AdS spacetime in warped spacetimes

M. Cvetič and H. Lü
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104

C. N. Pope
Center for Theoretical Physics, Texas A&M University, College Station, Texas 77843

J. F. Vázquez-Poritz
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 12 June 2000; published 28 November 2000)

We obtain a large class of AdS spacetimes warped with certain internal spaces in 11-dimensional and type IIA or IIB supergravities. The warp factors depend only on the internal coordinates. These solutions arise as the near-horizon geometries of more general semilocalized multi-intersections of p-branes. We achieve this by noting that any sphere (or AdS spacetime) of dimension greater than 3 can be viewed as a foliation involving S^3 (or AdS_3). Then the S^3 (or AdS_3) can be replaced by a three-dimensional lens space (or a BTZ black hole), which arises naturally from the introduction of a NUT (or a pp wave) to the M-branes or the D3-brane. We then obtain multi-intersections by performing a Kaluza-Klein reduction or Hopf T-duality transformation on the fiber coordinate of the lens space (or the BTZ black hole). These geometries provide further possible examples of the AdS/CFT correspondence and of consistent embeddings of lower-dimensional gauged supergravities in $D=11$ or $D=10$.

PACS number(s): 11.25.Mj, 04.65.+e, 11.10.Kk

I. INTRODUCTION

Anti–de Sitter (AdS) spacetimes naturally arise as the near-horizon geometries of non-dilatonic p-branes in supergravity theories. The metric for such a solution is usually the direct sum of AdS spacetime and an internal sphere. These geometries are of particular interest because of the conjecture that supergravity on such a background is dual to a conformal field theory on the boundary of the AdS spacetime [1–3]. Examples include all the anti–de Sitter spacetimes AdS_d with $2\leqslant d\leqslant 7$, with the exception of $d=6$. The origin of AdS_6 is a little more involved, and it was first suggested in [4] that it was related to the ten-dimensional massive type IIA theory. Recently, it was shown that the massive type IIA theory admits a warped-product solution of AdS_6 with S^4 [5], which turns out to be the near-horizon geometry of a semilocalized D4- or D8-brane intersection [6]. It is important that the warp factors depend only on the internal S^4 coordinates, since this implies that the reduced theory in $D=6$ has AdS spacetime as its vacuum solution. The consistent embedding of $D=6$, $N=1$ gauged supergravity in massive type IIA supergravity was obtained in [7]. Ellipsoidal distributions of the D4 or D8 system were also obtained, giving rise to AdS domain walls in $D=6$, supported by a scalar potential involving 3 scalars [8].

In fact, configurations with AdS spacetime in a warped spacetime are not rare occurrences. In [9], a semi-localized M5 or M5 system [6] was studied, and it was shown that the near-horizon geometry turns out to be a warped product of AdS_5 with an internal 6-space. This makes it possible to study AdS_5/four-dimensional conformal field theory (CFT_4) from the point of view of M theory. In this paper, we shall consider AdS with a warped spacetime in a more general context and obtain such geometries for all the AdS_d, as the near-horizon limits of semi-localized multiple intersections in both type IIA and type IIB theories.

The possibility of this construction is based on the following observations. As is well known, a non-dilatonic p-brane has the near-horizon geometry $AdS_d\times S^n$. The internal n-sphere can be described geometrically as a foliation of $S^p\times S^q$ surfaces with $n=p+q+1$ (see Appendix A), and so, in particular, if $n\geqslant 4$, the n-sphere can be viewed in terms of a foliation with $S^3\times S^{n-4}$ surfaces, viz.

$$d\Omega_n^2=d\alpha^2+\cos^2\alpha d\Omega_3^2+\sin^2\alpha d\Omega_{n-4}^2\,. \qquad (1)$$

In Appendix B, we show that when a non-dilatonic p-brane with an n-sphere in the transverse space intersects with a Kaluza-Klein monopole [a Taub-NUT (Newman-Unti-Tamburino) with charge $Q_{\rm N}$] in a semi-localized manner, the net result turns out to be effectively a coordinate transformation of a solution with a distribution of pure p-branes with no NUT present. The round S^3 in Eq. (1) becomes the cyclic lens space $S^3/Z_{Q_{\rm N}}$ with the metric

$$d\bar{\Omega}_3^2=\frac{1}{4}d\Omega_2^2+\frac{1}{4}\left(\frac{dy}{Q_{\rm N}}+\omega\right)^2, \qquad (2)$$

where $d\omega=\Omega_2$ is the volume form of the unit 2-sphere. This metric retains the same local structure as the standard round 3-sphere, and it has the same curvature tensor, but the y coordinate on the $U(1)$ fibers is now identified with a period which is $1/Q_{\rm N}$ of the period for S^3 itself. We can now perform a dimensional reduction, or a T-duality transformation, on the fiber coordinate y, and thereby obtain AdS spacetime in a warped spacetime. The warp factor depends only on the internal ‘‘latitude’’ coordinate α, but is independent of the lower-dimensional spacetime coordinates. In fact, the

M5/M5 system with AdS_5 found in [9] can be obtained in precisely such a manner from the D3-brane by using type IIA or IIB T duality. Note that an isotropic p-brane can be viewed as carrying a single unit of NUT charge. Although this semi-localized way of introducing a Taub-NUT seems trivial, in that it amounts to a coordinate transformation, performing a Kaluza-Klein reduction on the fiber coordinate does create a non-trivial intersecting component, since the Kaluza-Klein 2-form field strength now carries a non-trivial flux. This fact was used in [10] to construct multi-charge p-branes starting from flat spacetime.

An analogous procedure can instead be applied to the anti–de Sitter spacetime, rather than the sphere, in the near-horizon limit $\mathrm{AdS}_d \times \mathrm{S}^n$ of a non-dilatonic p-brane. As discussed in Appendix A, AdS_d can be described in terms of a foliation of $\mathrm{AdS}_p \times \mathrm{S}^q$ surfaces with $d=p+q+1$ and so, in particular, for $d \geqslant 4$ it can be expressed as a foliation of $\mathrm{AdS}_3 \times \mathrm{S}^{d-3}$:

$$ds^2_{\mathrm{AdS}_d} = d\rho^2 + \cosh^2\rho\, ds^2_{\mathrm{AdS}_3} + \sinh^2\rho\, d\Omega^2_{d-4}. \tag{3}$$

In the presence of a pp wave that is semi-localized on the world volume of the p-brane, the AdS_3 turns out to have the form of a $U(1)$ bundle over AdS_2 [11],

$$ds^2_{\mathrm{AdS}_3} = -r^2 W^{-1} dt^2 + \frac{dr^2}{r^2} + r^2 W[dy + (W^{-1}-1)dt]^2, \tag{4}$$

where $W = 1 + Q_w/r^2$, and Q_w is the momentum carried by the pp wave. This is precisely the structure of the extremal Bañados-Teitelboim-Zanelli (BTZ) black hole [12]. We can now perform a Kaluza-Klein reduction, or T-duality transformation, on the fiber coordinate y. In the near-horizon limit where the ''1'' in W can be dropped, we obtain AdS_2 in a warped spacetime with a warp factor that depends only on the foliation coordinate, ρ.

A T-duality transformation on such a fiber coordinate of AdS_3 or S^3 has been called the Hopf T duality [13]. It has the effect of (un)twisting the AdS_3 or S^3. The effect of this procedure on the six-dimensional dyonic string, whose near-horizon limit is $\mathrm{AdS}_3 \times \mathrm{S}^3$, was extensively studied in [11]. In this paper, we apply the same technique to AdS_3 or S^3 geometries that are themselves factors in the foliation surfaces of certain larger-dimensional AdS spacetimes or spheres.

In Sec. II, we consider the semi-localized D3-NUT system and show that the effect of turning on the NUT charge Q_{N} in the intersection is merely to convert the internal 5-sphere, viewed as a foliation of $\mathrm{S}^1 \times \mathrm{S}^3$, into a foliation of $\mathrm{S}^1 \times (\mathrm{S}^3/Z_{Q_{\mathrm{N}}})$, where $\mathrm{S}^3/Z_{Q_{\mathrm{N}}}$ is the cyclic lens space of order Q_{N}. We can then perform a T-duality transformation on the Hopf fiber coordinate of the lens space and thereby obtain an AdS_5 in a warped spacetime as a solution in M theory, as the near-horizon geometry of a semi-localized M5 or M5 system.

In Sec. III, we consider a semi-localized D3 or pp-wave system, for which the AdS_5 becomes a foliation of a circle with the extremal BTZ black hole, which is locally AdS_3 and can be viewed as a $U(1)$ bundle over AdS_2. We then perform a Hopf T-duality transformation on the fiber coordinate to obtain a solution with AdS_2 in a warped spacetime in M theory, as the near-horizon geometry of a semi-localized M2 or M2 system.

