Topological inflation in supergravity

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We investigate a topological inflation model in supergravity. By means of numerical simulations, it is confirmed that topological inflation can take place in supergravity. We also show that the condition for successful inflation depends not only on the vacuum-expectation value (VEV) of inflaton field but also on the form of its Kähler potential. In fact, it is found that the required VEV of the inflaton φ can be as small as $\langle\varphi\rangle \simeq 1\times M_G$, where M_G is the gravitational scale.

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I. INTRODUCTION

Superstring theories compactified on $(3+1)$-dimensional space-time have many discrete symmetries in the low-energy effective Lagrangian [1]. A spontaneous breakdown of such discrete symmetries creates topological defects, i.e., domain walls, in the early universe [2]. If the vacuum-expectation value (VEV) of a scalar field φ is larger than the gravitational scale $M_G \simeq 2\times 10^{18}$ GeV, the region inside the wall undergoes inflationary expansion and eventually becomes the present whole universe [3,4]. If the universe is open at the beginning, it expands and the spontaneous breakdown of the symmetries always takes place at some epoch in the early universe. It has been recently argued that the quantum creation of the open universe may take place with appropriate continuation from the Euclidean instanton [5]. Thus, topological inflation is a natural consequence of the dynamics of the system, and it does not require any fine-tuning of initial conditions for the beginning universe. Furthermore, it does not cause the ''graceful exit'' problem and the universe becomes homogeneously radiation dominated after reheating.

A simple and interesting model for topological inflation was proposed in the framework of supergravity [6].[1] However, it was not explicitly shown whether topological inflation really takes place. In this paper, we perform a numerical analysis on the above model and show that topological inflation indeed occurs in a wide range of parameter space. We also show that the condition for successful inflation depends not only on the superpotential, which determines the vacuum expectation value (VEV) of inflaton φ, but also on the form of its Kähler potential. We in fact find that the required VEV can be as small as $\langle\varphi\rangle \simeq 1\times M_G$, which is far below the lower bound of $\langle\varphi\rangle = \eta_{cr} \simeq 1.7 M_G$ derived in Ref. [9].

II. TOPOLOGICAL INFLATION MODEL

We begin with the topological inflation model proposed in Ref. [6], which is based on *R*-invariant supergravity. The gravitational scale M_G is set to be unity below. In this model the superpotential for the inflaton superfield $\phi(x,\theta)$ is given by

$$W = v^2 X(1 - g\phi^2). \tag{1}$$

Here, we have imposed $U(1)_R \times Z_2$ symmetry and omitted higher-order terms for simplicity. Under the $U(1)_R$ we assume

$$X(\theta) \to e^{-2i\alpha} X(\theta e^{i\alpha}), \quad \phi(\theta) \to \phi(\theta e^{i\alpha}). \tag{2}$$

We also assume that the superfield X is even and ϕ is odd under the Z_2. This discrete Z_2 symmetry is an essential ingredient for the topological inflation [3,4]. In the above superpotential (1), we always take v^2 and g to be real constants without loss of generality.

The *R*- and Z_2-invariant Kähler potential is given by

$$K(\phi,X) = |X|^2 + |\phi|^2 + k_1 |X|^2 |\phi|^2 + \frac{k_2}{4}|X|^4 + \frac{k_3}{4}|\phi|^4 + \cdots, \tag{3}$$

where k_1, k_2, and k_3 are constants of order unity.

The potential of a scalar component of the superfields $X(x,\theta)$ and $\phi(x,\theta)$ in supergravity is given by

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[1]Other topological inflation models were studied in the superstring inspired models [7,8].

$$V=e^K\left\{\left(\frac{\partial^2 K}{\partial z_i \partial z_j^*}\right)^{-1} D_{z_i} W D_{z_j^*} W^* - 3|W|^2\right\} \quad (z_i=\phi,X), \tag{4}$$

with

$$D_{z_i} W=\frac{\partial W}{\partial z_i}+\frac{\partial K}{\partial z_i} W. \tag{5}$$

This potential yields an R-invariant vacuum

$$\langle X\rangle=0, \quad \langle\phi\rangle=\frac{1}{\sqrt{g}}\equiv\frac{\eta}{\sqrt{2}}, \tag{6}$$

at which the potential energy vanishes. Here, the scalar components of the superfields are denoted by the same symbols as the corresponding superfields. If η is larger than the critical value $\eta_{\rm cr}$, which will be discussed in the next section, the topological inflation occurs.

