# Perturbative effects in the form factor $\gamma\gamma^* \to \pi^0$ and extraction of the pion distribution amplitude from CLEO data

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We study the pion form factor $F^{\pi\gamma\gamma^*}(Q^2)$ in the light-cone sum rule approach, accounting for radiative corrections and higher twist effects. Comparing the results to the CLEO experimental data on $F^{\pi\gamma\gamma^*}(Q^2)$, we extract the pion distribution amplitude of twist 2. The deviation of the distribution amplitude from the asymptotic one is small and is estimated to be $a_2(\mu) = 0.12 \pm 0.03$ at $\mu = 2.4$ GeV, in the model with one nonasymptotic term. The ansatz with two nonasymptotic terms gives some region of $a_2$ and $a_4$, which is consistent with the asymptotic distribution amplitude, but does not agree with some old models.

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## I. INTRODUCTION

The production process of one neutral pion by two virtual photons, $\gamma^*\gamma^* \to \pi^0$, plays a crucial role in the studies of exclusive processes in quantum chromodynamics. Being one of the simplest exclusive processes, it involves only one hadron and relates directly to the pion distribution amplitude [1]. At large photon virtualities, we can calculate the form factor using perturbative QCD and obtain important information on the shape of the pion distribution amplitude from the experimental data.

In general, the pion distribution amplitudes serve as input in the QCD sum rule method and allow the calculation of many form factors (for example heavy-to-light form factors $B \to \pi$ and $D \to \pi$) and hadronic coupling constants (for example $g_{B*B\pi}$ and $g_{D*D\pi}$). We refer the reader to reviews [6,7] and recent studies [8].

Recently, the CLEO collaboration has measured the $\gamma\gamma^* \to \pi^0$ form factor. In this experiment, one of the photons is nearly on-shell and the other one is highly off-shell, with a virtuality in the range 1.5 GeV$^2$–9.2 GeV$^2$ [4][1]. We study the possibility of extracting the twist-2 pion distribution amplitude from the CLEO data.

The pion-photon transition has been the subject of many studies in the framework of perturbative QCD [9], lattice calculations [10] and QCD sum rule method [11,12,9,13].

In the present paper, we use the light-cone sum rule (LCSR) method for calculating the form factor of the process $\gamma\gamma^* \to \pi^0$. The method of LCSR has been suggested in [14,15] and consists of operator product expansion (OPE) on the light-cone [3,2,1] combined with the QCD sum rule technique [11].

The first attempt to calculate this form factor by using the LCSR method has been reported in [13]. The twist-2 and twist-4 contributions were calculated to leading order without accounting for perturbative QCD effects. The radiative QCD effects are usually large (20%) and allow us to fix the normalization scale dependence of involved parameters.

In this paper we analyze the LCSR for the form factor $\gamma\gamma^* \to \pi^0$ at next-to-leading order in $\alpha_S$ (NLO). We derive $\mathcal{O}(\alpha_S)$ radiative corrections to the spectral density of the twist-2 operator. We combine the twist-2 contribution at NLO with higher twist contributions (twist-4) in order to analyze the LCSR for the form factor of the process $\gamma\gamma^* \to \pi^0$ numerically.

Using CLEO experimental data, we extract the parameters of the twist-2 distribution amplitude. The distribution amplitude appears to be very close to the asymptotic distribution amplitude. The deviation of the distribution amplitude from the asymptotic one is small and is estimated to be

$$a_2(\mu) = 0.12 \pm 0.03 \quad \text{at} \quad \mu = 2.4 \text{ GeV},$$

using pion distribution amplitude with one non-asymptotic term. This result on $a_2$ agrees well with recent analysis on electromagnetic pion form factor [16].

The pion distribution amplitude with two non-asymptotic terms is also extracted, giving some region in the $a_2, a_4$-plane. Theoretical systematical uncertainties are dominant over experimental statistical ones and systematical uncertainties are strongly correlated, defining the allowed parameter space of $a_2, a_4$. The extracted region for the distribution amplitude is in qualitative agreement with results derived in [17–19,16], where it has been claimed that the pion distribution amplitude is very close to the asymptotic form. However, our region does not overlap with the pion distribution amplitudes suggested in [20] and [3].

The paper is organized as follows. In the next section we discuss the general framework of calculating the $\pi\gamma\gamma^*$ form factor with LCSR. In Sec. III we calculate a spectral density at leading order (LO) and next LO (NLO) and present the final sum rule at NLO. In Sec. IV we perform a numerical

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[1]There exist also older results by the CELLO Collaboration [5].

analysis and discuss a procedure to extract the parameters $a_2$ and $a_4$ including the estimation of the systematic and statistical uncertainties.

## II. THE METHOD OF CALCULATION

We start with the correlator of two vector currents $j_{\mu/\nu} = (\frac{2}{3}\bar{u}\gamma_{\mu/\nu}u - \frac{1}{3}\bar{d}\gamma_{\mu/\nu}d)$:

$$\int d^4x e^{-iq_1x}\langle\pi^0(0)|T\{j_\mu(x)j_\nu(0)\}|0\rangle$$
$$= i\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta F^{\pi\gamma^*\gamma^*}(q_1,q_2), \quad (1)$$

where $q_1,q_2$ are the momenta of the photons. If both virtualities $s_1=q_1^2$ and $s_2=q_2^2$ are large and Euclidean, $-s_1,-s_2\gg\Lambda_{QCD}$, the correlator can be expanded near the light-cone ($x^2\to0$). The leading twist-2 contribution to the correlator (1) is [1]

$$F^{\pi\gamma^*\gamma^*}(s_1,s_2)=\frac{\sqrt{2}f_\pi}{3}\int_0^1\frac{du\,\varphi_\pi(u)}{s_2(u-1)-s_1u}, \quad (2)$$

where $\varphi_\pi(u)$ is the pion distribution amplitude of twist-2 defined through

$$\langle\pi(q)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle=-iq_\mu f_\pi\int_0^1 du\,\varphi_\pi(u)e^{i\,uqx}. \quad (3)$$

In principle, the higher twist contributions can be calculated using the OPE on the light cone.

