Gravitational collapse of cylindrical shells made of counterrotating dust particles

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The general formulas of a nonrotating dynamic thin shell that connect two arbitrary cylindrical regions are given using Israel's method. As an application of them, the dynamics of a thin shell made of counterrotating dust particles, which emit both gravitational waves and massless particles when it is expanding or collapsing, is studied. It is found that when the models represent a collapsing shell, in some cases the angular momentum of the dust particles is strong enough to halt the collapse, so that a spacetime singularity is prevented from forming, while in other cases it is not, and a linelike spacetime singularity is finally formed on the symmetry axis.

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I. INTRODUCTION

Gravitational collapse of a realistic body has been one of the most thorny and important problems in Einstein's theory of general relativity. Because of the complexity of the Einstein field equations, the problem, even in simple cases such as spacetimes with spherical symmetry, is still not well understood [1], and new phenomena keep emerging [2]. Particularly, in 1991 Shapiro and Teukolsky (ST) [3] studied numerically the problem of a dust spheroid, and found that only if the spheroid is compact enough can a black hole be formed. Otherwise, the collapse most likely ends with a naked singularity. Later, Barrabés, Israel, and Letelier constructed an analytical model of a collapsing convex thin shell and found that in certain cases no apparent horizons are formed [4]. Their results were soon generalized to more general cases [5]. However, since in all the cases considered by them, the external gravitational field of the collapsing shell is not known, one cannot exclude, similar to the ST case, the formation of an outer event horizon [4,5]. Since then, the gravitational collapse with nonspherical symmetry has been attracting more and more attention. In particular, by studying the collapse of a cylindrical shell that is made of counterrotating particles, Apostolatos and Thorne (AT) showed analytically that the centrifugal forces associated with an arbitrarily small amount of rotation, by themselves, without the aid of any pressure, can halt the collapse at some nonzero, minimum radius, and the shell will then oscillate until it settles down at some final, finite radius, whereby a spacetime singularity is prevented from forming on the symmetry axis [6]. Soon after AT's work, Shapiro and Teukolsky studied numerically the gravitational collapse of rotating spheroids, and found that the rotation indeed significantly modifies the evolution when it is sufficiently large. However, for small enough angular momentum, their simulations showed that spindle singularities appeared to arise without apparent horizons, too. Hence, it is possible that even spheroids with some angular momentum may still form naked singularities [7].

It should be noted that in the AT work it was considered the only case where the shell has zero total angular momentum and is momentarily static and radiation-free. In a realistic case, the spacetime has neither cylindrical symmetry nor zero angular momentum, and gravitational and particle radiations are always expected to occur. As a generalization of the AT work, in this paper we shall consider the case where cylindrical shell radiates gravitational waves and massless particles, as it is collapsing, while keeping the requirement that the total angular momentum of the shell be zero. Specifically, the paper is organized as follows: In Sec. II, the formulas for a general dynamic timelike thin shell that connects two arbitrary cylindrical regions are given, using Israel's formula [8], while in Sec. III, a collapsing thin shell made of counterrotating dust particles is studied. To model the particle radiation of the shell, we consider the case where the spacetime outside the shell is described by an out-going radiation fluid [9]. The paper ends with Sec. IV, where our main conclusions are presented.

II. DYNAMICS OF CYLINDRICAL THIN SHELLS WITHOUT ROTATION

Both static [10] and dynamic [11] cylindrical thin shells with zero total angular momentum have been studied previously. However, in most of these studies a specific form of metric was usually assumed, which is valid only in some particular cases, such as, the spacetime which is vacuum outside and inside the shell [6]. In this section, we shall give a general treatment that is valid for any dynamic timelike cylindrical thin shell, connecting two arbitrary cylindrical regions.

