PHYSICAL REVIEW D, VOLUME 62, 117301

Nuclear matrix element uncertainties in short range $0\nu\beta\beta$ decay

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(Received 5 May 2000; published 30 October 2000)

The evaluation of short range contributions to neutrinoless double beta decay has been challenged due to criticism of the ansatz of the nuclear matrix element calculations. We comment on the criticism and uncertainties of these calculations and the effect on the derived limits.

PACS number(s): 23.40.Hc, 21.30.−x, 21.60.−n

Neutrinoless double beta decay corresponds to the lepton-number converting process

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+2}X + 2e^{-}. \tag{1}$$

So far no positive signal for this decay has been observed, yielding the most stringent limit on the effective neutrino Majorana mass and neutrino-mediated contributions from R-parity violating supersymmetry (SUSY) and establishing this decay to be one of the most sensitive tools to search for particle physics beyond the standard model. In addition to these long range contributions, where the decay is triggered by the exchange of a light Majorana neutrino, also contributions due to heavy particle exchange (superheavy neutrinos and SUSY partners) have been discussed, and extremely stringent constraints on the effective superheavy neutrino mass $\langle m_H \rangle$ and R-parity violating coupling λ'_{111} have been published (for an overview and recent limits see [1,2]):

$$\langle m_H \rangle = \left| \sum_j \frac{U_{ej}^2}{m_j} \right|^{-1} > 9\times 10^7 \ \text{GeV}, \tag{2}$$

$$\lambda'_{111} \leqslant 4\times 10^{-4} \left(\frac{m_{\tilde{q}}}{100 \ \text{GeV}} \right)^2 \left(\frac{m_{\tilde{g}}}{100 \ \text{GeV}} \right)^{1/2}. \tag{3}$$

For comparison, a future linear collider with a center of mass energy of 1 TeV would be sensitive to 250 TeV$<\langle m_H \rangle <5000$ TeV, only [3] (for a serious discussion of the possibilities to observe inverse neutrinoless double beta decay at future colliders due to fine-tuned cancellations of mass eigenstates in the double beta decay observable see [4]). These latter conclusions from the $0\nu\beta\beta$ decay half-life limit have been challenged [5] concerning the matrix element calculations at short distances. In the following we will comment on these criticisms and uncertainties of these calculations and the effect on the derived limits.

The standard ansatz for nuclear matrix element calculations treats double beta decay in terms of nucleons of finite size with a hard core. The finite nucleon size effect is taken into account by nucleon form factors in momentum space [6]:

$$F(q^2) = F(0) \left(1 - \frac{q^2}{m_A^2} \right)^{-2}, \tag{4}$$

with $m_A = 0.85$ GeV. The form factors $F(0)$ used have been calculated treating the quarks in the MIT bag model [7]. The nucleon-nucleon repulsion at short distances is considered in two ways. First, the repulsion effect is included in the nucleon potential. In addition, to be conservative, the nucleon hard core is simulated with introducing a cutoff by multiplying the two particle wave functions by the correlation function [8]

$$1 - f(r) = 1 - e^{-ar^2}(1 - br^2). \tag{5}$$

The parameters a and b can be related to each other so that, effectively, there is one free parameter, the correlation length

$$l_c = -\int_0^{\infty} ds \{ [1 + f(r)]^2 - 1 \}. \tag{6}$$

The standard value of $l_c \simeq 0.7$ fm fits experimental data from nucleon-nucleon scattering. In this approach the total suppression of short range matrix elements compared to long range matrix elements with the same transition operator equals 1/20–1/30.

