

New constraints on multifield inflation with nonminimal coupling

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We study the dynamics and perturbations during inflation and reheating in a multifield model where a second scalar field χ is nonminimally coupled to the scalar curvature ($\frac{1}{2}\xi R\chi^2$). When ξ is positive, the usual inflationary prediction for large-scale anisotropies is hardly altered while the χ fluctuation in sub-Hubble modes can be amplified during preheating for large ξ . For negative values of ξ , however, long-wave modes of the χ fluctuation exhibit exponential increase during inflation, leading to the strong enhancement of super-Hubble metric perturbations even when $|\xi|$ is less than unity. This is because the effective χ mass becomes negative during inflation. We constrain the strength of ξ and the initial χ by the amplitude of produced density perturbations. One way to avoid nonadiabatic growth of super-Hubble curvature perturbations is to stabilize the χ mass through a coupling to the inflaton. Preheating may thus be necessary in these models to protect the stability of the inflationary phase.

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I. INTRODUCTION

The idea of inflation is remarkable in the sense that it cannot only solve the horizon and flatness problems of the standard big bang cosmology, but provides seeds of density perturbations relevant for the large scale structure [1]. The perturbations give an imprint on the cosmic microwave background (CMB) anisotropies, whose temperature fluctuations can be analyzed by present observations. The inflationary paradigm typically predicts the nearly scale-invariant primordial power spectrum [2,3], which is consistent with observations of the Cosmic Background Explorer (COBE) satellite. Since the accuracy of measurements is expected to be improved in future observations, it is very important to fully understand the primordial power spectrum predicted by the inflationary paradigm.

Generally, it is assumed that only one scalar field called the *inflaton* determines the dynamics of inflation, which leads to the exponential expansion of the universe when the inflaton slowly evolves along a sufficiently flat potential. In the single-field model, density perturbations are typically “frozen” when a physical scale crosses the Hubble radius during inflation. This makes it possible to evaluate the power spectrum at the end of inflation by equating it at the first horizon crossing. In the preheating era after inflation, the fluctuation of the inflaton can be enhanced by parametric resonance [4,5], which may stimulate the growth of metric perturbations. In the single-field case, however, the super-Hubble curvature perturbation is typically conserved during preheating [6], while sub-Hubble modes can be amplified in some models of inflation [7] including the nonminimally coupled inflationary model [8]. As long as the system is a single-field model, and the stress energy is conserved, nonadiabatic growth of the large-scale curvature perturbation cannot be expected during inflation and reheating including generalized Einstein theories [9].

Multifield inflationary scenarios have received much attention for the generality of inflation and preheating. In fact, density perturbations in multifield models were analytically derived by several authors in the scheme of the slow-roll approximation [10]. In the presence of more than two scalar fields, large-scale curvature perturbations are not necessarily conserved due to the existence of isocurvature perturbations. In the context of scalar-tensor gravity theories, several authors [11,12] studied density perturbations in the two-field system where there exists a Brans-Dicke or dilaton field in addition to inflaton. In particular, García-Bellido and Wands [12] constrained parameters of the gravity theories by comparing the predicted spectral index with observational data. In addition to this, since the higher-dimensional generalized Kaluza-Klein theories [13] also give rise to a dilaton field by reducing the effective four-dimensional theories, it is worth investigating to predict the primordial power spectrum in the presence of inflaton in generalized Einstein theories from a cosmological point of view. In this respect, Berkin and Maeda [14] studied the new and chaotic inflationary models with a dilaton potential $U(\sigma)=0$, and constrained the parameters of models by produced density perturbations. Multiple scalar fields also play important roles in the assisted inflation with exponential potentials [15]. This scenario was recently extended to the assisted chaotic inflation induced by higher-dimensional theories [16], and density perturbations were calculated in Ref. [17].

In the preheating era, scalar fields coupled to the inflaton can be resonantly amplified, which is typically more efficient than in the single-field case. It was also pointed out that there is also an interesting possibility that super-Hubble metric perturbations will be excited due to the growth of field perturbations [18–20]. Since growth of metric perturbations can be expected as long as scalar fields are not severely damped in the inflationary period [21,22] and are enhanced during preheating, it is important to take care the dynamics of scalar fields during inflation. When the effective mass of scalar fields is heavy relative to the Hubble parameter H , long wave modes of field fluctuations exhibit exponential decrease dur-

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ing inflation [21]. In contrast, “light” fields such as the inflaton whose masses are smaller than H are hardly affected by the inflationary suppression [19], and can lead to the enhancement of super-Hubble metric perturbations in preheating era if they are amplified by parametric resonance [20]. In this respect, one of the present authors recently investigated the evolution of field and metric perturbations in the presence of a dilaton field with quadratic inflaton potential [23], and found that the curvature perturbation in cosmological relevant modes remains almost constant in this model (Candelas Weinberg model, see Ref. [24]), including the backreaction effect of created particles.

From the viewpoint of quantum field theories in curved spacetime, nonminimal couplings naturally arise, with their own nontrivial renormalization group flows. The ultraviolet fixed point of these flows are often divergent, implying that nonminimal couplings may be important at high energies [25]. In the single-field case with a nonminimally coupled inflaton field, Futamase and Maeda [26] studied the dynamics of chaotic inflation, and found that the nonminimal coupling is constrained as $|\xi| \lesssim 10^{-3}$ in the quadratic potential, by the requirement of sufficient amount of the inflation. On the other hand, such a constraint is absent in the self-coupling potential for negative ξ , and as a bonus, the fine tuning problem of the self-coupling λ in the minimally coupled case can be relaxed by large negative values of ξ [27,28]. Several authors evaluated scalar and tensor perturbations generated during inflation [29–31] and preheating [8] in this model. Since the system is reduced to the single-field model with some modified inflaton potential by a conformal transformation, the super-Hubble curvature perturbation remains almost constant, while metric preheating is found to be vital on sub-Hubble scales [8].

In the multifield model with the inflaton and a nonminimally coupled scalar field χ , it was found that χ particles can be efficiently produced *during inflation* when ξ is negative in the unperturbed Friedmann-Robertson-Walker background [32]. The dynamics of scalar fields strongly depends on the coupling ξ . In fact, although the exponential suppression of super-Hubble χ modes will take place for positive ξ due to large effective mass relative to the Hubble rate, they can grow exponentially by negative instability for $\xi < 0$. Then it is expected that negative nonminimal coupling may lead to the enhancement of super-Hubble metric perturbations during inflation. In addition to this, it is of interest how metric preheating proceeds on large scales, since the χ fluctuation can also be amplified by parametric resonance when $|\xi|$ is greater than of order unity [33,34]. In this paper, motivated by above considerations, we will make precise analysis about the evolution of field and metric perturbations during inflation and preheating in the presence of a nonminimally coupled scalar field χ . We believe that our study will be important in the sense that we can constrain the strength of nonminimal coupling by the COBE normalization. In the case where the power spectrum exceeds the observational upper bound by negative nonminimal coupling, we will give one escape route from nonadiabatic growth of super-Hubble metric perturbations.

II. THE MODEL AND BASIC EQUATIONS

We investigate a model where a massless scalar field χ is nonminimally coupled with the scalar curvature R in the presence of an inflaton field ϕ :

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) - \frac{1}{2} (\nabla\chi)^2 - \frac{1}{2} \xi R \chi^2 \right], \quad (2.1)$$

where $G \equiv \kappa^2/8\pi = m_{\text{pl}}^{-2}$ is a gravitational coupling constant, and ξ is a nonminimal coupling. In this paper, we adopt the quadratic potential for the inflaton,

$$V(\phi) = \frac{1}{2} m^2 \phi^2. \quad (2.2)$$

The variation of the action Eq. (2.1) yields the following field equations:

$$\begin{aligned} \frac{1 - \xi \kappa^2 \chi^2}{\kappa^2} G_{\mu\nu} &= 2\xi \chi (g_{\mu\nu} \square \chi - \nabla_\mu \nabla_\nu \chi) - g_{\mu\nu} V(\phi) \\ &+ (\nabla_\mu \phi)(\nabla_\nu \phi) - \frac{1}{2} g_{\mu\nu} (\nabla^\lambda \phi)(\nabla_\lambda \phi) \\ &+ (1 - 2\xi)(\nabla_\mu \chi)(\nabla_\nu \chi) \\ &- \left(\frac{1}{2} - 2\xi \right) g_{\mu\nu} (\nabla^\lambda \chi)(\nabla_\lambda \chi), \end{aligned} \quad (2.3)$$

$$\square \phi - V'(\phi) = 0, \quad (2.4)$$

$$\square \chi - \xi \chi R = 0, \quad (2.5)$$

where a prime denotes the derivative with respect to ϕ .

