-\lambda) e^{ipx}] \tag{11}$$

one can express H_{int} in terms of creation and annihilation operators.

The coupling constant renormalization is obtained by calculating the matrix element of $H_{\text{int}}(0)$ between initial and final states [10]

$$\langle \psi_f | H_{int}(0) | \psi_i \rangle = \langle \Phi_f | H_{int}(0) | \Phi_i \rangle + \sum_{n1} \frac{\langle \Phi_f | H_{int}(0) | n_1 \rangle \langle n_1 | H_{int}(0) \Phi_i \rangle}{p_f^- - p_{n1}^- + i\epsilon} + \sum_{n1,n2} \frac{\langle \Phi_f | H_{int}(0) | n_1 \rangle \langle n_1 | H_{int}(0) | n_2 \rangle \langle n_2 | H_{int}(0) | \Phi_i \rangle}{(p_f^- - p_{n1}^- + i\epsilon)(p_f^- - p_{n2}^- + i\epsilon)} + \cdots. \tag{12}$$

In a standard LFQCD calculation, the initial and final states Φ_i and Φ_f are chosen to be the Fock states. Harindranath and Zhang [11] have calculated the lowest order radiative correction to quark-gluon coupling constant by using Fock states Φ_i and Φ_f in the perturbative expansion in Eq. (12). In the x^+ ordered Hamiltonian perturbation theory, one has the following contributions to $O(g^2)$ correction corresponding to diagrams in Figs. 2–9:

$$M_2 = \left\langle p', \lambda'; q\sigma \left| H_1 \frac{1}{p^- - H_0} H_1 \frac{1}{p^- - H_0} H_1 \right| p, \lambda \right\rangle, \tag{13}$$

$$M_3 = \left\langle p'\lambda'; q\sigma \left| H_2 \frac{1}{p^- - H_0} H_2 \frac{1}{p^- - H_0} H_1 \right| p\lambda \right\rangle, \tag{14}$$

$$M_4 = \left\langle p'\lambda'; q\lambda \left| H_1 \frac{1}{p^- - H_0} H_2 \frac{1}{p^- - H_0} H_1 \right| p\lambda \right\rangle, \tag{15}$$

$$M_5 = \left\langle p'\lambda'; q\sigma \left| H_2 \frac{1}{p^- - H_0} H_1 \frac{1}{p^- - H_0} H_1 \right| p\lambda \right\rangle, \tag{16}$$

$$M_6 = \left\langle p'\lambda'; q\sigma \left| H_1 \frac{1}{p^- - H_0} H_5 \right| p\lambda \right\rangle, \tag{17}$$

$$M_7 = \left\langle p'\lambda'; q\sigma \left| H_5 \frac{1}{p^- - H_0} H_1 \right| p\lambda \right\rangle, \tag{18}$$

$$M_8 = \left\langle p'\lambda' \left| H_1 \frac{1}{p^- - H_0} H_6 \right| p\lambda \right\rangle, \tag{19}$$

$$M_9 = \left\langle p'\lambda'; q\sigma \left| H_2 \frac{1}{p^- - H_0} H_5 \right| p\lambda \right\rangle. \tag{20}$$

$M_2 - M_9$ can be evaluated using the Feynman rules given in Ref. [11] or by directly substituting H_{int} in the above expressions and inserting appropriate number of complete sets of states. However, authors in Ref. [11] have avoided the infrared problems caused by massless gluons by choosing a suitable cutoff on transverse momentum. It is the aim of this work to show that the true IR divergences in $M_2 - M_9$ get canceled if one calculates the Hamiltonian matrix element in a coherent states basis which is obtained by using the method of asymptotic dynamics [12].

III. COHERENT STATES IN LFQCD

The coherent state method is based on the observation that in the limit $|x^+| \to \infty$, the total Hamiltonian does not reduce to the free field Hamiltonian, but to an asymptotic Hamiltonian H_{as}, which is obtained by putting $x^+ = 0$ in H_{int}. Each term in H_{int} has a light cone dependence of the form $\exp[-i(p_1^- + p_2^- + \cdots p_n^-)x^+/2]$ and therefore, if $p_1^- + p_2^- + \cdots + p_n^- = 0$ at some vertex, then the corresponding term in H_{int} will not vanish in large x^+ limit.

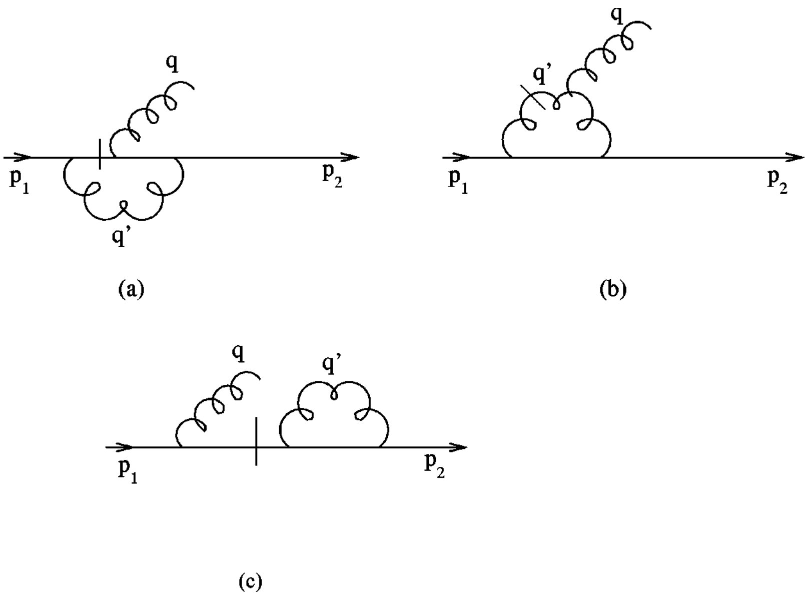

FIG. 6. $O(g^2)$ contribution to qqg vertex represented by $\mathcal{M}_6$.

Therefore, the total Hamiltonian

$$H=H_0+H_I \tag{21}$$

can also be written as

$$H=H_{\rm as}+H_I', \tag{22}$$

where

$$H_{\rm as}(x^+)=H_0+V_{\rm as}(x^+). \tag{23}$$

The associated x^+ evolution operator $U_{\rm as}(x^+)$ in the Schrödinger representation, which satisfies the equation

$$i\frac{dU_{\rm as}(x^+)}{dx^+}=H_{\rm as}(x^+)U_{\rm as}(x^+) \tag{24}$$

can then be used to generate an initial asymptotic states' space

$$\mathcal{H}_{\rm as}=\exp[-\Omega^A(x^+)]\mathcal{H}_F \tag{25}$$

from the usual Fock space $\mathcal{H}_F$, in the limit $x^+\to-\infty$. $\Omega^A(x^+)$ is asymptotic x^+-evolution operator defined by

$$U_{\rm as}(x^+)=\exp[-iH_0x^+]\exp[\Omega^A(x^+)]. \tag{26}$$

$\Omega^A(x^+)$ is determined by solving Eq. (24) using Magnus theorem [14] and is given by iterative integrations over an infinite series of Lie elements of the asymptotic potential $V_{\rm as}(x^+)$,

$$\begin{aligned}\Omega^A(x^+)=&-\int^{x^+}dx'^+H^I_{\rm as}(x'^+)\\&-\frac{1}{2}\int^{x^+}dx'^+\int^{x'^+}dx''^+[H^I_{\rm as}(x'^+),H^I_{as}(x''^+)]\\&+\cdots.\end{aligned} \tag{27}$$

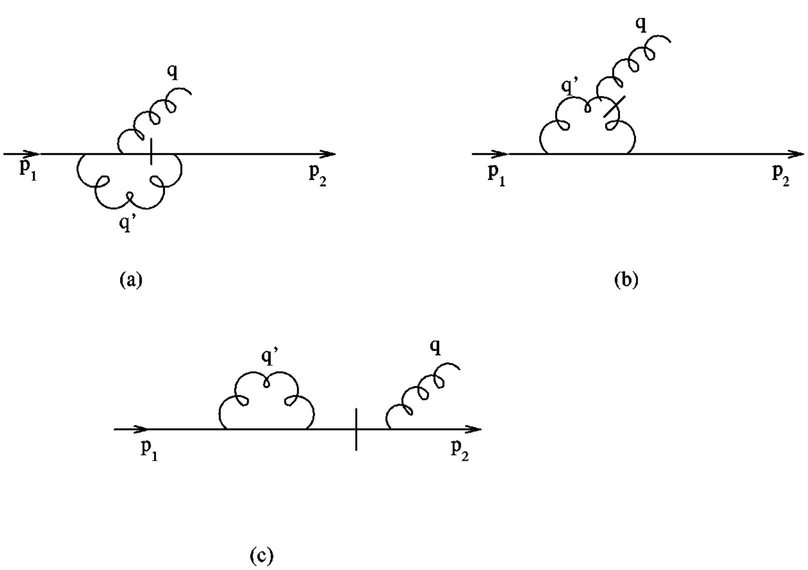

FIG. 7. $O(g^2)$ contribution to qqg vertex represented by $\mathcal{M}_7$.