In Secs. IV and V, we apply the same analysis to the M2-NUT and M2–pp-wave systems, and the M5-NUT and M5–pp-wave systems, respectively; we obtain various configurations of AdS spacetime in warped spacetimes by performing Kaluza-Klein reductions and Hopf T-duality transformations on the fiber coordinates.

In Sec. VI, we consider the D4-D8 system, which has the near-horizon geometry of a warped product of AdS_6 and S^4. We perform a Hopf T-duality transformation on the fiber coordinate of the foliating lens space of S^4, and thereby embed AdS_6 in a warped spacetime solution of type IIB theory.

We end with concluding remarks in Sec. VII. In Appendix A, we show how arbitrary-dimensional spheres and AdS spacetimes can be described in terms of foliations. In Appendix B, we show that the solution describing the semi-local intersection of a non-dilatonic p-brane with a Kaluza-Klein monopole (Taub-NUT) is equivalent, after a coordinate transformation, to a solution purely composed of distributed p-branes, with no NUT.

II. D3-NUT SYSTEMS AND AdS_5 IN M THEORY FROM T DUALITY

AdS_5 spacetime arises naturally from type IIB theory as the near-horizon geometry of the D3-brane. Its origin in M theory is more obscure. One way to embed the AdS_5 in M theory is to note that S^5 can be viewed as a $U(1)$ bundle over CP^2, and hence we can perform a Hopf T-duality transformation on the $U(1)$ fiber coordinate. The resulting M-theory solution becomes $\mathrm{AdS}_5 \times CP^2 \times T^2$ [13]. However, this solution is not supersymmetric at the level of supergravity, since CP^2 does not admit a spin structure. *Charged* spinors exist but, after making the T-duality transformation, the relevant electromagnetic field is described by the winding-mode vector and it is only in the full string theory that states charged with respect to this field arise. It was therefore argued in [13] that the lack of supersymmetry (and indeed of any fermions at all) is a supergravity artifact and that, when the full string theory is considered, the geometry is supersymmetric. Such a phenomenon was referred as ''supersymmetry without supersymmetry'' in [14].

Recently, AdS_5 in warped 11-dimensional spacetime was constructed in [9]. It arises as the near-horizon limit of the semi-localized M5-M5 intersecting system. After performing a T-duality transformation, the warped spacetime of the near-horizon limit becomes $\mathrm{AdS}_5 \times (S^5/Z_{Q_{\mathrm{N}}})$. In this section, we shall review this example in detail and show that the M5-M5 system originates from a semi-localized D3-NUT intersection in type IIB supergravity.

A. D3-NUT system

Any p-brane with a transverse space of sufficiently high dimension can intersect with a NUT. The D3-NUT solution of type IIB supergravity is given by

TABLE I. The D3-NUT brane intersection. Here $\times$ and $-$ denote the world volume and transverse space coordinates respectively, and * denotes the fiber coordinate of the Taub-NUT.

	t	w_1	w_2	w_3	x_1	x_2	z_1	z_2	z_3	y	
D3	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$-$	$-$	$-$	H
NUT	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	*	K

$$
\begin{aligned}
ds^2_{10\mathrm{IIB}} &= H^{-1/2}(-dt^2+dw_1^2+\cdots+dw_3^2)\\
&\quad +H^{1/2}[dx_1^2+dx_2^2K(dz^2+z^2d\Omega_2^2)\\
&\quad +K^{-1}(dy+Q_{\mathrm{N}}\omega)^2],\\
F_5 &= dt\wedge d^3w\wedge dH^{-1}+*(dt\wedge d^3w\wedge dH^{-1}),
\end{aligned}
\tag{5}
$$

where $z^2=z_1^2+z_2^2+z_3^2$, and ω is a 1-form satisfying $d\omega=\Omega_2$. The solution can be best illustrated by Table I.

The function K is associated with the NUT component of the intersection; it is a harmonic function in the overall transverse Euclidean 3-space represented as coordinates by z_i. The function H is associated with the D3-brane component. It satisfies the equation

$$
\partial_z^2 H + K\partial_{\vec{x}}^2 H = 0. \tag{6}
$$

Equations of this type were also studied in [15–25]. In the absence of NUT charge, i.e., $K=1$, the function H is harmonic in the the transverse 6-space of the D3-brane. When the NUT charge Q_{N} is non-zero, K is instead given by

$$
K=1+\frac{Q_{\mathrm{N}}}{z}, \tag{7}
$$

and the function H cannot be solved analytically, but only in terms of a Fourier expansion in $\vec{x}$ coordinates. The usual way to solve for the solution is to consider the zero modes in the Fourier expansion. In other words, one assumes that H is independent of $\vec{x}$. The consequence of this assumption is that the resulting metric no longer has an AdS structure in its near-horizon region. In [6], it was observed that an explicit closed-form solution for H can be obtained in the case where the "1" in function K is dropped. This solution is given by [6]

$$
K=\frac{Q_{\mathrm{N}}}{z}, \quad H=1+\sum_k \frac{Q_k}{(|\vec{x}-\vec{x}_{0k}|^2+4Q_{\mathrm{N}}z)^2}. \tag{8}
$$

In this paper, we shall consider the case where the D3-brane is located at the origin of the $\vec{x}$ space and so we have

$$
H=1+\frac{Q}{(x^2+4Q_{\mathrm{N}}z)^2}, \tag{9}
$$

where $x^2=x^ix^i$. Thus, the D3-brane is also localized in the space of the $\vec{x}$ as well. Let us now make a coordinate transformation

$$
\begin{aligned}
&x_1=r\cos\alpha\cos\theta, \quad x_2=r\cos\alpha\sin\theta,\\
&z=\tfrac{1}{4}Q_{\mathrm{N}}{}^{-1}r^2\sin^2\alpha.
\end{aligned}
\tag{10}
$$

In terms of the new coordinates, the metric for the solution becomes

$$
\begin{aligned}
ds^2_{10\mathrm{IIB}} &= H^{-1/2}(-dt^2+dw_1^2+dw_2^2+dw_3^2)\\
&\quad +H^{1/2}(dr^2+r^2dM_5^2),\\
H&=1+\frac{Q}{r^4},
\end{aligned}
\tag{11}
$$

where

$$
dM_5^2=d\alpha^2+c^2d\theta^2+\frac{1}{4}s^2\left[d\Omega_2^2+\left(\frac{dy}{Q_{\mathrm{N}}}+\omega\right)^2\right], \tag{12}
$$

and $s=\sin\alpha$, $c=\cos\alpha$. Thus, we see that dM_5^2 describes a foliation of S^1 times the lens space $S^3/Z_{Q_{\mathrm{N}}}$. For a unit NUT charge, $Q_{\mathrm{N}}=1$, the metric dM_5^2 describes the round 5-sphere and the solution becomes an isotropic D3-brane. It is interesting to note that the regular D3-brane can be viewed as a semi-localized D3-brane intersecting with a NUT with unit charge.[1] In the near-horizon limit $r\to 0$, where the constant 1 in the function H can be dropped, the metric becomes $\mathrm{AdS}_5\times M_5$:

$$
\begin{aligned}
ds^2_{10\mathrm{IIB}} &= Q^{-1/2}r^2(-dt^2+dw^idw^i)+Q^{1/2}\frac{dr^2}{r^2}\\
&\quad +Q^{1/2}\left\{d\alpha^2+c^2d\theta^2+\tfrac{1}{4}s^2\left[d\Omega_2^2+\left(\frac{dy}{Q_{\mathrm{N}}}+\omega\right)^2\right]\right\}.
\end{aligned}
\tag{13}
$$

B. M5-M5 system and AdS$_5$ in M theory

Since the near-horizon limit of a semi-localized D3-brane or NUT is a direct product of AdS$_5$ and an internal 5-sphere that is a foliation of a circle times a lens space, it follows that if we perform a T-duality transformation on the $U(1)$ fiber coordinate y, we shall obtain AdS$_5$ in a warped spacetime as a solution of the type IIA theory. The warp factor is associated with the scale factor s^2 of dy^2 in Eq. (13). This type of Hopf T duality has the effect of untwisting a 3-sphere into $\mathrm{S}^2\times\mathrm{S}^1$ [11]. If one performs the T-duality transformation on the original full solution (5), rather than concentrating on its near-horizon limit, then one obtains a semi-localized NS5-D4 system of the type IIA theory, which can be further lifted back to $D=11$ to become a semi-localized M5-M5 system, obtained in [6]. In [9], the near-horizon structures

[1]An analogous observation was also made in [10], where multi-charge solutions were obtained from flat space by making use of the fact that S^3 can be viewed as a $U(1)$ bundle over S^2. In other words, flat space can be viewed as a NUT, with unit charge, located on the $U(1)$ coordinate.

