$\nu\nu\gamma$ amplitude in an external homogeneous electromagnetic field

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Neutrino-photon interactions in the presence of an external homogeneous constant electromagnetic field are studied. The $\nu\nu\gamma$ amplitude is calculated in an electromagnetic field of the general type, when the two field invariants are nonzero.

I. INTRODUCTION

Neutrino-photon interactions are of interest in astrophysics and cosmology. It is well known that in the standard model neutrino-photon interactions appear at the one-loop level. It is also well known that in a vacuum the reaction $\nu \rightarrow \nu\gamma$ is forbidden. The processes with two photons, for example, neutrino-photon scattering $\gamma\,\nu \rightarrow \gamma\,\nu$, turn out to be highly suppressed. In Ref. [1], Gell-Mann showed that in the four-Fermi limit of the standard model the amplitude is exactly zero to order G_F because, according to Yang's theorem [2], two photons cannot couple to the $J=1$ state. Therefore the amplitude is suppressed by the additional factors of ω/m_W, where ω is the photon energy and m_W is the W mass [1,3–5]. For example, in the case of massless neutrinos, the amplitude for $\gamma\nu \rightarrow \gamma\nu$ in the standard model is suppressed by the factor $1/m_W^2$ [6]. As a result, the cross section is exceedingly small. So we see that neutrino-photon processes with one or two photons do not occur or are suppressed in the vacuum.

The presence of a medium or external electromagnetic field drastically changes the situation. It induces an effective coupling between photons and neutrinos, which contributes to the $\nu \rightarrow \nu\gamma$ process and cross-related reactions. Furthermore, it was shown in [7] that in the presence of an external magnetic field, cross sections for neutrino-photon processes such as $\gamma\gamma \rightarrow \nu\bar{\nu}$ and $\nu\gamma \rightarrow \nu\gamma$ are enhanced by the factor $\sim (m_W/m_e)^4 (B/B_c)^2$ for $\omega \ll m_e$ and $B \ll B_c$. Later this result was extended to very strong magnetic fields [8] and arbitrary ω [9].

In this paper we will deal with $\nu \rightarrow \nu\gamma$ and $\gamma \rightarrow \nu\,\bar{\nu}$ reactions in the presence of an external electromagnetic field. The photon decay process $\gamma \rightarrow \nu\,\bar{\nu}$ in presence of a magnetic field was studied by several authors [10,11]. The Cherenkov process $\nu \rightarrow \nu\gamma$ is the cross process to photon decay and was studied in a crossed field [10] and in a magnetic field in [12,13]. The aim of this paper is to consider the case of an arbitrary homogeneous electromagnetic field. This case was also considered recently in [14] but, from our point of view, the expression obtained is unfinished insofar as the very short tensorial structure of the V-A two-point amplitude, which we derive in Eq. (13), is nowhere visible in [14]. Therefore it was not possible to compare our expressions to

Schubert's formula (5.15). Since this topic abounds in errors and controversies (see [13] for a thorough discussion of the literature and its critique), we will devote this article mainly to a careful analysis of this expression and will postpone physical applications to another forthcoming paper.

II. THE $\nu\nu\gamma$ PROCESS IN PRESENCE OF EXTERNAL FIELDS

Let us begin with the effective Lagrangian. At energies very much smaller than the W- and Z-boson masses, both processes are described by an effective four-fermion interaction,

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} \bar{\nu}\gamma^\mu(1+\gamma_5)\nu \bar{E}\gamma_\mu(g_V + g_A\gamma_5)E. \qquad (1)$$

Here, E stands for the electron field, $\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$, $g_V = 1 - \frac{1}{2}(1-4\sin^2\theta_W)$ and $g_A = 1 - \frac{1}{2}$ for ν_e, where the first terms in g_V and g_A are the contributions from the W exchange diagram and the second one from the Z exchange diagram. Also, $g_V = 2\sin^2\theta_W - \frac{1}{2}$ and $g_A = -\frac{1}{2}$ for $\nu_{\mu,\tau}$. Then the amplitude for the diagram in Fig. 1 is given by

$$\mathcal{M} = \frac{G_F}{\sqrt{2}\,e}\,\varepsilon_\mu\bar{\nu}\gamma_\nu(1+\gamma_5)\nu\,(g_V\Pi^{\mu\nu} + g_A\Pi_5^{\mu\nu}), \qquad (2)$$