Let us consider the perturbed metric in the longitudinal gauge around a Friedmann-Lemaître-Robertson-Walker (FLRW) background

$$ds^2 = -(1 + 2\Phi) dt^2 + a^2(t) (1 - 2\Psi) \delta_{ij} dx^i dx^j, \quad (2.6)$$

with $a(t)$ the scale factor, and Φ, Ψ are gauge-invariant potentials [3]. Decomposing scalar fields into $\varphi_J(t, \mathbf{x}) \rightarrow \varphi_J(t) + \delta\varphi_J(t, \mathbf{x})$ ($J=1,2$), where $\varphi_J(t)$ are homogeneous parts and $\delta\varphi_J(t, \mathbf{x})$ are gauge-invariant fluctuations, we obtain the following background equations for the Hubble parameter $H \equiv \dot{a}/a$ and scalar fields $\varphi_J(t)$:

$$H^2 = \frac{\kappa^2}{3(1 - \xi \kappa^2 \chi^2)} \left[\frac{1}{2} \dot{\phi}^2 + V(\phi) + \frac{1}{2} \dot{\chi}^2 + 6\xi H \chi \dot{\chi} \right], \quad (2.7)$$

$$\dot{H} = - \frac{\kappa^2}{2(1 - \xi \kappa^2 \chi^2)} \left[\dot{\phi}^2 + (1 - 2\xi) \dot{\chi}^2 - 2\xi \chi (\ddot{\chi} - H \dot{\chi}) \right], \quad (2.8)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \quad (2.9)$$

$$\ddot{\chi} + 3H\dot{\chi} + \xi R\chi = 0, \quad (2.10)$$

where the scalar curvature R is given by

$$\begin{aligned} R &= 6(2H^2 + \dot{H}) \\ &= \frac{\kappa^2}{1 - \xi\kappa^2\chi^2} [-\dot{\phi}^2 + 4V(\phi) - \dot{\chi}^2 \\ &\quad + 18\xi H\chi\dot{\chi} + 6\xi(\dot{\chi}^2 + \chi\ddot{\chi})]. \end{aligned} \quad (2.11)$$

The Fourier modes of the linearized perturbed Einstein equations are written as [9]

$$\Psi_k = \Phi_k - \frac{2\xi\kappa^2\chi}{1 - \xi\kappa^2\chi^2} \delta\chi_k, \quad (2.12)$$

$$\begin{aligned} \dot{\Psi}_k + \left(H - \frac{\xi\kappa^2\chi\dot{\chi}}{1 - \xi\kappa^2\chi^2} \right) \Phi_k \\ = \frac{\kappa^2}{2(1 - \xi\kappa^2\chi^2)} [\dot{\phi}\delta\phi_k + (1 - 2\xi)\dot{\chi}\delta\chi_k \\ - 2\xi\chi(\delta\dot{\chi}_k - H\delta\chi_k)], \end{aligned} \quad (2.13)$$

$$\begin{aligned} \delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left[\frac{k^2}{a^2} + V''(\phi) \right] \delta\phi_k \\ = 2(\dot{\phi} + 3H\phi)\Phi_k + \dot{\phi}(\dot{\Phi}_k + 3\dot{\Psi}_k), \end{aligned} \quad (2.14)$$

$$\begin{aligned} \delta\ddot{\chi}_k + 3H\delta\dot{\chi}_k + \left(\frac{k^2}{a^2} + \xi R \right) \delta\chi_k \\ = 2(\ddot{\chi} + 3H\dot{\chi})\Phi_k + \dot{\chi}(\dot{\Phi}_k + 3\dot{\Psi}_k) - \xi\chi\delta R_k, \end{aligned} \quad (2.15)$$

with

$$\begin{aligned} \delta R_k = - \left[12(2H^2 + \dot{H})\Phi_k + 6H(\dot{\Phi}_k + 4\dot{\Psi}_k) \right. \\ \left. + 6\dot{\Psi}_k - \frac{2k^2}{a^2}\Phi_k + \frac{4k^2}{a^2}\Psi_k \right]. \end{aligned} \quad (2.16)$$

Note that Φ_k and Ψ_k do not coincide in the nonminimally coupled case, due to the nonvanishing anisotropic stress. In the absence of the nonminimally coupled χ field (i.e., single-field case), there exists a conserved quantity $\zeta_k \equiv -\Psi_k + (\Psi_k + \dot{\Psi}_k/H)H^2/\dot{H}$ for super-Hubble k modes in the linear perturbations [3]. During reheating phase, although entropy perturbations can be produced when $\dot{\phi}$ periodically passes through zero, curvature perturbations on super-Hubble scales are typically conserved in single-field models [6] including the nonminimally coupled inflaton case [8]. Even in generalized Einstein theories including scalar-tensor and higher-curvature gravity theories, it was found that the conserved

structure on large scales still holds in the single-field case [9]. Because of this adiabaticity of the curvature perturbation, we only take care the perturbation at the first horizon crossing in order to evaluate the inflationary power spectrum.

In the multifield case, however, the curvature perturbation on uniform-density hypersurfaces [9],

$$\zeta_k \equiv -\Psi_k + \frac{H^2(1 - \xi\kappa^2\chi^2)}{\dot{H}(1 - \xi\kappa^2\chi^2) + 2H\xi\kappa^2\chi\dot{\chi}} \left(\Psi_k + \frac{\dot{\Psi}_k}{H} \right), \quad (2.17)$$

includes the isocurvature perturbation [10–12], which can vary nonadiabatically during inflation and reheating. In fact, in the present model, since the homogeneous χ and the $\delta\chi_k$ fluctuation in small k modes can be strongly excited for negative ξ as is found in Eqs. (2.10) and (2.15), this will stimulate the growth of super-Hubble metric perturbations and produce entropy perturbations. On the other hand, for positive ξ , it is expected that long-wave χ modes will exponentially decrease during inflation due to the large effective χ mass relative to the Hubble rate [21,22], which may not lead to the nonadiabatic growth of curvature perturbations on super-Hubble scales even if field fluctuations will exhibit parametric amplifications in reheating phase. In the next section, we will make a detailed analysis about the dynamics of field and metric perturbations during inflation and reheating.

III. COSMOLOGICAL PERTURBATIONS DURING INFLATION AND REHEATING

Let us first review the dynamics of inflation with potential (2.2) in the absence of the nonminimally coupled χ field. Neglecting the $\dot{\phi}$ term in Eq. (2.7) and the $\dot{\phi}$ term in Eq. (2.9), we obtain the following approximate relation during inflation:

$$H \approx \sqrt{\frac{4\pi}{3}} \frac{m}{m_{\text{pl}}} \phi, \quad \dot{\phi} \approx -\frac{m^2}{3H} \phi. \quad (3.1)$$

Combining these relations is to give

$$\phi = \phi(0) - \frac{m_{\text{pl}}}{\sqrt{12}\pi} mt, \quad (3.2)$$

$$a = a(0) \exp \left[\sqrt{\frac{4\pi}{3}} \frac{m}{m_{\text{pl}}} \left\{ \phi(0)t - \frac{m_{\text{pl}}}{\sqrt{48}\pi} mt^2 \right\} \right], \quad (3.3)$$

where $\phi(0)$ and $a(0)$ are initial values of the inflaton and the scale factor, respectively. In the initial stage of the inflation, the scale factor evolves exponentially as $a \sim a(0) \exp[\sqrt{4\pi/3}(m/m_{\text{pl}})\phi(0)t]$. With the increase of the last term in Eq. (3.3), the expansion rate slows down, which is followed by the oscillating stage of the inflaton. In order to solve several cosmological puzzles of the standard big bang cosmology, the number of e-foldings $N \equiv \ln(a/a(0))$ is required to be $N \gtrsim 55$, by which the initial value of the inflaton is constrained as $\phi(0) \gtrsim 3m_{\text{pl}}$. The inflationary period ends

when the slow-roll parameter $\epsilon \equiv (V'/V)^2/2\kappa^2$ grows of order unity, which corresponds to $\phi \sim 0.3m_{\text{pl}}$.

If nonminimal coupling is taken into account, the dynamics of the inflation can be changed. In fact, growth of the χ field affects the Hubble parameter by Eq. (2.7), which also alters the evolution of the inflaton by Eq. (2.9). Let us first investigate the evolution of the χ fluctuation approximately. Neglecting the contribution of metric perturbations in Eq. (2.15) and introducing a new scalar field $\delta X_k \equiv a \delta \chi_k$ and a conformal time $\eta \equiv \int a^{-1} dt$, Eq. (2.15) yields

$$\delta X_k'' + \left[k^2 - (1 - 6\xi) \frac{a''}{a} \right] \delta X_k = 0, \quad (3.4)$$

where a prime denotes the derivative with respect to the conformal time. This solution can be expressed by the combinations of the Hankel functions $H_\nu^{(J)}$ ($J=1,2$) [32]:

$$\delta \chi_k = a^{-1} [c_1 \sqrt{\eta} H_\nu^{(2)}(k\eta) + c_2 \sqrt{\eta} H_\nu^{(1)}(k\eta)], \quad (3.5)$$

where the order ν of the Hankel functions is given by

$$\nu^2 = \frac{9}{4} - 12\xi. \quad (3.6)$$

Note that the choice of $c_1 = \sqrt{\pi}/2$ and $c_2 = 0$ corresponds to the state of the Bunch-Davies vacuum. The solution of the homogeneous χ field also looks the form of Eq. (3.5) with $k=0$.

