Masses, branching ratios, and full widths of heavy ρ', ρ'' and ω', ω'' resonances

N. N. Achasov* and A. A. Kozhevnikov†

Laboratory of Theoretical Physics, Sobolev Institute for Mathematics, 630090, Novosibirsk-90, Russia

(Received 20 July 2000; published 10 November 2000)

Based on previous and recent fits of multiple data on the reactions of e^+e^- annihilation, τ lepton decay, and the reaction $K^-p\to\pi^+\pi^-\Lambda$, the magnitude of the branching ratios and total widths of the isovector ρ', ρ'', and the isoscalar ω', ω'' resonances are calculated. Some topics on the spectroscopy of the $\rho(1450)$ and $\omega(1420)$ states are discussed.

PACS number(s): 13.65.+i, 13.25.Jx, 14.40.Cs

The situation with the resonances $\rho'\equiv\rho_1'\equiv\rho(1450)$, $\rho''\equiv\rho_2'\equiv\rho(1700)$, with isospin one and $\omega'\equiv\omega_1'\equiv\omega(1420)$, $\omega''\equiv\omega_2'\equiv\omega(1600)$ with isospin zero is still far from clear. Although the characteristic peaks corresponding to these resonances were observed in a number of channels of one-photon e^+e^- annihilation, τ lepton decays, $N\bar{N}$ annihilation, photoproduction, etc., the specific masses and branching ratios are not properly established and hence are not given in the Particle Data Group (PDG) tables [1]. Recently, the present authors undertook an attempt to fit then existing data on e^+e^- annihilation, τ lepton decays, and the reaction $K^-p\to\pi^+\pi^-\Lambda$, where the above heavy resonances were observed, in the framework of the unified approach. The approach is based on a scheme that takes into account both the energy dependence of the partial widths and the mixing via common decay modes among the latter and the ground state $\rho(770)$ and $\omega(782)$ resonances, in the respective isovector [2] and isoscalar [3] channels. Here we calculate the branching ratios and total widths of the $\rho_{1,2}'$ and $\omega_{1,2}'$ resonances using new data on the cross section of the reactions $e^+e^-\to\pi^+\pi^-\pi^0$ [4,5], $e^+e^-\to\omega\pi^0$ [6], and data on the spectral function v_1 measured in the decays $\tau^-\to\omega\pi^-\nu_\tau$, $\tau^-\to\pi^+\pi^-\pi^-\pi^0\nu_\tau$ [7]. We also comment on the issue of why the resonances with masses greater than 1400 MeV *should be* wide in the conventional $q\bar{q}$ model, and why their bare masses get shifted towards the greater values as compared to the visible peak positions. The latter point, as will be shown below, is of direct relevance to the hadronic spectroscopy of the $\rho(1300)$ resonance reported by the LASS group [8] and the $\omega(1200)$ resonance reported by the SND team [5].

As is known [1], the indications on the existence of the $\rho_{1,2}'$ and $\omega_{1,2}'$ resonances were obtained, in particular, in the VP channels, where V and P stand, respectively, for the vector and pseudoscalar mesons. It is also known that the typical magnitude of the VVP coupling constant is $g_{\omega\rho\pi}\simeq14.3$ GeV^{-1}. All other VVP coupling constants can be expressed through $g_{\omega\rho\pi}$ via the SU(3) Clebsch-Gordan coefficients or, equivalently, the quark model relations. As far as $V_{1,2}'VP$ coupling constants are concerned, the theoretical situation is unclear. Indeed, the QCD sum rule approach [9] used to evaluate $g_{\omega\rho\pi}$ [10] cannot give any prediction, since the contributions of heavy resonances are suppressed due to the Borel transformation. The quark model predictions are rather uncertain; e.g., Ref. [11] predicts $|g_{\rho_1'\omega\pi}|=4$–24 GeV^{-1}, $|g_{\rho_2'\omega\pi}|=3$–5 GeV^{-1}, and [12] predicts $|g_{\rho_1'\omega\pi}|\approx6$ GeV^{-1}, $|g_{\rho_2'\omega\pi}|\approx2$ GeV^{-1}, while [13] gives $|g_{\rho_1'\omega\pi}|\approx5$ GeV^{-1}. Hence, we did not rely on any specific value of the $V_{1,2}'VP$ coupling constants in Refs. [2,3] and took them to be free parameters. Their extracted values turned out to be in the intervals $|g_{\rho_1'\omega\pi}|=10$–$18$ GeV^{-1}, and $|g_{\rho_2'\omega\pi}|=2$–13 GeV^{-1}, see the details in [2,3]. Qualitatively, the relation $|g_{\rho_{1,2}'\omega\pi}|\sim|g_{\omega_{1,2}'\rho\pi}|$ was found to be satisfied. One can see that there are no reasons to expect, both theoretically and phenomenologically, that the $V_{1,2}'VP$ coupling constants should be drastically suppressed as compared to the VVP ones. Hence, taking $g_{\rho_{1,2}'\omega\pi}\sim g_{\omega_{1,2}'\rho\pi}\simeq10$ GeV^{-1} and $m_{\rho_1'}\approx m_{\omega_1'}=1400$ MeV, one finds