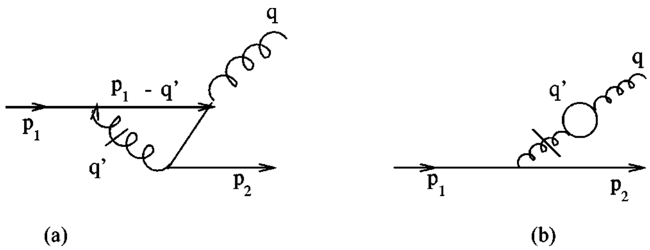

FIG. 8. $O(g^2)$ contribution to qqg vertex represented by $\mathcal{M}_8$.

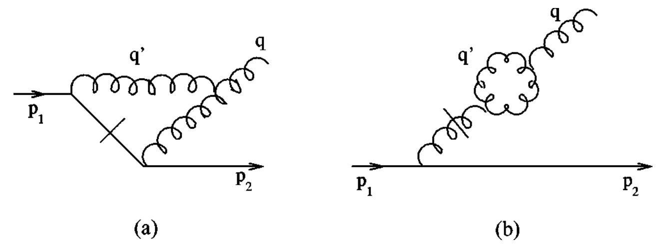

FIG. 9. $O(g^2)$ contribution to qqg vertex represented by $\mathcal{M}_9$.

(The lower end points of the integrations are dropped because H_0 is modified to $H_{\rm as}$ only at large $|x^+|$ [14].) $H^I_{\rm as}(x^+)$ is obtained from $H_I(x^+)$ by putting $x^+=0$.

Alternatively, one can also solve Eq. (24) as in time-ordered perturbation theory,

$$\exp[\Omega^A(x^+)]=1+\Sigma(-i)^n\int^{x^+}dx_1^+\int^{x_1^+}dx_2^+\cdots \times\int^{x_{n-1}^+}dx_n^+\, V_{\rm as}(x_1^+)\cdots V_{\rm as}(x_n^+). \qquad (28)$$

The above equation can then be used to define the coherent states [16]

$$|n{:}{\rm coh}\rangle=\lim_{x^+\to\infty}\exp[-\Omega^A]|n\rangle. \qquad (29)$$

Up to $0(g^2)$, Ω^A is given by

$$\exp[\Omega^A(x^+)]=\Omega_f(x^+)+\Omega_c(x^+)+\Omega_q(x^+)+\Omega_{qqgg}(x^+)+\Omega_{gggg}(x^+)+\Omega_{qqqq}+\Omega^{(2)}_{fg}+\Omega^{(2)}_{gf}+\Omega^{(2)}_{ff}+\Omega^{(2)}_{gg}, \qquad (30)$$

where

$$\begin{aligned}\Omega_f(x^+)=&-i\int H_{qqg}\,dx^+=-g\sum_s\sum_{s'}\sum_\lambda\int\frac{dp^+\,d^2p_\perp}{2(2\pi)^3}\int\frac{dq^+\,d^2q_\perp}{2(2\pi)^3[q^+]}\int\frac{dp'^+\,d^2p'_\perp}{2(2\pi)^3}\\
&\times\xi^+_{s'}\Bigg[b^\dagger(p',s')T^ab(p,s)a_a(q,\lambda)\epsilon^i_\lambda\left[\frac{2q^i}{q^+}-\frac{\sigma_i(\sigma_\perp\cdot p_\perp-im)}{p^+}-\frac{(\sigma_\perp\cdot(p_\perp+q_\perp)+im)\sigma^i}{p^++q^+}\right]\\
&\times\delta^3(p'-p-q)\Theta_\Delta(p'^--p^--q^-)\frac{e^{i/2(p'^--p^--q^-)x^+}}{p'^--p^--q^-}+b^\dagger(p',s')T^ab(p,s)a^\dagger_a(q,\lambda)\\
&\times\epsilon^{i*}_\lambda\left[\frac{2q^i}{q^+}-\frac{\sigma_i(\sigma_\perp\cdot p_\perp-im)}{p^+}-\frac{(\sigma_\perp\cdot(p_\perp-q_\perp)+im)\sigma^i}{p^+-q^+}\right]\delta^3(p'-p+q)\Theta_\Delta(p'^--p^-+q^-)\frac{e^{i/2(p'^--p^--q^-)x^+}}{p'^--p^-+q^-}\\
&+d(p',s')T^ad^\dagger(p,s)a_a(q,\lambda)\epsilon^i_\lambda\left[\frac{2q^i}{q^+}-\frac{\sigma_i(\sigma_\perp\cdot p_\perp+im)}{p^+}-\frac{(\sigma_\perp\cdot(p_\perp+q_\perp)-im)\sigma^i}{q^++p^+}\right]\\
&\times\delta^3(p'-p+q)\Theta_\Delta(p'^--p^-+q^-)\frac{e^{i/2(p'^--p^-+q^-)x^+}}{p'^--p^-+q^-}+d(p',s')T^ad^\dagger(p,s)a^\dagger_a(q,\lambda)\\
&\times\epsilon^{i*}_\lambda\left[\frac{2q^i}{q^+}-\frac{\sigma_i(\sigma_\perp\cdot p_\perp+im)}{p^+}-\frac{(\sigma_\perp\cdot(p_\perp+q_\perp)-im)\sigma^i}{p^++q^+}\right]\delta^3(p'-p-q)\Theta_\Delta(p'^--p^--q^-)\\
&\times\frac{e^{i/2(p'^--p^--q^-)x^+}}{p'^--p^--q^-}\Bigg]\xi_s. \qquad (31)\end{aligned}$$

Here $\Theta_\Delta(k^-)$ is a step function which takes value one only when the denominator in the corresponding term $1/k^-$ is smaller than some light cone energy cutoff Δ, which can be fixed by experimental resolution [19]. $\Theta_\Delta(k^-)$ defines the asymptotic region and also what is meant by a soft gluon in the definition of coherent state. For example, for the qqg vertex, the condition

$$p^-+k^--p'^-<\Delta \qquad (32)$$

leads to the following limits on the momentum of the soft gluon [9]:

$$k_\perp^2 < \frac{k^+\Delta}{p^+}, k^+ < \frac{p^+\Delta}{m^2}. \tag{33}$$

Similarly one can define the other terms in Ω^A also:

$$\Omega_c(x^+) = -i\int^{x^+} H_{ggg}\, dx'^+ = 2gif^{abc}\int \frac{dq^+\, d^2q_\perp}{2(2\pi^3)[q^+]}\frac{dq'^+\, d^2q'_\perp}{2(2\pi)^3[q'^+]}\frac{dq''^+\, d^2q''_\perp}{2(2\pi)^3[q''^+]}$$
$$\times\Bigg[\frac{1}{2}a^\dagger_{ai}(q)a_{bj}(q')a_{ck}(q'')V_{ijk}(q_1,-q',-q'')\Theta_\Delta(q^- - q'^- - q''^-)e^{\frac{i(q^- - q'^- - q''^-)x^+}{2}}\delta^3(q-q'-q'')\frac{q'^+ + q''^+}{q'\cdot q''}$$
$$+\text{H.c.} - \frac{1}{6}a_{ai}(q)a_{bj}(q')a_{ck}(q'')V'_{ijk}(q,q',q'')\Theta_\Delta(q^- + q'^- + q''^-)\frac{q^+ + q''^+}{q\cdot q'}\delta^3(q+q'+q'')$$
$$\times e^{-i(q^- + q'^- + q''^-)x^+/2} + \text{H.c.} - iq_i\frac{q''^+}{q'\cdot q''}\{a_{ai}(q)a_{bj}(q')a_{cj}(q'')\Theta_\Delta(q'^- + q''^- + q^-)\delta^3(q+q'+q'')$$
$$\times e^{-i(q'^- + q''^- + q^-)x^+/2} + \text{H.c.} + a_{ai}(q)a_{bj}(q')a^\dagger_{cj}(q'')\delta^3(q-q'+q'')e^{i(q''^- - q^- - q'^-)x^+/2}\Theta_\Delta(q''^- - q^- - q'^-)$$
$$+\text{H.c.} + a^\dagger_{ai}(q)a_{bj}(q')a_{cj}(q'')\delta^3(q-q'-q'')\Theta_\Delta(q'^- + q''^- - q^-)e^{-i(q'^- + q''^- - q^-)x^+/2}\}\Bigg], \tag{34}$$

where

$$V'_{ijk}(q,q',q'') = \delta_{ij}(q-q')_k + \delta_{jk}(q'-q'')_i + \delta_{ik}(q''-q)_j \tag{35}$$

for $q+q'+q''=0$.