However, in the CLEO experimental data, one of the virtualities is small, i.e. $s_2\to0$. A straightforward operator product expansion (OPE) calculation is not possible and we have to use analyticity and duality arguments.

Since the form factor $F^{\pi\gamma^*\gamma^*}(s_1,s_2)$ is an analytical function in both variables, we can write the form factor as a dispersion relation in $s_2$:

$$F^{\pi\gamma^*\gamma^*}(s_1,s_2)=\frac{\sqrt{2}f_\rho F^{\rho\pi}(s_1)}{m_\rho^2-s_2}+\int_{s_0}^\infty ds\frac{\rho^h(s_1,s)}{s-s_2}. \quad (4)$$

The physical ground states are $\rho$ and $\omega$ vector mesons. We use the zero-width approximation and define the matrix elements of electromagnetic currents, assuming isospin symmetry: $m_\rho\simeq m_\omega$; $\frac{1}{3}\langle\pi^0(p)|j_\mu|\omega(q_2)\rangle\simeq\langle\pi^0(p)|j_\mu|\rho^0(q_2)\rangle$ $=(1/m_\rho)\epsilon_{\mu\nu\alpha\beta}e^\nu q_1^\alpha q_2^\beta F^{\rho\pi}(s_1)$; $3\langle\omega|j_\nu|0\rangle\simeq\langle\rho^0|j_\nu|0\rangle$ $=(f_\rho/\sqrt{2})m_\rho e_\nu^*$. Here $e_\nu$ is the polarization vector of the $\rho$ meson; $f_\rho$ is its decay constant.

The spectral density of higher energy states $\rho^h(s_1,s)$ is derived from the QCD-calculated expression for $F_{QCD}^{\pi\gamma^*\gamma^*}(s_1,s)$ by using usual semilocal quark-hadron duality for $s>s_0$

$$\int ds(\text{any function})\rho^h(s_1,s)$$
$$=\int ds(\text{any function})\frac{1}{\pi}\text{Im}\,F_{QCD}^{\pi\gamma^*\gamma^*}(s_1,s). \quad (5)$$

We equate the dispersion relation (4) with the QCD expression at large $s_2$. Using the dispersion relation for the QCD function $F_{QCD}^{\pi\gamma^*\gamma^*}(s_1,s_2)$, we obtain

$$\frac{\sqrt{2}f_\rho F^{\rho\pi}(s_1)}{m_\rho^2-s_2}=\frac{1}{\pi}\int_0^{s_0}ds\frac{\text{Im}\,F_{QCD}^{\pi\gamma^*\gamma^*}(s_1,s)}{s-s_2}. \quad (6)$$

The next step is to perform a Borel transformation in $s_2$. We finally get the LCSR for the form factor $F^{\rho\pi}(s_1)$:

$$\sqrt{2}f_\rho F^{\rho\pi}(s_1)=\frac{1}{\pi}\int_0^{s_0}ds\,\text{Im}\,F_{QCD}^{\pi\gamma^*\gamma^*}(s_1,s)e^{(m_\rho^2-s)/M^2}, \quad (7)$$

where $M$ is a Borel parameter.

Substituting Eq. (7) and the duality approximation (5) into Eq. (4) and taking the $s_2\to0$ limit we obtain the sum rule for the form factor $F^{\gamma\gamma^*\pi}(s_1)$ [13]:

$$F^{\pi\gamma\gamma^*}(s_1)=\frac{1}{\pi m_\rho^2}\int_0^{s_0}ds\,\text{Im}\,F_{QCD}^{\pi\gamma^*\gamma^*}(s_1,s)e^{(m_\rho^2-s)/M^2}$$
$$+\frac{1}{\pi}\int_{s_0}^\infty\frac{ds}{s}\text{Im}\,F_{QCD}^{\pi\gamma^*\gamma^*}(s_1,s). \quad (8)$$

We will use this expression as our basic sum rule for the numerical analysis.

## III. BORN TERM AND QCD RADIATIVE CORRECTION

The next step is to calculate the spectral density at LO and NLO.

Calculating twist-2 contributions to $F^{\pi\gamma^*\gamma^*}(s_1,s_2)$, only one distribution amplitude enters, the pion distribution amplitude, $\varphi_\pi(u)$, defined by (3). As a result, the $F^{\pi\gamma^*\gamma^*}(s_1,s_2)$ can be written as the convolution of the hard amplitude $T(s_1,s_2,u)$ and the distribution amplitude $\varphi_\pi(u)$:

$$F^{\pi\gamma^*\gamma^*}(s_1,s_2)=f_\pi\int_0^1 du\,\varphi_\pi(u)T(s_1,s_2,u). \quad (9)$$

The hard amplitude—it plays the role of the Wilson coefficient in OPE—is calculable within perturbative theory, the pion distribution amplitude $\varphi_\pi(u)$ contains the long-distance effects.