$$\Gamma_{\rho_1'\to\omega\pi}=g^2_{\rho_1'\omega\pi}q^3_{\omega\pi}(m_{\rho_1'})/12\pi\sim280\text{ MeV},$$
$$\Gamma_{\omega_1'\to\rho\pi}=g^2_{\omega_1'\rho\pi}q^3_{\rho\pi}(m_{\omega_1'})/4\pi\sim820\text{ MeV}.\qquad(1)$$

Here $q_{bc}(m_a)$ is the momentum of the final particle b or c in the rest frame system of particle a, in the decay $a\to b+c$, and the three isotopic modes in the $\omega_1'\to\rho\pi$ decay are taken into account. To appreciate the rapid growth of the VP widths with energy, their evaluation, assuming the masses to be 1200 MeV, gives, respectively, 92 and 295 MeV. Analogously, assuming that $m_{\rho_2'}\approx m_{\omega_2'}=1750$ MeV, one finds

$$\Gamma_{\rho_2'\to\omega\pi}\sim880\text{ MeV},\quad\Gamma_{\omega_2'\to\rho\pi}\sim2600\text{ MeV}.\qquad(2)$$

Since the VP decay modes are not the only ones to which heavy resonances can decay [1], the resonances $\rho_{1,2}'$ and $\omega_{1,2}'$, in fact, *should* be rather wide. The large width of a resonance is one of the obstacles in its identification, because such a resonance often reveals itself as rather smooth feature in the energy behavior of the cross section. Of course, dividing the above adopted magnitude of coupling constants by a factor of 2 does not result in changing our qualitative conclusions concerning the $V_{1,2}'VP$ partial widths.

The second obstacle, as was pointed out in [2,3], is the shift of the resonance peak position from the input value of the resonance mass. Indeed, let us consider, for simplicity, the single resonance R with bare mass m_R observed in some channel f of e^+e^- annihilation, $e^+e^-\to R\to f$. Then the cross section of the above process can be written as

*Electronic address: achasov@math.nsc.ru

†Electronic address: kozhev@math.nsc.ru

TABLE I. Masses, total widths (in the units of MeV), leptonic widths (in the units of keV), and branching ratios (percent) of the ρ'_1 resonance, calculated using the coupling constants extracted from the fits of the specific channel [2]. The symbol $\sim$ means that the central value is given, with the errors exceeding it considerably. The ρ'_1 resonance does not reveal itself in the $e^+e^-\to\omega\pi^0$ reaction and τ^- decay.

Channel	$\pi^+\pi^-$ [a]	$\rho\eta$ [a]	$2\pi^+2\pi^-$ [a]	$\pi^+\pi^-2\pi^0$ [a]	$J/\psi\to 3\pi$	$K^-p\to\pi^+\pi^-\Lambda$
$m_{\rho'_1}$	1370^{+90}_{-70}	1460 ± 400	1350 ± 50	1400^{+220}_{-140}	1570^{+250}_{-190}	1360^{+180}_{-160}
$B_{\rho'_1\to\pi^+\pi^-}$	1.1 ± 1.1	~3.7	~1.4	~8.0	~0	~0.7
$B_{\rho'_1\to\omega\pi^0}$	86.5 ± 41.5	~56.9	93.6 ± 60.0	77.8 ± 62.2	66.5 ± 65.5	93.3 ± 82.7
$B_{\rho'_1\to\rho\eta}$ [b]	~5.6	~3.6	~5.0	~6.6	~13.2	~5.5
$B_{\rho'_1\to K^*K\bar{K}+cc}$ [b]	0	~4.3	0	~0.4	~15.0	0
$B_{\rho'_1\to 4\pi}$	~6.8	~31.5	~0.2	~7.2	~4.4	~0.7
$\Gamma_{\rho'_1\to l^+l^-}$	$6.4^{+1.2}_{-1.4}$	~13	$5.4^{+2.6}_{-1.8}$	$6.3^{+3.3}_{-2.5}$	–	–
$\Gamma_{\rho'_1}$	763 ± 500	~2222	~518	~970	~3444	~460