Similarly $\Omega_q(x^+)$, $\Omega_{gggg}(x^+)$, $\Omega_{qqqq}(x^+)$, and $\Omega_{qqgg}(x^+)$ are defined by the following equations:

$$\Omega_q(x^+) = -i\int^{x^+} H_3(x'^+)dx'^+ \tag{36}$$

is the contribution to $\Omega^A(x^+)$ due to usual quartic coupling of QCD

$$\Omega_{gggg}(x^+) = -i\int^{x^+} H_4(x'^+)dx'^+ \tag{37}$$

is due to instantaneous four gluon interaction

$$\Omega_{qqgg}(x^+) = -i\int^{x^+} H_5(x'^+)dx'^+ \tag{38}$$

results due to instantaneous $qqgg$ interaction, and

$$\Omega_{qqqq}(x^+) = -i\int^{x^+} H_6(x'^+)dx'^+ \tag{39}$$

results due to instantaneous four fermion interaction. Expressions for Ω_q, Ω_{gggg}, Ω_{qqgg}, and Ω_{qqqq} are given in Appendix A.

$\Omega^{(2)}_{fg}$, $\Omega^{(2)}_{gf}$, $\Omega^{(2)}_{ff}$, and $\Omega^{(2)}_{gg}$ are the second order terms given by

$$\Omega^{(2)}_{ff} = \int^{x^+} dx_1^+ \int^{x_1^+} dx_2^+\, H_1^{\text{as}}(x_1^+)H_1^{as}(x_2^+), \tag{40}$$

$$\Omega^{(2)}_{fg} = \int^{x^+} dx_1^+ \int^{x_1^+} dx_2^+\, H_1^{\text{as}}(x_1^+)H_2^{\text{as}}(x_2^+), \tag{41}$$

$$\Omega^{(2)}_{gf} = \int^{x^+} dx_1^+ \int^{x_1^+} dx_2^+\, H_2^{\text{as}}(x_1^+)H_1^{\text{as}}(x_2^+), \tag{42}$$

$$\Omega^{(2)}_{gg} = \int^{x^+} dx_1^+ \int^{x_1^+} dx_2^+\, H_2^{\text{as}}(x_1^+)H_2^{\text{as}}(x_2^+). \tag{43}$$

In the coherent state approach, the Hamiltonian matrix elements are calculated between asymptotic states defined by Eq. (29). We will show, in the next section, that the coherent state contributions to matrix elements are similar to loop integrals of x^+-ordered perturbation theory and if one uses x^+-ordered perturbation theory to calculate matrix elements then the coherent state operator automatically generates extra diagrams with signs and weights such that the matrix elements are finite in $O(g^2)$.

IV. HAMILTONIAN MATRIX ELEMENTS IN COHERENT STATES BASIS

We will now show that there is an explicit cancellation of IR divergences in the matrix elements in lowest nontrivial order in perturbation theory, if one uses the coherent state basis constructed in Sec. III to calculate the matrix elements. We calculate here the $O(g^2)$ corrections to the qqg vertex by collecting the $O(g^2)$ terms in the following matrix element:

$$\mathcal{M}=\left\langle qg\colon\mathrm{coh}\right|\left[H_I+H_I\frac{1}{p^- - H_0}H_I + H_I\frac{1}{p^- - H_0}H_I\frac{1}{p^- - H_0}H_I\right]\left|q\colon\mathrm{coh}\right\rangle, \tag{44}$$

where

$$|A\colon\mathrm{coh}\rangle=[1-\Omega_f-\Omega_c-\Omega_q-\Omega_{qqgg}-\Omega_{gggg}-\Omega_{qqqq}-\Omega_{ff}^{(2)} - \Omega_{fg}^{(2)}-\Omega_{gf}^{(2)}-\Omega_{gg}^{(2)}]|A\rangle \tag{45}$$

with $|A\rangle$ being a Fock state.

One may notice that in Eq. (44), we are using x^+-ordered perturbation theory by taking H_0 as the unperturbed Hamiltonian and not H_{as} whereas it is the latter which we had used to obtain the expressions for coherent states. The idea is to work within conventional perturbation theory by using Eq. (21) to define the unperturbed Hamiltonian as H_0 and to take (only) the initial and final states as coherent states. The intermediate states are taken to be Fock states only. One could have used H_{as} as the unperturbed Hamiltonian in Eq. (44), but that would not make any difference in the following calculations as up to the order in which we are working, i.e., $O(g^2)$ both the methods will lead to the same result.

On using the coherent state basis, there will be extra diagrams in addition to those shown in Sec. II. These additional diagrams correspond to processes in which emission or absorption of soft gluons by initial or final state has been taken into account. The full matrix element is represented by diagrams in Figs. 2–9 plus the processes involving soft gluon emission (absorption) by initial (final) state and is given by

$$\mathcal{M}=\mathcal{M}_1+\mathcal{M}_2'+\mathcal{M}_3'+\mathcal{M}_4'+\mathcal{M}_5'+\mathcal{M}_6'+\mathcal{M}_7'+\mathcal{M}_8'+\mathcal{M}_9', \tag{46}$$

where

$$\mathcal{M}_1=\langle qg|H_I|q\rangle \tag{47}$$

and

$$\mathcal{M}_i'=\mathcal{M}_i+\mathcal{M}_i'', \tag{48}$$

for $i=2,\ldots,9$. Here $\mathcal{M}_i$ is the corresponding matrix element between Fock states given by Eqs. (13)–(20) and $\mathcal{M}_i''$ is the additional contribution due to coherent state basis. Exact expressions for $\mathcal{M}_i'$ and $\mathcal{M}_i''$ are given in the Appendix . Here, we will give only the expressions for $\mathcal{M}_4'+\mathcal{M}_5'$ and will show that this sum is free of IR divergences. We have chosen $\mathcal{M}_4'$ and $\mathcal{M}_5'$ rather than $\mathcal{M}_2'$ because these involve triple gluon vertex also which is not present in QED. Thus the following calculation is not just a trivial extension of earlier work on QED but involves more complicated calculations due to non-Abelian nature of QCD.

Recall that in Fock basis $\mathcal{M}_4$ and $\mathcal{M}_5$ are represented by Figs. 4 and 5, respectively. These are the only two diagrams which involve two qqg vertices and one triple gluon vertex. Replacing the Fock states in $\mathcal{M}_4$ and $\mathcal{M}_5$ by their corresponding coherent states, we arrive at the following expressions for $\mathcal{M}_4'$ and $\mathcal{M}_5'$:

$$\begin{aligned}\mathcal{M}_4'=&\left\langle qg\left|H_1\frac{1}{p^- - H_0}H_2\frac{1}{p^- - H_0}H_1\right|q\right\rangle\\ &-\left\langle qg\left|H_1\frac{1}{p^- - H_0}H_2\Omega_f\right|q\right\rangle\\ &+\left\langle qg\left|\Omega_f H_2\frac{1}{p^- - H_0}H_1\right|q\right\rangle-\langle qg|\Omega_f H_2\Omega_f|q\rangle\\ &-\langle qg|H_1\Omega_{gf}^{(2)}|q\rangle+\langle qg|\Omega_{fg}^{(2)}H_1|q\rangle,\end{aligned} \tag{49}$$

$$\begin{aligned}\mathcal{M}_5'=&\left\langle qg\left|H_2\frac{1}{p^- - H_0}H_1\frac{1}{p^- - H_0}H_1\right|q\right\rangle\\ &-\left\langle qg\left|H_2\frac{1}{p^- - H_0}H_1\Omega_f\right|q\right\rangle\\ &+\left\langle qg\left|\Omega_c H_1\frac{1}{p^- - H_0}H_1\right|q\right\rangle-\langle qg|\Omega_c H_1\Omega_f|q\rangle\\ &+\langle qg|\Omega_{gf}^{(2)}H_1|q\rangle-\langle qg|H_2\Omega_{ff}^{(2)}|q\rangle.\end{aligned} \tag{50}$$