The theoretical spectral density at $s_2>0$ and $s_1<0$ can be calculated from

$$\rho_{QCD}(s_1,s_2) = \frac{1}{\pi} \mathrm{Im}_{s_2} F^{\pi\gamma^*\gamma^*}(s_1,s_2)$$

$$= f_\pi \int_0^1 du\, \varphi_\pi(u) \frac{1}{\pi} \mathrm{Im}_{s_2} T(s_1,s_2,u). \tag{10}$$

The contribution of twist-2 at LO approximation (2) [1] is

$$T_{QCD}^{LO}(s_1,s_2,u) = \frac{\sqrt{2}}{3} \frac{1}{s_2(u-1)-s_1 u} \quad \text{and}$$

$$\rho_{QCD}^{LO} = \frac{\sqrt{2} f_\pi}{3(s_2-s_1)} \varphi_\pi(u_0), \tag{11}$$

where $u_0 = s_2/(s_2-s_1)$.

The $\mathcal{O}(\alpha_s)$ correlation function for $\gamma^*\gamma^* \to \pi^0$ was considered in [21] (see also [22,23]). Here we use this result and present it in a form which is useful for our further calculations ($c = [\sqrt{2} f_\pi/3(s_2-s_1)][\alpha_s(\mu) C_F/2\pi]$):

$$T_{QCD} = T_{QCD}^{LO} + T_{QCD}^{NLO}; \tag{12}$$

$$T_{QCD}^{NLO}(s_1,s_2,u) = c(a_0 L_0 + a_1 L_1 + a_2 L_2), \quad \text{with}$$

$$L_n = \frac{\log^n(u-u_0)}{u-u_0}. \tag{13}$$

The coefficients $a_0$ are expanded in terms of $\log^n(-u_0)$:

$$a_0 = b_0 l_0 + b_1 l_1 + b_2 l_2 \quad \text{with } l_n = \log^n(-u_0). \tag{14}$$

The coefficients are

$$b_2 = \frac{-s_2^2 u + s_2 s_1(1+u)}{2u(s_2-s_1)^2}, \tag{15}$$

$$b_1 = -2 b_2 (L_\mu+1) - \frac{s_2}{2(s_2-s_1)u},$$

$$b_0 = L_u^2 b + L_u \left( \frac{s_1}{2(s_2-s_1)(u-1)} - 2(L_\mu+1) b \right)$$

$$- \frac{3}{2}(3+L_\mu),$$

$$a_2 = \frac{1}{2} + \frac{s_2 s_1}{2(s_2-s_1)^2 u(u-1)},$$

$$a_1 = -2 a_2 L_\mu$$

$$+ \frac{s_2^2(1-u)^2 + s_1^2 u^2 + s_2 s_1(-3+2u-2u^2)}{2(s_2-s_1)^2(u-1)u},$$

with

$$L_\mu = \log\left(\frac{\mu^2}{s_2-s_1}\right); \quad L_u = \log(1-u_0);$$

$$b = \frac{s_1^2(1-u) + s_2 s_1(u-2)}{2(s_2-s_1)^2(u-1)}.$$

Now we take the imaginary part of this expression in the energy region $s_1 < 0$ and $s_2 > 0$. The nontrivial imaginary part comes from the functions $L_n$ and $l_n$. We collect all useful formulas in Appendix A. The combined result is

$$\frac{1}{\pi} \mathrm{Im}\, T_{QCD}^{NLO}(s_1,s_2,u) = -c \left( \mathrm{P}\left(\frac{1}{u-u_0}\right) (b_1 + 2 \log(u_0) b_2) - \delta(u-u_0) \right.$$

$$\times \left[ b_0 + \log(u_0)(b_1+a_1) + \log^2(u_0)(b_2+a_2) - \pi^2\left(b_2 + \frac{1}{3} a_2\right) \right]$$

$$\left. + \Theta(u_0-u)\left[ a_1\left(\frac{1}{u-u_0}\right)_+ + a_2\left(\frac{2\log|u_0-u|}{u-u_0}\right)_+ \right] \right), \tag{16}$$

where we use the $()_+$ operation, which is defined inside the integral as

$$\int du\, f(u)(g(u,u_0))_+ = \int du\, (f(u)-f(u_0))g(u,u_0).$$

We have checked that the dispersion integral with the imaginary part (16) reproduces the hard amplitude (13).

The expression (16) is universal and could be used with any distribution amplitude. In the present paper we expand the distribution amplitude in Gegenbauer polynomials, keeping the first three terms (see also next section for details):

$$\varphi_\pi^{tw2}(u,\mu) = 6 u(1-u)[1 + a_2(\mu) C_2^{3/2}(2u-1) + a_4(\mu) C_4^{3/2}(2u-1) + \cdots]. \tag{17}$$

The direct integration over variable $u$ gives the theoretical spectral density

$$\rho_{QCD}^{NLO}(s_2,s_1,\mu)=\frac{\sqrt{2}f_\pi}{3}\frac{\alpha_s(\mu)C_F}{2\pi}[A_0(s_2,s_1)+a_2(\mu)A_2(s_2,s_1,\mu)+a_4(\mu)A_4(s_2,s_1,\mu)], \tag{18}$$

with the coefficients

$$A_0=\frac{s_2 s_1}{(s_2-s_1)^3}\left[-15+\pi^2-3\log\left(-\frac{s_2}{s_1}\right)^2\right], \tag{19}$$

$$A_2=-\frac{s_2}{4(s_2-s_1)^5}\left(-25s_2^3-8(95+3\pi^2)s_2^2 s_1-36(25+2\pi^2)s_2 s_1^2-12(5+2\pi^2)s_1^3\right.$$
$$\left.+12s_1(s_2^2+3s_2s_1+s_1^2)\left[-25\log\left(\frac{\mu^2}{s_2}\right)+6\log\left(-\frac{s_2}{s_1}\right)^2\right]\right),$$