To begin with, let us consider the cylindrical spacetimes described by the metric,

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$$ds^2_- = f^-(t,r)dt^2 - g^-(t,r)dr^2 - h^-(t,r)dz^2 - l^-(t,r)d\varphi^2, \tag{1}$$

where $\{x^{-\mu}\} \equiv \{t,r,z,\varphi\}$, $(\mu=0,1,2,3)$ are the usual cylindrical coordinates. For the spacetimes to be cylindrical, several criteria have to be satisfied [12]. When the symmetry axis is regular, those conditions are easily imposed. However, when it is singular, it is still not clear which kind of conditions should be imposed [13].

In general the spacetimes described by Eq. (1) have two Killing vectors. One is associated with the invariant translations along the symmetry axis, $\xi_{(z)} = \partial z$, where z is the Killing coordinate length with $-\infty < z < +\infty$, and the other is associated with the invariant rotations about the axis, $\xi_{(\varphi)} = \partial\varphi$ with $0 \le \varphi \le 2\pi$, where the hypersurface $\varphi=0$ is identical with the one $\varphi = 2\pi$. Clearly, for the metric given above, the two Killing vectors are orthogonal. Consequently, the metric represents spacetimes without rotation, and the polarization of gravitational waves has only one degree of freedom [14,15].

Assume that a given spacetime is divided by a hypersurface Σ into two regions, say, $V^\pm$, where the region V^- is described by the metric (1), while the region V^+ is described by the metric

$$ds^2_+ = f^+(T,R)dT^2 - g^+(T,R)dR^2 - h^+(T,R)dz^2 - l^+(T,R)d\varphi^2, \tag{2}$$

where $\{x^{+\mu}\} \equiv \{T,R,z,\varphi\}$, $(\mu=0,1,2,3)$, is another set of the cylindrical coordinates. The hypersurface Σ in the coordinates $x^{\pm\mu}$ is given, respectively, by

$$r = r_0(t), \quad R = R_0(T). \tag{3}$$

On the surface, the metrics (1) and (2) reduce, respectively, to

$$\begin{aligned} ds^2_-|_{r=r_0(t)} &= [f^-(t,r_0(t)) - g^-(t,r_0(t))r'^2_0(t)]dt^2 \\ &\quad - h^-(t,r_0(t))dz^2 - l^-(t,r_0(t))d\varphi^2, \\ ds^2_+|_{R=R_0(T)} &= [f^+(T,R_0(T)) \\ &\quad - g^+(T,R_0(T))R'^2_0(T)]dT^2 \\ &\quad - h^+(T,R_0(T))dz^2 - l^+(T,R_0(T))d\varphi^2, \end{aligned} \tag{4}$$

where a prime denotes the ordinary differentiation with respect to the indicated argument. In this paper, we shall consider only the case where Σ is timelike. Then, if we choose the intrinsic coordinates of the hypersurface as $\{\xi^a\} = \{\tau,z,\varphi\}$, $(a=1,2,3)$, where τ denotes the proper time of the surface, we find that the metric on the hypersurface can be written as

$$ds^2|_\Sigma = \gamma_{ab}d\xi^a d\xi^b = d\tau^2 - h(\tau)dz^2 - l(\tau)d\varphi^2, \tag{5}$$

where

$$\begin{aligned} d\tau &= [f^-(t,r_0(t)) - g^-(t,r_0(t))r'^2_0(t)]^{1/2}dt \\ &= [f^+(T,R_0(T)) - g^+(T,R_0(T))R'^2_0(T)]^{1/2}dT, \\ h(\tau) &\equiv h^-(t,r_0(t)) = h^+(T,R_0(T)), \\ l(\tau) &\equiv l^-(t,r_0(t)) = l^+(T,R_0(T)), \end{aligned} \tag{6}$$

where the function dependence of τ on t and T is given by the first equation. Note that in writing the above expressions, we had chosen $d\tau$, dT, and dt, without loss of generality, to have the same sign, and already applied the first junction conditions,

$$ds^2_-|_{r=r_0(t)} = ds^2_+|_{R=R_0(T)}. \tag{7}$$

It can be shown that the unit spacelike normal vector to the hypersurface Σ in the coordinates $x^{\pm\mu}$ is given, respectively, by