The Hankel functions take the following form in the limit of $k\eta \rightarrow 0$ [35]:

$$H_\nu^{(2,1)}(k\eta) \rightarrow \pm \frac{i}{\pi} \Gamma(\nu) \left(\frac{k\eta}{2} \right)^{-\nu}, \quad (3.7)$$

where $\Gamma(\nu)$ is the Gamma function with order ν . Taking notice of the relation $\eta \approx -1/(aH)$ during inflation, we easily find from Eq. (3.5) that long-wave $\delta \chi_k$ modes exponentially increase as $\delta \chi_k \sim a^{\nu-3/2}$ when $\nu > 3/2$. This case corresponds to the negative ξ by Eq. (3.6), leading to the particle creation during inflation by negative instability as noted in Ref. [32]. This efficient particle production for low momentum modes is expected to enhance metric perturbations for wavelengths larger than the Hubble radius, due to the increase of the right-hand side (RHS) of Eq. (2.13). This will also stimulate the growth of field perturbations as expected by Eqs. (2.14) and (2.15). In contrast, for positive ξ , long-wave $\delta \chi_k$ modes decay exponentially in de Sitter background. Especially for the case of $\xi > 3/16$ where ν takes complex values, $\delta \chi_k$ decreases as $\sim a^{-3/2}$. This makes the χ term in the RHS of Eq. (2.13) unimportant and super-Hubble metric perturbations will not be strongly amplified even taking into account the parametric amplification of χ and $\delta \chi_k$ modes during preheating as shown in Ref. [21] in the model of $V(\phi, \chi) = \frac{1}{2} m^2 \phi^2 + \frac{1}{2} g^2 \phi^2 \chi^2$.

Before analyzing the dynamics of the system, we should mention initial conditions of field fluctuations. For positive frequency $\omega_k^2 > 0$, we typically choose the conformal vacuum state $\delta \varphi_k = 1/\sqrt{2\omega_k}$ and $\delta \dot{\varphi}_k = (-i\omega_k - H)\delta \varphi_k$. However, in

de Sitter background with nonminimal coupling, the frequency of scalar fields can take negative values. Hence we adopt the de Sitter invariant vacuum state given by [36]

$$\begin{aligned} \delta X_k(\eta_0) &= \frac{1}{k^{3/2}} \left[ik + \frac{a'(\eta_0)}{a(\eta_0)} \right] \exp(ik\eta_0), \\ \delta X_k'(\eta_0) &= \frac{1}{k^{1/2}} \left[ik + \frac{a'(\eta_0)}{a(\eta_0)} - \frac{i}{k} \left(\frac{a'(\eta_0)}{a(\eta_0)} \right)' \right] \exp(ik\eta_0). \end{aligned} \quad (3.8)$$

In the case of $k \gg a'(\eta_0)/a(\eta_0)$, this takes the similar form as the conformal vacuum state, while mode functions depend on the choice of the vacuum for small k . However, in the context of the inflationary power spectrum, it is known that different choice of initial conditions has little effect on the results [3].

In what follows, we numerically study the evolution of scalar fields and super-Hubble metric perturbations for positive and negative values of ξ during inflation and reheating, and also investigate the case where the coupling between ϕ and χ ($\frac{1}{2} g^2 \phi^2 \chi^2$) is introduced at the end of this section.

A. Case of $\xi > 0$

Let us first consider the minimally coupled case ($\xi=0$) before analyzing the positive ξ case. In this case, the inflationary period proceeds as in the single-field case, as long as χ is initially small relative to the inflaton. We plot in Fig. 1 the evolution of the metric perturbation $\Psi_k (= \Phi_k)$ and the curvature perturbation ζ_k for a cosmological mode $k = a_0 H_0$ where a_0 denotes the scale factor about 55 e-foldings before the end of inflation. The initial value of inflaton is chosen as $\phi(0) = 3m_{\text{pl}}$, in which case the inflationary period continues until $mt \approx 20$, leading to about 57 e-foldings at the end of inflation. The curvature perturbation,

$$\zeta_k = -\Psi_k + \frac{H^2}{\dot{H}} \left(\Psi_k + \frac{\dot{\Psi}_k}{H} \right), \quad (3.9)$$

remains almost constant on large scales as is clearly seen in Fig. 1.

When the system enters the reheating stage, $\dot{\phi}$ periodically passes through zero, during which entropy perturbations can be produced [18]. Nevertheless, this process is not strong enough to lead to the overall increase of ζ_k and Φ_k in super-Hubble modes (see Fig. 1). Not only scalar field fluctuations for low momentum modes are not relevantly amplified, but metric preheating is inefficient on large scales for $\xi=0$.

Let us define the power-spectrum of ζ_k as

$$\mathcal{P}_{\zeta_k} \equiv \frac{k^3}{2\pi^2} |\zeta_k|^2 = \frac{|\tilde{\zeta}_k|^2}{2\pi^2}, \quad (3.10)$$

where $\tilde{\zeta}_k$ is defined by $\tilde{\zeta}_k \equiv k^{3/2} \zeta_k$. Assuming that ζ_k remains conserved on cosmological scales until it reenters the Hubble

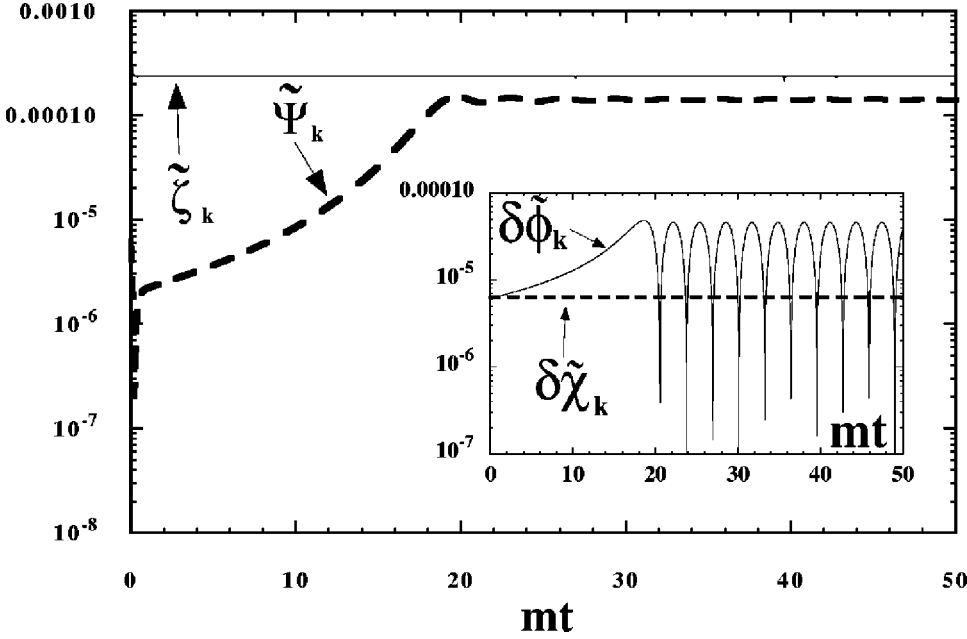


FIG. 1. The evolution of $\tilde{\zeta}_k \equiv k^{3/2}\zeta_k$ and $\tilde{\Psi}_k \equiv k^{3/2}\Psi_k$ during inflation and reheating for a super-Hubble mode $k = a_0 H_0$ in the case of $\xi = 0$ with $m = 10^{-6} m_{\text{pl}}$ and initial values $\phi(0) = 3m_{\text{pl}}$, $\chi(0) = 10^{-3} m_{\text{pl}}$. The inflationary period ends at $mt \approx 20$, after which the system enters the reheating stage. In this case, since the system is effectively identical with the single-field case, $\tilde{\zeta}_k$ is conserved. *Inset*: The evolution of field perturbations $\delta\tilde{\phi}_k \equiv k^{3/2}\delta\phi_k/m_{\text{pl}}$ and $\delta\tilde{\chi}_k \equiv k^{3/2}\delta\chi_k/m_{\text{pl}}$.

length in the matter-dominant stage, the power spectrum at the end of inflation [$=\mathcal{P}_{\zeta_k}(t_e)$] can be related with the density perturbation $\delta_H(k)$ at the horizon reentry as [3,12]

$$\delta_H^2(k) \approx \frac{4}{25} \mathcal{P}_{\zeta_k}(t_e). \quad (3.11)$$

For the inflaton mass $m \sim 10^{-6} m_{\text{pl}}$ regulated by CMB observations, numerical calculations give $\tilde{\zeta}_k \approx 2.4 \times 10^{-4}$ at the end of inflation. Then we obtain the density perturbation as $\delta_H(k) \approx 2 \times 10^{-5}$ by the relation (3.11).

For positive ξ , χ and long-wave $\delta\chi_k$ modes exponentially decrease during inflation. This decreasing rate strongly depends on the strength of the coupling ξ . When $0 < \xi < 3/16$, the order ν of the Hankel function is in the range of $0 < \nu < 3/2$, and long-wave $\delta\chi_k$ modes evolve as $\delta\chi_k \sim a^{-(3/2-\nu)}$.