$\mathcal{M}_4'$ and $\mathcal{M}_5'$ can be calculated by using the Feynmann rules given in Ref. [11] and the expression for Ω_f, Ω_c, and $\Omega^{(2)'s}$ given in Sec. III. After some simple algebra, one obtains

$$\mathcal{M}_4'=\mathcal{M}_{4a}+\mathcal{M}_{4b}+\mathcal{M}_{4c}+\mathcal{M}_{4d}+\mathcal{M}_{4e}+\mathcal{M}_{4f}, \tag{51}$$

where

$$\mathcal{M}_{4a}=\left\langle qg\left|H_1\frac{1}{p^- - H_0}H_2\frac{1}{p_- - H_0}H_1\right|q\right\rangle$$
$$=\int[dq']\theta(q'^+)\Gamma^l_{q0}(p_1-q',q'-q,p_2)$$
$$\times\frac{1}{p_1^- -(p_1-q')^- -(q'-q)^- -q^-}$$
$$\times\Gamma^i_{jl}(q,-q',q-q')$$
$$\times\frac{1}{p_1^- -q'^- -(p_1-q')^-}\Gamma^j_{q0}(p_1,p_1-q',q'), \tag{52}$$

where

$$[dq']=\frac{dq'^+d^2q'_\perp}{16\pi^3[q'^+]}, \tag{53}$$

$$\Gamma^i_{q0}(p,p-k,k)=2\frac{k^i}{[k^+]}-\frac{\sigma^j(p^j-k^j)-im}{[p^+-k^+]}\sigma^i$$
$$-\sigma^i\frac{\sigma^jp^j+im}{[p^+]}, \tag{54}$$

and

$$\Gamma^i_{jl}(p,-k,p-k)=\left[(p-2k)^i-\frac{p^i}{[p^+]}(p^+-2k^+)\right]\delta_{jl}$$
$$+\left[(k-2p)^j-\frac{k^j}{[k^+]}(k^+-2p^+)\right]\delta_{il}$$
$$+\left[(p+k)^l-\frac{p^l-k^l}{[p^+-k^+]}(p^++k^+)\right]\delta_{ij}, \tag{55}$$

$$\mathcal{M}_{4b}=-\left\langle qg\left|H_1\frac{1}{p^- - H_0}H_2\Omega_f\right|q\right\rangle=-\int[dq']\theta(q'^+)\Gamma^l_{q0}(p_1-q',q'-q,p_2)\Gamma^i_{jl}(q,-q',q-q')$$
$$\times\Gamma^j_{q0}(p_1,p_1-q',q')\frac{1}{p_1^- -(p_1-q')^- -(q'-q)^- -q^-}\Theta_\Delta(p_1^- -q'^- -(p_1-q')^-)\frac{1}{p_1^- -q'^- -(p_1-q')^-}, \tag{56}$$

$$\mathcal{M}_{4c}=\left\langle qg\left|\Omega_fH_2\frac{1}{p^- - H_0}H_1\right|q\right\rangle=-\int[dq']\theta(q'^+)\Gamma^l_{q0}(p_1-q',q'-q,p_2)\Gamma^i_{jl}(q,-q',q-q')\Gamma^j_{q0}(p_1,p_1-q',q')$$
$$\times\frac{\Theta_\Delta(p_2^- -(q'-q)^- -(p_1-q')^-)}{p_2^- -(q'-q)^- -(p_1-q')^-}\frac{1}{p_1^- -q'^- -(p_1-q')^-}, \tag{57}$$

$$\mathcal{M}_{4d}=-\langle qg|\Omega_fH_2\Omega_f|q\rangle=+\int[dq']\theta(q'^+)\Gamma^l_{q0}(p_1-q',q'-q,p_2)\Gamma^i_{jl}(q,-q',q-q')\Gamma^j_{q0}(p_1,p_1-q',q')$$
$$\times\frac{\Theta_\Delta(p_2^- -(q'-q)^- -(p_1-q')^-)}{p_2^- -(q'-q)^- -(p_1-q')^-}\frac{\Theta_\Delta(p_1^- -q'^- -(p_1-q'))}{p_1^- -q'^- -(p_1-q')^-}, \tag{58}$$

$$\mathcal{M}_{4e}=-\langle qg|H_1\Omega^{(2)}_{gf}|q\rangle=-\int[dq']\theta(q'^+)\Gamma^l_{q0}(p_1-q',q'-q,p_2)\Gamma^i_{jl}(q,-q',q-q')\Gamma^j_{q0}(p_1,p_1-q',q')$$
$$\times\frac{\Theta_\Delta(p_1^- -q^- -(q'-q)^- -(p_1-q')^-)}{p_1^- -q^- -(q'-q)^- -(p_1-q')^-}\frac{1}{p_1^- -q'^- -(p_1-q')^-}, \tag{59}$$

$$\mathcal{M}_{4f}=\langle qg|\Omega^{(2)}_{fg}H_1|q\rangle=-\int[dq']\theta(q'^+)\Gamma^l_{q0}(p_1-q',q'-q,p_2)\Gamma^i_{jl}(q,-q',q-q')\Gamma^j_{q0}(p_1,p_1-q',q')$$
$$\times\frac{1}{p_2^- -(q'-q)^- -(p_1-q')^-}\frac{\Theta_\Delta(p_2^- +q^- -(q'-q)^- -(p_1-q')^-)}{p_2^- +q^- -(q'-q)^- -(p_1-q')^-}. \tag{60}$$

Similarly, $\mathcal{M}'_5$ can be written as

$$\mathcal{M}_5' = \mathcal{M}_{5a} + \mathcal{M}_{5b} + \mathcal{M}_{5c} + \mathcal{M}_{5d} + \mathcal{M}_{5e} + \mathcal{M}_{5f}, \tag{61}$$

where

$$\mathcal{M}_{5a} = \left\langle qg \left| H_2 \frac{1}{p^- - H_0} H_1 \frac{1}{p^- - H_0} H_1 \right| q \right\rangle = \int [dq'] \theta(q'^+) \Gamma^i_{jl}(q,-q',q-q') \Gamma^j_{q0}(p_1,p_1-q',q') \Gamma^l_{q0}(p_1-q',q'-q,p_2)$$
$$\times \frac{1}{p_1^- - q'^- + (q'-q)^- - p_2^-} \frac{1}{p_1^- - q'^- - (p_1-q')^-}, \tag{62}$$

$$\mathcal{M}_{5b} = -\langle qg | H_2 H_1 \Omega_f | q \rangle = -\int [dq'] \theta(q'^+) \Gamma^i_{jl}(q,-q',q-q') \Gamma^j_{q0}(p_1,p_1-q',q') \Gamma^l_{q0}(p_1-q',q'-q,p_2)$$
$$\times \frac{1}{p_1^- - q'^- + (q'-q)^- - p_2^-} \frac{\Theta_\Delta(p_1^- - q'^- - (p_1-q')^-)}{p_1^- - q'^- - (p_1-q')^-}, \tag{63}$$

$$\mathcal{M}_{5c} = \left\langle qg \left| \Omega_c H_1 \frac{1}{p^- - H_0} H_1 \right| q \right\rangle = \int [dq'] \theta(q'^+) \Gamma^i_{jl}(q,-q',q-q') \Gamma^j_{q0}(p_1,p_1-q',q') \Gamma^l_{q0}(p_1-q',q'-q,p_2)$$
$$\times \frac{\Theta_\Delta(q'^- - q^- - (q'-q)^-)}{q'^- - q^- - (q'-q)^-} \frac{1}{p_1^- - q'^- - (p_1-q')^-}, \tag{64}$$

$$\mathcal{M}_{5d} = -\langle qg | \Omega_c H_1 \Omega_f | q \rangle = -\int [dq'] \theta(q'^+) \Gamma^i_{jl}(q,-q',q-q') \Gamma^j_{q0}(p_1,p_1-q',q') \Gamma^l_{q0}(p_1-q',q'-q,p_2)$$
$$\times \frac{\Theta_\Delta(q'^- - q^- - (q'-q)^-)}{q'^- - q^- - (q'-q)^-} \frac{\Theta_\Delta(p_1^- - q'^- - (p_1-q')^-)}{p_1^- - q'^- - (p_1-q')^-}, \tag{65}$$

$$\mathcal{M}_{5e} = -\langle qg | H_2 \Omega^{(2)}_{ff} | q \rangle = -\int [dq'] \theta(q'^+) \Gamma^i_{jl}(q,-q',q-q') \Gamma^j_{q0}(p_1,p_1-q',q') \Gamma^l_{q0}(p_1-q',q'-q,p_2)$$
$$\times \frac{\Theta_\Delta(p_1^- - q'^- + (q'-q)^- - p_2^-)}{p_1^- - q'^- + (q'-q)^- - p_2^-} \frac{1}{p_1^- - q'^- - (p_1-q')^-}, \tag{66}$$

$$\mathcal{M}_{5f} = \langle qg | \Omega^{(2)}_{gf} H_1 | q \rangle = -\int [dq'] \theta(q'^+) \Gamma^i_{jl}(q,-q',q-q') \Gamma^j_{q0}(p_1,p_1-q',q') \Gamma^l_{q0}(p_1-q',q'-q,p_2)$$
$$\times \frac{1}{q'^- - q^- - (q'-q)^-} \frac{\Theta_\Delta(q'^- - q^- + (p_1-q')^- - p_2^-)}{q'^- - q^- + (p_1-q')^- - p_2^-}. \tag{67}$$

Now, we make the following observations: (1) $p_1^- - q'^- - (p_1-q')^- \to 0$ as $q'^+ \to q^+, q_i' \to q_i$, (2) $p_1^- - (q'^- - q^-)^- - (p_1-q')^-$ and $q'^- - q^- - (q'-q)^- \to 0$ as $q'^+ \to 0, q_i' \to 0$, (3) In the limit $q'^+ \to q^+, q_i' \to q_i$,

$$p_2^- - (q'-q)^- - (p_1-q')^- = q'^- - (q'-q)^- - q^-, \tag{68}$$

and therefore

$$\mathcal{M}_{4c} + \mathcal{M}_{5c} = 0 \tag{69}$$

and

$$\mathcal{M}_{4d} + \mathcal{M}_{5d} = 0, \tag{70}$$

(4) $q'^- - q^- + (p_1-q')^- - p_2^- \to 0$ when $q'^+ \to q^+, q_i' \to q_i$, therefore, as before,

$$\mathcal{M}_{4f} + \mathcal{M}_{5f} = 0, \tag{71}$$

hence,

$$\mathcal{M}_4' + \mathcal{M}_5' = \mathcal{M}_{4a} + \mathcal{M}_{4b} + \mathcal{M}_{4e} + \mathcal{M}_{5a} + \mathcal{M}_{5b} + \mathcal{M}_{5e}, \tag{72}$$

which does not have any vanishing energy denominators and, hence, is free of true IR divergences.