For $|X|$ and $|\phi|\ll 1$, we approximately rewrite the potential (4) as

$$V\simeq v^4|1-g\phi^2|^2+v^4(1-k_1)|\phi|^2-k_2 v^4|X|^2. \tag{7}$$

If $k_2\lesssim -1$, X field quickly settles down to the origin and we set $X=0$ in our analysis taking $k_2\lesssim -1$. For $g>0$, we can identify the inflaton field $\varphi(x)/\sqrt{2}$ with the real part of the field $\phi(x)$ since the imaginary part of $\phi(x)$ has a positive mass and the real part has a negative mass. Because the positive mass of the imaginary part is larger than the size of the negative mass, the imaginary part is irrelevant for the inflation dynamics and hence we neglect it. Then, we obtain a potential for the inflaton for $\varphi\ll 1$:

$$V(\varphi)\simeq v^4-\frac{\kappa}{2}v^4\varphi^2, \tag{8}$$

where

$$\kappa\equiv 2g+k_1-1. \tag{9}$$

The slow-roll condition for the inflaton φ is satisfied for $0<\kappa<1$ and $0\lesssim\varphi\lesssim 1$.[2] The Hubble parameter during the inflation is given by $H\simeq v^2/\sqrt{3}$. The scale factor of the universe increases by a factor of e^N when the inflaton φ rolls slowly down the potential from φ_N to 1. The e-fold number N is given by

$$N\simeq -\frac{1}{\kappa}\ln\varphi_N. \tag{10}$$

The amplitude of primordial density fluctuations $\delta\rho/\rho$ due to this inflation is written as

$$\frac{\delta\rho}{\rho}\simeq\frac{1}{5\sqrt{3}\pi}\frac{v^2}{\kappa\varphi_N}\sim 2.0\times 10^{-5}. \tag{11}$$

The normalization is given by the data of anisotropies of the cosmic microwave background radiation (CMB) by the COBE satellite [10]. Since the e-fold number N corresponding to the COBE scale is about 60, which leads to

$$v\simeq 2.3\times 10^{-2}\sqrt{\kappa}e^{-\kappa N/2}|_{N=60}\simeq 1.8\times 10^{-3}-3.6\times 10^{-4}, \tag{12}$$

for $0.02\leqslant\kappa\leqslant 0.1$.

The interesting point on the above density fluctuations is that it results in the tilted spectrum whose spectrum index n_s is given by

$$n_s\simeq 1-2\kappa. \tag{13}$$

We may expect a possible deviation from the Harrison-Zeldvich scale-invariant spectrum $n_s=1$. Observational constraint on n_s is $|n_s-1|<0.2$ [10], which implies $0<\kappa<0.1$.

After inflation ends, the inflaton φ may decay into ordinary particles as discussed in Ref. [6] and the reheating temperature is low enough to avoid overproduction of gravitinos [11] which are thermally produced at the reheating epoch. Recently, nonthermal production at the preheating stage was found to be important in some inflation models [12]. For the present model, nonthermal production of gravitinos at the preheating phase is roughly estimated as

$$\left(\frac{n_{3/2}}{s}\right)_{\text{non-TH}}\sim\frac{m_\varphi^3}{v^4/T_R}\lesssim 10^{-14}\left(\frac{T_R}{10^{10}\ \text{GeV}}\right), \tag{14}$$

where $m_\varphi(\simeq v^2)$, $n_{3/2}$, s, and T_R are the mass of the inflaton, the number density of gravitinos, entropy density, and reheating temperature, respectively. This is much less than the thermal production given by $(n_{3/2}/s)_{\rm TH}\sim 10^{-11}(T_R/10^{10}\ \text{GeV})$ and hence we can neglect the nonthermal production of gravitinos.

III. NUMERICAL SIMULATION

We perform numerical simulations to decide whether topological inflation takes place in supergravity and determine the condition for successful topological inflation. For the purpose, we follow time evolution of the domain wall and investigate whether it inflates or not. Since we consider a planar domain wall whose width is order of the horizon scale, we cannot adopt the Friedmann Robertson-Walker metric. Instead, we assume that the spacetime has a reflection symmetry of the coordinate x perpendicular to the wall and adopt the metric given by

$$ds^2=-dt^2+A^2(t,|x|)dx^2+B^2(t,|x|)(dy^2+dz^2), \tag{15}$$

where A and B correspond to the scale factors in the direction of x and y-z, respectively. If the inflation occurs and the proper width of the wall becomes much larger than the horizon scale, A and B expand as $A\sim B\propto e^{Ht}$ (as shown later)

[2]We can always take φ positive since we have the Z_2 symmetry ($\phi\to-\phi$).

and the universe approaches the de Sitter spacetime. Thus, we examine whether the proper width of the wall becomes much larger than the horizon scale. Once it is realized, the universe expands exponentially [3,4].