TABLE II. The D3–pp-wave brane intersection. Here $\sim$ denotes the wave coordinate.

	t	y	x_1	x_2	z_1	z_2	z_3	z_4	z_5	z_6	
D3	$\times$	$\times$	$\times$	$\times$	$-$	$-$	$-$	$-$	$-$	$-$	H
wave	$\times$	$\sim$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	W

of these semi-localized branes of M theory were analyzed, and AdS_5 was obtained as a warped spacetime solution. We refer the readers to Ref. [9] and shall not discuss this solution further, but only mention that, from the above analysis, it can be obtained by implementing the T-duality transformation on the coordinate y in Eq. (13).

III. D3–pp-WAVE SYSTEM AND EXTREMAL BTZ BLACK HOLE

In this section, we study the semi-localized pp wave intersecting with a D3-brane. The solution is given by

$$ds^2_{10\mathrm{IIB}}=H^{-1/2}[-W^{-1}dt^2+W(dy+(W^{-1}-1)dt)^2 +dx_1^2+dx_2^2]+H^{1/2}(dz_1^2+\cdots+dz_6^2),$$

$$F_{(5)}=dt\wedge dy\wedge dx_1\wedge dx_2\wedge dH^{-1} +*(dt\wedge dy\wedge dx_1\wedge dx_2\wedge dH^{-1}). \tag{14}$$

The solution can be illustrated by Table II.

In the usual construction of such an intersection, the harmonic functions H and W depend only on the overall transverse space coordinates $\vec{z}$. The near-horizon limit of the solution then becomes $K_5\times S^6$, where K_5 is the generalized Kaigorodov metric in $D=5$, and the geometry is dual to a conformal field theory in the infinite momentum frame [26]. On the other hand, the semi-localized solution is given by [6]

$$H=\frac{Q}{|\vec{z}|^4},\quad W=1+Q_w\left(|\vec{x}|^2+\frac{Q}{|\vec{z}|^2}\right). \tag{15}$$

We now let

$$x_1=\frac{1}{r}\cos\alpha\cos\theta,\quad x_2=\frac{1}{r}\cos\alpha\sin\theta,\quad z_i=\frac{rQ^{1/2}}{\sin\alpha}\nu_i, \tag{16}$$

where ν_i coordinates, satisfying $\nu_i\nu_i=1$, define a 5-sphere with the unit sphere metric $d\Omega_5^2=d\nu_i d\nu_i$. Using these coordinates, the metric of the semi-localized D3-wave system becomes

$$ds^2_{10\mathrm{IIB}}=Q^{1/2}s^{-2}(ds^2_{\mathrm{AdS}_3}+d\alpha^2+c^2d\theta^2+s^2d\Omega_5^2), \tag{17}$$

where $ds^2_{\mathrm{AdS}_3}$ is given by

$$ds^2_{\mathrm{AdS}_3}=-r^2W^{-1}dt^2+r^2W[dy+(W^{-1}-1)dt]^2+\frac{dr^2}{r^2}, \tag{18}$$
$$W=1+\frac{Q_w}{r^2}.$$

Note that the above metric is exactly the extremal BTZ black hole [12], and hence it is locally AdS_3. Thus we have demonstrated that the semi-localized D3–pp-wave system is in fact a warped product of AdS_3 (the extremal BTZ black hole) with a 7-sphere, where S^7 is described as a foliation of $S^1\times S^5$ surfaces.[2] Note that the metric (17) can also be expressed as a direct product of $AdS_5\times S^5$, with the AdS_5 metric written in the following form:

$$ds_5^2=s^{-2}(ds^2_{\mathrm{AdS}_3}+d\alpha^2+c^2d\theta^2). \tag{19}$$

Making a coordinate transformation $\tan(\alpha/2)=e^\rho$, the metric becomes

$$ds_5^2=d\rho^2+\sinh^2\rho\, d\theta^2+\cosh^2\rho\, ds^2_{\mathrm{AdS}_3}, \tag{20}$$

which is precisely the AdS_5 metric written as a foliation of a circle times AdS_3 (see Appendix A).

The extremal BTZ black hole occurs [28] as the near-horizon geometry of the boosted dyonic string in six dimensions, which can be viewed as an intersection of a string and a 5-brane in $D=10$. The boosted D1-D5 system was used to obtain the first stringy interpretation [29] of the microscopic entropy of the Reissner-Nordström black hole in $D=5$. The boosted dyonic string has three parameters, namely the electric and magnetic charges Q_e, Q_m, and the boost momentum parameter Q_w. On the other hand, the extremal BTZ black hole itself has only two parameters: the cosmological constant, proportional to $\sqrt{Q_eQ_m}$, and the mass (which is equal to the angular momentum in the extremal limit), which is related to Q_w. (Analogous discussion applies to $D=4$ [30].) In our construction of the BTZ black hole in warped spacetime, the original configuration also has only two parameters, namely the D3-brane charge Q, related to the cosmological constant of the BTZ black hole, and the pp-wave charge, associated with the mass.

A. NS1-D2 and M2-M2 systems and AdS_2

We can perform a T-duality transformation on the coordinate y in the previous solution. The D3-brane is T dual to the D2-brane, and the wave is T dual to the NS-NS string. Thus the D3–pp-wave system of the type IIB theory becomes an NS1-D2 system in the type IIA theory, given by

$$ds^2_{10\mathrm{IIA}}=W^{1/4}H^{3/8}[-(WH)^{-1}dt^2+H^{-1}(dx_1^2+dx_2^2) +W^{-1}dy_1^2,+dz_1^2+\cdots+dz_6^2], \tag{21}$$
$$e^\phi=W^{-1/2}H^{1/4},$$

[2]A D3-brane with an $S^3\times\mathbb{R}$ world volume was obtained in [27]. In that solution, which was rather different from ours, the dilaton was not constant.

TABLE III. The NS1-D2 brane intersection.

	t	x_1	x_2	y_1	z_1	z_2	z_3	z_4	z_5	z_6	
D2	×	×	×	−	−	−	−	−	−	−	H
NS1	×	−	−	×	−	−	−	−	−	−	W

$$F_{(4)}=dt\wedge dx_1\wedge dx_2\wedge dH^{-1},\quad F_{(3)}=dt\wedge dy_1\wedge dW^{-1}.$$

This solution can be represented diagrammatically as Table III.

In the near-horizon limit where the 1 in W is dropped, the metric of the NS1-D2 system (21), in terms of the new coordinates (16), becomes

$$ds_{10}^2=Q_w^{1/4}Q^{5/8}s^{-5/2}[ds_{\mathrm{AdS}_2}^2+d\alpha^2+c^2d\theta^2+s^2d\Omega_5^2+(Q_wQ)^{-1}s^4dy_1^2],\tag{22}$$

where

$$ds_{\mathrm{AdS}_2}^2=-\frac{r^4dt^2}{Q_w}+\frac{dr^2}{r^2}.\tag{23}$$

Thus we see that the near-horizon limit of the NS1-D2 system is a warped product of AdS_2 with a certain internal 8-space, which is a warped product of a 7-sphere with a circle.

We can further lift the solution back to $D=11$, where it becomes a semi-localized M2-M2 system,

$$\begin{aligned}ds_{11}^2&=(WH)^{1/3}[-(WH)^{-1}dt^2+H^{-1}(dx_1^2+dx_2^2)\\&\quad+W^{-1}(dy_1^2+dy_2^2),+dz_1^2+\cdots+dz_6^2],\\F_{(4)}&=dt\wedge dx_1\wedge dx_2\wedge dH^{-1}\\&\quad+dt\wedge dy_1\wedge dy_2\wedge dW^{-1}.\end{aligned}\tag{24}$$

The configuration for this solution can be summarized in Table IV.

It is straightforward to verify that the near-horizon geometry of this system is a warped product of AdS_2 with a certain 9-space, namely

$$ds_{11}^2=Q_w^{1/3}Q^{2/3}s^{-8/3}[ds_{\mathrm{AdS}_2}^2+d\alpha^2+c^2d\theta^2+s^2d\Omega_5^2+(Q_wQ)^{-1}s^4(dy_1^2+dy_2^2)],\tag{25}$$

where $ds_{\mathrm{AdS}_2}^2$ is an AdS_2 metric given by Eq. (23), and the internal 9-space is a warped product of a 7-sphere and a 2-torus.