where $\Pi^{\mu\nu}$ is the well-known polarization operator of QED. It was calculated earlier in [15–19]. Our aim is to calculate $\Pi_5^{\mu\nu}$. For this purpose we will use the technique developed in [20–23,18]. In this approach we begin with

$$\Pi_5^{\mu\nu} = -ie^2 M_5^{\mu\nu}, \qquad (3)$$
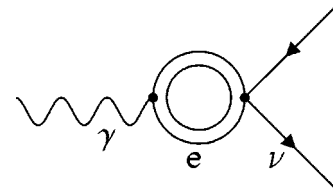
where

FIG. 1. Neutrino-photon process in an external electromagnetic field. The double line represents the electron propagator in the presence of an external field.

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$$M_5^{\mu\nu} \equiv Sp\langle 0| \gamma^\mu \frac{1}{\mathcal{P}-m+i\varepsilon} \gamma^\nu \gamma^5 \frac{1}{\mathcal{P}-k-m+i\varepsilon}|0\rangle, \quad (4)$$

with $\mathcal{P}_\mu = i\partial_\mu - eA_\mu$, and we have to calculate the mean value over the states $x=0$: $\langle 0| \ldots |0\rangle$. For the analysis it is convenient to use a special representation for the field tensor:

$$F_{\mu\nu} = aC_{\mu\nu} + bB_{\mu\nu}, \quad (5)$$

with the tensors $C_{\mu\nu}$ and $B_{\mu\nu}$ defined by

$$C_{\mu\nu} = \frac{1}{a^2+b^2}(aF_{\mu\nu}+bF_{\mu\nu}^*);$$

$$B_{\mu\nu} = \frac{1}{a^2+b^2}(bF_{\mu\nu}-aF_{\mu\nu}^*), \quad (6)$$

where

$$a,b = \sqrt{(\mathcal{F}^2+\mathcal{G}^2)^{1/2} \pm \mathcal{F}}; \quad \mathcal{F} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

$$\mathcal{G} = -\frac{1}{4}F_{\mu\nu}^* F^{\mu\nu}. \quad (7)$$

Then[1]

$$(C^2)_{\mu\nu} = \frac{1}{a^2+b^2}(F_{\mu\nu}^2 + b^2 g_{\mu\nu});$$

$$(B^2)_{\mu\nu} = \frac{1}{a^2+b^2}(F_{\mu\nu}^2 - a^2 g_{\mu\nu}). \quad (8)$$

These tensors satisfy very useful identities:

$$(CB)_{\mu\nu} = 0; \quad (C^3)_{\mu\nu} = C_{\mu\nu}; \quad (B^3)_{\mu\nu} = -B_{\mu\nu}. \quad (9)$$

Now the result of our calculation is given by

$$M_5^{\mu\nu} = 2i\frac{\pi^2}{(2\pi)^4}\left\{ 2ebC^{\mu\nu} - 2eaB^{\mu\nu} + \int_{-1}^{1}dv\int_0^\infty sds\frac{ie^2ab}{\sin(ebs)\sinh(eas)}\right.$$

$$\times \exp[i\Psi]e^{-ism^2}[(Ck)^\mu(C^2k)^\nu M_1 + (Bk)^\mu(B^2k)^\nu M_2 + [(C^2k)^\mu(Bk)^\nu + (Bk)^\mu(C^2k)^\nu + (kC^2k)B^{\mu\nu}]$$

$$\left. \times M_3 + [(B^2k)^\mu(Ck)^\nu + (Ck)^\mu(B^2k)^\nu + (kB^2k)C^{\mu\nu}]M_4\right\}, \quad (10)$$