We adopt the numerical technique developed in Ref. [9]. The Einstein-Hilbert action is given by

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{R}-\frac{1}{2}(\partial_\mu\varphi)^2-V(\varphi)\right]. \tag{16}$$

Variating the above action with respect to the metric $g_{\mu\nu}$, we obtain the Einstein equations,

$$G_{\mu\nu}\equiv\mathcal{R}_{\mu\nu}-\frac{1}{2}\mathcal{R}=T_{\mu\nu}, \tag{17}$$

where $T_{\mu\nu}$ is the energy-momentum tensor,

$$T_{\mu\nu}=\partial_\mu\varphi\partial_\nu\varphi-g_{\mu\nu}\left[\frac{1}{2}(\partial_\mu\varphi)^2+V(\varphi)\right]. \tag{18}$$

Variation with respect to the scalar field φ gives the equation of motion for the scalar field φ,

$$\Box\varphi=\frac{dV(\varphi)}{d\varphi}. \tag{19}$$

In order to make it easier to follow time evolution of the system, we choose certain combinations of Einstein equations, which read

$$-G_0^0=\mathcal{K}_2^2(2\mathcal{K}-3\mathcal{K}_2^2)-\frac{2B''}{A^2B}-\frac{B'^2}{A^2B^2}+\frac{2A'B'}{A^3B}$$
$$=\frac{\dot\varphi^2}{2}+\frac{\varphi'^2}{2A^2}+V(\varphi), \tag{20}$$

$$\frac{1}{2}G_{01}=\dot{\mathcal{K}}_2^{2\prime}+\frac{B'}{B}(3\mathcal{K}_2^2-\mathcal{K})$$
$$=\frac{1}{2}\dot\varphi\varphi', \tag{21}$$

$$\frac{1}{2}(G_1^1+G_2^2+G_3^3-G_0^0)$$
$$=\dot{\mathcal{K}}-(\mathcal{K}_1^1)^2-2(\mathcal{K}_2^2)^2$$
$$=\dot\varphi^2-V(\varphi), \tag{22}$$

$$-\mathcal{R}_2^2-\frac{1}{2}G_0^0=\dot{\mathcal{K}}_2^2+\frac{B'^2}{2A^2B^2}-\frac{3}{2}(\mathcal{K}_2^2)^2$$
$$=\frac{\dot\varphi^2}{4}+\frac{\varphi'^2}{4A^2}-\frac{V(\varphi)}{2}, \tag{23}$$

where an overdot denotes the time derivative and a dash the spatial derivative. $\mathcal{K}_{ij}$ are the extrinsic curvature tensors of constant time hypersurface, given by

$$\mathcal{K}_1^1=-\frac{\dot A}{A},\quad \mathcal{K}_2^2=\mathcal{K}_3^3=-\frac{\dot B}{B}, \tag{24}$$

and $\mathcal{K}$ denotes its trace $\mathcal{K}\equiv\mathcal{K}_i^i$. The equation of motion for the scalar field φ becomes

$$\ddot\varphi-\mathcal{K}\dot\varphi-\frac{\varphi''}{A^2}-\left(-\frac{A'}{A}+\frac{2B'}{B}\right)\frac{\varphi'}{A^2}+\frac{dV(\varphi)}{d\varphi}=0. \tag{25}$$

We set the initial condition for numerical simulations. First, we consider an initial configuration of a domain wall. For the convenience of numerical calculations, we take only the region $|x|/\delta\leqslant 2^3$ where $\delta(=\eta/\sqrt{2}v^2)$ is the width of the domain wall and η is the VEV of φ. We impose the free boundary condition, that is, $\varphi'=A'=B'=0$ at the boundaries $x/\delta=-2$ and $x/\delta=2$. Then, we adopt the following initial configuration for the domain wall so that the gradient of the field disappears at the boundaries,

$$\varphi(t=0,x)=\begin{cases}\eta\left[\dfrac{x}{\delta}-\dfrac{5}{4}\left(\dfrac{8}{15}\dfrac{x}{\delta}\right)^3+\dfrac{3}{8}\left(\dfrac{8}{15}\dfrac{x}{\delta}\right)^5\right] & \left(0\leqslant\dfrac{x}{\delta}\leqslant\dfrac{15}{8}\right),\\ \eta & \left(\dfrac{15}{8}\leqslant\dfrac{x}{\delta}\leqslant 2\right),\end{cases} \tag{26}$$