TABLE IV. The M2-M2 brane intersection.

	t	x_1	x_2	y_1	y_2	z_1	z_2	z_3	z_4	z_5	z_6	
M2	×	×	×	−	−	−	−	−	−	−	−	H
M2	×	−	−	×	×	−	−	−	−	−	−	W

TABLE V. The M2-M2-NUT brane intersection.

	t	x_1	x_2	y_1	y_2	z_1	z_2	z_3	y	u_1	u_2	
M2	×	×	×	−	−	−	−	−	−	−	−	H
M2	×	−	−	×	×	−	−	−	−	−	−	W
NUT	×	×	×	×	×	−	−	−	*	×	×	K

B. Further possibilities

Note that in the above examples, we can replace the round sphere $d\Omega_5^2$ by a lens space of the following form:

$$d\Omega_5^2=d\tilde{\alpha}^2+\tilde{c}^2d\tilde{\theta}^2+\tilde{s}^2\left[d\tilde{\Omega}_2^2+\left(\frac{d\tilde{y}}{\tilde{Q}_{\mathrm{N}}}+\tilde{\omega}\right)^2\right],\tag{26}$$

where $\tilde{c}\equiv\cos\tilde{\alpha}$, $\tilde{s}\equiv\sin\tilde{\alpha}$ and $d\tilde{\omega}=\tilde{\Omega}_2$. As we have discussed in Appendix B, this can be viewed as an additional NUT with charge $\tilde{Q}_{\mathrm{N}}$ intersecting with the system. We can now perform a Kaluza-Klein reduction or T-duality transformation on the fiber coordinate $\tilde{y}$, leading to many further examples of warped products of AdS_2 or AdS_3 with certain internal spaces. The warp factors again depend only on the coordinates of the internal space. These geometries can be viewed as the near-horizon limits of three intersecting branes, with charges Q, Q_{N} and $\tilde{Q}_{\mathrm{N}}$. Of course, this system can equally well be obtained by replacing the horospherical AdS_5 in Eq. (13) with Eq. (19).

For example, let us consider the M2-M2 system with an additional NUT component. The solution of this semi-localized intersecting system is given by

$$\begin{aligned}ds_{11}^2&=(WH)^{1/3}[-(WH)^{-1}dt^2+H^{-1}(dx_1^2+dx_2^2)\\&\quad+W^{-1}(dy_1^2+dy_2^2),+K(dz^2+z^2d\Omega_2^2)\\&\quad+K^{-1}(dy+Q_{\mathrm{N}}\omega)^2+du_1^2+du_2^2],\\F_{(4)}&=dt\wedge dx_1\wedge dx_2\wedge dH^{-1}\\&\quad+dt\wedge dy_1\wedge dy_2\wedge dW^{-1}.\end{aligned}\tag{27}$$

where the functions H, W and K are given by

$$\begin{aligned}H&=\frac{Q}{(|\vec{u}|^2+4Q_{\mathrm{N}}z)^2},\\W&=1+Q_w\left(|\vec{x}|^2+\frac{Q}{|\vec{u}|^2+4Q_{\mathrm{N}}z}\right),\\Ka&=\frac{Q_{\mathrm{N}}}{z}.\end{aligned}\tag{28}$$

We illustrate this solution in Table V.

The near-horizon structure of this solution is basically the same as that of the M2-M2 system with the round S^3 in the foliation replaced by the lens space $S^3/Z_{Q_{\mathrm{N}}}$. We can now

TABLE VI. The D2-D2-D6 brane intersection.

	t	x_1	x_2	y_1	y_2	z_1	z_2	z_3	u_1	u_2	
D2	×	×	×	−	−	−	−	−	−	−	H
D2	×	−	−	×	×	−	−	−	−	−	W
D6	×	×	×	×	×	−	−	−	×	×	K

perform Kaluza-Klein reduction on the fiber coordinate y and the solution becomes the semi-localized D2-D2-D6 brane intersection, given by

$$
\begin{aligned}
ds_{10\mathrm{IIA}}^2 &= (WH)^{3/8}K^{-1/8}[-(WH)^{-1}dt^2+H^{-1}(dx_1^2+dx_2^2)\\
&\quad +W^{-1}(dy_1^2+dy_2^2),\\
&\quad +K(dz^2+z^2d\Omega_2^2)+du_1^2+du_2^2],\\
F_{(4)} &= dt\wedge dx_1\wedge dx_2\wedge dH^{-1}+dt\wedge dy_1\wedge dy_2\wedge dW^{-1}.
\end{aligned}
\tag{29}
$$

$$
e^{\phi}=(WH)^{1/4}K^{-3/4},\quad F_{(2)}=Q_{\mathrm{N}}\Omega_2. \tag{30}
$$

The solution can be illustrated by Table VI.

IV. M2-NUT AND M2−*pp*-WAVE SYSTEMS

In this section, we apply an analogous analysis to the M2-brane. We show that the semi-localized M2-brane intersecting with a NUT is in fact an isotropic M2-brane with the internal 7-sphere itself being described as a foliation of a regular S^3 and lens space $S^3/Z_{Q_{\mathrm{N}}}$, where Q_{N} is the NUT charge. Reducing the system to $D=10$, we obtain a semi-localized D2-D6 system whose near-horizon geometry is a warped product of AdS_4 with an internal 6-space. We also show that a semi-localized pp wave intersecting with the M2-brane is in fact a warped product of AdS_3 (the BTZ black hole) and an 8-space. The system can be reduced to $D=10$ to become a semi-localized D0-Neveu-Schwarz-1-brane (NS1) intersection.

A. M2-brane−NUT system

The solution for the intersection of an M2-brane and a NUT is given by

$$
\begin{aligned}
ds_{11}^2 &= H^{-2/3}(-dt^2+dw_1^2+dw_2^2)+H^{1/3}[dx_1^2+\cdots+dx_4^2\\
&\quad +K(dz^2+z^2d\Omega_2^2)+K^{-1}(dy+Q_{\mathrm{N}}\omega)^2],\\
F_{(4)} &= dt\wedge dw_1\wedge dw_2\wedge dH^{-1},
\end{aligned}
\tag{31}
$$

where $z^2=z_1^2+z_2^2+z_3^2$ and $d\omega=\Omega_2$. The solution can be illustrated by Table VII.

TABLE VII. The M2-NUT brane intersection.

	t	w_1	w_2	x_1	x_2	x_3	x_4	z_1	z_2	z_3	y	
M2	×	×	×	−	−	−	−	−	−	−	−	H
NUT	×	×	×	×	×	×	×	−	−	−	*	K

If the function K associated with the NUT components of the intersection takes the form $K=Q_{\mathrm{N}}/z$, then the function H associated with the M2-brane component can be solved in the semi-localized form

$$
H=1+\frac{Q}{(|\vec{x}|^2+4Q_{\mathrm{N}}z)^3}. \tag{32}
$$

Thus, the solution is also localized on the space of the $\vec{x}$ coordinates. Let us now make a coordinate transformation

$$
x_i=r\cos\alpha\mu_i,\quad z=\tfrac{1}{4}Q_{\mathrm{N}}{}^{-1}r^2\sin^2\alpha, \tag{33}
$$

where $\mu_i\mu_i=1$, defining a 3-sphere, with the unit 3-sphere metric given by $d\Omega_3^2=d\mu_i d\mu_i$. In terms of the new coordinates, the metric for the solution becomes

$$
\begin{aligned}
ds_{11}^2 &= H^{-2/3}(-dt^2+dw_1^2+dw_2^2)+H^{1/3}(dr^2+r^2dM_7^2),\\
H &= 1+\frac{Q}{r^6},
\end{aligned}
\tag{34}
$$

where

$$
dM_7^2=d\alpha^2+c^2d\Omega_3^2+\tfrac{1}{4}s^2\left[d\Omega_2^2+\left(\frac{dy}{Q_{\mathrm{N}}}+\omega\right)^2\right]. \tag{35}
$$

Thus we see that dM_7^2 is a foliation of a regular 3-sphere, together with a lens space $S^3/Z_{Q_{\mathrm{N}}}$. When $Q_{\mathrm{N}}=1$ the metric dM_7^2 describes a round 7-sphere and the solution becomes an isotropic M2-brane. Interestingly, the regular M2-brane can be viewed as an intersecting semi-localized M2-brane with a NUT of unit charge. In the near-horizon limit $r\to 0$, where the 1 in the function H can be dropped, the metric becomes $\mathrm{AdS}_4\times M_7$.

B. D2-D6 system

In the M2-brane and NUT intersection (31), we can perform a Kaluza-Klein reduction on the y coordinate. This gives rise to a semi-localized intersection of D2-branes and D6-branes:

$$
\begin{aligned}
ds_{10\mathrm{IIA}}^2 &= H^{-5/8}K^{-1/8}(-dt^2+dw_1^2+dw_2^2)\\
&\quad +H^{3/8}K^{-1/8}(dx_1^2+\cdots+dx_4^2)H^{3/8}K^{7/8}\\
&\quad \times(dz_1^2+dz_2^2+dz_3^2),\\
e^{\phi} &= H^{1/4}K^{-3/4},\\
F_{(4)} &= dt\wedge d^2w\wedge dH^{-1},\\
F_2 &= e^{-3/2\phi}*(dt\wedge d^2w\wedge d^4x\wedge dK^{-1}).
\end{aligned}
\tag{36}
$$

The solution can be illustrated by Table VIII.