The dependence of short range nuclear matrix element calculations in the proton-neutron quasiparticle random phase approximation (pn-QRPA) model on the quantities m_A and l_C has been discussed extensively in [9] (for another recent calculation of the matrix elements involved, confirming the calculation in [9] with an accuracy of a factor of 2, see [10]). It has been shown that in this approach the main contribution to the matrix element comes from nuclear distances larger than 1 fm. The matrix elements are stable to variations of m_A and l_C, changes up to 50% of the standard values yield only comparable variations of the nuclear matrix elements. Although no guarantee—in the sense that the nucleon cannot be derived from QCD and no direct experimental test apart from comparison with data from nucleon-nucleon scattering is possible—exists that this approach is applicable for the case of heavy particle exchange, it was successful in predicting the matrix element of the (long range) standard model mode of double beta decay (two neutrino emitting decay) with an accuracy of $\sqrt{2}$ (compare Refs. [11,12]).

The criticism of Ref. [5] is based on the argument that for intermediate particle masses such as a heavy mass as discussed here the correct picture would be the quark rather than the nucleon picture. One should keep in mind, however, that the heavy exchanged particles are virtual, that the momenta transferred are much smaller, and that the quark dynamics are simulated by the effective treatment of nucleons with a form factor, hard core, and nucleon-nucleon interaction. The total suppression of short range transitions compared to long range transitions due to the quark-quark repulsion has been estimated in Ref. [5] to yield a suppression by a factor of 1/40 or less. This estimation is based on a spin singlet requirement to achieve an overall antisymmetric wave function ($\simeq 2/3$), the color Coulomb repulsion of the involved d quarks ($\simeq 1/3$ estimated by a WKB evaluation of the color Coulomb barrier) and a similar factor from the interaction of the remaining two quarks in the nucleus, which is justified by the picture that each of the two decaying d quarks is ''pulled on by a u and a d quark from its own nucleon,'' the latter being estimated to be $\simeq (1/3)^2$ or less. Whether attracting interactions between quarks belonging to the other nucleon changes this picture is not discussed in Ref. [5]. Also effects of the nuclear environment may change this picture and are totally ignored in this estimation. While this estimation is not based on an approach which is generally accepted (Ref. [5] from 1996 is not published yet), the total suppression factor of 1/40 argued confirms the order of magnitude of the suppression of short range matrix elements compared to long range matrix elements in the pn-QRPA approach, 1/20–1/30. However, Ref. [5] incorrectly applied this suppression factor to the limits derived with the pn-QRPA short range matrix elements and in this way considered the suppression factor 2 times. Moreover, old experimental limits have been used in the comparison of double beta decay and the inverse process.

In fact to our knowledge the only serious attempt at a calculation based on a relativistic quark model (see Ref. [13]) confirms matrix element calculations in the standard approach with an accuracy of a factor of 3. It should be stressed also that other decay modes, e.g., with pion exchange between the nucleons [14] and multiquark clusters in the nucleus [15], have been considered, yielding similar results. We therefore assume it to be rather premature to classify (as in [3]) all matrix elements calculated for heavy particle exchange as ''old'' in the sense of them being no longer valid.

If one in spite of these facts assumes (incorrectly) the estimated suppression of short range matrix elements from Ref. [3], the limit on the superheavy Majorana neutrino becomes 2000 TeV, still being competitive with a 1 TeV linear collider. For supersymmetric contributions in addition one has to take into account that the bound on the coupling scales with the square root of the nuclear matrix element, so that the estimated suppression would lead to a limit on λ'_{111} being worse only by a factor of order 5.

Summarizing, we commented on the criticisms of short range matrix element calculations for neutrinoless double beta decay. Since a real alternative based on a treatment in the quark picture is missing and in view of the lack of any reasonable estimation leading to considerably worse limits (i.e., more than a factor of 3), we find it useful to present as limits furthermore the results of the calculations in the nucleon picture. Moreover, even if one assumes the—clearly incorrect—estimation of Ref. [5], limits on SUSY are only worse by a factor of 5 and limits on superheavy neutrinos are still compatible with what could be obtained at future linear colliders. It should be stressed further that these criticisms do not concern the neutrinoless double beta decay contributions with light particle exchange yielding limits on light neutrino masses [16], R-parity violating SUSY [17], and leptoquarks [18] as well as violations of the equivalence principle and Lorentz invariance [19].

We thank M. Hirsch for useful discussions.

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