[a]This is the final state in e^+e^- annihilation.

[b]Calculated assuming SU(3) relations among the VVP coupling constants.

$$\sigma(s)=12\pi m_R^3\Gamma_{Rl^+l^-}(m_R)g^2_{Rf}\frac{s^{-3/2}W_{Rf}(s)}{(s-m_R^2)^2+s\Gamma_R^2(s)},\qquad(3)$$

where s is the square of the total center-of-mass energy, and $\Gamma_{Rl^+l^-}(m_R)$ is the leptonic width of the resonance evaluated at $\sqrt{s}=m_R$. The partial hadronic width of the decay $R\to f$ is represented in the form $\Gamma_{Rf}(s)=g^2_{Rf}W_{Rf}(s)$, where g_{Rf} is the coupling constant of R with final state f, and $W_{Rf}(s)$ is the dynamical phase space factor of the decay $R\to f$ that includes the possible resonance intermediate states as, for example, in the decay $\omega\to\rho\pi\to3\pi$. The total width of the resonance is $\Gamma_R(s)=\Sigma_f\Gamma_{Rf}(s)$. The peak position is given by the condition of the vanishing derivative of $\sigma(s)$ with respect to s:

$$s-m_R^2=(G(s))^{-1}\{1\pm[1+s\Gamma_R^2F(s)G(s)]^{1/2}\},\qquad(4)$$

where $G(s)=[\ln(s^{-3/2}W_{Rf})]'$, $F(s)=[\ln(s^{5/2}\Gamma_R^2/W_{Rf})]'$. Hereafter, $\Gamma_R\equiv\Gamma_R(s)$, $W_{Rf}\equiv W_{Rf}(s)$, and the prime denotes differentiation with respect to s. Equation (4) should match the usual expression $s-m_R^2=0$ in the limit of slow varying narrow width; hence the lower minus sign should be chosen in Eq. (4). The latter is still a very complicated equation for the determination of the peak position, so numerical methods should be invoked for its solution. However, all necessary qualitative conclusions can be drawn upon approximating the right hand side of this equation by taking its value at $s=m_R^2$. One can convince oneself that the dominant decay modes of heavy resonances have phase space factors growing faster than the decrease of the leptonic width as $s^{-3/2}$. Then the function $G(s)$ is positive. Also positive is the function $F(s)$. Hence, the factor following $G^{-1}(s)$ in Eq. (4) is negative. To first order in the derivative of the phase space volume, one finds from Eq. (4) the peak position s_R:

$$s_R\approx m_R^2-\tfrac{1}{2}s\Gamma_R^2F(s)|_{s=m_R^2}.\qquad(5)$$

One can see that in the case of a sufficiently narrow ($\Gamma_R<200$ MeV) resonance such as $\rho(770)$, $\omega(782)$, and $\phi(1020)$, the peak position, with good accuracy, coincides with the bare mass m_R.

The situation changes when the resonance is wide, as takes place in the case of the $\rho'_{1,2}$ and the $\omega'_{1,2}$ one. See Eqs. (1) and (2). The peak position is shifted towards the lower value as compared to the magnitude of the bare mass. This is just what was revealed in the fits [2,3]. The greater the width of the resonance width, the more it is shifted, so that, say, the ρ'_2 and ω'_2 resonances with bare masses around 1900 MeV are revealed as peaks at 1500–1600 MeV.

After these preliminary remarks let us present the results of the calculation of the branching ratios and full widths. As compared to Refs. [2,3], recent data on the reactions $e^+e^-\to\pi^+\pi^-\pi^0$ [4,5], $e^+e^-\to\omega\pi^0$ [6], and τ lepton decays [7] are used to extract the necessary resonance parameters. The

TABLE II. The same as in Table I, but in the case of the ρ'_2 resonance. The latter does not reveal itself in the $K^-p\to\pi^+\pi^-\Lambda$ reaction.