$$\begin{aligned} n^+_\mu &= \left[\frac{f^+g^+}{f^+ - g^+R'^2_0(T)}\right]^{1/2}\{-R'_0(T)\delta^T_\mu + \delta^R_\mu\}, \\ n^-_\mu &= \left[\frac{f^-g^-}{f^- - g^-r'^2_0(t)}\right]^{1/2}\{-r'_0(t)\delta^t_\mu + \delta^r_\mu\}. \end{aligned} \tag{8}$$

Then, the nonvanishing components of the extrinsic curvature tensor $K^\pm_{ab}$, defined by[1]

$$K_{ab} = n_\alpha\left(\frac{\partial^2 x^\alpha}{\partial\xi^a\partial\xi^b} + \Gamma^\alpha_{\beta\delta}\frac{\partial x^\beta}{\partial\xi^a}\frac{\partial x^\delta}{\partial\xi^b}\right), \tag{9}$$

are given by

$$\begin{aligned} K^+_{\tau\tau} &= -\frac{(f^+g^+)^{1/2}}{2[f^+ - g^+R'^2_0(T)]^{3/2}}\left\{-\frac{f^+_{,R}}{g^+} + \left(\frac{f^+_{,T}}{f^+} - 2\frac{g^+_{,T}}{g^+}\right)R'_0(T)\right. \\ &\quad \left. + \left(2\frac{f^+_{,R}}{f^+} - \frac{g^+_{,R}}{g^+}\right)R'^2_0(T) + \frac{g^+_{,T}}{f^+}R'^3_0(T) - 2R''_0(T)\right\}, \\ K^+_{zz} &= -\frac{1}{2}\left[\frac{f^+g^+}{f^+ - g^+R'^2_0(T)}\right]^{1/2}\left\{\frac{h^+_{,R}}{g^+} + \frac{h^+_{,T}}{f^+}R'_0(T)\right\}, \\ K^+_{\varphi\varphi} &= -\frac{1}{2}\left[\frac{f^+g^+}{f^+ - g^+R'^2_0(T)}\right]^{1/2}\left\{\frac{l^+_{,R}}{g^+} + \frac{l^+_{,T}}{f^+}R'_0(T)\right\}, \end{aligned} \tag{10}$$

where $f^+_{,T} \equiv \partial f^+(T,R)/\partial T$, etc., and K^-_{ab} can be obtained from the above expressions by the replacement

$$\begin{aligned} &f^+,\ g^+,\ h^+,\ l^+,\ R_0(T),\ T,\ R \\ &\quad \to f^-,\ g^-,\ h^-,\ l^-,\ r_0(t),\ t,\ r. \end{aligned} \tag{11}$$

[1]Note that in this paper the definition for the extrinsic curvature tensor is different from that of Israel by a ‘‘−’’ sign [8].

In terms of $K^{\pm}_{ab}$ and γ_{ab}, the surface energy-momentum tensor, τ_{ab}, is defined as [8],

$$\tau_{ab}=\frac{1}{\kappa}\{[K_{ab}]^{-}-\gamma_{ab}[K]^{-}\}, \tag{12}$$

where $\kappa[\equiv 8\pi G/c^4]$ is the Einstein constant, and

$$[K_{ab}]^{-}\equiv K^{+}_{ab}-K^{-}_{ab}, \quad [K]^{-}\equiv\gamma^{ab}[K_{ab}]^{-}. \tag{13}$$