with $\varphi(t=0,-x)=-\varphi(t=0,\ x)$ for $-2\leqslant x/\delta\leqslant 0$. This is a deformed version of the static domain wall solution in a flat spacetime, $\varphi_{\rm flat}=\eta\tanh(x/\delta)$. The function $\varphi(t=0,x)$ is decided so as to satisfy the following three conditions: (1) it is a fifth-order odd polynomial function of x, (2) the first term coincides with that of the expansion of $\varphi_{\rm flat}$; (3) it is smooth at $x/\delta=15/8$, that is, $\varphi'=\varphi''=0$ at $x/\delta=15/8$. Also, $\dot\varphi$ is set to be 0.

Next, on the initial hypersurface, we determine $A,B,\mathcal{K}_2^2,\mathcal{K}$ so as to satisfy the Hamiltonian constraint (20) and the momentum constraint (21). We have freedom for the initial hypersurface to have homogeneous and isotropic curvature, which automatically satisfies the momentum constraint (21). This choice leads to

$$\frac{\mathcal{K}}{3}=\mathcal{K}_1^1=\mathcal{K}_2^2=\text{``negative'' const}, \tag{27}$$

where ''negative'' implies that the universe is in an expanding phase. Furthermore, we can take the conformally flat spatial gauge, $A=B$, on the initial hypersurface and set $A=B=1$ at $x=0$. Finally, we determine the negative value of

[3]We have confirmed that the results do not change even if we take wider ranges of the direction x (e.g., $|x|/\delta=3$).

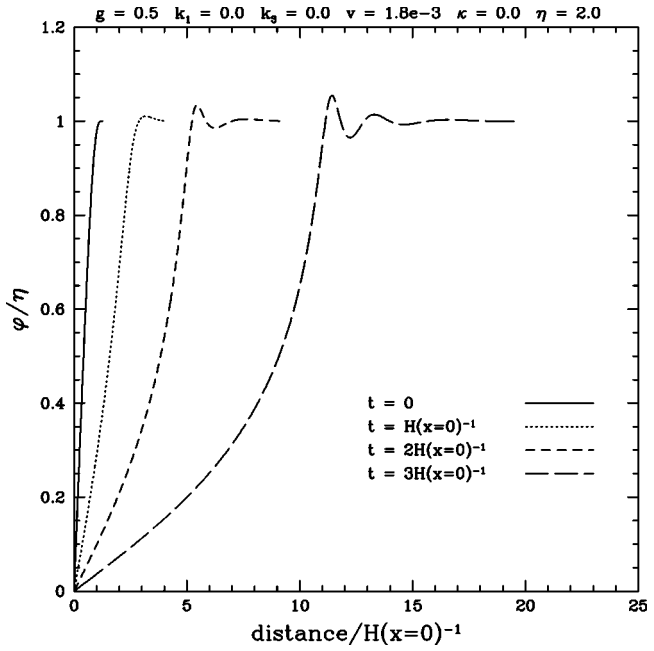

FIG. 1. Time evolution of the domain wall in the case of $k_1=0$ and $g=0.5$ ($\eta=2.0$). The vertical axis represents the value of the scalar field. The horizontal axis represents the proper distance from the domain wall core. Note that the proper width of the wall becomes much larger than the Horizon scale. As time elapses, the domain wall expands and topological inflation takes place. Here we set $v=1.8\times10^{-3}$. But the result does not depend on the energy scale v as shown later.