In Fig. 2, we plot the evolution of $\delta\tilde{\chi}_k \equiv k^{3/2}\delta\chi_k/m_{\text{pl}}$ for several ξ with initial values $\phi(0) = 3m_{\text{pl}}$ and $\chi(0) = 10^{-3} m_{\text{pl}}$. For $\xi = 0.01$, $\delta\tilde{\chi}_k$ decreases only by one order of magnitude during inflation. With the increase of ξ , the inflationary suppression becomes relevant and the decreasing rate is getting larger. When $\xi > 3/16$ (i.e., $\nu^2 < 0$), super-Hubble $\delta\chi_k$ fluctuations decay as $\delta\chi_k \sim a^{-3/2}$ irrespective of the coupling ξ . This means that the amplitude of $\delta\chi_k$ at the end of inflation depends on the total amount of inflation. In the simulation of initial values $\phi(0) = 3m_{\text{pl}}$ and $\chi(0) = 10^{-3} m_{\text{pl}}$, $\delta\tilde{\chi}_k$ decreases of order $\delta\tilde{\chi}_k \approx 10^{-40}$ since the number of e-foldings is about $N \sim 57$ in this case. The homogeneous χ field is also affected by this suppression. Hence χ dependent terms in the RHS of Eq. (2.13) can be negligible relative to the ϕ dependent term, leading to the conservation of the super-Hubble curvature perturbation during inflation.

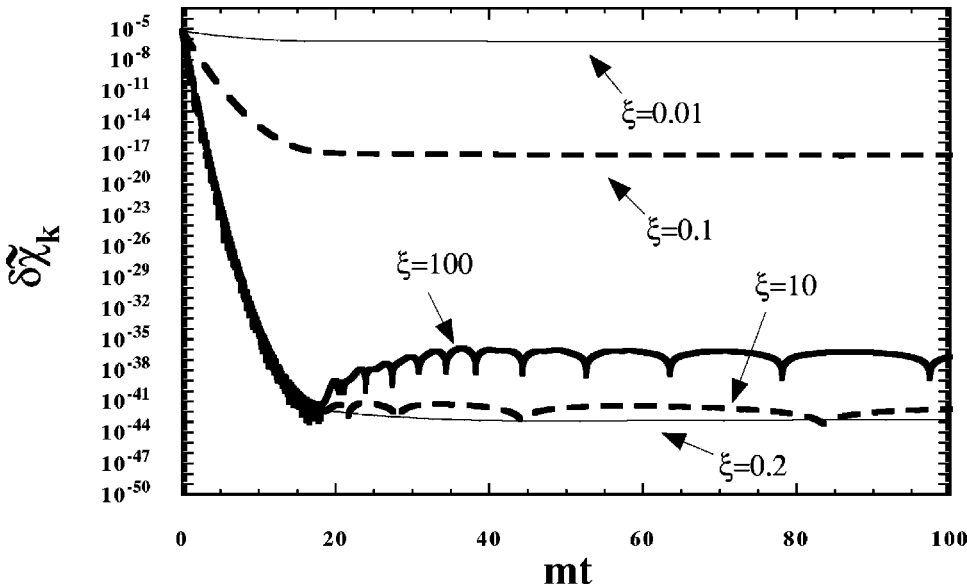


FIG. 2. Suppression of the field fluctuation $\delta\tilde{\chi}_k$ during inflation and reheating for a super-Hubble mode $k = a_0 H_0$ in the case of $\xi = 0.01, 0.1, 0.2, 10, 100$ with $m = 10^{-6} m_{\text{pl}}$ and initial values $\phi(0) = 3m_{\text{pl}}$, $\chi(0) = 10^{-3} m_{\text{pl}}$. When $0 < \xi < 3/16$, $\delta\tilde{\chi}_k$ decreases as $\sim a^{-(3/2-\nu)}$, while $\delta\tilde{\chi}_k \sim a^{-3/2}$ for $\xi > 3/16$ independent of the strength of ξ . The homogeneous χ field exhibits the similar behavior.

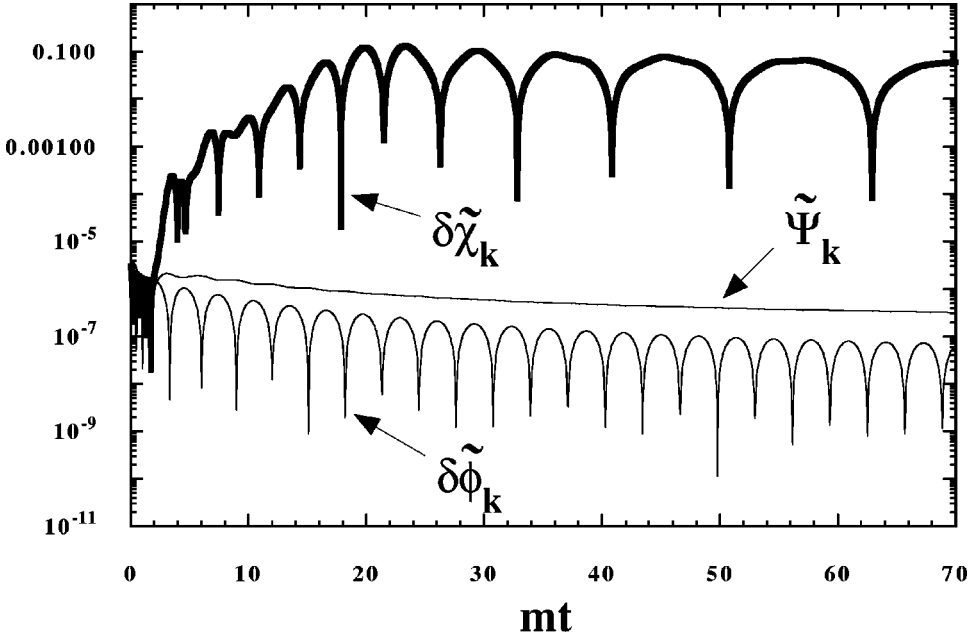


FIG. 3. The evolution of $\delta\tilde{\chi}_k$, $\delta\tilde{\phi}_k$, and $\tilde{\Psi}_k$ during preheating for a sub-Hubble mode $\bar{k}=3$ with $\xi=100$ and $m=10^{-6}m_{\text{pl}}$. Note that we start integrating from the end of inflation, and choose initial values as $\phi(0)=0.28m_{\text{pl}}$ and $\chi(0)=10^{-40}m_{\text{pl}}$ with the $\delta\chi_k$ fluctuation (3.12). Φ_k almost coincides with Ψ_k in this case.

As is found in Eq. (2.11), the scalar curvature reduces with the decrease of the inflaton, unless the χ field is amplified. When the kinetic term of the inflaton becomes comparable to its potential energy, the scalar curvature begins to oscillate due to the oscillation of the inflaton. It is well known that χ particles coupled to ϕ can be nonperturbatively produced by parametric resonance during preheating era [5]. In the present model, we can also expect the amplification of the $\delta\chi_k$ fluctuation due to the oscillating scalar curvature. This geometric preheating scenario was studied in Refs. [33,34] neglecting metric perturbations. In Fig. 2, we find that the $\delta\chi_k$ fluctuation undergoes the parametric excitation during the reheating era ($mt \gtrsim 20$) for the coupling $\xi \gtrsim 10$, while there is no growth for $\xi \lesssim 1$. With the increase of ξ , the growth rate of $\delta\chi_k$ gets gradually larger, leading to the efficient particle production. Nevertheless, as deeply studied in Ref. [34], larger values of ξ do not necessarily result in the larger amount of the final fluctuation, due to the suppression effect of the χ particle production itself. In fact, the final variance takes the maximum value at $\xi=100\text{--}200$ [34]. This indicates that long-wave $\delta\chi_k$ fluctuations are still much smaller relative to inflaton fluctuations even in the case of $\xi \gtrsim 100$ because of the very small amplitude at the end of inflation. As a result, the existence of the preheating era does not lead to the enhancement of super-Hubble metric perturbations, whose source terms in the RHS of Eq. (2.13) are completely dominated by the inflaton-dependent term. We have numerically checked that the curvature perturbation ζ_k and metric perturbations Ψ_k, Φ_k on large scales exhibit the same behavior as shown in Fig. 1, as long as the initial value of χ at the onset of inflation is much smaller than ϕ . As a result, the adiabatic picture of large-scale cosmological perturbations in the single-field case still holds even during preheating in the presence of the positive nonminimal coupling.