Inserting Eq. (10) and the corresponding expressions for K^{-}_{ab} into Eq. (12), we find that τ_{ab} can be written in the form

$$\tau_{ab}=\rho w_a w_b+p_z z_a z_b+p_{\varphi}\varphi_a\varphi_b, \quad (a,b=\tau,\ z,\ \varphi), \tag{14}$$

where

$$\rho=\frac{1}{\kappa}\left\{\frac{[K_{zz}]^{-}}{h(\tau)}+\frac{[K_{\varphi\varphi}]^{-}}{l(\tau)}\right\},$$

$$p_z=\frac{1}{\kappa}\left\{[K_{\tau\tau}]^{-}-\frac{[K_{\varphi\varphi}]^{-}}{l(\tau)}\right\},$$

$$p_{\varphi}=\frac{1}{\kappa}\left\{[K_{\tau\tau}]^{-}-\frac{[K_{zz}]^{-}}{h(\tau)}\right\}, \tag{15}$$

and w_a, z_a, and φ_a are unit vectors, defined as

$$w_a=\delta^{\tau}_{a}, \quad z_a=h^{1/2}(\tau)\delta^{z}_{a}, \quad \varphi_a=l^{1/2}(\tau)\delta^{\varphi}_{a}. \tag{16}$$

Clearly, the surface energy-momentum tensor given by Eq. (14) can be interpreted as representing a massive thin shell with its velocity w_a, and principal pressures p_z and p_{φ}, respectively, in the direction, z_a and φ_a, provided that it satisfies some energy conditions [16].

Using Eq. (5) and Eqs. (14)–(16), one can show that the conservation law on the hypersurface Σ [8],

$$\tau^{b}_{a|b}=-[T^{+}_{\alpha\beta}n^{+\alpha}e^{+\beta}_{(a)}-T^{-}_{\alpha\beta}n^{-\alpha}e^{-\beta}_{(a)}], \tag{17}$$

has only one nonvanishing component, which can be written as

$$\frac{d\rho}{d\tau}+\frac{(\rho+p_z)}{2h(\tau)}\frac{dh(\tau)}{d\tau}+\frac{(\rho+p_{\varphi})}{2l(\tau)}\frac{dl(\tau)}{d\tau}$$
$$=-[T^{+}_{\alpha\beta}n^{+\alpha}e^{+\beta}_{(\tau)}-T^{-}_{\alpha\beta}n^{-\alpha}e^{-\beta}_{(\tau)}], \tag{18}$$

where "$|_b$" denotes the covariant differentiation with respect to the three-metric γ_{ab}, and $T^{\pm}_{\alpha\beta}$ are the energy-momentum tensors calculated, respectively, in V^{+} and V^{-}, and

$$e^{+\mu}_{(\tau)}\equiv\frac{\partial x^{+\mu}}{\partial\tau}=(f^{+}-g^{+}R'^{2}_{0}(T))^{-1/2}\{\delta^{\mu}_{T}+R'^{2}_{0}(T)\delta^{\mu}_{R}\},$$

$$e^{+\mu}_{(z)}\equiv\frac{\partial x^{+\mu}}{\partial z}=\delta^{\mu}_{z}, \quad e^{+\mu}_{(\varphi)}\equiv\frac{\partial x^{+\mu}}{\partial\varphi}=\delta^{\mu}_{\varphi},$$

$$e^{-\mu}_{(\tau)}\equiv\frac{\partial x^{-\mu}}{\partial\tau}=(f^{-}-g^{-}r'^{2}_{0}(t))^{-1/2}\{\delta^{\mu}_{t}+r'^{2}_{0}(t)\delta^{\mu}_{r}\},$$

$$e^{-\mu}_{(z)}\equiv\frac{\partial x^{-\mu}}{\partial z}=\delta^{\mu}_{z}, \quad e^{-\mu}_{(\varphi)}\equiv\frac{\partial x^{-\mu}}{\partial\varphi}=\delta^{\mu}_{\varphi}. \tag{19}$$

When no matter shell appears on the hypersurface Σ, we have $\tau_{ab}=0$, and the hypersurface represents a boundary surface [8], with the junction conditions being given by Eq. (6) and Eq. (18). The latter can be written in the form

$$T^{+}_{\alpha\beta}n^{+\alpha}e^{+\beta}_{(\tau)}|_{\Sigma}=T^{-}_{\alpha\beta}n^{-\alpha}e^{-\beta}_{(\tau)}|_{\Sigma}, \quad (\tau_{ab}=0). \tag{20}$$

Once we have the general formulas, let us turn to consider their applications to some specific cases.