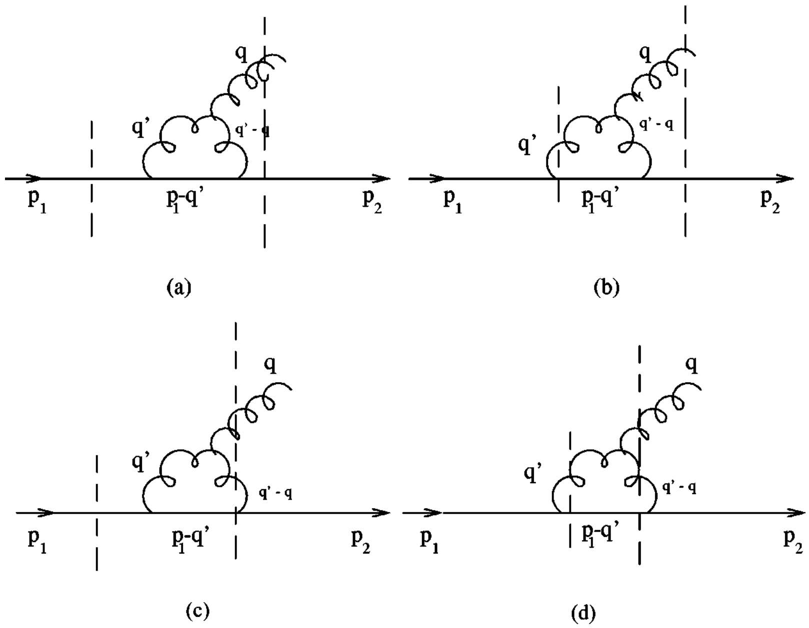

FIG. 10. Set of diagrams representing $\mathcal{M}_4'$. States to the left (right) line represent the incoming (outgoing) state. Region between the dashed lines represents the interaction region.

This result can be expressed diagrammatically by replacing each of the diagrams in Figs. 4 and 5 by a set of four diagrams and by using the diagrammatic notation of Nelson [16] to represent the additional contributions due to emission and absorption of soft gluons by initial and final states. For example, $\mathcal{M}_{4a}$, $\mathcal{M}_{4b}$, $\mathcal{M}_{4e}$, and $\mathcal{M}_{4d}$ will be represented by Figs. 10(a)–10(d) respectively. In each of these diagrams, states to the left and right of the dashed lines represent the initial and final states respectively.

In a similar manner, each of the diagrams in Figs. 2–9 will be replaced by a set of four diagrams, each one of the diagrams in a set having the same vertex structure and therefore one can show, in an identical manner, that the sum of diagrams in a given topological set will be free of vanishing energy denominators. Thus, the $O(g^2)$ corrections to the qqg vertex do not have any true IR divergences.

V. SUMMARY

True infrared divergences of equal time theory appear as vanishing energy denominators in light front Hamiltonian perturbation theory. However, if the Hamiltonian matrix elements are calculated between states of a suitably chosen coherent state basis, these divergences are expected to cancel, at least, in the lowest nontrivial order. We have applied the method of asymptotic dynamics to LFQCD to obtain a set of coherent states and have used this set to calculate $O(g^2)$ corrections to qqg vertex. The method of Kulish and Faddeev leads, in a natural way, to asymptotic states which contain integrals involving energy denominator similar to those appearing in loop integrals of x^+-ordered perturbation theory. This fact leads to a cancellation of IR divergences in diagrams with the same topological structure. For the particular form of coherent states chosen here, the argument used in this work to demonstrate the cancellation of IR divergences can be generalized to other cases also in a straightforward manner and IR finite matrix elements can be obtained in $O(g^2)$ of LFQCD.

In an earlier work [9], we have obtained the form of coherent states for light-front QED and have used these states to show that $O(e^2)$ vertex corrections are free of true IR divergences. A similar calculation in case of LFQCD is much more complicated due to the following reasons: First, the form of coherent states obtained by applying the method of Kulish and Faddeev (KF) to LFQCD is much more complicated due to the presence of cubic and quartic couplings. In other words, a physical incoming/outgoing state consisting of one quark and one gluon is much more complicated than a corresponding state consisting of one electron and one photon because in the former case, there is additional possibility of the incoming-outgoing gluon also emitting-absorbing soft gluons apart from the usual soft gluons accompanying the quark. This possibility is not there in QED. Secondly, $O(g^2)$ corrections to the qqg vertex involve many more diagrams due to the presence of cubic and quartic couplings. The example that we have chosen for our calculation involves a triple gluon vertex. This indicates that, in spite of the differences in QED and QCD, the $O(g^2)$ true IR divergences get canceled in both the theories. Actually, up to this order, there are no contributions to the qqg vertex involving the 4-gluon vertex and the sum of contributions involving the instantaneous interactions is also zero [11]. However, it is apparent from the derivation presented here that one can show the absence of vanishing energy denominators for each set of diagrams represented by Figs. 2–9 separately.

It is worth mentioning that in this work we have used the conventional definition of transition matrix by using the free particle Hamiltonian H_0 as the unperturbed Hamiltonian. The picture that we have is that at asymptotic limits the dynamics is described by an asymptotic Hamiltonian obtained by taking the $x^+ \to \pm\infty$ limit of the full Hamiltonian but inside the interaction region the intermediate states are Fock states. If one uses $H_{\rm as}$ in place of H_0 in the perturbative expansion, then one will have to use coherent states as the intermediate states as well. However, at the order in which we are working both forms for the unperturbed Hamiltonian lead to the same result. The choice of unperturbed Hamiltonian may be important in higher orders as far as cancellation or noncancellation of divergences is concerned. The issue of whether one should replace H_0 by H_{as} in Eq. (44) and what are the implications of this is under study and will be addressed in a future work.

It is well known that in equal time QCD, the asymptotic states obtained by KF method do not lead to IR finite matrix elements beyond leading order [20]. There is no reason to expect otherwise in LFQCD. The reason why IR divergences cancel out in QED but not in QCD lies in confinement property of QCD.

The basic assumption in the Lehmann-Symanzik-Zimmerman (LSZ) formalism is that in a scattering event, the free particle Hamiltonian describes the dynamics of incoming and outgoing particles. However, this picture breaks down for charged particles interacting via gauge theories of the standard model [21]. The KF method of asymptotic dynamics takes into account the long range interaction between incoming and outgoing states by replacing the free Hamiltonian for a theory by an asymptotic Hamiltonian.In QED, the potential between static charges falls off as $1/r$ and masslessness of photon causes IR divergences. A massless photon can travel over a large distance and an infinite number of soft photons can be created for any finite amount of energy. Block-Nordsieck theorem takes into account these soft photons by summing over all processes in which the electron is accompanied by an infinite number of such photons. In an actual experiment also, due to finite resolution of the detector, there is no restriction on the number of photons which may accompany a charged particle. The method of asymptotic dynamics incorporates this fact in the formalism by replacing the free particle states by coherent states. These coherent states take into account emission and absorption of soft photons at large distances. The Hamiltonian that is used to obtain these states is the large time limit of QED Hamiltonian. It is well known that this method leads to IR finite matrix elements in QED.

The procedure followed in case of QED can be applied to QCD also as has been done in this paper and in the work by Nelson *et al.*, but there it leads only to coherent states which take into account the large distance limit of QCD potential. However, in QCD the incoming and outgoing systems are bound states of quarks and gluons and, therefore, if the method of asymptotic dynamics has to be applied, one must add to the free particle Hamiltonian not just the large distance limit of the QCD potential but also the confining potential which is responsible for binding of quarks and gluons in the hadrons. This is the reason why IR divergences do not cancel in higher orders of QCD even when coherent state method is used.