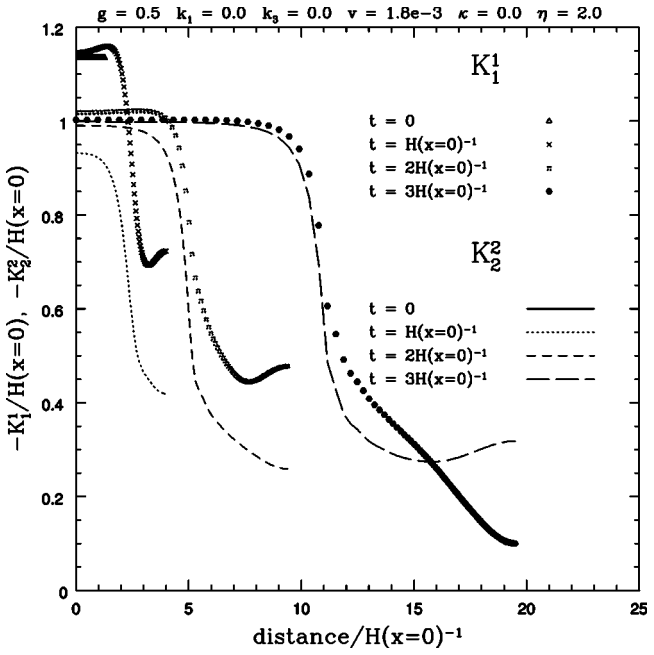

FIG. 2. The expansion rates of the scale factors, A and B, that is, $-\mathcal{K}_1^1=\dot{A}/A$ and $-\mathcal{K}_2^2=\dot{B}/B$ are shown for the case of Fig. 1. As the universe expands enough, $-\mathcal{K}_1^1$ and $-\mathcal{K}_2^2$ approach the same value, the Hubble parameter $H(x=0)$, inside the wall.

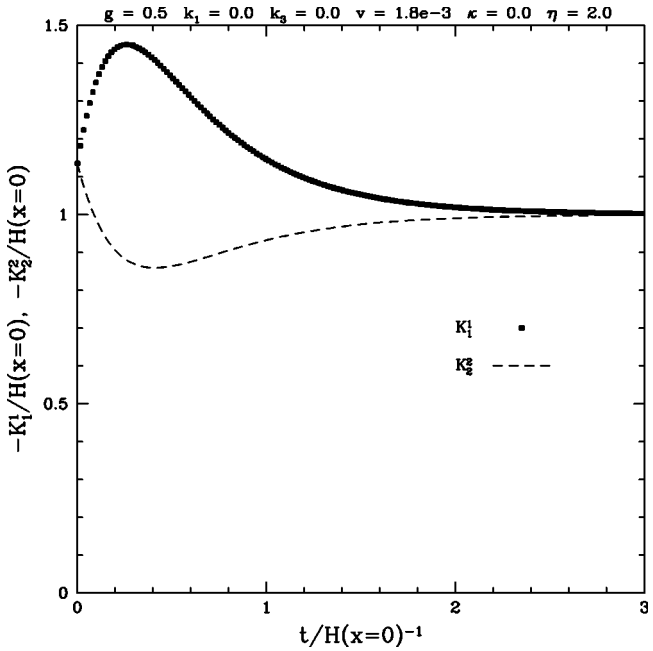

FIG. 3. Time evolution of $-\mathcal{K}_1^1$ and $-\mathcal{K}_2^2$ at the origin $x=0$ is shown for the case of Fig. 1.

$\mathcal{K}$. Since we adopt the reflection symmetry of the coordinate x and the free boundary condition, the condition $A'=B'=0$ at $x=0$ and $x=\pm2\delta$ must be satisfied. $\mathcal{K}$ is determined so that the Hamiltonian constraint (20) satisfies the above conditions.

Now the initial settings are completed and hence we have only to follow the time evolution of five variables, A, B, $\mathcal{K}$, $\mathcal{K}_2^2$, and φ. Note that we have introduced five variables, A, B, $\mathcal{K}$, $\mathcal{K}_2^2$, and φ though only three variables

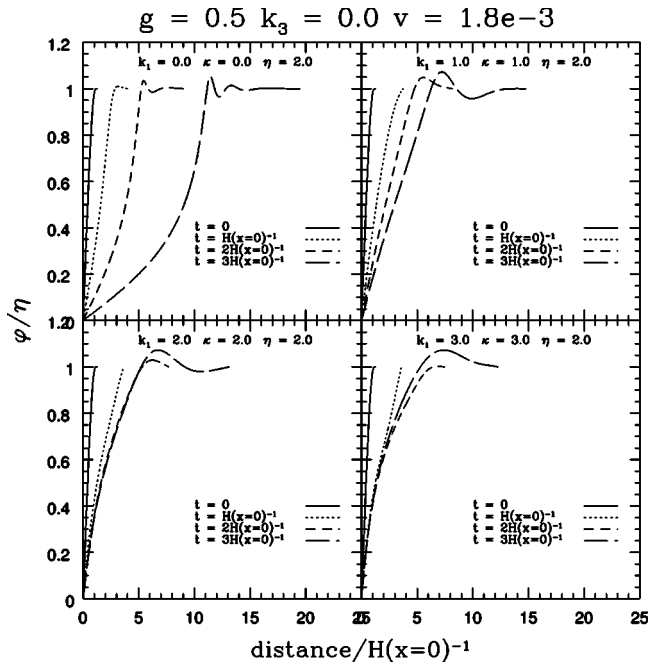

FIG. 4. Time evolution of the domain wall in the cases of different κ for the fixed VEV, $\eta=2.0$ ($g=0.5$), with $v=1.8\times10^{-3}$.