We should mention the evolution of $\delta\chi_k$ and Ψ_k, Φ_k in sub-Hubble modes during preheating. For large k -modes, the adiabatic solution for the $\delta\chi_k$ fluctuation is estimated by Eq. (3.8) as

$$|\delta\tilde{\chi}_k| \approx \bar{k} \frac{m}{m_{\text{pl}}}, \quad |\delta\dot{\tilde{\chi}}_k|/m \approx \bar{k}^2 \frac{m}{m_{\text{pl}}}, \quad (3.12)$$

where $\bar{k} \equiv k/(ma_I)$ with a_I the scale factor at the beginning of preheating. Since sub-Hubble modes correspond to $\bar{k} \gtrsim 1$, the amplitude of the $\delta\chi_k$ fluctuation at the end of inflation is found to be $|\delta\tilde{\chi}_k| \gtrsim m/m_{\text{pl}} \sim 10^{-6}$, which is not strongly suppressed compared with the super-Hubble case. Then the growth of the total variance

$$\langle \delta\chi^2 \rangle \equiv \frac{1}{2\pi^2} \int k^2 |\delta\chi_k|^2 dk = \frac{m_{\text{pl}}^2}{2\pi^2} \int |\delta\tilde{\chi}_k|^2 d(\log k), \quad (3.13)$$

is typically governed by sub-Hubble modes. This situation is similar to the preheating scenario with the $\frac{1}{2}g^2\phi^2\chi^2$ coupling [21]. We depict in Fig. 3 the evolution of $\delta\chi_k$, $\delta\phi_k$, and Ψ_k in preheating phase for a sub-Hubble mode $\bar{k}=3$ with $\xi=100$. While the $\delta\chi_k$ fluctuation exhibits parametric excitation by the oscillating scalar curvature, we find that sub-Hubble metric perturbations are hardly enhanced during preheating. In spite of the unsuppressed initial conditions for sub-Hubble $\delta\chi_k$ and $\delta\dot{\chi}_k$ modes in the RHS of Eq. (2.13), the homogeneous components χ and $\dot{\chi}$ are strongly damped as in the case of super-Hubble $\delta\chi_k$ modes. Then we cannot expect the strong amplification of sub-Hubble metric perturbations, due to the suppression of χ -dependent source terms in Eq. (2.13). However, this result may change if we take into account second order metric perturbations [38] as in Ref. [21], which we do not consider in this paper. Since the RHS of Eq. (2.14) can be negligible and the resonant term is absent in the LHS, the inflaton fluctuation is also hardly amplified as is found in Fig. 3 even in the presence of metric perturbations.

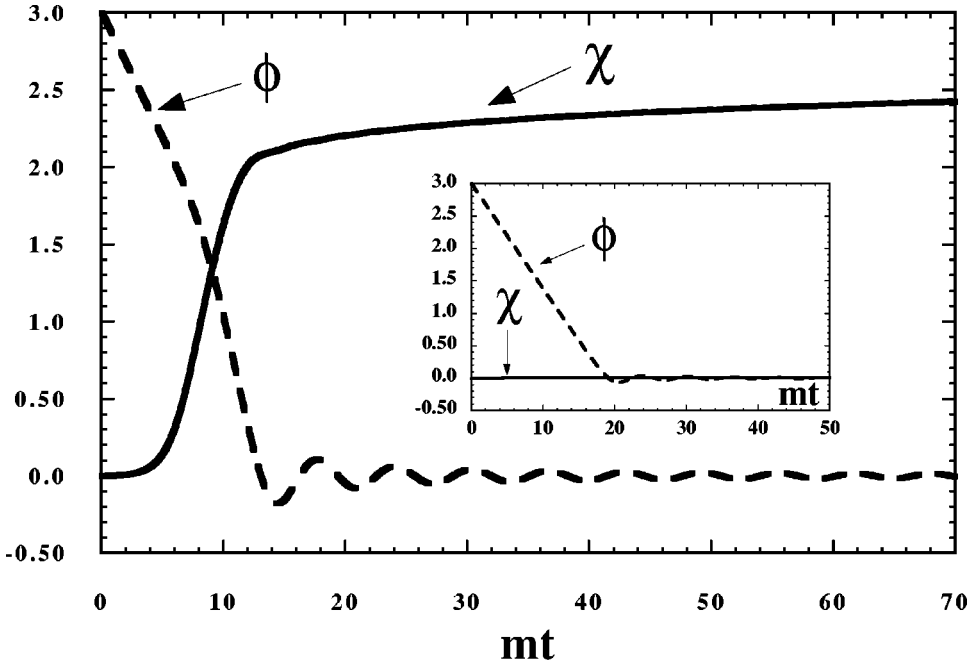


FIG. 4. The evolution of ϕ and χ fields during inflation and reheating for a super-Hubble mode $k = a_0 H_0$ in the case of $\xi = -0.05$ and initial values $\phi(0) = 3m_{\text{pl}}$, $\chi(0) = 10^{-3}m_{\text{pl}}$ with $m = 10^{-6}m_{\text{pl}}$. The χ field rapidly grows by negative instability, whose effect terminates inflation at $mt \approx 13$. *Inset:* ϕ and χ vs t for $\xi = -0.005$. In this case, the χ field is hardly enhanced.

For positive ξ , we conclude that the curvature perturbation in cosmologically relevant scales is conserved during inflation and preheating as long as χ is initially small relative to ϕ .

B. Case of $\xi < 0$

Let us next proceed to the case of negative ξ . In this case, χ and long-wave $\delta\chi_k$ modes can be enhanced during inflation by negative instability as is found by Eqs. (2.10) and (2.15). The analytic estimation of Eq. (3.5) which neglects metric perturbations includes the following growing solution in small k modes:

$$|\delta\chi_k| \propto a^c, \quad \text{with } c = \frac{3}{2} \left(\sqrt{1 + \frac{16}{3}|\xi|} - 1 \right). \quad (3.14)$$

This growth rate gets larger with the increase of $|\xi|$. The exponential increase of the $\delta\chi_k$ fluctuation also stimulates the growth of super-Hubble metric perturbations as is found in Eq. (2.13). Then metric perturbations will strengthen field resonances by Eqs. (2.14) and (2.15) in the perturbed metric case, as we numerically study it later. What we are mainly concerned with is how the dynamics and produced perturbations are modified during inflation and preheating by negative nonminimal coupling.

Let us first consider the case of $\phi(0) = 3m_{\text{pl}}$ where the number of e-foldings reaches $N \sim 57$ in the absence of the χ field. If the negative nonminimal coupling is taken into account, the total amount of inflation is not necessarily sufficient to solve cosmological puzzles. For example, when $\chi(0) = 10^{-3}m_{\text{pl}}$, the dynamics of inflation is modified due to the enhancement of the χ field for $|\xi| \geq 0.02$.

In Fig. 4, we plot the evolution of the homogeneous ϕ and χ fields for $\xi = -0.05$ with initial values $\phi(0) = 3m_{\text{pl}}$ and $\chi(0) = 10^{-3}m_{\text{pl}}$. The negative nonminimal coupling leads to the growth of the χ field, which catches up the ϕ

field for $mt \approx 10$. In the initial stage where ϕ is larger than χ , the dynamics of inflation is approximately described by Eqs. (3.2) and (3.3). However, after χ exceeds ϕ for $mt \geq 10$, these relations can no longer be applied. With the increase of χ , the potential $V(\phi)$ becomes gradually unimportant relative to χ -dependent terms in Eq. (2.7) and the $1/(1 - \xi\kappa^2\chi^2)$ factor also gets smaller, the Hubble expansion rate decreases faster than in the $\xi = 0$ case. Then inflation ends at $mt_e \approx 13$ with the e-folding number $N \approx 42$. Note that these values are smaller than in the minimally coupled case, $mt_e \approx 20$ and $N \approx 58$. Since the decrease of the scalar curvature $R = 6(2H^2 + \dot{H})$ is accompanied by the decrease of H , the growth of the χ field typically becomes irrelevant after the inflationary period terminates (see Fig. 4). We show in the inset of Fig. 4 the evolution of ϕ and χ for $\xi = -0.005$. In this case, the χ field is hardly amplified, which results in almost the same dynamics of inflation as in the $\xi = 0$ case.

In Fig. 5 we show the evolution of large-scale $\delta\chi_k$ and $\delta\phi_k$ fluctuations for $\xi = -0.05$ with $\phi(0) = 3m_{\text{pl}}$ and $\chi(0) = 10^{-3}m_{\text{pl}}$. We numerically found that the growth of field perturbations is relevant for $|\xi| \geq 0.02$. In the case of $\xi = -0.05$, $\delta\chi_k$ fluctuations in small k -modes are enhanced from the beginning as described in Eq. (3.14) with $c \approx 0.188$. Recalling that the total amount of inflation is about $N \approx 42$, $|\delta\chi_k|$ is amplified about 10^3 times during inflation by the analytic estimation of Eq. (3.14) neglecting metric perturbations. We plot in the inset of Fig. 5 the evolution of long-wave field perturbations in the unperturbed metric case [i.e., setting $\Psi_k = \Phi_k = 0$ in Eqs. (2.14) and (2.15)]. We can easily confirm that numerical calculations coincide with the analytic result (3.14) fairly well. In the perturbed metric case, the evolution of the $\delta\chi_k$ fluctuation is almost the same as in the unperturbed metric case except for the final short stage of inflation, which indicates that the ξR term in the LHS of Eq. (2.15) mainly determines the growth of $\delta\chi_k$ even taking into account metric perturbations.