Channel	$\pi^+\pi^-$ [a]	$\omega\pi^0$ [a]	$\rho\eta$ [a]	$2\pi^+2\pi^-$ [a]	$\pi^+\pi^-2\pi^0$ [a]	$J/\psi\to3\pi$	$\tau^-\to(4\pi)^-\nu_\tau$
$m_{\rho'_2}$	1900^{+170}_{-130}	1710 ± 90	1910^{+1000}_{-370}	1851^{+270}_{-240}	1790^{+110}_{-70}	2080^{+160}_{-900}	1860^{+260}_{-160}
$B_{\rho'_2\to\pi^+\pi^-}$	~0	~0	~0.4	~1.2	~0.4	~0	1.5 ± 1.4
$B_{\rho'_2\to\omega\pi^0}$	~16.7	22.3 ± 8.0	~1.6	13.4 ± 3.9	31.0 ± 18.6	~28.4	18.9 ± 2.8
$B_{\rho'_2\to\rho\eta}$ [b]	~5.9	6.3 ± 2.0	~0.3	4.6 ± 1.4	9.6 ± 8.6	~11.5	10.3 ± 2.2
$B_{\rho'_2\to K^*\bar{K}+cc}$ [b]	~8.9	8.7 ± 2.8	~0.9	6.7 ± 2.0	14.0 ± 11.2	~17.8	6.8 ± 1.5
$B_{\rho'_2\to4\pi}$	~68.5	61.2 ± 7.8	~96.9	74.0 ± 32.1	45.0 ± 18.0	~42.2	62.6 ± 5.0
$\Gamma_{\rho'_2\to l^+l^-}$	1.8 ± 1.5	5.2 ± 1.5	~1.1	$4.02^{+0.28}_{-0.27}$	4.5 ± 1.3	–	9.3 ± 0.6 [c]
$\Gamma_{\rho'_2}$	~303.9	1886 ± 613	~3284	3123 ± 296	3151 ± 1281	~9386	3255 ± 388

[a]This is the final state in e^+e^- annihilation.

[b]Calculated assuming SU(3) relations among the VVP coupling constants.

[c]Found assuming the conserved vector current (CVC) relation between the spectral function and the combination of the e^+e^- annihilation cross sections; see Ref. [18].

TABLE III. Masses, total widths (in the units of MeV), leptonic widths (in the units of eV), and branching ratios (percent) of the $\omega'_{1,2}$ resonances, calculated using the coupling constants extracted from the fits of the specific channel of e^+e^- annihilation [3]. The symbol $\sim$ means that the central value is given, with the errors exceeding it considerably.

Channel	$\pi^+\pi^-\pi^0$	$\omega\pi^+\pi^-$	K^+K^-	$K_S^0K^\pm\pi^\mp$	$K^{*0}K^\mp\pi^\pm$
$m_{\omega'_1}$	1430^{+110}_{-70}	~ 1400	~ 1460	~ 1500	~ 1380
$B_{\omega'_1\to 3\pi}$	~ 21.6	~ 8	~ 67	~ 96	~ 34
$B_{\omega'_1\to K^*\bar{K}+cc}$	~ 0.2	~ 0	~ 1	~ 4	0
$B_{\omega'_1\to K^*\bar{K}\pi}$	~ 0	0	0	0	0
$B_{\omega'_1\to\omega\pi^+\pi^-}$	~ 78.2	~ 92	~ 31.2	~ 0	~ 65.8
$\Gamma_{\omega'_1\to l^+l^-}$	144^{+94}_{-58}	~ 0.2	~ 8	~ 8	~ 48
$\Gamma_{\omega'_1}$	~ 903	~ 129	~ 173	~ 1252	~ 112
$m_{\omega'_2}$	1940^{+170}_{-130}	2000 ± 180	1780^{+170}_{-300}	~ 2120	1880^{+600}_{-1000}
$B_{\omega'_2\to 3\pi}$	~ 22.1	~ 34.2	~ 88.8	~ 91.2	~ 60.1
$B_{\omega'_2\to K^*\bar{K}+cc}$	~ 3.5	~ 5.8	~ 11.2	~ 15.8	~ 8.9
$B_{\omega'_2\to K^*\bar{K}\pi}$	~ 68.2	~ 53.4	0	0	~ 30.9
$B_{\omega'_2\to\omega\pi^+\pi^-}$	~ 6.2	~ 6.6	0	0	~ 0
$\Gamma_{\omega'_2 l^+l^-}$	109^{+58}_{-46}	531 ± 225	0	~ 189	1162 ± 922
$\Gamma_{\omega'_2}$	~ 14000	~ 5757	~ 2420	~ 9854	~ 13820