$$A_4=-\frac{s_2}{10(s_2-s_1)^7}\left(-91s_2^5-2(5413+75\pi^2)s_2^4 s_1-125(541+12\pi^2)s_2^3 s_1^2-100(901+30\pi^2)s_2^2 s_1^3\right.$$
$$-150(193+10\pi^2)s_2 s_1^4-15(109+10\pi^2)s_1^5+30s_1(s_2^4+10s_2^3 s_1+20s_2^2 s_1^2+10s_2 s_1^3+s_1^4)$$
$$\left.\times\left[-91\log\left(\frac{\mu^2}{s_2}\right)+15\log\left(-\frac{s_2}{s_1}\right)^2\right]\right).$$

As a result the asymptotic contribution of twist-2 operators at NLO has a very simple form

$$\rho_{QCD}=\frac{2\sqrt{2}f_\pi s_2 s_1}{(s_2-s_1)^3}\left(1+\frac{\alpha_s(\mu)C_F}{12\pi}\right.$$
$$\left.\times\left[-15+\pi^2-3\log\left(-\frac{s_2}{s_1}\right)^2\right]\right). \tag{20}$$

We note here that the spectral density contains double logarithms $\alpha_s\log(-s_2/s_1)^2$. Numerically, these are moderate in our LCSR with $s_2\approx M^2\approx s_0=\mathcal{O}(1\text{ GeV}^2)$ and $-s_1<10$ GeV$^2$ (as in the case of CLEO data). For higher virtualities, $-s_1\gg10$ GeV$^2$, the resummation of the double logarithms will be necessary.

Finally, we obtain LCSR by combining the general expression (8) with the formulas for the twist-2 spectral density at NLO derived in this paper (16), (18), (19) with the twist-4 contribution taken from the literature [13].

### IV. NUMERICAL RESULTS

We use the following parameters from the Particle Data Group [24] for the numerical analysis: $f_\pi=132$ MeV, $f_\rho=216$ MeV and $m_\rho=770$ MeV [24]. For the running of the QCD coupling constant $\alpha_S$, we use the two-loop expression with $N_f=3$ and $\Lambda^{(4)}=380$ MeV which corresponds to $\alpha_S(M_Z)=0.118$ [24], after matching twice the QCD coupling constant at the quark-antiquark thresholds $\mu_{c\bar{c}}=2.4$ GeV and $\mu_{b\bar{b}}=10$ GeV.

The distribution amplitude $\varphi_\pi$ can be expanded in terms of Gegenbauer polynomials $\Psi_n(u)=6u(1-u)C_n^{3/2}(2u-1)$. Arguments based on conformal spin expansion [20] allow us

to neglect higher terms in this expansion. We adopt here the ansatz which consists of three terms, assuming that the terms $a_{n>4}$ are small

$$\varphi_\pi(u,\mu_0)=\Psi_0(u)+a_2(\mu_0)\Psi_2(u)+a_4(\mu_0)\Psi_4(u). \tag{21}$$

The asymptotic distribution amplitude $\varphi_\pi(u)=6u(1-u)$ is unambiguously fixed [1]. The terms $n>0$ describe non-asymptotic corrections.

We may consider two different approaches to confront the sum rule with the experimental data. The first possibility is to use the existing values for $a_2$, $a_4$ [20,3] in our sum rule and to compare the calculated form factor with the CLEO experimental data. The second approach is to treat $a_2$, $a_4$ as unknown parameters and to extract them from the CLEO data. We shall discuss both possibilities, starting with the first one.

#### A. The asymptotic, BF- and CZ-distribution amplitudes

Previous extractions of the parameters $a_2$, $a_4$ can be divided into three classes: (1) Chernyak and Zhitnitsky (CZ) obtained the coefficients $a_2(\mu_0)=2/3$ and $a_4(\mu_0)=0$ [3] at the scale $\mu_0=0.5$ GeV; (2) Braun and Filyanov (BF) extracted the coefficients $a_2(\mu_0)=0.44$ and $a_4(\mu_0)=0.25$ [20] at the scale $\mu_0=1$ GeV; (3) some groups argued that the wave function is very close to the asymptotic one, the coefficients are very close to $a_2=0$ and $a_4=0$ (see for example [12,9,17–19]).

The QCD evaluation of $a_2$ and $a_4$ at different scales $\mu$ gives the values in Table I. Now we calculate the form factor using these values. We use the normalization point $\mu=2.4$ GeV, which corresponds to the virtuality of the photons in the central region of the experimental data. This scale

TABLE I. The pion wave function parameters $a_2, a_4$ at different scales $\mu$, for the Braun-Filyanov (BF) and Chernyak-Zhitnitsky (CZ) models.

| $\mu$ | 0.7 GeV | | 1.0 GeV | | 1.5 GeV | | 2.4 GeV | |
|---|---|---|---|---|---|---|---|---|
| | $a_2$ | $a_4$ | $a_2$ | $a_4$ | $a_2$ | $a_4$ | $a_2$ | $a_4$ |
| BF [11] | — | — | 0.44 | 0.25 | 0.33 | 0.17 | 0.28 | 0.13 |
| CZ [3] | 2/3 | 0 | — | — | 0.33 | -0.008 | 0.28 | -0.009 |

is also used in many $B \to \pi$ calculations [8]. The lower scale $\mu = 1.5$ GeV is taken in order to check the sensitivity of the results to the variation of $\mu$.