Again, in the usual construction of a D2-D6 system, the harmonic functions H and K are taken to depend only on the overall transverse space coordinates $\vec{z}$. In the semi-localized

TABLE VIII. The D2-D6 brane intersection.

	t	w_1	w_2	x_1	x_2	x_3	x_4	z_1	z_2	z_3	
D2	×	×	×	−	−	−	−	−	−	−	H
D6	×	×	×	×	×	×	×	−	−	−	K

construction, the function H depends on $\vec{x}$ as well. In terms of the new coordinates defined in Eq. (33), the metric becomes

$$ds^2_{10\mathrm{IIA}} = \left(\frac{rs}{2Q_\mathrm{N}}\right)^{1/4}[H^{-5/8}(-dt^2+dw_1^2+dw_2^2) + H^{3/8}(dr^2+r^2)(d\alpha^2+c^2 d\Omega_3^2+\tfrac{1}{4}s^2 d\Omega_2^2)]. \tag{37}$$

Thus, in the near-horizon limit where the 1 in H can be dropped, the solution becomes a warped product of AdS_4 with an internal 6-space:

$$ds^2_{10\mathrm{IIA}} = (2Q_\mathrm{N})^{-1/4}Q^{3/8}s^{1/4} \times (ds^2_{\mathrm{AdS}_4}+d\alpha^2+c^2 d\Omega_3^2+\tfrac{1}{4}s^2 d\Omega_2^2), \tag{38}$$

where ds_4^2 is the metric on AdS_4, given by

$$ds^2_{\mathrm{AdS}_4} = \frac{r^4}{Q}(-dt^2+dw_1^2+dw_2^2)+\frac{dr^2}{r^2}. \tag{39}$$

The internal 6-space is a warped product of a 4-sphere with a 2-sphere.

C. AdS_4 in type IIB from *T* duality

In the above discussion, we found that our starting point is effectively to replace the round 7-sphere of the M2-brane by the foliation of a round 3-sphere together with a lens space S^3/Z_{Q_N}. We can also replace the round 3-sphere by another lens space $S^3/Z_{\tilde{Q}_\mathrm{N}}$, given by

$$d\bar{\Omega}_3^2 = \tfrac{1}{4}\left[d\tilde{\Omega}_2^2+\left(\frac{d\tilde{y}}{\tilde{Q}_\mathrm{N}}+\omega\right)^2\right]. \tag{40}$$

As discussed in Appendix A, the lens space arises from introducing a NUT around the fiber coordinate $\tilde{y}$, with NUT charge $\tilde{Q}_\mathrm{N}$. The system can then be viewed as the near-horizon limit of three intersecting branes, with charges Q, Q_N and $\tilde{Q}_\mathrm{N}$. For example, with this replacement the D2-D6 system becomes a D2-D6-NUT system. Performing a *T*-duality transformation on the fiber coordinate $\tilde{y}$, the S^3 untwists to become $\mathrm{S}^2\times\mathrm{S}^1$. The resulting type IIB metric is given by

$$ds^2_{10\mathrm{IIB}} = \left(\frac{Qsc}{4Q_\mathrm{N}}\tilde{Q}_\mathrm{N}\right)^{1/2}\left(ds^2_{\mathrm{AdS}_4}+d\alpha^2+\tfrac{1}{4}c^2 d\tilde{\Omega}_2^2+\tfrac{1}{4}s^2 d\Omega_2^2 + \frac{(4Q_\mathrm{N}\tilde{Q}_\mathrm{N})^2}{Qs^2c^2}d\tilde{y}^2\right). \tag{41}$$

TABLE IX. The D2-D6-NUT system.

	t	w_1	w_2	x_1	x_2	x_3	y	z_1	z_2	z_3	
D2	×	×	×	−	−	−	−	−	−	−	H
D6	×	×	×	×	×	×	×	−	−	−	K
NUT	×	×	×	−	−	−	*	×	×	×	$\tilde{K}$

This metric can be viewed as describing the near-horizon geometry of a semi-localized D3-D5-NS5 system in the type IIB theory. This metric (41) provides a background for consistent reduction of type IIB supergravity to give rise to four-dimensional gauged supergravity with AdS background.

In order to construct the semi-localized D3-D5-NS5 intersecting system in the type IIB theory, we start with the D2-D6-NUT system, given by

$$\begin{aligned} ds^2_{10\mathrm{IIA}} &= H^{-5/8}K^{-1/8}(-dt^2+dw_1^2+dw_2^2) \\ &\quad + H^{3/8}K^{7/8}(dz_1^2+dz_2^2+dz_3^2) \\ &\quad + H^{3/8}K^{-1/8}[\tilde{K}(dx^2+x^2 d\tilde{\Omega}_2^2)+\tilde{K}^{-1}(dy+\tilde{Q}_\mathrm{N}\tilde{\omega})^2], \\ e^{\phi} &= H^{1/4}K^{-3/4}, \\ F_{(4)} &= dt\wedge d^2w\wedge dH^{-1}, \\ F_2 &= e^{-3/2\phi}{*}(dt\wedge d^2w\wedge d^4x\wedge dK^{-1}), \end{aligned} \tag{42}$$

where $x^2=x_1^2+x_2^2+x_3^2$ and the functions H, K and $\tilde{K}$ are given by

$$H = 1+\frac{Q}{(4\tilde{Q}_\mathrm{N}x+4Q_\mathrm{N}z)^3}, \quad K=\frac{Q_\mathrm{N}}{z}, \quad \tilde{K}=\frac{\tilde{Q}_\mathrm{N}}{x}. \tag{43}$$

It is instructive to illustrate the solution in Table IX.

We can now perform the *T* duality on the coordinate y, and obtain the semi-localized D3-D5-NS5 intersection of the type IIB theory, given by

$$ds^2_{10\mathrm{IIB}} = H^{-1/2}(K\tilde{K})^{-1/4}[-dt^2+dw_1^2+dw_2^2 H\tilde{K} \times(dx_1^2+dx_2^2+dx_3^2)+K\tilde{K}dy^2 + HK(dz_1^2+dz_2^2+dz_3^2)]. \tag{44}$$

It is straightforward to verify that the near-horizon structure of the above D3-D5-NS5 system is of the form (41). The solution can be illustrated by Table X.

TABLE X. The D3-D5-NS5 system.

	t	w_1	w_2	x_1	x_2	x_3	y	z_1	z_2	z_3	
D3	×	×	×	−	−	−	×	−	−	−	H
D5	×	×	×	×	×	×	−	−	−	−	K
NS5	×	×	×	−	−	−	−	×	×	×	$\tilde{K}$

TABLE XI. The M2–pp-wave brane intersection.

	t	y_1	x_1	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	
M2	×	×	×	–	–	–	–	–	–	–	–	H
wave	×	∼	–	–	–	–	–	–	–	–	–	W

TABLE XII. The NS1-D0 brane intersection.

	t	x_1	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	
NS1	×	×	–	–	–	–	–	–	–	–	H
D0	×	–	–	–	–	–	–	–	–	–	W

D. M2–*pp*-wave system

The M2–pp-wave solution is given by

$$ds_{11}^2=H^{-2/3}\{-W^{-1}dt+W[dy+(W^{-1}-1)dt]^2+dx^2\}+H^{1/3}(dz^2+z^2d\Omega_7^2),$$

$$F_{(4)}=dt\wedge dy\wedge dx\wedge dH^{-1}. \tag{45}$$

The solution can be illustrated by Table XI.

When both functions H and W are harmonic on the overall transverse space of the z^i coordinates, the metric becomes a direct product of the Kaigorodov metric with a 7-sphere in the near-horizon limit. Here, we instead consider a semi-localized solution, with H and K given by

$$H=\frac{Q}{z^6},\quad W=1+Q_w\left(x^2+\frac{Q/4}{z^4}\right). \tag{46}$$

Making the coordinate transformation

$$x=\frac{\cos\alpha}{r},\quad z^2=\frac{rQ^{1/2}}{2\sin\alpha}, \tag{47}$$

the metric becomes $AdS_4\times S^7$, with

$$ds_{11}^2=\frac{Q^{1/3}}{4s^2}(ds_{AdS_3}^2+d\alpha^2)+Q^{1/3}d\Omega_7^2. \tag{48}$$

Here $ds_{AdS_3}^2$ is the metric of AdS_3 (the BTZ black hole), given by Eq. (18). Thus, we have demonstrated that the semi-localized M2–pp-wave system is a warped product of AdS_3 and an 8-space. Making the coordinate transformation $\tan(\alpha/2)=e^{\rho}$, the first part of Eq. (48) can be expressed as

$$ds_4^2=d\rho^2+\cosh^2\rho\, ds_{AdS_3}^2. \tag{49}$$

This is AdS_4 expressed as a foliation of AdS_3 (see Appendix A).