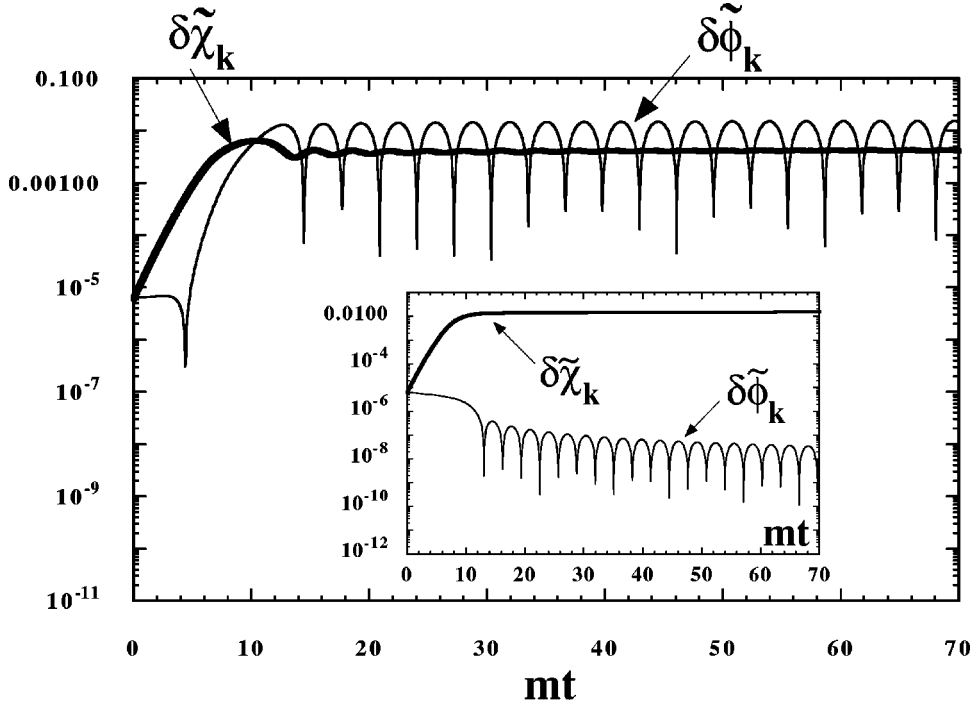


FIG. 5. Growth of field fluctuations $\delta\tilde{\chi}_k$ and $\delta\tilde{\phi}_k$ during inflation and reheating for a super-Hubble mode $k=a_0H_0$ in the case of $\xi=-0.05$ and initial values $\phi(0)=3m_{\text{pl}}$, $\chi(0)=10^{-3}m_{\text{pl}}$ with $m=10^{-6}m_{\text{pl}}$. Both field fluctuations are amplified during inflation. *Inset:* $\delta\tilde{\chi}_k$ and $\delta\tilde{\phi}_k$ vs t for $\xi=-0.05$ neglecting metric perturbations. Although $\delta\tilde{\chi}_k$ is enhanced as in the perturbed metric case, $\delta\tilde{\phi}_k$ does not grow in the absence of metric perturbations.

In contrast, the difference appears in the inflaton fluctuation. If we neglect metric perturbations, the $\delta\phi_k$ fluctuation does not grow nonperturbatively in the massive inflaton model. Including metric perturbations, the enhancement of χ and $\delta\chi_k$ fluctuations in small k -modes stimulates the growth of super-Hubble metric perturbations by Eq. (2.13). Then the RHS of Eq. (2.14) leads to the amplification of the $\delta\phi_k$ fluctuation, which is absent in the rigid spacetime case. This difference is clearly seen in Fig. 5.

Let us investigate the evolution of the curvature perturbation ζ_k and metric perturbations Ψ_k and Φ_k on large scales. Since the growth of metric perturbations is accompanied by the excitement of the χ field fluctuation, ζ_k increases during

inflation when $|\xi| \geq 0.02$ with initial values $\phi(0)=3m_{\text{pl}}$ and $\chi(0)=10^{-3}m_{\text{pl}}$. In Fig. 6, we show the evolution of Ψ_k and Φ_k for $\xi=-0.05$ and $\xi=-0.005$. When $\xi=-0.005$, Ψ_k almost coincides with Φ_k as in the case of $\xi \geq 0$. For $\xi=-0.05$, however, super-Hubble metric perturbations are strongly amplified during inflation, leading to the distortions in the CMB spectrum. Note that the difference of Ψ_k and Φ_k appears in this case, due to the enhancement of the χ field fluctuation. While the super-Hubble curvature perturbation is conserved for $\xi=-0.005$, it exhibits rapid growth during inflation for $\xi=-0.05$. This means that the standard picture of adiabatic perturbations in the single-field case can no

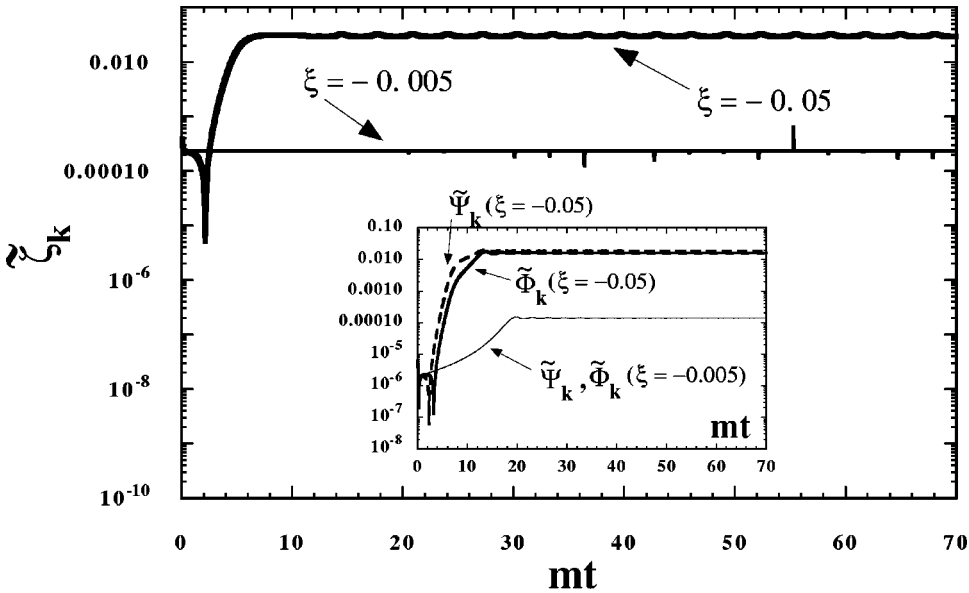


FIG. 6. The evolution of the curvature perturbation ζ_k during inflation and reheating for a super-Hubble mode $k=a_0H_0$ in the cases of $\xi=-0.05$ and $\xi=-0.005$ with $\phi(0)=3m_{\text{pl}}$, $\chi(0)=10^{-3}m_{\text{pl}}$, and $m=10^{-6}m_{\text{pl}}$. We find that ζ_k grows nonperturbatively during inflation for $\xi=-0.05$, while it is conserved for $\xi=-0.005$. *Inset:* The evolution of super-Hubble metric perturbations $\tilde{\Psi}_k$ and $\tilde{\Phi}_k$ for $\xi=-0.05$ and $\xi=-0.005$.

TABLE I. The time mt_f , the number of e-foldings N , the value χ_f , and the super-Hubble curvature perturbation $\tilde{\zeta}_k$ at the end of inflation with initial values $\phi(0)=3m_{\text{pl}}$ and $\chi(0)=10^{-3}m_{\text{pl}}$. With the increase of $|\xi|$, the duration of inflation becomes shorter, which results in the smaller amount of e-foldings. We also find that large values of $|\xi|$ ($|\xi|>0.02$) leads to the nonadiabatic increase of the curvature perturbation due to the enhancement of the χ field.

ξ	mt_f	N	χ_f/m_{pl}	$\tilde{\zeta}_k$
-0.005	18	57	0.0031	0.00023
-0.01	18	57	0.0094	0.00023
-0.05	12	41	2.0	0.031
-0.1	8	23	2.7	0.018
-1	3	4	1.7	0.0028

longer be applied in the presence of the nonminimally coupled χ field with negative coupling.

In Table I, we show the number of e-foldings, the homogeneous χ field, and the super-Hubble curvature perturbation at the end of inflation for several values of ξ with initial conditions $\phi(0)=3m_{\text{pl}}$ and $\chi(0)=10^{-3}m_{\text{pl}}$. Since the enhancement of the χ field is weak for $|\xi|\leq 0.02$, ζ_k remains constant during inflation. For $|\xi|\geq 0.02$, the rapid growth of χ makes the inflationary period terminate earlier as confirmed in Table I. This leads to the smaller amount of inflation with the increase of $|\xi|$. For example, when $\xi=-1$, the system soon enters the reheating stage after only 4 e-foldings. We have to caution that large $|\xi|$ does not necessarily yield the larger values of χ and ζ_k at the end of inflation, because the duration of inflation gets shorter. In fact, ζ_k decreases with the increase of $|\xi|$ for $|\xi|\geq 0.05$, although large $|\xi|$ also leads to the distortions in the anisotropies of CMB. The important point is that the successful inflationary scenario can be completely violated with the existence of the large negative nonminimal coupling.