The sum rule depends on the Borel mass $M$ and the threshold energy $s_0$. The dependence on both parameters is small. The variation of $s_0$ by 20% gives deviations in the form factor of less than 2%. Figure 1 shows the Borel mass dependence of the form factor at various $Q^2$ ($Q^2 = -s_1$). We note that the dependence is very small for virtualities around $Q^2 \approx 2$–3 GeV$^2$, where the experimental data is concentrated. For other $Q^2$ the Borel dependence is still moderate ($\pm 2$–4%), showing a good quality of the sum rule. We use in the calculations $s_0 = 1.5$ GeV$^2$ and $M^2 = 0.7 \pm 0.2$ GeV.

Now we are ready to compute the form factor using BF, CZ and asymptotic distribution amplitudes. In Fig. 2 we show the contributions of the asymptotic and nonasymptotic parts of the distribution amplitude at $\mu = 1.5$ and 2.4 GeV; the parameters $a_2$ and $a_4$ are normalized to 1 in the whole region of $Q^2$. The twist-4 parameter $\delta^2(\mu)$ is fixed at $\mu = 1$ GeV to be 0.2 and scaled by the renormalization group equation with the one-loop anomalous dimension. We see that the asymptotic contribution is too small in order to fit experimental data.

The results with BF- and CZ- distribution amplitudes are presented in Fig. 3 and are too large to describe CLEO data.

**B. Extraction of the distribution amplitude from CLEO data**

We use a numerical nonlinear fit procedure to estimate best values for Gegenbauer coefficients.
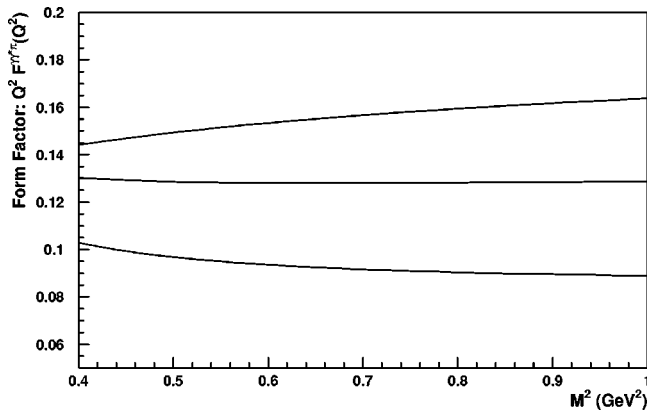


FIG. 1. The form factor $Q^2 F^{\gamma\gamma^*\pi}(Q^2)$ as a function of the Borel parameter $M^2$ for $Q^2 = 1,2,9$ GeV$^2$ (from lower to upper line).
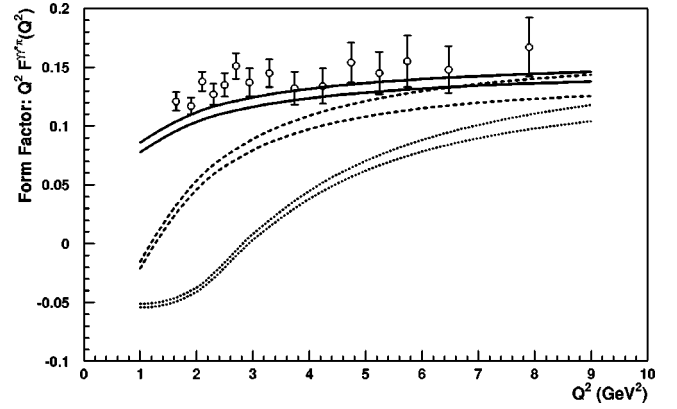


FIG. 2. The contributions of the asymptotic distribution amplitude (solid line) and non-asymptotic terms of the first two Gegenbauer polynomials with $a_2 = 1$ in the whole $Q^2$-region (dashed line) and $a_4 = 1$ (dotted line) to the form factor $Q^2 F^{\gamma\gamma^*\pi}(Q^2)$ as a function of $Q^2$ at the normalization point $\mu = 2.4, 1.5$ GeV (upper, lower curves from a bunch of two curves correspondingly). The experimental data are taken from [4].

First, we study the ansatz for the pion distribution amplitude with only one non-asymptotic term $a_2$. Using this ansatz and comparing our results with CLEO results we obtain

$$a_2 = 0.12 \pm 0.03 \quad \text{at} \quad \mu = 2.4 \text{ GeV.} \tag{22}$$

This result shows that the pion distribution amplitude is very close to the asymptotic form. This conclusion is in agreement with recent analysis on electromagnetic pion form factor [16] and also with results based on QCD sum rules presented in [17–19].

Let us comment on the normalization scale dependence of our result, which enters through the QCD coupling constant and logarithms in the radiative correction. For the extraction of $a_2(\mu)$, we may choose any reasonable normalization point in the interval $1 < \mu < 3$ GeV. Different input values of $\mu$ will give different pairs of $a_2(\mu)$. We have checked that
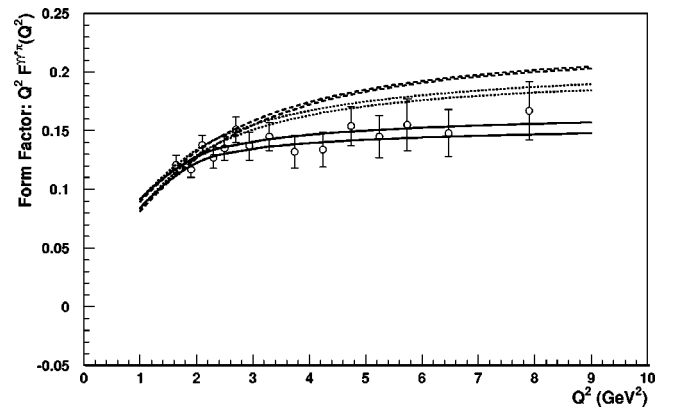