E. The NS1-D0 system

Reducing the above solution on the coordinate y_1, it becomes an intersecting NS1-D0 system, with

$$ds_{10IIA}=H^{-3/4}W^{-7/8}[-dt^2+Wdx^2+WH(dz_1^2+\cdots+dz_8^2)],$$

$$F_{(3)}=dt\wedge dx\wedge dH^{-1},\quad F_{(2)}=dt\wedge dW^{-1},$$

$$e^{\phi}=H^{-1/2}W^{3/4}. \tag{50}$$

The metric of the near-horizon region describes a warped product of AdS_2 with an 8-space:

$$ds_{10IIA}^2=8^{-3/4}Q^{3/8}Q_w^{1/8}s^{-9/4}(ds_{AdS_2}^2+d\alpha^2+4s^2d\Omega_7^2), \tag{51}$$

where $ds_{AdS_2}^2$ is the metric of AdS_2, given by Eq. (23). The NS1-D0 system can be illustrated by Table XII.

In the M2–pp-wave and NS1-D0 systems, the internal space has a round 7-sphere. We can replace it by foliating two lens spaces S^3/Z_{Q_N} and $S^3/Z_{\tilde{Q}_N}$. As discussed in Appendix B, this can be achieved by introducing two NUTs in the intersecting system. We can then perform Kaluza-Klein reductions or T-duality transformations on the two associated fiber coordinates of the lens spaces. The resulting configurations can then be viewed as the near-horizon geometries of four intersecting p-branes, with charges Q, Q_w, Q_N and $\tilde{Q}_N$.

V. M5-NUT AND M5–*pp*-WAVE SYSTEMS

A. M5-NUT and NS5-D6 systems

The solution of an M5-brane intersecting with a NUT is given by

$$ds_{11}^2=H^{-1/3}(-dt^2+dw_1^2+\cdots+dw_5^2)+H^{2/3}[dx_1^2+K(dz^2+z^2d\Omega_2^2)+K^{-1}(dy+\omega)^2],$$

$$F_{(4)}={}^*(dt\wedge d^5w\wedge dH^{-1}). \tag{52}$$

The solution can be illustrated by Table XIII.

In the usual construction where the harmonic functions H and K depend only the z coordinate, the metric does not have an AdS structure in the near-horizon region. Here, we instead consider a semi-localized solution, given by

$$H=1+\frac{Q}{(x^2+4Q_Nz)^{3/2}},\quad K=\frac{Q_N}{z}. \tag{53}$$

After an analogous coordinate transformation, we find that the metric can be expressed as

$$ds_{11}^2=H^{-1/3}(-dt^2+dw_idw_i)+H^{2/3}(dr^2+r^2dM_4^2),$$

TABLE XIII. The M5-NUT brane intersection.

	t	w_1	w_2	w_3	w_4	w_5	x_1	z_1	z_2	z_3	y	
M5	×	×	×	×	×	×	–	–	–	–	–	H
NUT	×	×	×	×	×	×	×	–	–	–	*	K

TABLE XIV. The NS5-D6 brane intersection.

	t	w_1	w_2	w_3	w_4	w_5	x_1	z_1	z_2	z_3	
NS5	×	×	×	×	×	×	−	−	−	−	H
D6	×	×	×	×	×	×	×	−	−	−	K

$$dM_4^2 = d\alpha^2 + \tfrac{1}{4}s^2\left[d\Omega_2^2 + \left(\frac{dy}{Q_{\rm N}} + \omega\right)^2\right]. \tag{54}$$

Thus, in the near-horizon limit, the metric is $\mathrm{AdS}_7 \times M_4$, where M_4 is a foliation of a lens space $S^3/Z_{Q_{\rm N}}$.

We can dimensionally reduce the solution (52) on the fiber coordinate y. The resulting solution is the NS-NS 5-brane intersecting with a D6-brane (see Table XIV).

The solution is given by

$$\begin{aligned} ds_{10\mathrm{IIA}}^2 &= H^{-1/4}K^{-1/8}(-dt^2 + dw_i dw_i) \\ &\quad + H^{3/4}K^{-1/8}dx^2 + H^{3/4}K^{7/8}dz_i dz_i\,, \\ e^{\phi} &= H^{1/2}K^{-3/4}, \\ F_{(3)} &= e^{\phi/2} * (dt\wedge d^5w \wedge dH^{-1}), \\ F_{(2)} &= e^{-3\phi/2} * (dt\wedge d^5 w\wedge dx \wedge dK^{-1}). \end{aligned} \tag{55}$$

In the near-horizon limit, the metric becomes a warped product of AdS_7 with a 3-space

$$\begin{aligned} ds_{10\mathrm{IIA}}^2 = \frac{Q^{3/4}}{(2Q_{\rm N})^{1/4}} s^{1/4}\bigg(&\frac{r}{Q}(-dt^2 + dw_i dw_i) + \frac{dr^2}{r^2} \\ &+ d\alpha^2 + \tfrac{1}{4}s^2 d\Omega_2^2\bigg). \end{aligned} \tag{56}$$

B. M5−*pp*-wave and D0-D4 systems

The solution of an M5-brane with a pp wave is given by

$$\begin{aligned} ds_{11}^2 &= H^{-1/3}\{-W^{-1}dt^2 + W[dy_1 + (W^{-1}-1)dt]^2 \\ &\quad + dx_1^2 + \cdots + dx_4^2\} + H^{2/3}(dz_1^2 + \cdots + dz_5^2), \\ F_4 &= *(dt\wedge dy_1 \wedge d^4x \wedge dH^{-1}). \end{aligned} \tag{57}$$

The solution can be illustrated by Table XV.

We shall consider semi-localized solutions, with the functions H and W given by

$$H = \frac{Q}{z^3}, \quad W = 1 + Q_w\left(x^2 + \frac{4Q}{z}\right). \tag{58}$$

TABLE XV. The M5-*pp*-wave brane intersection.

	t	y_1	x_1	x_2	x_3	x_4	z_1	z_2	z_3	z_4	z_5	
M5	×	×	×	×	×	×	−	−	−	−	−	H
wave	×	∼	−	−	−	−	−	−	−	−	−	W

TABLE XVI. The D0-D4 brane intersection.

	t	x_1	x_2	x_3	x_4	z_1	z_2	z_3	z_4	z_5	
D4	×	×	×	×	×	−	−	−	−	−	H
D0	×	−	−	−	−	−	−	−	−	−	W

Using analogous coordinate transformations, we find that the metric of the semi-localized M5−pp-wave system becomes

$$ds_{11}^2 = 4Q^{2/3}s^{-2}(ds_{\mathrm{AdS}_3}^2 + d\alpha^2 + c^2 d\Omega_3^2) + Q^{2/3}d\Omega_4^2, \tag{59}$$

where $ds_{\mathrm{AdS}_3}^2$, given by Eq. (18), is precisely the extremal BTZ black hole and hence is locally AdS_3. After making the coordinate transformation $\tan(\alpha/2) = e^{\rho}$, the first part of the metric (59) can be expressed as

$$ds_7^2 = d\rho^2 + \sinh^2\rho d\Omega_3^2 + \cosh^2 \rho ds_3^2. \tag{60}$$

This is AdS_7 written as a foliation of AdS_3 and S^3.

Performing a dimensional reduction of the solution (57) on the coordinate y_1, we obtain a D0-D4 intersecting system, given by

$$\begin{aligned} ds_{10\mathrm{IIA}}^2 &= H^{-3/8}W^{-7/8}(-dt^2 + Wdx_i dx_i + HWdz_i dz_i), \\ e^{\phi} &= H^{-1/4}W^{3/4}, \quad F_{(2)} = dt \wedge dW^{-1}, \\ F_4 &= e^{-\phi/2} * (dt\wedge d^4x \wedge dH^{-1}). \end{aligned} \tag{61}$$

The near-horizon limit of the semi-localized D0-D4 system is a warped product of AdS_2 with an 8-space:

$$ds_{10\mathrm{IIA}}^2 = 2^{9/4}Q^{3/4}Q_w^{1/8}s^{-9/4}(ds_2^2 + d\alpha^2 + c^2 d\Omega_3^2 + \tfrac{1}{4}s^2 d\Omega_4^2), \tag{62}$$

where ds_2^2 is given by Eq. (23). We illustrate this intersecting system with Table XVI.

In this example in the internal space the round S^3 and S^4 can be replaced by a lens space $S^3/Z_{Q_{\rm N}}$ and the foliation of a lens space $S^3/Z_{\tilde{Q}_{\rm N}}$, respectively. We can then perform Kaluza-Klein reductions or T-duality transformations on the fiber coordinates of the lens spaces, leading to four-component intersections with charges Q, Q_w, $Q_{\rm N}$ and $\tilde{Q}_{\rm N}$.