results of the evaluation of the branching ratios and full widths are shown in Tables I, II, and III. One can see that the simple qualitative estimates displayed in Eqs. (1) and (2), assuming modes besides the *VP* one are included, agree with the results presented there.

Now, some remarks on the spectroscopy of the heavier vector mesons are in order. First, the $\rho(1300)$ state reported by the LASS detector team [8], who studied the reaction $K^-p\to\pi^+\pi^-\Lambda$, revived an old discussion concerning the possible existence of the $\rho(1250)$ meson, in addition to the $\rho(1450)$ claimed to be observed in e^+e^- annihilation. The results presented in Table I show that the corresponding peak observed by the LASS group should be attributed to the same state $\rho(1450)$ as that presented by the PDG [1].

A similar situation is with the state $\omega(1200)$ observed recently by the SND team in the reaction $e^+e^-\to\pi^+\pi^-\pi^0$ [5]. The new data on this reaction [4,5] are included in the fit done to consider the $\omega(1200)$ state in the framework of the approach [2,3], however, upon neglecting the contributions of the $\phi'_{1,2}$ resonances, because their couplings in the above reaction were found to be consistent with zero [3]. The parameters of the $\omega'_{1,2}$ resonances extracted from this new fit coincide within errors with those reported earlier [3]. They are used in filling the corresponding entries in Table III. The corresponding curves are plotted in Fig. 1. Our conclusion is that the $\omega(1200)$ state observed by SND [5] is the same state as $\omega(1420)\equiv\omega'_1$ presented by the PDG [1]. The shift of the visible peak as compared to the bare mass of the resonance should be attributed, as is explained above, to the rather large width and the rapid growth of the partial widths with the energy increase. As far as the ω'_2 resonance is concerned, its huge width (see Table II) results, in accordance with Eq. (5), in a shift of the peak to $\simeq 1600$ MeV from the bare mass $\simeq 1900$–2000 MeV. The large partial widths found in the analysis [2,3] are also in qualitative agreement with the expectations shown in Eqs. (1) and (2).

The recent data on the reaction $e^+e^-\to\omega\pi^0$ [6] have been included in the fit. The resonance parameters are found to agree within errors with those obtained in Ref. [2]. Specifically, the ρ'_1 resonance is not revealed, since its extracted parameters are consistent with zero within very large errors. Hence we exclude ρ'_1 from the fit, leaving only the $\rho(770)+\rho'_2$ contribution. The corresponding curve is shown in Fig. 2. When comparing each individual resonance contribution with the total one in Figs. 1, 2, and 3, one should bare in mind that the latter is not directly connected to the former ones, because the mixing among resonances via their common decay modes is very strong [2,3].

The joint fit of the CLEO data [7] results in a considerably improved accuracy of the determination of the ρ'_2 resonance parameters as compared to the earlier fit [2] of the ARGUS data [17]. Here also the ρ'_1 resonance is unnecessary, and the data are well described by the only ρ'_2 resonance in addition to the $\rho(770)$. The results are shown in

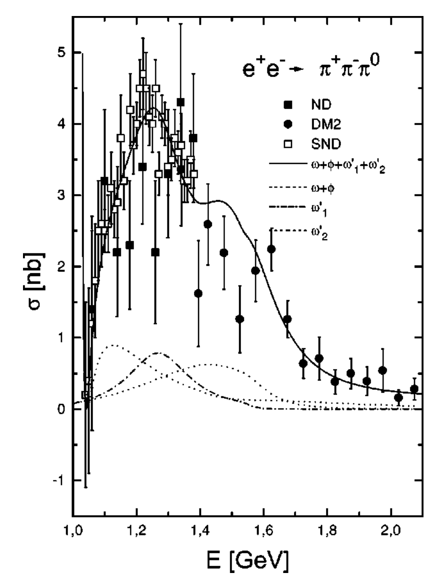

FIG. 1. The cross section of the reaction $e^+e^-\to\pi^+\pi^-\pi^0$ above 1 GeV. The data are SND [5], ND [14], DM2 [15].