Let us consider the case where initial values of χ are changed. With the decrease of $\chi(0)$, larger $|\xi|$ is required for the growth of curvature perturbations. When $\chi(0)$ is close to the order of m_{pl} , χ soon catches up inflaton even for not so large values of $|\xi|$, which prevents successful inflation. In this case, inflation typically terminates with small amount of e-foldings before ζ_k begins to grow significantly. We present two-dimensional plots of ξ and $\chi(0)$ which divide the ‘‘allowed’’ and ‘‘ruled out’’ regions in Fig. 7 for the case of $\phi(0)=3m_{\text{pl}}$. The ‘‘allowed’’ regions mean that conditions of $\delta_H(k)<2\times 10^{-5}$ and $N>55$ are both satisfied at the end of inflation for the inflaton mass $m=10^{-6}m_{\text{pl}}$. When $\chi(0)\leq 0.1m_{\text{pl}}$, the separating curve is mainly determined by the condition of $\delta_H(k)<2\times 10^{-5}$ (i.e., ζ_k remains almost constant on super-Hubble scales). For $\chi(0)\geq 0.1m_{\text{pl}}$, the condition of $N>55$ plays the dominant role rather than that of the density perturbation. It is important to note that wide ranges of parameters are ruled out even in the case of $|\xi|\leq 0.1$ unless we take smaller values of $\chi(0)$. When $|\xi|\geq 1$, we find that nonlinear growth of super-Hubble curvature perturbations is inevitable even for very small initial χ as $\chi(0)=10^{-50}m_{\text{pl}}$.

One may consider that allowed regions may become wider if initial values of inflaton are larger. However, this is not generally true. Since larger values of ϕ correspond to the larger potential energy, the inflationary period during which the ϕ field dominates the dynamics of the system is longer. This prolonged inflation leads to the amplification of super-Hubble ζ_k as well as the χ field fluctuation. For example, when $\phi(0)=4m_{\text{pl}}$ and $\chi(0)=10^{-3}m_{\text{pl}}$ with $\xi=-0.02$, ζ_k grows up to $\tilde{\zeta}_k\approx 0.07$, while for the smaller value $\phi(0)=3m_{\text{pl}}$, ζ_k remains almost constant for the same value of ξ . For $\phi(0)=4m_{\text{pl}}$ and $\chi(0)=10^{-3}m_{\text{pl}}$, the allowed values of ξ are found to be $|\xi|\leq 0.01$, whose condition is tighter than in the case of $\phi(0)=3m_{\text{pl}}$. When $\chi(0)$ is close to of order unity, larger values of $\phi(0)$ typically make the e-folding

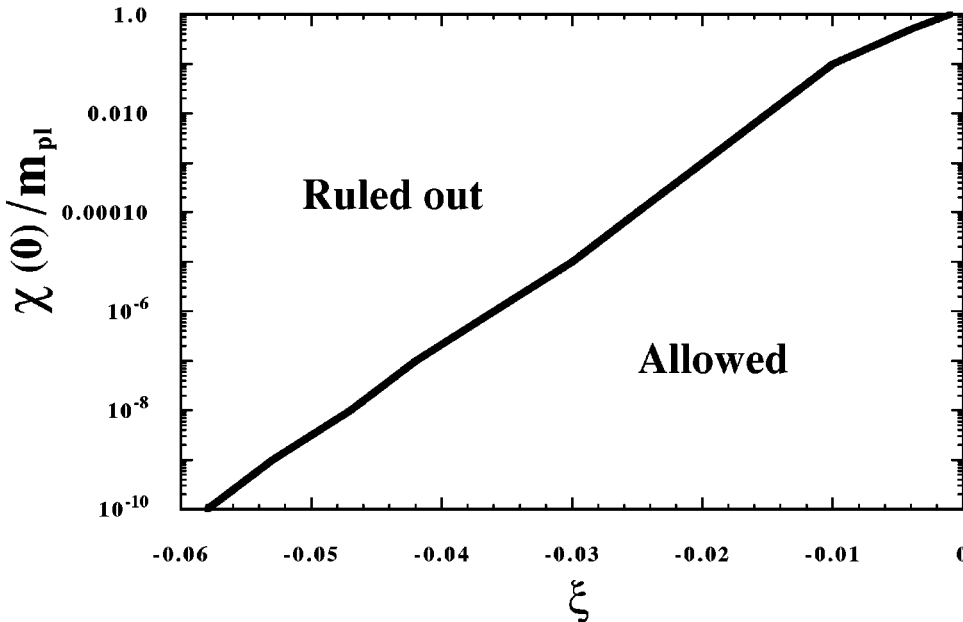


FIG. 7. The parameter regions of the coupling ξ and the initial $\chi(0)$ where the inflationary scenario proceeds in successful way or not, for the case of $\phi(0)=3m_{\text{pl}}$ and $m=10^{-6}m_{\text{pl}}$. We find that large negative coupling prevents successful inflation unless we choose small values of $\chi(0)$.

number larger, which can broaden allowed regions in some cases. For example, when $\phi(0)=4m_{\text{pl}}$ and $\chi(0)=m_{\text{pl}}$, the allowed values are $|\xi|\leq 0.007$, while $|\xi|\leq 0.001$ for $\phi(0)=3m_{\text{pl}}$ and $\chi(0)=m_{\text{pl}}$. However, when $\phi(0)$ takes further large values as $\phi(0)\geq 10m_{\text{pl}}$, ζ_k grows nonadiabatically even for $\xi=-0.001$. This indicates that large $\phi(0)$ does not necessarily result in the successful inflation in the presence of negative nonminimal coupling.

In the reheating phase, parametric amplification of the χ fluctuation is relevant only for the case of $\xi\lesssim -1$ [33,34], because the scalar curvature gradually decreases during inflation. However, such large values of $|\xi|$ generally prevents the successful inflationary scenario as explained above, which will be ruled out even if $\chi(0)$ is initially very small. This is in contrast with the positive ξ case where exponential suppression of long-wave modes in the $\delta\chi_k$ fluctuation do not affect on the dynamics of inflation. In the absence of other interactions, negative nonminimal coupling leads to the strong distortions on CMB in wide ranges of parameters.

C. The $\frac{1}{2}g^2\phi^2\chi^2$ coupling is taken into account

So far, we have not considered the interaction between ϕ and χ fields. Taking into account the simple four-point coupling $\frac{1}{2}g^2\phi^2\chi^2$ provides a way to escape nonadiabatic growth of super-Hubble curvature perturbations. The background equations for the scale factor and inflaton are obtained by changing $V(\phi)=\frac{1}{2}m^2\phi^2$ to $V(\phi,\chi)=\frac{1}{2}m^2\phi^2+\frac{1}{2}g^2\phi^2\chi^2$ in Eqs. (2.7) and (2.9). The homogeneous χ and the $\delta\chi_k$ fluctuation satisfy

$$\ddot{\chi}+3H\dot{\chi}+(g^2\phi^2+\xi R)\chi=0, \quad (3.15)$$

$$\begin{aligned} & \delta\ddot{\chi}_k+3H\delta\dot{\chi}_k+\left(\frac{k^2}{a^2}+g^2\phi^2+\xi R\right)\delta\chi_k \\ & =2(\ddot{\chi}+3H\dot{\chi})\Phi_k+\dot{\chi}(\Phi_k+3\Psi_k)-\xi\chi\delta R_k \\ & \quad -2g^2\phi\chi\delta\phi_k. \end{aligned} \quad (3.16)$$

Then the effective mass of the χ and super-Hubble $\delta\chi_k$ modes is given by

$$m_{\text{eff}}^2=g^2\phi^2+\xi R. \quad (3.17)$$

Neglecting the contribution of the χ field relative to inflaton in Eq. (2.11), the scalar curvature is approximately written as $R\approx 2\kappa^2 m^2\phi^2$ during inflation, which yields the relation

$$m_{\text{eff}}^2\approx\left[g^2+16\pi\xi\left(\frac{m}{m_{\text{pl}}}\right)^2\right]\phi^2. \quad (3.18)$$

When $\xi<0$, the negative effective mass leads to the exponential increase of χ and long-wave $\delta\chi_k$ modes during inflation. This effect is weakened in the presence of the g coupling. Especially for the positive effective mass, which corresponds to

$$g>4\sqrt{\pi}\frac{m}{m_{\text{pl}}}\sqrt{|\xi|}, \quad (3.19)$$

the χ particle production is shut off in inflationary phase.

Neglecting metric perturbations, we have the analytic solution (3.5), where the order of the Hankel functions is given by [32]

$$\nu^2=\frac{9}{4}-12\xi-\frac{g^2\phi^2}{H^2}. \quad (3.20)$$

Since the Hubble expansion rate is approximately written as $H^2\approx 4\pi/3(m/m_{\text{pl}})^2\phi^2$ during inflation, we obtain

$$\nu^2\approx\frac{9}{4}-12\xi-\frac{3g^2}{4\pi}\left(\frac{m_{\text{pl}}}{m}\right)^2. \quad (3.21)$$

Eq. (3.19) corresponds to cancelling the second term in Eq. (3.21) by the g term. When $\nu^2<0$, i.e.,

$$g>4\sqrt{\pi}\frac{m}{m_{\text{pl}}}\sqrt{|\xi|+\frac{3}{16}}, \quad (3.22)$$

χ modes exponentially decrease as $\propto a^{-3/2}$ in the similar way as in the large positive ξ case. When g ranges in the region of $4\sqrt{\pi}(m/m_{\text{pl}})\sqrt{|\xi|}<g<4\sqrt{\pi}(m/m_{\text{pl}})\sqrt{|\xi|+3/16}$, χ decays more slowly as $\propto a^{-(3/2-\nu)}$. However, as long as the condition (3.19) is satisfied and χ is initially small relative to ϕ , the χ field hardly affects the dynamics of inflation, which results in the conservation of the curvature perturbation ζ_k on super-Hubble scales.