FIG. 3. The form factor $Q^2 F^{\gamma\gamma^*\pi}(Q^2)$ calculated with different distribution amplitudes: the Braun-Filyanov (dashed lines), Chernyak-Zhitnitsky (dotted lines) and our results extracted from CLEO data at $\mu = 2.4$ GeV (upper lines from bunches of two lines) and $\mu = 1.5$ GeV (lower lines). The experimental data are taken from [4].
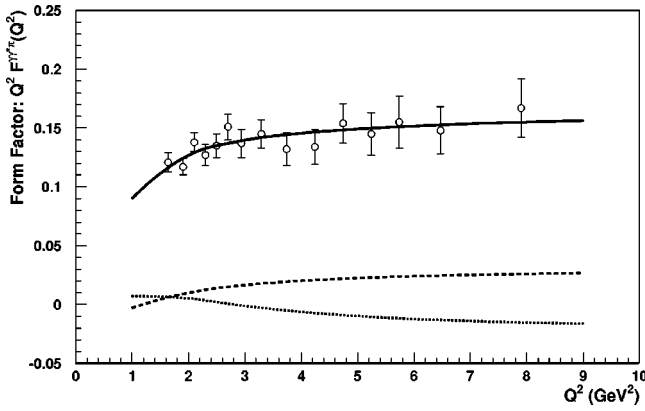
FIG. 4. The form factor $Q^2 F^{\gamma\gamma^*\pi}(Q^2)$ with the distribution amplitude extracted from CLEO data at $\mu = 2.4$ GeV (solid lines). The contribution of $a_2 = 0.19$ (dashed line) and $a_4 = -0.14$ (dotted line) are the central values of our nonlinear fit. The experimental data are taken from [4].

all extracted values of $a_2$ are in agreement with the renormalization group equation. The details about the running of $a_n(\mu)$ with $\mu$ are discussed in Appendix B.

We have also studied the ansatz for the pion distribution amplitude with two non-asymptotic terms $a_2$ and $a_4$. Using this ansatz and comparing our results with CLEO results we obtain: $a_2 = 0.19$ and $a_4 = -0.14$ at $\mu = 2.4$ GeV as the fit parameters. The form factor calculated with the central values of the extracted parameters $a_2(2.4)$ and $a_4(2.4)$ is shown in Fig. 4.

Now let us discuss systematic uncertainties. There are many sources of systematic uncertainties. One is the high order QCD perturbative corrections. Taking into account the size of one loop QCD correction (20%), we estimate two loop (and higher order correction) to be of order $0.2 \times 0.2 = 0.04(4\%)$. Additionally, there are power corrections of twist higher than 4. We assume them to be 20% of the twist-4 contribution, which itself contributes about 20–30 % to the twist-2 term. The effect of the twist-4 contribution is shown in Fig. 5. We also adopt zero width approximation for
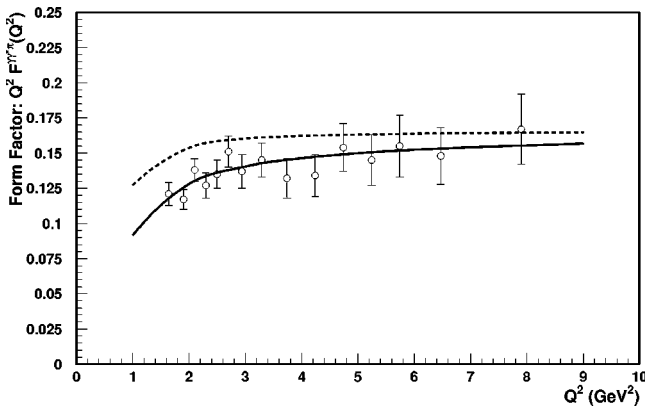


FIG. 5. The form factor $Q^2 F^{\gamma\gamma^*\pi}(Q^2)$ as a function of $Q^2$ at best extracted values of $a_2$ and $a_4$ at $\mu = 2.4$ GeV with (solid line) and without (dashed line) twist-4 contribution. The experimental data are taken from [4].
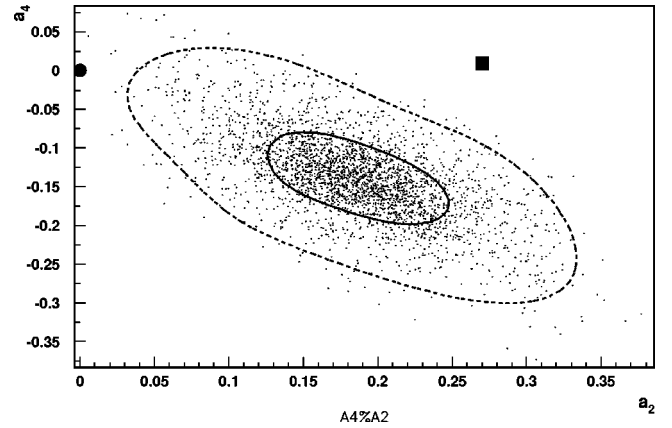


FIG. 6. The parameter space of $(a_2, a_4)$ pairs extracted from 3000 randomly chosen sets of data allowed by the experimental statistical uncertainties [4] as well as by the theoretical systematical uncertainties (23). Countor lines show 68% (solid line) and 95% (dashed line) confidential regions. Bold dots show the parameter pairs for asymptotic (circle) and Chernyak-Zhitnitsky (square) distribution amplitudes.