III. GRAVITATIONAL COLLAPSE OF CYLINDRICAL SHELLS MADE OF COUNTERROTATING DUST PARTICLES

In this section, we shall consider the gravitational collapse of a cylindrical shell made of counterrotating dust particles. The shell emits gravitational and particle radiations, when it is collapsing. The metric inside the shell will be chosen as that of Minkowski,

$$ds^2_{-}=dt^2-dr^2-dz^2-r^2d\varphi^2, \tag{21}$$

so that the symmetry axis is well defined and the local-flatness condition is satisfied [12]. The metric outside the shell will be chosen as that representing out-going radiation fluid, given by [9]

$$ds^2_{+}=e^{-b(\xi)}(dT^2-dR^2)-dz^2-R^2d\varphi^2, \tag{22}$$

where $b(\xi)$ is an arbitrary function of ξ with $\xi\equiv T-R$. Corresponding to the metric (22), the energy-momentum tensor is given by

$$T^{+}_{\mu\nu}=\frac{b'(\xi)}{R}k_{\mu}k_{\nu}, \tag{23}$$

where k_{μ} is a null vector, defined as

$$k_{\mu}=\frac{1}{\sqrt{2}}(\delta^{T}_{\mu}-\delta^{R}_{\mu}), \tag{24}$$

which is the generator of the out-going radial null geodesic congruence [9]. The presence of the out-going gravitational waves is indicated by the only nonvanishing component of the Weyl tensor, $C_{\mu\nu\lambda\sigma}$, given by [17]

$$\Psi_0\equiv-C_{\mu\nu\lambda\sigma}L^{\mu}M^{\nu}L^{\lambda}M^{\sigma}=-\frac{b'(\xi)}{2R}e^{b(\xi)}, \tag{25}$$

where L^{μ} and M^{μ} are null vectors, the definitions of which are given by Eq. (8) in Ref. [17].

From Eqs. (21) and (22) we find that the first junction conditions (6) now reduce to

$$d\tau = [1 - r'^2_0(t)]^{1/2} dt = e^{-b(\xi_0)/2}[1 - R'^2_0(T)]^{1/2} dT,$$
$$r_0(t) = R_0(T), \tag{26}$$

where ξ_0 is defined as $\xi_0 = T - R_0(T)$. From the above expressions we find

$$\left(\frac{dT}{dt}\right)^2 = \frac{1}{\Delta} \equiv [R'^2_0(T) + e^{-b(\xi_0)}(1 - R'^2_0(T))]^{-1}, \tag{27}$$

which results in

$$\frac{d^2T}{dt^2} = -\frac{1}{2\Delta^2}\{2R'_0R''_0 - e^{-b(\xi_0)}[b'(\xi_0)(1-R'_0)(1-R'^2_0) + 2R'_0R''_0]\},$$

$$r''_0(t) = \frac{d^2T}{dt^2}R'_0 + \left(\frac{dT}{dt}\right)^2 R''_0 = \frac{e^{-b(\xi_0)}}{2\Delta^2}\{2R''_0 + b'(\xi_0)R'_0(1-R'_0)(1-R'^2_0)\}. \tag{28}$$