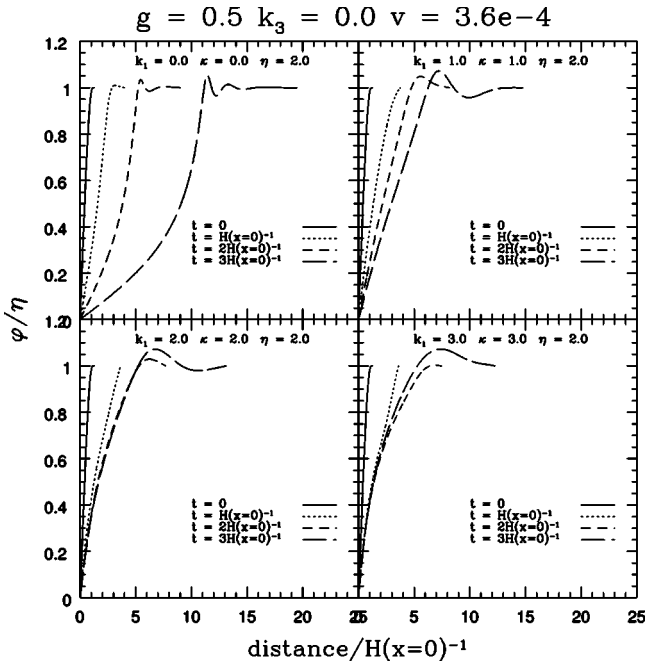

FIG. 5. The results with $v=3.6\times10^{-4}$. The energy scale v is different from that in Fig. 4.

are independent. This is partly because the second order differential equations have been reduced to the first order differential equations. Moreover, as for time evolution of $\mathcal{K}_2^2$, we use Eq. (23) only at $x=0$ and acquire the value at $x\neq0$ by integrating Eq. (21) in the direction of x in order to avoid numerical instability.

When inflation takes place, A and B grow exponentially so that the proper distance from the domain wall core ($x=0$) also increases exponentially. In order to see whether this happens or not, we follow time evolution of the width of the wall for a given potential $V(\varphi)$.

To fix the potential $V(\varphi)$, we first consider the Kähler potential with only terms up to the fourth order,

$$K(\phi,X)=|X|^2+|\phi|^2+k_1|X|^2|\phi|^2+\frac{k_2}{4}|X|^4+\frac{k_3}{4}|\phi|^4. \tag{28}$$

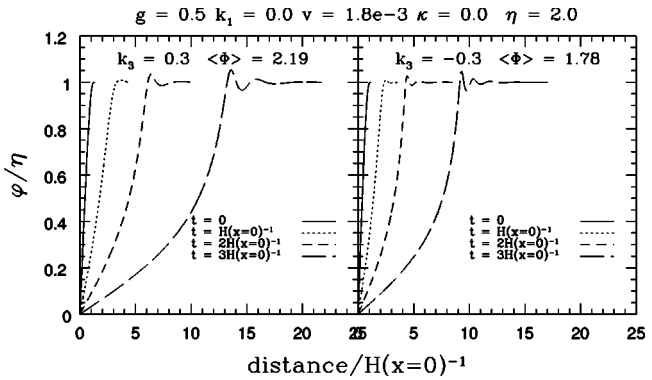

FIG. 6. The results for the same parameters in Fig. 1 except $k_3=\pm0.3$.

Then, the Lagrangian density is given by

$$\begin{aligned}\mathcal{L}(\phi)&=\mathcal{L}_{kin}(\phi)-V(\phi)\\&=-(1+k_3|\phi|^2)\partial_\mu\phi\partial^\mu\phi^*\\&\quad-v^4|1-g\phi^2|^2\frac{\exp\left(|\phi|^2+\frac{k_3}{4}|\phi^4|\right)}{1+k_1|\phi|^2}.\end{aligned} \tag{29}$$