The study of coherent state formalism in QCD is interesting because of this ‘‘noncancellation’’ of IR divergences. It has long been speculated that the clue to understanding color confinement in QCD may lie in the noncancellation of IR divergences [3,22–24]. The question to be addressed now is whether the suitability of coherent state formalism to light front Hamiltonian perturbation theory can be exploited to gain insight into the form of the artificial confining potential that is needed for the LFFT program of Wilson *et al.*

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APPENDIX A

In this appendix we give the expressions for Ω_q, Ω_{gggg}, Ω_{qqgg}, and Ω_{qqqq}:

$$\Omega_q(x^+) = -i\int^{x^+} H_3(x'^+)dx'^+ = \frac{g^2}{2}\int \frac{dq_1^+ d^2q_{1\perp}}{2(2\pi)^3[q_1^+]}\frac{dq_2^+ d^2q_{2\perp}}{2(2\pi)^3[q_2^+]}\frac{dq_3^+ d^2q_{3\perp}}{2(2\pi)^3[q_3^+]}\frac{dq_4^+ d^2q_{4\perp}}{2(2\pi)^3[q_4^+]}\Bigg[a^\dagger_{ai}(q_3)a^\dagger_{ej}(q_4)a_{bk}(q_1)a_{cl}(q_2)$$
$$\times V^{debc}_{ijkl}(q_3,q_4,-q_1,-q_2)\times\left(\frac{q_1^+ + q_2^+ - q_3^+}{q_1\cdot q_2 - q_2\cdot q_3 - q_3\cdot q_4}\right)e^{-(i/2)(q_1^- + q_2^- - q_3^- - q_4^-)x^+}\delta^3(q_3+q_4-q_1-q_2)$$
$$\times\Theta_\Delta(q_1^- + q_2^- - q_3^- - q_4^-) - \frac{2}{3}a^\dagger_{bi}(q_1)a_{cj}(q_2)a_{dk}(q_3)a_{el}(q_4)V^{bcde}_{ijkl}\left(\frac{q_1^+ - q_2^+ - q_3^+}{q_2\cdot q_3 - q_1\cdot q_2 - q_3\cdot q_1}\right)\Theta_\Delta(q_1^- - q_2^- - q_3^-$$
$$- q_4^-)\times\delta^3(q_1-q_2-q_3-q_4)e^{(i/2)(q_1^- - q_2^- - q_3^- - q_4^-)x^+} + \frac{1}{6}a_{ai}(q_1)a_{bj}(q_2)a_{ck}(q_3)a_{de}(q_4)V^{abcd}_{ijkl}(q_1,q_2,q_3,q_4)$$
$$\times\left(\frac{q_1^+ + q_2^+ + q_3^+}{q_1\cdot q_2 + q_2\cdot q_3 + q_3\cdot q_1}\right)\delta^3(q_1+q_2+q_3+q_4)e^{-(i/2)(q_1^- + q_2^- + q_3^- + q_4^+)x^+}\Bigg]\Theta_\Delta(q_1^- + q_2^- + q_3^- + q_4^-) \tag{A1}$$

with

$$V^{abcd}_{ijkl}(q_1,q_2,q_3,q_4) = f_{abc}f_{cde}(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}) + f_{ace}f_{bde}(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{jk}) + f_{ade}f_{cbe}(\delta_{ik}\delta_{jl} - \delta_{ij}\delta_{kl}). \tag{A2}$$

$\Omega_{gggg}(x^+)$ is due to 4-gluon instantaneous interaction:

$$
\begin{aligned}
\Omega_{gggg}(x^+) = & -i\int^{x^+} H_4(x'^+)dx'^+ = \frac{g^2}{2} f^{abc} f^{ade} \int \frac{dq_1^+ d^2q_{1\perp}}{2(2\pi)^3[q_1^+]} \frac{dq_2^+ d^2q_{2\perp}}{2(2\pi)^3[q_2^+]} \frac{dq_3^+ d^2q_{3\perp}}{2(2\pi)^3[q_3^+]} \frac{dq_4^+ d^2q_{4\perp}}{2(2\pi)^3[q_4^+]} q_2^+ q_4^+ \\
& \times \Bigg[a_{bi}(q_1)a_{ci}(q_2)a_{dj}(q_3)a_{ej}(q_4) \frac{q_1^+ + q_2^+ + q_3^+}{(q_1^+ + q_2^+)(q_3^+ + q_4^+)} \frac{e^{i(q_1^- + q_2^- + q_3^- + q_4^-)x^+/2}}{q_1\cdot q_2 + q_2\cdot q_3 + q_3\cdot q_1} \Theta_\Delta(q_1^- + q_2^- + q_3^- + q_4^-) + \text{H.c.} \\
& + a_{bi}(q_1)a_{ci}(q_2)a_{dj}^\dagger(q_3)a_{ej}(q_4) \frac{q_1^+ + q_2^+ + q_3^+}{(q_1^+ + q_2^+)(q_3^+ + q_4^+)} \frac{e^{-i(q_1^- + q_2^- - q_3^- - q_4^-)x^+/2}}{q_1\cdot q_2 - q_1\cdot q_3 - q_2\cdot q_3} \Theta_\Delta(q_1^- + q_2^- - q_3^- - q_4^-) \\
& \times \delta^3(q_1 + q_2 - q_3 - q_4) + \text{H.c.} - a_{bi}(q_1)a_{ci}(q_2)a_{dj}(q_3)a_{ej}^\dagger(q_4) \frac{q_1^+ + q_2^+ + q_3^+}{(q_1^+ + q_2^+)(q_3^+ - q_4^+)} \frac{e^{-i(q_1^- + q_2^- + q_3^- - q_4^-)x^+/2}}{q_1\cdot q_2 + q_2\cdot q_3 + q_1\cdot q_3} \\
& \times \Theta_\Delta(q_1^- + q_2^- + q_3^- - q_4^-) \times \delta^3(q_1 + q_2 + q_3 - q_4) - \text{H.c.} - a_{bi}(q_1)a_{ci}(q_2)a_{dj}^\dagger(q_3)a_{ej}(q_4) \\
& \times \frac{q_1^+ + q_2^+ - q_3^+}{(q_1^+ + q_2^+)(q_3^+ - q_4^+)} \frac{e^{-i(q_1^- + q_2^- - q_3^- + q_4^-)x^+/2}}{q_1\cdot q_2 - q_2\cdot q_3 - q_1\cdot q_3} \Theta_\Delta(q_1^- + q_2^- - q_3^- + q_4^-) \times \delta^3(q_1 + q_2 - q_3 + q_4) + \text{H.c.} \\
& - a_{bi}(q_1)a_{ci}^\dagger(q_2)a_{dj}(q_3)a_{ej}(q_4) \frac{q_1^+ - q_2^+ - q_3^+}{(q_1^+ - q_2^+)(q_3^+ + q_4^+)} \frac{e^{-i(q_1^- - q_2^- + q_3^- + q_4^-)x^+/2}}{q_1 q_3 - q_1 q_2 - q_2 q_3} \Theta_\Delta(q_1^- - q_2^- + q_3^- + q_4^-) \\
& \times \delta^3(q_1 - q_2 + q_3 + q_4) - \text{H.c.} - a_{bi}(q_1)a_{ci}^\dagger(q_2)a_{dj}^\dagger(q_3)a_{ej}^\dagger(q_4) \frac{q_1^+ - q_2^+ - q_3^+}{(q_1^+ - q_2^+)(q_3^+ + q_4^+)} \frac{e^{-i(q_1^- - q_2^- - q_3^- - q_4^-)x^+/2}}{q_2\cdot q_3 - q_1\cdot q_2 - q_1\cdot q_3} \\
& \times \Theta_\Delta(q_1^- - q_2^- - q_3^- - q_4^-) \times \delta^3(q_1 - q_2 - q_3 - q_4) - \text{H.c.} \\
& + a_{bi}(q_1)a_{ci}^\dagger(q_2)a_{dj}(q_3)a_{ej}^\dagger(q_4) \frac{q_1^+ - q_2^+ + q_3^+}{(q_1^+ - q_2^+)(q_3^+ - q_4^+)} \frac{e^{-i(q_1^- - q_2^- + q_3^- - q_4^-)x^+/2}}{q_1\cdot q_3 - q_1\cdot q_2 - q_2\cdot q_3} \Theta_\Delta(q_1^- - q_2^- + q_3^- - q_4^-) \\
& \times \delta^3(q_1 - q_2 + q_3 - q_4) + \text{H.c.} + a_{bi}(q_1)a_{cj}^\dagger(q_2)a_{dj}^\dagger(q_3)a_{ej}(q_4) \frac{q_1^+ - q_2^+ - q_3^+}{(q_1^+ - q_2^+)(q_3^+ - q_4^+)} \frac{e^{-i(q_1^- - q_2^- - q_3^- + q_4^-)x^+/2}}{q_2\cdot q_3 - q_1\cdot q_2 - q_1\cdot q_3} \\
& \times \Theta_\Delta(q_1^- - q_2^- - q_3^- + q_4^-) \times \delta^3(q_1 - q_2 - q_3 + q_4) + \text{H.c.} \Bigg]. \qquad \text{(A3)}
\end{aligned}
$$