VI. AdS$_6$ IN TYPE IIB FROM *T* DUALITY

So far in this paper we have two examples of intersecting Dp/D$(p+4)$ systems in the type IIA theory that give rise to warped products of AdS_{p+2} with certain internal spaces, namely for $p=0$ and $p=2$. It was observed [5] also that the D4-D8 system, arising from massive type IIA supergravity, gives rise to the warped product of AdS_6 with a 4-sphere in the near-horizon limit:

$$ds_{10\mathrm{IIA}}^2 = s^{1/12}[ds_{\mathrm{AdS}_6}^2 + g^{-2}(d\alpha^2 + c^2 d\Omega_3^2)]. \tag{63}$$

TABLE XVII. The D5-D7-NS5 brane intersection.

	t	w_1	w_2	w_3	w_4	x_1	x_2	x_3	y	z	
D5	×	×	×	×	×	–	–	–	–	–	H_1
D7	×	×	×	×	×	×	×	×	–	–	H_2
NS5	×	×	×	×	×	–	–	–	–	×	K

Note that the D4-D8 system is less trivial than the previous examples, in the sense that it cannot be mapped by T duality to a non-dilatonic p-brane intersecting with a NUT or a wave.

We can now introduce a NUT in the intersecting system which has the effect, in the near-horizon limit, of replacing the round 3-sphere by a lens space, given in Eq. (2). We can then perform a Hopf T-duality transformation and obtain an embedding of AdS_6 in type IIB theory:

$$ds_{10}^2 = c^{1/2}[ds_{\mathrm{AdS}_6}^2 + g^{-2}(d\alpha^2 + \tfrac{1}{4}c^2 d\Omega_2^2) + s^{2/3}c^{-2}dy^2]. \tag{64}$$

This solution can be viewed as the near-horizon geometry of an intersecting D5-D7-NS5 system. It provides a background for the exact embedding of six-dimensional gauged supergravity in type IIB theory.

The D5-D7-NS5 semi-localized solution can be obtained by performing the T duality on the D4-D8-NUT system. The solution is given by

$$\begin{aligned} ds_{10\mathrm{IIB}}^2 = (H_1K)^{-1/4}[& -dt^2 + dw_1^2 + \cdots + dw_4^2 \\ & + H_1K(dx_1^2 + dx_2^2 + dx_3^2) + H_2Kdy^2 + H_1H_2dz^2]. \end{aligned} \tag{65}$$

The functions H_1, H_2 and K are given by

$$H_1 = 1 + \frac{Q_1}{\left(4Q_{\mathrm{N}}|\vec{x}| + \dfrac{4Q_2}{9}z^3\right)^{5/3}}, \quad H_2 = Q_2 z, \quad K = \frac{Q_{\mathrm{N}}}{|\vec{x}|}. \tag{66}$$

It is straightforward to verify that the near-horizon structure of this system is of the form (64). The solution can be illustrated by Table XVII.

VII. CONCLUSION

In this paper, we obtain various AdS spacetimes warped with certain internal spaces in 11-dimensional and type IIA and IIB supergravities. These solutions arise as the near-horizon geometries of more general semi-localized multi-intersections of M-branes in $D=11$ or NS-NS branes or D-branes in $D=10$. We achieve this by noting that any bigger sphere (AdS spacetime) can be viewed as a foliation involving S^3 (AdS_3). Then the S^3 (AdS_3) can be replaced by a three-dimensional lens space (BTZ black hole), which arises naturally from the introduction of a NUT (pp wave). We can then perform a Kaluza-Klein reduction or Hopf T-duality transformation on the fiber coordinate of the lens space (BTZ black hole).

It is important to note that the warp factor depends only on the internal foliation coordinate but not on the lower-dimensional spacetime coordinates. This implies the possibility of finding a larger class of consistent dimensional reduction of 11-dimensional or type IIA and IIB supergravities on the internal space, giving rise to gauged supergravities in lower dimensions with AdS vacuum solutions. The first such example was obtained in [7]. In this paper, we obtain further examples for possible consistent embeddings of lower-dimensional gauged supergravity in $D=11$ and $D=10$. For example, we obtain vacuum solutions for the embedding of the six- and four-dimensional gauged AdS supergravities in type IIB theory and for the embedding of the seven-dimensional gauged AdS supergravity in type IIA theory.

ACKNOWLEDGMENTS

C.N.P. would like to thank the Caltech-USC Center for Theoretical Physics and Imperial College, London, for hospitality during the course of this work. The research of M.C., H.L., and J.F.V.-P. was supported in part by DOE Grant No. DE-FG02-95ER40893. The research of C.N.P. was supported in part by DOE Grant No. DE-FG03-95ER40917.

APPENDIX A: SPHERES AND AdS SPACETIME FROM FOLIATIONS

There are two closely parallel constructions which arise in the various intersections involving NUTs and waves. The former involves a construction of the unit metric on the sphere S^{p+q+1} as a foliation of $S^p \times S^q$ surfaces, while the latter involves an analogous construction of the unit metric on AdS_{p+q+1}, as a foliation of $\mathrm{AdS}_p \times S^q$ surfaces.

Consider first the construction of the unit S^{p+q+1} metric. We start from the unit metrics $d\Omega_p^2 = dX^i dX^i$ and $d\Omega_q^2 = dY^a dY^a$ on the spheres S^p and S^q, defined as the surfaces

$$X^i X^i = 1, \quad Y^a Y^a = 1 \tag{A1}$$

in $\mathbb{R}^{p+1}$ and $\mathbb{R}^{q+1}$ respectively. We now introduce Cartesian coordinates $Z^A = (Z^i, Z^a)$ in $\mathbb{R}^{p+q+2}$, defined by

$$Z^i = X^i \cos\alpha, \quad Z^a = Y^a \sin\alpha, \tag{A2}$$

and so $Z^A Z^A = 1$, thus defining a unit sphere S^{p+q+1} in $\mathbb{R}^{p+q+2}$. Clearly Eq. (A2) defines a complete parametrization of points in $\mathbb{R}^{p+q+2}$, with $0 \leq \alpha \leq \frac{1}{2}\pi$, and so α and the constrained coordinates x^i and y^a on the spheres S^p and S^q provide coordinates for the unit sphere S^{p+q+1} with a manifest $SO(p+q+2)$ isometry group action on the Z^A coordinates. The metric on S^{p+q+1} is given by $d\Omega_{p+q+1}^2 = dZ^A dZ^A$, and so from the above definitions we obtain

$$d\Omega_{p+q+1}^2 = d\alpha^2 + \cos^2\alpha\, d\Omega_p^2 + \sin^2\alpha\, d\Omega_q^2. \tag{A3}$$

The foliating surfaces at a fixed value of the "latitude" coordinate α are $S^p \times S^q$, with radii $\cos\alpha$ and $\sin\alpha$ for the two factors. The construction is a generalization of the Clifford torus $S^1 \times S^1$ foliating S^3.

In a similar manner, one can construct a metric $d\omega_{p+q+1}^2$ on the unit AdS_{p+q+1} as follows. We start from a unit AdS_p, with metric $d\omega_p^2 = dX^\mu dX^\nu \eta_{\mu\nu}$, and a unit S^q with metric $d\Omega_q^2 = dY^a dY^a$, where the coordinates X^μ on $\mathbb{R}^{p+1}$ satisfy the indefinite-signature condition

$$X^\mu X^\nu \eta_{\mu\nu} = -1, \quad \eta_{\mu\nu} = \mathrm{diag}(-1,-1,1,1,\ldots,1), \tag{A4}$$

while the coordinates Y^a on $\mathbb{R}^{q+1}$ satisfy $Y^a Y^a = 1$ as before. We now define coordinates $Z^A = (Z^\mu, Z^a)$ by

$$Z^\mu = X^\mu \cosh\rho, \quad Z^a = Y^a \sinh\rho, \tag{A5}$$

which therefore satisfy

$$Z^A Z^B \eta_{AB} = -1, \quad \eta_{AB} = \mathrm{diag}(-1,-1,1,1,\ldots,1). \tag{A6}$$

The coordinates Z^A, subject to this constraint, therefore define AdS_{p+q+1}, with a manifest $SO(p+q-1,2)$ isometry. The metric $d\omega_{p+q+1}^2 = dZ^A dZ^B \eta_{AB}$ is given by

$$d\omega_{p+q+1}^2 = d\rho^2 + \cosh^2\rho\, d\omega_p^2 + \sinh^2\rho\, d\Omega_q^2. \tag{A7}$$

APPENDIX B: NUTs WITHOUT NUTs

In this appendix, we show explicitly that the semi-localized intersection of a p-brane with a Kaluza-Klein monopole (a NUT) can be recast, after appropriate coordinate transformations, as a restricted class of ordinary distributed p-branes. For definiteness, we take the case of a semi-localized intersection of the M2-brane with a NUT as an example. The analysis for the other cases is essentially identical.