Here we have set $X=0$. Identifying the inflaton field $\varphi(x)/\sqrt{2}$ with the real part of the field $\phi(x)$, the Lagrangian density becomes

$$\begin{aligned}\mathcal{L}(\varphi)&=\mathcal{L}_{kin}(\varphi)-V(\varphi)\\&=-\frac{1}{2}\left(1+\frac{1}{2}k_3\varphi^2\right)(\partial_\mu\varphi)^2\\&\quad-v^4\left(1-\frac{g}{2}\varphi^2\right)^2\frac{\exp\left(\frac{1}{2}\varphi^2+\frac{k_3}{16}\varphi^4\right)}{1+\frac{k_1}{2}\varphi^2},\end{aligned} \tag{30}$$

with the VEV $\eta=\sqrt{2/g}$. In the present model we have four free parameters k_1, k_2, k_3 and g. However, $k_2(\lesssim-1)$ only works as a stabilizer of the X field as explained before and it is not important for the dynamics of topological inflation itself. Once the X field is stabilized at $X=0$, the potential $V(\varphi)$ does not depend on k_2. k_3 is almost irrelevant for the dynamics of ϕ field and only changes its VEV due to the redefinition of φ with a canonical kinetic term. Then, we set $k_3=0$ first and later consider the case of nonzero k_3. Thus, we have only two relevant parameters, k_1 and g. The potential $V(\varphi)$ has a pole at $\varphi=\sqrt{2/|k_1|}$ for $k_1<0$. But, we are

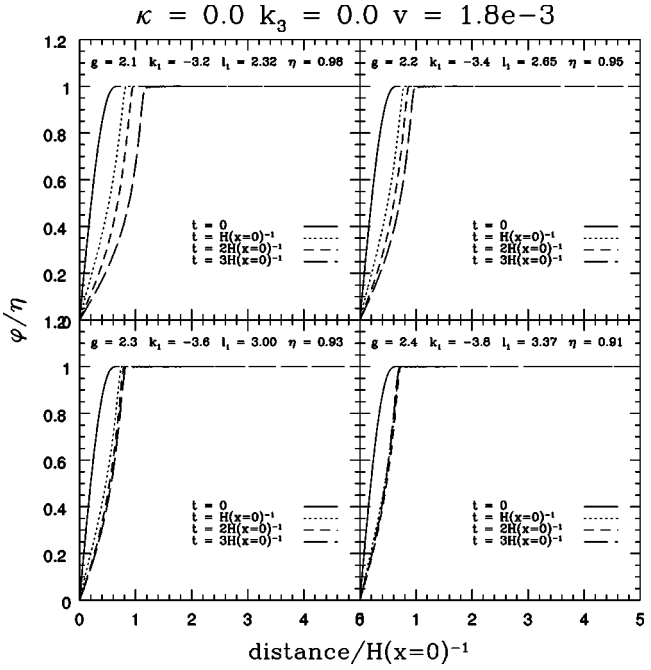

FIG. 7. The results with $\eta\sim\eta_{cr}$ for $\kappa=0.0$. They show $\eta_{cr}\simeq0.95$ for $\kappa=0.0$.

only interested in the dynamics of φ up to the VEV η so that there is no problem if $|k_1|<g$ for $k_1<0$.

We introduce dimensionless quantities, $\bar{\varphi}=\varphi/M_G$, $\bar{x}=xH(x=0)$, $\bar{t}=tH(x=0)$, $\bar{\mathcal{K}}_{ij}=\mathcal{K}_{ij}/H(x=0)$, and $\bar{\delta}=\delta H(x=0)=1/(\sqrt{3g})$ where $H(x=0)=v^2/\sqrt{3}$. As the first step, we consider the simplest case of $k_1=0$ and $g=0.5$ (i.e., $\eta=2.0$), which leads to the spectral index $n=1$. Time evolution of the domain wall is depicted in Fig. 1. The vertical axis represents the value of the scalar field. The horizontal axis represents the proper distance from the domain wall core. As time elapses, the domain wall (the region for $\varphi\lesssim 0.8$) expands and topological inflation really takes place. As shown in Figs. 2 and 3, once the domain wall expands enough, the scale factors A and B increase at the same rate, that is, $-\mathcal{K}_1^1\sim-\mathcal{K}_2^2\sim H(x=0)$ inside the wall. Thus, the universe expands exponentially and approaches the de Sitter spacetime.

We consider the dependence on κ for the VEV fixed, $\eta=2.0$ $(g=0.5)$. The result is depicted in Fig. 4. For large κ, the domain wall cannot inflate enough. κ controls the mass scale of the scalar field φ near the origin. As κ becomes larger, the scalar field φ rolls down faster so that only the small region of the original domain wall earns the vacuum energy and cannot overcome the gradient energy.