$\Omega_{qqqq}(x^+)$ results due to four fermion instantaneous interaction

$$
\begin{aligned}
\Omega_{qqqq}(x^+) = & -i\int^{x^+} dx'^+ H_6(x'^+) = 2g^2 \sum_{\text{spins}} \sum \int \frac{dp_1^+ d^2p_{1\perp}}{2(2\pi^3)} \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{dp_3^+ d^2p_{3\perp}}{2(2\pi)^3} \frac{dp_4^+ d^2p_{4\perp}}{2(2\pi)^3} \\
& \times \xi_{s1}^+ \xi_{s2} \xi_{s3}^+ \xi_{s4} [b^\dagger(p_1,s_1)T^a b(p_2,s_2) b^\dagger(p_3,s_3) T^a b(p_4,s_4) \frac{e^{i(p_1^- - p_2^- + p_3^- - p_4^-)x^+/2}}{p_1^- - p_2^- + p_3^- - p_4^-} \frac{\delta^3(p_1 - p_2 + p_3 - p_4)}{(p_1^+ - p_2^+)^2} \\
& \times \Theta_\Delta(p_1^- - p_2^- + p_3^- - p_4^-) - d(p_1,-s_1)T^a d^\dagger(p_2,s_2) d(p_3,-s_3) T^a d^\dagger(p_4,-s_4) \\
& \times \frac{e^{-i(p_1^- - p_2^- + p_3^- - p_4^-)x^+/2}}{p_1^- - p_2^- + p_3^- - p_4^-} \frac{\delta^3(p_1 - p_2 + p_3 - p_4)}{(p_1^+ - p_2^+)^2} \Theta_\Delta(p_1^- - p_2^- + p_3^- - p_4^-) + b^\dagger(p_1,s_1)T^a b(p_2,s_2) d(p_3,-s_3) \\
& \times T^a d^\dagger(p_4,-s_4) \frac{e^{i(p_1^- - p_2^- - p_3^- + p_4^-)x^+/2}}{p_1^- - p_2^- - p_3^- + p_4^-} \frac{\delta^3(p_1 - p_2 - p_3 + p_4)}{(p_1^+ - p_2^+)^2} \Theta_\Delta(p_1^- - p_2^- - p_3^- + p_4^-) - d(p_1,-s_1)T^a d^\dagger \\
& (p_2,-s_2) b^\dagger(p_3,s_3) T^a b(p_4,s_4) \frac{e^{(-p_1^- - p_2^- - p_3^- + p_4^-)x^+/2}}{p_1^- - p_2^- - p_3^- + p_4^-} \frac{\delta^3(p_1 - p_2 - p_3 + p_4)}{(p_1^+ - p_2^+)^2} \Theta_\Delta(p_1^- - p_2^- - p_3^- + p_4^-)
\end{aligned}
$$

$$
\begin{aligned}
&+b^\dagger(p_1,s_1)T^a d^\dagger(p_2,-s_2)b^\dagger(p_3,s_3)T^a d^\dagger(p_4,s_4)\frac{e^{i(p_1^-+p_2^-+p_3^-+p_4^-)x^+/2}}{p_1^-+p_2^-+p_3^-+p_4^-}\,\frac{\delta^3(p_1+p_2+p_3+p_4)}{(p_1^++p_2^+)^2}\\
&\times\Theta_\Delta(p_1^-+p_2^-+p_3^-+p_4^-)-d(p_1,-s_1)T^a b(p_2,s_2)d(p_3,-s_3)T^a b(p_4,s_4)\\
&\times\frac{e^{i(p_1^-+p_2^-+p_3^-+p_4^-)x^+/2}}{p_1^-+p_2^-+p_3^-+p_4^-}\,\frac{\delta^3(p_1+p_2+p_3+p_4)}{(p_1^++p_2^+)^2}\Theta_\Delta(p_1^-+p_2^-+p_3^-+p_4^-)\\
&+b^\dagger(p_1,s_1)T^a b(p_2,s_2)b^\dagger(p_3,s_3)T^a d(p_4,-s_4)\,\frac{e^{i(p_1^--p_2^-+p_3^-+p_4^-)x^+/2}}{p_1^--p_2^-+p_3^-+p_4^-}\,\frac{\delta^3(p_1-p_2+p_3+p_4)}{(p_1^+-p_2^+)^2}\\
&\times\Theta_\Delta(p_1^--p_2^-+p_3^-+p_4^-)+b^\dagger(p_1,s_1)T^a b(p_2,s_2)d(p_3,-s_3)T^a b(p_4,s_4)\\
&\times\frac{e^{i(p_1^--p_2^--p_3^--p_4^-)x^+/2}}{p_1^--p_2^--p_3^--p_4^-}\,\frac{\delta^3(p_1-p_2-p_3-p_4)}{(p_1^+-p_2^+)^2}\Theta_\Delta(p_1^--p_2^--p_3^--p_4^-)\\
&-d(p_1,-s_1)T^a d^\dagger(p_2,-s_2)b^\dagger(p_3,s_3)T^a b(p_4,s_4)\,\frac{e^{i(p_1^--p_2^--p_3^--p_4^-)x^+/2}}{p_1^--p_2^--p_3^--p_4^-}\,\frac{\delta^3(p_1-p_2-p_3-p_4)}{(p_1^+-p_2^+)^2}\\
&\times\Theta_\Delta(p_1^--p_2^--p_3^--p_4^-)-d(p_1,-s_1)T^a d^\dagger(p_2,-s_2)d(p_3,-s_3)T^a b(p_4,s_4)\\
&\times\frac{e^{-i(p_1^--p_2^-+p_3^-+p_4^-)x^+/2}}{p_1^--p_2^-+p_3^-+p_4^-}\,\frac{\delta^3(p_1-p_2+p_3-p_4)}{(p_1^+-p_2^+)^2}\Theta_\Delta(p_1^-+p_2^-+p_3^--p_4^-)+b^\dagger(p_1,s_1)T^a d^\dagger(p_2,-s_2)\\
&\times b^\dagger(p_3,s_3)T^a b(p_4,s_4)\,\frac{e^{i(p_1^-+p_2^-+p_3^--p_4^-)x^+/2}}{p_1^-+p_2^-+p_3^--p_4^-}\,\frac{\delta^3(p_1+p_2+p_3-p_4)}{(p_1^++p_2^+)^2}\Theta_\Delta(p_1^-+p_2^-+p_3^--p_4^-)\\
&+b^\dagger(p_1,s_1)T^a d^\dagger(p_2,-s_2)d(p_3,-s_3)T^a b(p_4,s_4)\,\frac{e^{i(p_1^-+p_2^--p_3^--p_4^-)x^+/2}}{p_1^-+p_2^--p_3^--p_4^-}\,\frac{\delta^3(p_1+p_2-p_3-p_4)}{(p_1^++p_2^+)^2}\\
&\times\Theta_\Delta(p_1^-+p_2^--p_3^--p_4^-)+b^\dagger(p_1,-s_1)T^a b(p_2,-s_2)d(p_3,-s_3)T^a d^\dagger(p_4,-s_4)\\
&\times\frac{e^{i/2(p_1^-+p_2^--p_3^-+p_4^-)x^+/2}}{p_1^-+p_2^--p_3^-+p_1^-}\,\frac{\delta^3(p_1+p_2-p_3+p_4)}{(p_1^++p_2^+)^2}\Theta_\Delta(p_1^-+p_2^--p_3^--p_4^-)\\
&+d(p_1,-s_1)T^a b(p_2,s_2)b^\dagger(p_3,s_3)T^a b(p_4,s_4)\,\frac{e^{-i/2(p_1^-+p_2^--p_3^-+p_4^-)x^+}}{p_1^-+p_2^--p_3^-+p_4^-}\,\frac{\delta^3(p_1+p_2-p_3+p_4)}{(p_1^++p_2^+)^2}\\
&\times\Theta_\Delta(p_1^-+p_2^--p_3^-+p_4^-)-d(p_1,-s_1)T^a b(p_2,s_2)b^\dagger(p_3,s_3)T^a d^\dagger(p_4,-s_4)\\
&\times\frac{e^{-i(p_1^-+p_2^--p_3^--p_4^-)x^+/2}}{p_1^-+p_2^--p_3^--p_4^-}\,\frac{\delta^3(p_1+p_2-p_3-p_4)}{(p_1^++p_2^+)^2}\Theta_\Delta(p_1^-+p_2^--p_3^--p_4^-)-d(p_1,-s_1)T^a b(p_2,s_2)\\
&\times d(p_3,-s_3)T^a d^\dagger(p_4,-s_4)\frac{e^{-i(p_1^-+p_2^-+p_3^--p_4^-)x^+/2}}{p_1^-+p_2^-+p_3^--p_4^-}\,\frac{\delta^3(p_1+p_2+p_3-p_4)}{(p_1^++p_2^+)^2}\Theta_\Delta(p_1^-+p_2^-+p_3^--p_4^-).
\end{aligned}
\tag{A4}
$$