$\rho$ and $\omega$. The uncertainty induced by this assumption is small, a few percent,[2] since the form factor $F^{\rho\pi}$ does not appear in the final sum rule. The duality assumption introduces another source of uncertainties which is difficult to estimate. We assume that the variation of $M^2$ and $s_0$ gives us a rough idea about the size.

To fix statistical uncertainties we apply the fit procedure on a statistical set of data tables, where the experimental points are randomly displaced within the given errors [4].

Our results are presented in Fig. 6, where we show a parameter space of the twist-2 distribution amplitude, which is obtained by comparing CLEO results with our sum rule. Besides of statistical uncertainties, we account for the possible theoretical uncertainties. We combine theoretical uncertainties together and write the theoretical form factor as (with 95% CL)

$$F^{\pi\gamma\gamma^*}(s_1) = F_{As}^{tw-2}(s_1)(1 \pm 5\%) + F_{As}^{tw-4}(s_1)(1 \pm 20\%)$$
$$+ a_2^{fit} F_{a_2}(s_1) + a_4^{fit} F_{a_4}(s_1). \tag{23}$$

The $F_{As}^{tw-2/4}$ denote the asymptotic contribution of twist-2/ twist-4 distribution amplitudes, $F_{a_2}, F_{a_4}$ are the normalized contributions of the higher Gegenbauer polynomials.

The width of the allowed region (Fig. 6) is due to experimental-statistical uncertainties, whereas the length is due to theoretical-systematical ones.

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[2]In fact, at the $-s_1 > 4$ GeV$^2$ the resonance part in Eq. (8) (dual to the form factor $F^{\rho\pi}$) contributes less then 20–30 %. The finite width of $\rho$ meson gives a deviation of order 10% [13], therefore, we are left with 2–3 % uncertainty in the region $-s_1 > 4$ GeV$^2$. It is worth to note that this energy region gives the same values for $a_2$, $a_4$ as the overall sample of data.
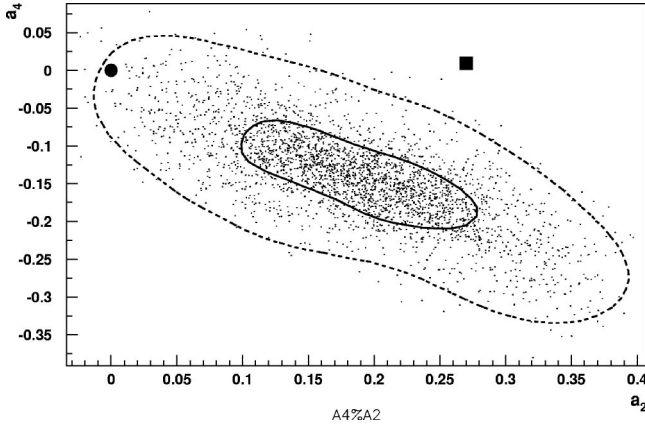
FIG. 7. The parameter space of $(a_2, a_4)$ pairs extracted from 3000 randomly chosen sets of data allowed by the experimental statistical uncertainties [4] as well as by the theoretical systematical uncertainties estimated *in a very conservative way*. Countor lines show 68% (solid line) and 95% (dashed line) confidential regions. Bold dots show the parameter pairs for asymptotic (circle) and Chernyak-Zhitnitsky (square) distribution amplitudes.

As we see from Fig. 6, theoretical-systematical uncertainties are dominant over experimental statistical ones, giving a correlation between the parameters $a_2, a_4$.

Figure 7 shows a more conservative scenario where we assume an uncertainty of $\pm 8\%$ in the twist-2 term and again $\pm 20\%$ for the twist-4 contribution.

It is worth mentioning that the allowed space of $a_2$ and $a_4$ does not overlap with Braun-Filyanov and Chernyak-Zhitnitsky distribution amplitudes [3,20]. These distribution amplitudes are beyond the 95% C.L. level region of the allowed parameter space for $a_2$, $a_4$. At the same time our region for $a_2, a_4$ is consistent with results presented in [17–19,16], where it has been claimed that the pion distribution amplitude is very close to the asymptotic form.

The region presented in Fig. 7 does not favor any central value. But in order to quantify the region, we may roughly identify the following values for $a_2$ and $a_4$:

$$a_2 = 0.19 \pm 0.04(\text{stat}) \pm 0.09(\text{syst});$$

$$a_4 = -0.14 \pm 0.03(\text{stat}) \mp 0.09(\text{syst}).$$

We also observe that $F^{\pi\gamma\gamma^*}(Q^2)$ form factor is more sensitive to the sum of $a_2$ and $a_4$, than to the $a_2 - a_4$. Really, we found that only one linear combination of $a_2$ and $a_4$ is determined well, it is $a_2 + 0.6a_4 = 0.11 \pm 0.03$ at $\mu = 2.4$ GeV.

It would be very useful to study some observable, which is sensitive to $a_2 - a_4$, for example, the coupling constant $g_{\rho\omega\pi}$. LCSR for this coupling is related to $\varphi(1/2)$, which is sensitive to the combination $a_2 - a_4$.

In general, the result for the model with one non-asymptotic term (22) is very reliable. At the same time, we have very useful constraint on the model with two nonasymptotic terms. However, an additional constraint on $a_2 - a_4$ would be extremely useful.