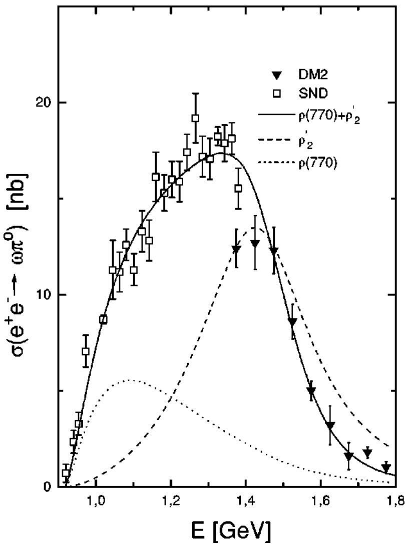

FIG. 2. The cross section of the reaction $e^+e^-\to\omega\pi^0$. The data are SND [6], DM2 [16].

Table II and Fig. 3. The shift of the visible peak towards the lower invariant mass of the 4π system around 1500 MeV is explained by the discussed effect exemplified by Eq. (5).

Our conclusions are as follows. First, the states $\rho'_{1,2}$ and $\omega'_{1,2}$ turn out to be wide resonance structures, as if the conventional quark picture of them as radial excitations is implied. In this respect, the present results, having in mind their significant uncertainties, do not contradict the assignment of ρ'_1 and ω'_1 resonances to the state 2^3S_1 [1,12]. In the meantime, the resonances ρ'_2 and ω'_2 are found to be wide, which contradicts assigning them to the state $1\,^3D_1$ predicted to be relatively narrow [12]. Second, one should be careful in attributing the specific peak or structure in the cross section to the specific spectroscopy state, because the large width, the rapid growth of the phase space with the energy increase, and the mixing among the resonances result in a shift of the visible peaks in the cross sections. Third, the very large widths of resonances found in this paper may indirectly give evidence in favor of some nonresonant contributions to the amplitudes. The accuracy of the existing data is still poor to isolate such contributions reliably. The forthcoming improvement of the accuracy of the data in the energy range 1400–2000 MeV will hopefully permit one to specify the above nonresonant contributions (if any) and to test the whole resonance interpretation of the high mass states.

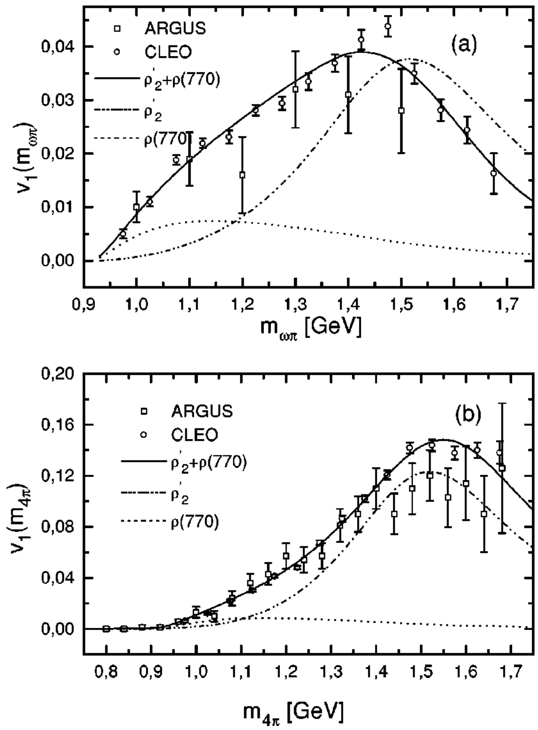

FIG. 3. The spectral function in the decay $\tau^-\to\omega\pi^-\nu_\tau$ (a) and $\tau^-\to\pi^+2\pi^-\pi^0\nu_\tau$ (b). The data are CLEO [7], ARGUS [17].

We are grateful to V. P. Druzhinin, S. I. Eidelman, E. V. Pakhtusova, and S. I. Serednyakov for discussions. The present work is supported in part by grant RFBR-INTAS IR-97-232.

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