Inserting Eqs. (10) and the corresponding expressions for K^-_{ab} into Eq. (15), and considering Eq. (28), we find

$$\rho = \frac{e^{b(\xi_0)/2}}{\kappa R_0(1-R'^2_0)^{1/2}}(\Delta - 1),$$

$$p_z = \frac{e^{b(\xi_0)/2}}{\kappa \Delta R'_0(1-R'^2_0)^{3/2}}\Big\{\Delta(1-\Delta)(1-R'^2_0) - (1-\Delta)R'_0R''_0 - \frac{1}{2}b'(\xi_0)R'_0(R'_0-\Delta) \times (1-R'_0)(1-R'^2_0)\Big\},$$

$$p_\varphi = \frac{e^{b(\xi_0)/2}}{\kappa\Delta(1-R'^2_0)^{3/2}}\Big\{(\Delta-1)R''_0 - \frac{1}{2}b'(\xi_0) \times (R'_0-\Delta)(1-R'_0)(1-R'^2_0)\Big\}. \tag{29}$$

When the cylindrical thin shell is made of counterrotating dust particles, where half of the dust particles orbit around the symmetry axis in a right-handed direction with angular momentum per unit rest mass p, and the other half orbit in the opposite, left-handed direction with angular momentum per unit rest mass $-p$, the surface energy-momentum tensor is given by Eq. (14) with $p_z = 0$ [6]. Thus, setting $p_z = 0$ in Eq. (29), we find

$$R''_0 = \frac{1-R'^2_0}{R'_0}\left\{\Delta + \frac{1}{2}b'(\xi_0)R'_0(1-R'_0)\frac{R'_0-\Delta}{\Delta-1}\right\}, \quad (p_z = 0). \tag{30}$$

Substituting the above expression into Eq. (29), we obtain

$$\rho = p_\varphi = \frac{e^{b(\xi_0)/2}}{\kappa R_0(1-R'^2_0)^{1/2}}(\Delta - 1), \quad (p_z = 0). \tag{31}$$

To further study the dynamics of the thin shell with $p_z = 0$, we need to solve Eq. (30), which is found too difficult to be done in the general case. Therefore, in the following we shall consider a particular case in which

$$R''_0 = \frac{1-R'^2_0}{R'_0}\beta, \tag{32}$$

$$\beta = \Delta + \frac{1}{2}b'(\xi_0)R'_0(1-R'_0)\frac{R'_0-\Delta}{\Delta-1}, \tag{33}$$

where β is an arbitrary constant. Once $R'_0(T)$ is known, the function $b'(\xi)$ can be obtained from Eq. (33) by quadrature. Since we are mainly interested in the dynamics of the shell, in the following we shall concentrate on Eq. (32). Integrating it we find that

$$R'_0(T) = \pm(1 - e^{-2\beta T})^{1/2}, \tag{34}$$

where the "+" sign corresponds to an expanding shell, while the "−" sign corresponds to a contracting shell. In the following let us consider the two cases separately.

A. Expanding thin shells

When the "+" sign in Eq. (34) is chosen, the integration of it yields

$$R_0(T) = (T + R_{min}) - \frac{1}{\beta}\{(1-e^{-2\beta T})^{1/2} - \ln[1 + (1-e^{-2\beta T})^{1/2}]\}, \tag{35}$$

where R_{min} is an integration constant. When $\beta > 0$, we find that

$$R_0(T) = \begin{cases} R_{min}, & T = 0, \\ +\infty, & T \to +\infty, \end{cases}$$

$$R'_0(T) = \begin{cases} 0, & T = 0, \\ 1, & T \to +\infty, \end{cases} \quad (\beta > 0), \tag{36}$$

which shows that in this case the corresponding solution represents the expansion of a thin shell made of counterrotating dust particles. The expansion starts from the moment $T = 0$ with the radius of the shell given by $R(0) = R_{min}$. At this moment the shell has zero radial velocity but infinitely large acceleration pointing outwards, as one can see from Eq. (32). Thus, the shell will expand until $T = +\infty$, where it reaches its maximal radius $R_0(+\infty)=+\infty$, with its radial velocity $R_0'(+\infty)=+1$ and acceleration $R_0''(+\infty)=0$.