## V. CONCLUSIONS

We have studied the pion form factor $F^{\pi\gamma\gamma^*}(Q^2)$ in the light-cone sum rule approach, including radiative corrections and higher twist effects. Comparing the results to the CLEO experimental data on $F^{\pi\gamma\gamma^*}(Q^2)$, we have extracted the pion distribution amplitude of twist-2. The deviation of the distribution amplitude from the asymptotic one is small and is estimated to be $a_2(\mu) = 0.12 \pm 0.03$ at $\mu = 2.4$ GeV, in the model with one nonasymptotic term. This result on $a_2$ is in agreement with very recent analysis on electromagnetic pion form factor [16]. The ansatz with two non-asymptotic terms gives some region of $a_2$ and $a_4$, which is in qualitative agreement with results derived in [18,17,19,16], but does not agree with CZ and BF models [3,20].

Since the theoretical systematical uncertainties are dominant over experimental statistical ones it will be very useful to estimate the power corrections and the higher order radiative corrections to the correlator in future. Also, it would be very useful to calculate LCSR for some observable, which is sensitive to $a_2 - a_4$, for example, the coupling constant $g_{\rho\omega\pi}$, and to confront LCSR results with experiment.

## APPENDIX A

To calculate the imaginary part, we use the following identities:

$$\frac{1}{\pi} \text{Im} L_0 = \delta(u - u_0);$$

$$\text{Re} L_0 = \text{P} \frac{1}{u - u_0};$$

$$\frac{1}{\pi} \text{Im} L_1 = -\Theta(u_0 - u)\left(\frac{1}{u - u_0}\right)_+ + \delta(u - u_0)\log|u_0|;$$

$$\frac{1}{\pi} \text{Im} L_2 = -\Theta(u_0 - u)\left(\frac{2\log|u_0 - u|}{u - u_0}\right)_+$$
$$- \delta(u - u_0)\left(\frac{\pi^2}{3} - \log^2|u_0|\right);$$

$$\frac{1}{\pi} \text{Im} l_1 = -\Theta(u_0);$$

$$\text{Re} l_1 = \log|u_0|;$$

$$\frac{1}{\pi}\mathrm{Im}\,l_2 = -2\Theta(u_0)\log(u_0);$$

$$\mathrm{Re}\,l_2 = \log^2|u_0| - \pi^2\Theta(u_0).$$

## APPENDIX B

The distribution amplitude $\varphi_\pi$ can be expanded in terms of Gegenbauer polynomials $\Psi_n(u) = 6u(1-u)C_n^{3/2}(2u-1)$. In NLO, the evolution of the distribution amplitude is given by [23]

$$\varphi_\pi(u,\mu) = \sum_n a_n(\mu_0)\exp\left(-\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} d\alpha \frac{\gamma^n(\alpha)}{\beta(\alpha)}\right)$$
$$\times\left(\Psi_n(u) + \frac{\alpha_s(\mu)}{4\pi}\sum_{k>n} d_n^k(\mu)\Psi_k(u)\right)$$

(B1)

with $a_0 = 1$. The coefficients $d_n^k(\mu)$ are due to mixing effects, induced by the fact that the polynomials $\Psi_n(u)$ are the eigenfunctions of the LO, but not of the NLO evolution kernel. The QCD beta-function $\beta$ [24] and the anomalous dimension $\gamma^n$ of the $n$th moment $a_n(\mu)$ of the distribution amplitude have to be taken in NLO. Explicitly at NLO, the exponent in Eq. (B1) is

$$U(\mu,\mu_0) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_0^n/2\beta_0}$$
$$\times\left(\frac{\beta_0 + \beta_1\dfrac{\alpha_s(\mu)}{4\pi}}{\beta_0 + \beta_1\dfrac{\alpha_s(\mu_0)}{4\pi}}\right)^{(1/2)(\gamma_1^n/\beta_1 - \gamma_0^n/\beta_0)}.$$

(B2)

The anomalous dimensions [25] are

$$\gamma^n = \frac{\alpha_s}{4\pi}\gamma_0^n + \left(\frac{\alpha_s}{4\pi}\right)^2\gamma_1^n$$

(B3)

with

$$\gamma_0^0 = 0, \quad \gamma_1^0 = 0,$$

$$\gamma_0^2 = \frac{100}{9}, \quad \gamma_1^2 = \frac{34450}{243} - \frac{830}{81}N_F,$$

$$\gamma_0^4 = \frac{728}{45}, \quad \gamma_1^4 = \frac{662846}{3375} - \frac{31132}{2025}N_F.$$

(B4)

$N_F$ is a number of active flavors. The beta-function coefficients are defined in a standard way [24]. The NLO mixing coefficients are [23,26]

$$d_n^k(\mu) = \frac{M_{nk}}{\gamma_0^k - \gamma_0^n - 2\beta_0}\left(1 - \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{(\gamma_0^k - \gamma_0^n - 2\beta_0)/2\beta0}\right),$$

(B5)

where the numerical values of the first few elements of the matrix $M_{nk}$ are

$$M_{02} = -11.2 + 1.73N_F, \quad M_{04} = -1.41 + 0.565N_F,$$

$$M_{24} = -22.0 + 1.65N_F.$$

(B6)

The QCD evaluation of $a_2, a_4$ to the scale $\mu = 2.4$ GeV gives

$$a_2(2.4) = 0.28, \quad a_4(2.4) = 0.13$$

for the Braun-Filyanov distribution amplitude and

$$a_2(2.4) = 0.28, \quad a_4(2.4) = -0.009$$

for the Chernyak-Zhitnitsky distribution amplitude.

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