Let us consider concrete cases. In Fig. 8 we plot the evolution of a long-wave $\delta\chi_k$ mode for $\xi=-0.05$ and several values of g with initial conditions $\phi(0)=3m_{\text{pl}}$ and $\chi(0)=10^{-3}m_{\text{pl}}$. When $g=0$, the $\delta\chi_k$ fluctuation exhibits exponential increase during inflation, leading to the nonadiabatic growth of super-Hubble curvature perturbations. In the presence of the g coupling, the conditions of Eqs. (3.19) and (3.22) yield $g>1.6\times 10^{-6}$ and $g>3.5\times 10^{-6}$, respectively, for the typical mass scale $m=10^{-6}m_{\text{pl}}$. As is found in Fig. 8, $\delta\chi_k$ decreases very rapidly as $\propto a^{-3/2}$ for $g=5.0\times 10^{-6}$, while its decreasing rate is smaller for $g=2.0\times 10^{-6}$. In both cases, however, large-scale curvature perturbations remain almost constant during inflation and reheating (see the inset of Fig. 8).

In the case of $|\xi|\gg 1$, Eq. (3.22) approximately takes the form of Eq. (3.19), which reads $g\geq 7.1\times 10^{-6}\sqrt{|\xi|}$ for the inflaton mass $m=10^{-6}m_{\text{pl}}$. Even for very large values of $|\xi|$ such as $\xi=-10^4$, the ξ effect can be removed for $g\geq 7.1\times 10^{-4}$. If the coupling between ϕ and χ is greater than of order 10^{-3} , the χ particle production during inflation discussed in Ref. [32] will be irrelevant. When $g\geq 10^{-3}$, since the g effect is typically dominant relative to the negative nonminimal coupling unless $|\xi|$ is unnaturally large, χ particles are created during preheating in the usual manner due

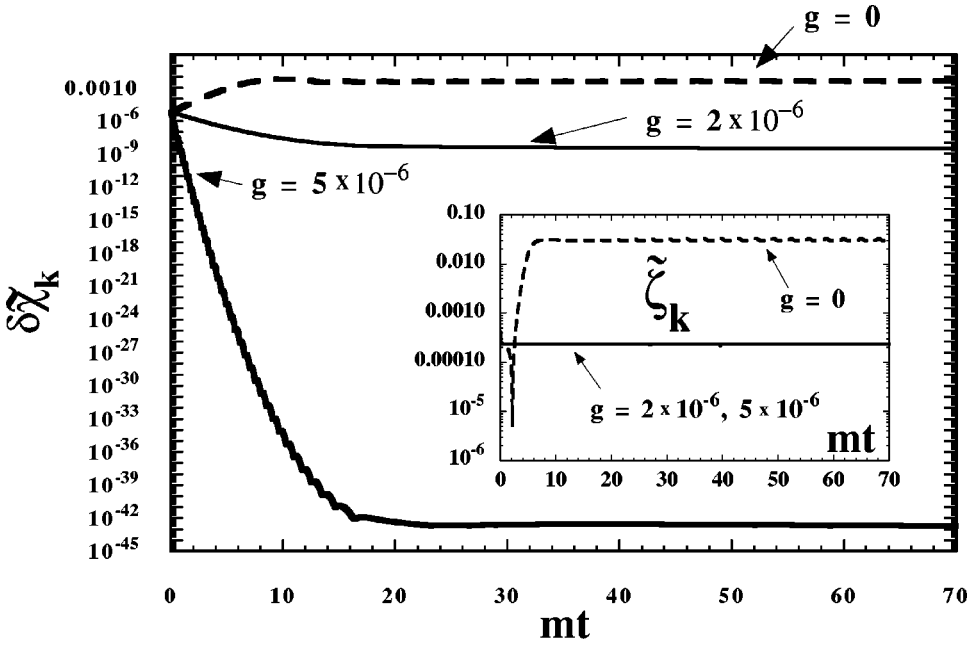


FIG. 8. The evolution of the field fluctuation $\delta\tilde{\chi}_k$ during inflation and reheating for a super-Hubble mode $k=a_0H_0$ in the cases of $g=0$, 2×10^{-6} , and 5×10^{-6} with $\xi=-0.05$, $m=10^{-6}m_{\text{pl}}$, and initial values $\phi(0)=3m_{\text{pl}}$, $\chi(0)=10^{-3}m_{\text{pl}}$. Inset: $\tilde{\zeta}_k$ vs t for $g=0$, 2×10^{-6} , and 5×10^{-6} . For the values of g which satisfy the relation (3.19), i.e. $g>1.6\times 10^{-6}$, the curvature perturbation is conserved in large scales.

to the g resonance [5]. Since long-wave χ modes are exponentially suppressed during inflation for the case of $g \gg 7.1\times 10^{-6}\sqrt{|\xi|}$, the existence of the preheating era does not lead to the amplification of super-Hubble metric perturbations, which provides the standard conservation law of large-scale curvature perturbations.

IV. CONCLUSIONS

We have studied the dynamics and perturbations in the multifield inflation with a nonminimally coupled χ field. When the coupling ξ is positive, χ and long-wave $\delta\chi_k$ fluctuations are exponentially suppressed in de Sitter background. In this case, the existence of the χ field hardly affects the dynamics of inflation, and the ordinary adiabatic scenario in large-scale curvature perturbations is not modified as long as χ is initially small relative to inflaton. Although χ fluctuations grow by parametric resonance during preheating after inflation for large values of ξ ($\gg 1$), this process is not sufficient to enhance super-Hubble metric perturbations, since the inflationary suppression is strong.

In contrast, negative nonminimal coupling can lead to the strong inflationary χ particle production in long-wave modes. This exponential increase of the χ fluctuation makes super-Hubble metric perturbations grow too, which violates the standard conservation property of large-scale curvature perturbations in adiabatic inflation models. We find that even the coupling $|\xi|$ less than unity yields the exponential growth of the χ fluctuation in small k modes, which terminates the inflationary period earlier than in the minimally coupled case. This effect reduces the total amount of inflation (e-foldings), in addition to the nonadiabatic increase of super-Hubble curvature perturbations. Large values of $|\xi|$ greater than unity make the inflationary phase very short, whose amount of inflation is typically insufficient for the success of the inflationary scenario. We have constrained the strength of negative nonminimal coupling by two requirements that the

large-scale curvature perturbation is almost conserved and the number of e-foldings satisfies $N>55$. Since the evolution of the χ fluctuation depends on its initial value at the beginning of inflation, we examined the allowed regions in two-dimensional plots of ξ and $\chi(0)$. With the increase of $|\xi|$, we require smaller values of $\chi(0)$ for the successful inflationary scenario. When $|\xi|\geq 1$, we find that strong enhancement of large-scale curvature perturbations is inevitable even for very small values of $\chi(0)$.

As one escape route from nonadiabatic growth of super-Hubble metric perturbations, we considered the interaction $\frac{1}{2}g^2\phi^2\chi^2$ between ϕ and χ fields. Introducing this coupling makes the effective χ mass heavy, which suppresses the inflationary χ particle production by negative nonminimal coupling. If two couplings satisfy the condition (3.19), the χ fluctuation does not exhibit exponential increase during inflation. This protects super-Hubble curvature perturbations from being amplified, because the system is effectively dominated by inflaton in this case.

Although we have considered the simple massive inflationary model, strong enhancement of long-wave χ fluctuations by negative nonminimal coupling will occur in potential-independent way in de Sitter background. Since the scalar curvature is approximately written as $R\approx 4\kappa^2V(\phi)$ during inflation, super-Hubble curvature perturbations as well as χ fluctuations will also grow nonperturbatively in other models of inflation while the potential energy $V(\phi)$ slowly decreases. In addition to this, exponential increase of cosmological perturbations leads to the production of inflaton particles. In spinodal inflation models [37] where the second derivative of $V(\phi)$ changes sign, the inflaton fluctuation in small k modes exhibits exponential increase when $V''(\phi)<0$ even in the single-field case. It is of interest to study the dynamics and perturbations in this model with a nonminimally coupled χ field, in which amplifications of the inflaton fluctuation may further strengthen super-Hubble

metric perturbations.

There are other issues which we did not address in this paper. Since negative nonminimal coupling works to violate the scale-invariance of the CMB spectrum, we should also consider the spectral index to constrain the strength of ξ by observations. In the single-field case, the spectral tilts are evaluated in generalized Einstein theories in Ref. [30], which may be interesting to extend to the multifield case including the nonminimally coupled χ field. In addition to this, although we did not consider backreaction effects in this paper, detailed studies including second order metric perturbations [38] will be needed toward complete understanding of

the multifield inflation and preheating. These issues are left to future works.

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