The semi-localized solution obtained in [6] is given by

$$ds_{11}^2 = H^{-2/3} dw^\mu dw_\mu + H^{1/3}[(dx_1^2 + \cdots + dx_4^2) + K(dz_1^2 + dz_2^2 + dz_3^2) + K^{-1}(dy + A_i dz_i)^2],$$

$$K = \frac{Q_{\mathrm{N}}}{|\vec{z}|}, \quad A_i dz_i = Q_{\mathrm{N}} \cos\theta\, d\varphi,$$

$$H = 1 + \sum_k \frac{Q_k}{(|\vec{x} - \vec{x}_{0k}|^2 + 4Q_{\mathrm{N}}|\vec{z}|)^3}, \tag{B1}$$

where Q_k denotes the M2-brane charge located at $\vec{x}_{0k}$, Q_{N} is the NUT charge, and we take

$$(z_1, z_2, z_3) = \frac{R^2}{4Q_{\mathrm{N}}}(\sin\theta\cos\varphi, \sin\theta\sin\varphi\cos\theta). \tag{B2}$$

It now follows that the part of the metric

$$d\overline{s^2} \equiv K(dz_1^2 + dz_2^2 + dz_3^2) + K^{-1}(dy + A_i dz_i)^2 \tag{B3}$$

is nothing but the locally flat metric

$$d\overline{s^2} = dR^2 + R^2 d\overline{\Omega}_3^2, \tag{B4}$$

where

$$d\overline{\Omega}_3^2 \equiv \frac{1}{4} d\Omega_2^2 + \frac{1}{4}\left(\frac{dy}{Q_{\mathrm{N}}} + \cos\theta\, d\varphi\right)^2 \tag{B5}$$

is the metric on the cyclic lens space $S^3/Z_{Q_{\mathrm{N}}}$. Locally, this is just the standard metric on the unit 3-sphere. Viewed as a $U(1)$ bundle over S^2 the coordinate y on the $U(1)$ fibers is taken always to have the period 4π. When $Q_{\mathrm{N}} = 1$, the topology is therefore precisely S^3. However, if Q_{N} is a larger integer, the fiber coordinate has a period that is smaller by the fraction $1/Q_{\mathrm{N}}$ than the period that would be needed for S^3 itself, and consequently the topology is $S^3/Z_{Q_{\mathrm{N}}}$.

The solution (B1) can therefore be recast as

$$ds_{11}^2 = H_2^{-2/3} dw^\mu dw_\mu + H_2^{1/3}(dx_1^2 + \cdots + dx_4^2 + d\tilde{z}_1^2 + \cdots + d\tilde{z}_4^2), \tag{B6}$$

with the harmonic function given by

$$H_2 = 1 + \sum_k \frac{Q_k}{(|\vec{x} - \vec{x}_{0k}|^2 + |\vec{\tilde{z}}|^2)^3}. \tag{B7}$$

The coordinates $\tilde{z}_i$ reside on $\mathbb{R}^4/Z_{Q_{\mathrm{N}}}$, and are related to R and the coordinates (θ,φ,y) on the lens space $S^3/Z_{Q_{\mathrm{N}}}$ by

$$\tilde{z}_1 + \mathrm{i}\tilde{z}_2 = R\sin\tfrac{1}{2}\theta\, e^{i(y/Q_{N+\varphi})/2},$$

$$\tilde{z}_3 + \mathrm{i}\tilde{z}_4 = R\cos\tfrac{1}{2}\theta\, e^{i(y/Q_{\mathrm{N}} - \varphi)/2}. \tag{B8}$$

In other words, if we make the following coordinate transformation from (z_1, z_2, z_3, y) to $(\tilde{z}_1, \tilde{z}_2, \tilde{z}_3, \tilde{z}_4)$,

$$\tilde{z}_1 + \mathrm{i}\tilde{z}_2 = \left[\frac{2Q_{\mathrm{N}}(r + z_3)(z_1 + \mathrm{i}z_2)}{\sqrt{z_1^2 + z_2^2}}\right]^{1/2} e^{(i/2Q_{\mathrm{N}})y},$$

$$\tilde{z}_3 + \mathrm{i}\tilde{z}_4 = \left[\frac{2Q_{\mathrm{N}}(r - z_3)(z_1 - \mathrm{i}z_2)}{\sqrt{z_1^2 + z_2^2}}\right]^{1/2} e^{(i/2Q_{\mathrm{N}})y}, \tag{B9}$$

where $r^2 \equiv z_1^2 + z_2^2 + z_3^2$, then the metric (B3) is seen to be nothing but

$$d\overline{s^2} = d\tilde{z}_1^2 + d\tilde{z}_2^2 + d\tilde{z}_3^2 + d\tilde{z}_4^2. \tag{B10}$$

The semi-localized M2-brane–NUT intersection (B1) can therefore be obtained by starting from a standard distribution of pure M2-branes (B6), with charges spread over only four of the eight transverse directions as in Eq. (B7). This is precisely equivalent to the semi-localized M2-brane–NUT intersection (B1) with unit NUT charge, $Q_{\mathrm{N}} = 1$. To obtain higher values of the NUT charge, one simply has to factor the $\mathbb{R}^4$ space of the $\tilde{z}_i$ coordinates by Z_{Q_N}, as defined above. Note that although this semi-localized way of introducing a NUT seems trivial, in that it amounts to a coordinate transformation, performing Kaluza-Klein reduction on the fiber coordinate does create a non-trivial intersecting component, since the Kaluza-Klein 2-form field strength now carries a non-trivial flux.

The above discussion carries over, *mutatis mutandis*, to the cases of the semi-localized M5-brane–NUT and D3-brane–NUT.

[1] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).
[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998).
[3] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[4] S. Ferrara, A. Kehagias, H. Partouche, and A. Zaffaroni, Phys. Lett. B **431**, 57 (1998).
[5] A. Brandenhuber and Y. Oz, Phys. Lett. B **460**, 307 (1999).
[6] D. Youm, ''Localized intersecting BPS branes,'' hep-th/9902208.
[7] M. Cvetič, H. Lü, and C. N. Pope, Phys. Rev. Lett. **83**, 5226 (1999).
[8] M. Cvetič, S. S. Gubser, H. Lü, and C. N. Pope, Phys. Rev. D **62**, 086003 (2000).
[9] Y. Oz, ''Warped compactifications and AdS/CFT,'' hep-th/0004009.
[10] M. Cvetič, H. Lü, and C. N. Pope, Nucl. Phys. **B549**, 194 (1999).
[11] M. J. Duff, H. Lü, and C. N. Pope, Nucl. Phys. **B544**, 145 (1999).
[12] M. Banados, C. Teitelboim, and J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992).
[13] M. J. Duff, H. Lü, and C. N. Pope, Nucl. Phys. **B532**, 181 (1998).
[14] M. J. Duff, H. L. Lü, and C. N. Pope, Phys. Lett. B **409**, 136 (1997).
[15] M. Cvetič and A. A. Tseytlin, Phys. Lett. B **366**, 95 (1996).
[16] M. Cvetič and A. A. Tseytlin, Phys. Rev. D **53**, 5619 (1996).
[17] A. A. Tseytlin, Mod. Phys. Lett. A **11**, 689 (1996).
[18] G. Horowitz and D. Marolf, Phys. Rev. D **55**, 3654 (1997).
[19] C. G. Callan, S. S. Gubser, I. R. Klebanov, and A. A. Tseytlin, Nucl. Phys. **B489**, 65 (1997).
[20] A. A. Tseytlin, Class. Quantum Grav. **14**, 2085 (1997).
[21] H. Lü and C. N. Pope, Int. J. Mod. Phys. A **13**, 4425 (1998).
[22] N. Itzhaki, A. A. Tseytlin, and S. Yankielowicz, Phys. Lett. B **432**, 298 (1998).
[23] H.-S. Yang, ''Localized intersecting brane solutions of $D=11$ supergravity,'' hep-th/9902128.
[24] A. Fayyazuddin and D. J. Smith, J. High Energy Phys. **04**, 030 (1999).
[25] A. Loewy, Phys. Lett. B **463**, 41 (1999).
[26] M. Cvetič, H. Lü, and C. N. Pope, Nucl. Phys. **B545**, 309 (1999).
[27] G. Papadopoulos, J. G. Russo, and A. A. Tseytlin, Class. Quantum Grav. **17**, 1713 (2000).
[28] K. Sfetsos and K. Skenderis, Nucl. Phys. **B517**, 179 (1998).
[29] A. Strominger and C. Vafa, Phys. Lett. B **379**, 99 (1996).
[30] M. Cvetič and F. Larsen, Phys. Rev. Lett. **82**, 484 (1999).