$\Omega_{qqgg}(x^+)$ results due to instantaneous $qqgg$ interaction

$$\Omega_{qqgg}(x^+)=-i\int^{x^+}H_5(x^+)dx^+=g^2\sum_{s,s'}\sum_{\lambda,\lambda'}\int\frac{dp^+d^2p_\perp}{2(2\pi)^3}\int\frac{dp'd^2p_\perp}{2(2\pi)^3}$$
$$\times\int\frac{dq^+d^2q_\perp}{2(2\pi)^3[q^+]}\int\frac{dq'^+d^2q'_\perp}{2(2\pi)^3[q'^+]}b^\dagger(p',s')a(q,\lambda)a(q',\lambda')b(p,s)\epsilon^i_\lambda\epsilon^j_\lambda\frac{e^{i(p^{-\prime}-q^--q'-p^-)x^+/2}}{p^{-\prime}-q^--q'-p^-}$$
$$\times\frac{\delta^3(p'-q-q'-p')}{q'+p^+}\Theta_\Delta(p^{-\prime}-q^--q^{-\prime}-p^{-\prime})$$
$$+b^+(p',s')a^+(q,\lambda)a^+(q',\lambda')b(p,s)\epsilon^{i*}_\lambda\epsilon^{j*}_{\lambda'}\frac{e^{i(p'^-+q^-+q^{-\prime}-p^-)x^+/2}}{p^{-\prime}+q^-+q^{-\prime}-p^-}\ \frac{\delta^3(p'+q+q'-p)}{p^+-q'^+}$$
$$\times\Theta_\Delta(p^{-\prime}+q^-+q^{-\prime}-p^{-\prime})-b^+(p',s')a(q,\lambda)a^+(q^+,\lambda)b(p,s)\epsilon^i_\lambda\epsilon^{j*}_{\lambda'}\frac{e^{-i(p'^--q^-+q'^--p^-)x^+/2}}{p'^--q^-+q'^--p^-}$$
$$\times\frac{\delta^3(p'-q+q'-p)}{p^+-q^+}\Theta_\Delta(p^{-\prime}-q^-+q^{-\prime}-p^{-\prime})-b^+(p',s')a^+(q,\lambda)a(q',\lambda')b(p,s)\epsilon^i_\lambda\epsilon^j_{\lambda'},$$
$$\times\frac{e^{-i(p'^-+q'^--q^--p^-)x^+/2}}{p'^-+q^--q'^--p^-}\ \frac{\delta^3(p'+q-q'-p)}{p'^+-q'+}\Theta_\Delta(p^{-\prime}+q^--q^{-\prime}-p^{-\prime}). \tag{A5}$$

APPENDIX B

We give below the expressions for $\mathcal{M}_2',\mathcal{M}_3',\ldots,\mathcal{M}_9'$ defined in Sec. IV

$$\mathcal{M}_2'=\left\langle qg\left|H_1\frac{1}{p^--H_0}H_1\frac{1}{p^--H_0}H_1\right|q\right\rangle-\left\langle qg\left|H_1\frac{1}{p^--H_0}H_1\Omega_f\right|q\right\rangle-\left\langle qg\left|\Omega_f^\dagger H_1\frac{1}{p_--H_0}H_1\right|q\right\rangle+\langle qg|\Omega_f^\dagger H_1\Omega_f|q\rangle, \tag{B1}$$

$$\mathcal{M}_3'=\left\langle qg\left|H_2\frac{1}{p^--H_0}H_2\frac{1}{p^--H_0}H_1\right|q\right\rangle-\left\langle qg\left|H_2\frac{1}{p^--H_0}H_2\Omega_f\right|q\right\rangle-\left\langle qg\left|\Omega_c^\dagger H_2\frac{1}{p^--H_0}H_1\right|q\right\rangle+\langle qg|\Omega_c^\dagger H_2\Omega_f|q\rangle, \tag{B2}$$

$$\mathcal{M}_4'=\left\langle qg\left|H_1\frac{1}{p^--H_0}H_2\frac{1}{p^--H_0}H_1\right|q\right\rangle-\left\langle qg\left|H_1\frac{1}{p^--H_0}H_2\Omega_f\right|q\right\rangle-\left\langle qg\left|\Omega_f^\dagger H_2\frac{1}{p^--H_0}H_1\right|q\right\rangle+\langle qg|\Omega_f^\dagger H_2\Omega_f|q\rangle, \tag{B3}$$

$$\mathcal{M}_5'=\left\langle qg\left|H_2\frac{1}{p^--H_0}H_1\frac{1}{p^--H_0}H_1\right|q\right\rangle-\left\langle qg\left|H_2\frac{1}{p^--H_0}H_1\Omega_f\right|q\right\rangle-\left\langle qg\left|\Omega_c^\dagger H_1\frac{1}{p^--H_0}H_1\right|q\right\rangle+\langle qg|\Omega_c^\dagger H_1\Omega_f|q\rangle, \tag{B4}$$

$$\mathcal{M}_6'=\left\langle qg\left|H_1\frac{1}{p^--H_0}H_5\right|q\right\rangle+\langle qg|\Omega_f^\dagger H_5|q\rangle-\langle qg|H_1\Omega_{qqgg}|q\rangle, \tag{B5}$$

$$\mathcal{M}_7'=\left\langle qg\left|H_5\frac{1}{p^--H_0}H_1\right|q\right\rangle+\langle qg|(\Omega_{qqgg}^\dagger H_1|q\rangle-\langle qg|H_5\Omega_f|q\rangle, \tag{B6}$$

$$\mathcal{M}_8'=\left\langle qg\left|H_1\frac{1}{p^--H_0}H_6\right|q\right\rangle-\langle qg|H_1\Omega_{qqqq}|q\rangle, \tag{B7}$$

$$\mathcal{M}_9'=\left\langle qg\left|H_2\frac{1}{p^--H_0}H_5\right|q\right\rangle-\langle qg|H_2\Omega_{qqgg}|q\rangle+\langle qg|\Omega_c^\dagger H_5|q\rangle. \tag{B8}$$

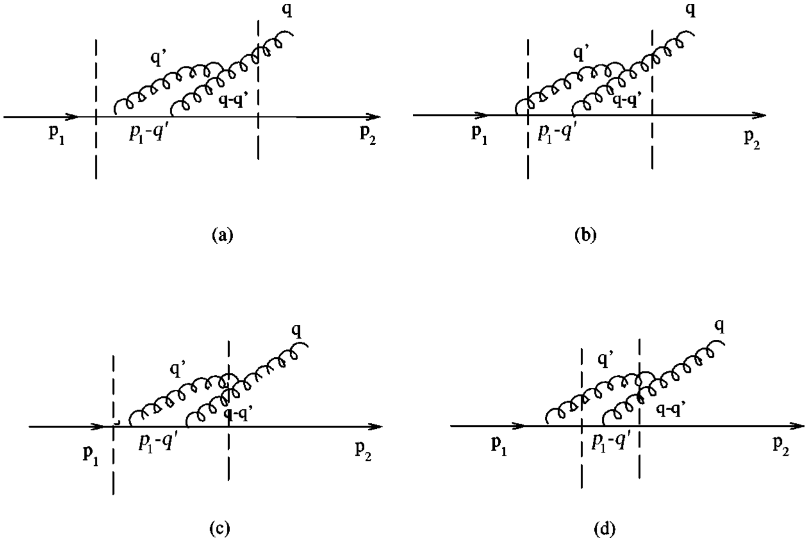

FIG. 11. Set of diagrams representing $\mathcal{M}_5'$.

The first term in $\mathcal{M}_i'(i=2,\ldots,9)$ is $\mathcal{M}_i$ given in Sec. II and represented by Figs. 2–9. We use the diagrammatic notation of Nelson [15,16] to represent the additional contributions due to emission and absorption of soft gluons. For example, diagrams in Fig. 5 will now be grouped together with three additional diagrams shown in Fig. 11. In each of these diagrams, states to the left and right of the dashed lines represents the initial and final states respectively. For example, in Fig. 11(b), the initial state consists of a quark and a soft gluon and it is obtained from the one quark state (the Fock state) by applying the asymptotic operator Ω_f to the one quark state:

$$|q(p)\!:\!\mathrm{coh}\rangle=\Omega_f|q(p)\rangle. \tag{B9}$$

Contribution of each of these diagrams to the quark gluon vertex can be evaluated by using the Feynman rules given in Ref. [10] and the form of the asymptotic states in Sec. III.

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