The dependence on v is studied, which determines the energy scale of the domain wall. The result with the same parameters in Fig. 4 except for $v=3.6\times10^{-4}$ is shown in Fig. 5. The results are quite the same and have no dependence on v. This can be interpreted as follows: First, the dependence on v only appears through the Hubble parameter during the inflation given by $H\simeq v^2/\sqrt{3}$. But, since the width of the domain wall $\delta\sim1/(\sqrt{3g}H)$, the rough criterion $\delta>H^{-1}$ becomes independent of H, that is, v. Also, rewriting the potential as $V(\varphi)\sim\lambda(\varphi^2-\eta^2)$ with $\lambda=(v/\eta)^4$, the independence of v is equivalent to that of λ. The independence of v is also found in Ref. [9].

We now consider the cases of the nonzero k_3. Since the kinetic term $\mathcal{L}_{kin}$ is not canonical in these cases, we define the scalar field Φ with the canonical kinetic term as

$$\Phi=\int_0^\varphi d\varphi\sqrt{1+\frac{1}{2}k_3\varphi^2}=\begin{cases}\dfrac{1}{2}\varphi\sqrt{1+\dfrac{1}{2}k_3\varphi^2}+\dfrac{\operatorname{arcsinh}\left(\sqrt{\dfrac{k_3}{2}}\varphi\right)}{\sqrt{2k_3}} & (k_3\geqslant 0),\\[2ex] \dfrac{1}{2}\varphi\sqrt{1-\dfrac{1}{2}|k_3|\varphi^2}+\dfrac{\arcsin\left(\sqrt{\dfrac{|k_3|}{2}}\varphi\right)}{\sqrt{2|k_3|}} & (k_3<0).\end{cases}\tag{31}$$

The results for the same parameters in Fig. 1 except for $k_3=\pm0.3$ are depicted in Fig. 6. The positive k_3 encourages the occurrence of topological inflation while the negative k_3 discourages. This is because the VEV of Φ is larger than η for the positive k_3 while smaller for the negative k_3.

Finally, we discuss the criterion for the VEV of φ for successful topological inflation within $0\leqslant\kappa\lesssim0.1$. In previous analyses, we search for only the parameter region satisfying $|k_1|<g$ for negative k_1 because of the appearance of a pole of the potential. The condition $|k_1|<g$ leads to $g<1+\kappa$ so that $\eta>1.41(1.35)$ for $\kappa=0.0(0.1)$, where we have confirmed the occurrence of topological inflation. In order to obtain the lower limit of η, we add to the Kähler potential the sixth order terms,

$$\Delta K=l_1|X|^2|\phi|^4+l_2|X|^4|\phi|^2+\frac{l_3}{9}|X|^6+\frac{l_4}{9}|\phi|^6,\tag{32}$$

where there are four parameters but only l_1 is relevant. If we take l_1 satisfying $l_1>g(g-1)$, the pole smaller than the VEV does not appear in the whole k_1-g parameter space. The result for smaller η with $\kappa=0.0$ is depicted in Fig. 7. We find the critical value of the breaking scale, $\eta_{cr}\simeq0.95$ (1.00) for $\kappa=0.0$ (0.1), which corresponds to $\langle\phi\rangle\simeq1/\sqrt{2}$.

IV. CONCLUSIONS AND DISCUSSIONS

We have studied a topological inflation in supergravity. First, we have shown that topological inflation really takes place in supergravity. Also, the criterion of successful topological inflation depends not only on the breaking scale of the discrete symmetry but also on the mass of the inflaton near the origin. This is because the inflaton rolls down rapidly from the origin if its mass is large. For a very flat case favored by the observation of the spectral index, $n_s\simeq1-0.8$ (i.e., $0<\kappa<0.1$), we have found that the critical breaking scale η_{cr} becomes as small as M_G, which is smaller than the critical value, $\eta_{cr}\cong1.7M_G$ observed in Ref. [9]. Finally we have discussed the primordial spectrum produced by the topological inflation. In general, the topological inflation predicts the tilted spectrum $n_s<1$ depending on κ.[4]

The present topological inflation model is free from the thermal and nonthermal overproduction of gravitinos since

[4]It is possible to produce more exotic spectrum including blue one. This is due to the exponential blow of the potential, which is significant for the region $\varphi\gtrsim M_G$ and makes the potential more complex than the simple double-well potential.

the reheating temperature can be as low as 10^8 GeV. Furthermore, as pointed out in Ref. [13], this model is consistent with a leptogenesis scenario in which heavy Majorana neutrinos are produced in the inflaton decay and successive decays of the Majorana neutrinos result in lepton asymmetry enough to explain the observed baryon asymmetry in the present universe.

ACKNOWLEDGMENTS

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