When $\beta<0$, we find

$$R_0(T)=\begin{cases} R_{min}, & T=0,\\ 0, & T=-|T_1|,\end{cases}$$

$$R_0'(T)=\begin{cases} 0, & T=0,\\ \text{finite}, & T=-|T_1|,\end{cases} \quad (\beta<0). \tag{37}$$

Thus, now the solution represents the expansion of a thin shell, starting from zero radius at the moment $T=-|T_1|$. It expands until the moment $T=0$, where its radial velocity and acceleration are given, respectively, by $R_0'(T=0)=0$ and $R_0''(T=0)=-\infty$. Because of its huge acceleration that now points inwards, the shell will collapse from this moment on, by following a process similar to that to be described below.

B. Collapsing thin shells

When the ''$-$'' sign in Eq. (34) is chosen, we find that

$$R_0(T)=(R_{min}-T)+\frac{1}{\beta}\{(1-e^{-2\beta T})^{1/2} -\ln[1+(1-e^{-2\beta T})^{1/2}]\}, \tag{38}$$

where R_{min} is another integration constant. Thus, when $\beta>0$, from Eq. (38) we find that

$$R_0(T)=\begin{cases} R_{min}, & T=0,\\ 0, & T=|T_1|,\end{cases}$$

$$R_0'(T)=\begin{cases} 0, & T=0,\\ -(1-e^{-2\beta|T_1|})^2, & T=|T_1|,\end{cases} \quad (\beta>0), \tag{39}$$

which shows that now the shell starts to collapse at the moment $T=0$ with zero radial velocity. The collapse ends up at the moment $T=|T_1|$, where the whole shell contracts into a linelike spacetime singularity, as Eqs. (23) and (31) show. Therefore, unlike the case studied by AT [6], in the present case the centrifugal forces of the counterrotating dust particles are not strong enough to prevent the collapse from forming a spacetime singularity.

When $\beta<0$, from Eqs. (34) and (38), we find

$$R_0(T)=\begin{cases} +\infty & T\to-\infty,\\ R_{min}, & T=0,\end{cases}$$

$$R_0'(T)=\begin{cases} -1, & T\to-\infty,\\ 0, & T=0,\end{cases} \quad (\beta<0). \tag{40}$$

Thus, in the present case the shell starts to collapse at the moment $T=-\infty$ with its radius $R_0(-\infty)=+\infty$ and its radial velocity $R_0'(-\infty)=-1$. As it collapses, it is radiating massless particles and gravitational waves, as one can see from Eqs. (23) and (25). At the moment $T=0$, it collapses to its minimal radius $R_0(0)=R_{min}$, where its velocity becomes zero. As far as $R_{min}\neq 0$, in this case no spacetime singularity is formed, and the centrifugal forces of the dust particles now are strong enough to halt the collapse. On the other hand, from Eq. (32) we can see that at $T=0$ the acceleration of the shell becomes infinitely large and points outwards. So, from this moment on, the shell will expand, by following a process similar to that described in the last subsection. When $R_{min}=0$, the centrifugal forces are still not strong enough to prevent the shell from collapsing into a zero radius, whereby a spacetime singularity is formed.

IV. CONCLUSIONS

In this paper, the general formulas of a nonrotating dynamic thin shell that connects two arbitrary cylindrical regions have been given in terms of the metric coefficients and their first derivatives, using Israel's method. As an application of these formulas, the dynamics of a thin shell made of counterrotating noninteracting particles, which emits both gravitational waves and massless particles, has been studied. It has been found that in some cases the models represent an expanding shell and others a collapsing shell. For the collapsing shell, two possible final states exist. In one case, after the shell collapses to a minimal nonzero radius, it starts to expand, that is, the angular momentum of the dust particles is strong enough to halt the collapse, so that a spacetime singularity is prevented from forming on the symmetry axis. However, in the other case, the rotation is not strong enough to halt the collapse at a finite nonzero radius, and as a result a spacetime singularity is formed finally. These results are different from the ones obtained by AT in the radiation-free case [6], but similar to the ones obtained by ST for rotating spheroids with radiation [7].

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