where

$$\Psi = \frac{(kC^2k)}{2ea}\frac{\cosh(eas)-\cosh(easv)}{\sinh(eas)}$$

$$+ \frac{(kB^2k)}{2eb}\frac{\cos(ebs)-\cos(ebsv)}{\sin(ebs)}, \quad (11)$$

with

$$M_1 = \sin(ebs)(\cosh(eas)-\cosh(easv))\frac{1}{\sinh^2(eas)} \quad (12)$$

$$M_2 = \sinh(eas)(-\cos(ebs)+\cos(ebsv))\frac{1}{\sin^2(ebs)}$$

$$M_3 = \frac{1}{2}\left(-\cos(ebsv)\cosh(easv)\coth(eas)\right.$$

$$+ \cos(ebsv)\frac{1}{\sinh(eas)}$$

$$\left. + \cot(ebs)\sin(ebsv)\sinh(easv)\right)$$

$$M_4 = \frac{1}{2}\left(\cos(ebsv)\cosh(easv)\cot(ebs)\right.$$

$$- \cosh(easv)\frac{1}{\sin(ebs)}$$

$$\left. + \coth(eas)\sin(ebsv)\sinh(easv)\right).$$

As in the previous calculations in presence of a magnetic field [11,13], this result is not gauge invariant. We also see that our amplitude is ultraviolet convergent. Nevertheless, in order to preserve gauge invariance, we must regularize it using a gauge invariant regularization. This phenomenon should be familiar from quantum electrodynamics. For example, the photon-photon scattering amplitude, though formally converging, must be regularized to get a gauge invariant result [24,25]. A convenient way of restoring gauge invariance in diagrams with pseudovector coupling is to use the Pauli-Villars regularization. Then it is easy to show that to obtain the final gauge invariant expression for the amplitude we must just omit the first two terms in Eq. (10), because they are cancelled by regulator terms. Finally, we obtain

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[1]We always use the metric $g = \text{diag}(+---)$.

$$M_5^{\mu\nu} = -\frac{1}{8\pi^2}\int_{-1}^{1}dv\int_{0}^{\infty}sds\,\frac{e^2ab}{\sin(ebs)\sinh(eas)}$$

$$\times\exp[i\Psi]e^{-ism^2}\{(Ck)^{\mu}(C^2k)^{\nu}M_1+(Bk)^{\mu}(B^2k)^{\nu}$$

$$\times M_2+[(C^2k)^{\mu}(Bk)^{\nu}+(Bk)^{\mu}(C^2k)^{\nu}+(kC^2k)B^{\mu\nu}]$$

$$\times M_3+[(B^2k)^{\mu}(Ck)^{\nu}+(Ck)^{\mu}(B^2k)^{\nu}$$

$$+(kB^2k)C^{\mu\nu}]M_4\}. \tag{13}$$

This method of getting a gauge invariant expression allows us, unlike the authors of [13], to do without an anomaly cancellation mechanism at this stage of calculation. Let us note that the whole contribution of the triangle diagram is still present in Eq. (13). This fact can be easily confirmed by an independent calculation of the triangle graph contribution. To deal with unwanted anomaly terms we must, as usual, take into account all fermions in a generation. In the following this will always be assumed.

Equation (13) is the main result of this article. Let us now consider various checks of this expression. First, in the limit of zero electric field it reproduces the results of [11,13]. A second nontrivial check of formula (13) is to consider the following transformation:

$$a\to ib;b\to -ia;C_{\mu\nu}\to -iB_{\mu\nu};B_{\mu\nu}\to iC_{\mu\nu};F_{\mu\nu}\to F_{\mu\nu}. \tag{14}$$

Because of $F_{\mu\nu}\to F_{\mu\nu}$, $F_{\mu\nu}^*\to F_{\mu\nu}^*$, the tensor $M_5^{\mu\nu}$ must be invariant under this transformation. From Eq. (13) we see that it is indeed invariant and that under this transformation, the first term is interchanged with the second, and the third term is interchanged with the fourth, thus demonstrating the nontriviality of this check.

The next interesting limit is the case of a crossed field, where $\vec{E}$ and $\vec{B}$ are not only orthogonal but are also of equal modulus. This causes the two invariants $\mathcal{F}$ and $\mathcal{G}$ to vanish. The field dependence is therefore completely described by the invariant $(F^{\mu\nu}k_\nu)^2$. The correct procedure of finding the amplitude is to set $a=b$ first, and then take the limit $a=b\to 0$. Using this procedure we find the following expression:

$$M_5^{\mu\nu} = -\frac{e}{8\pi^2}\left\{\frac{1}{2m^2}[k^2F^{*\mu\nu}+(F^*k)^{\mu}k^{\nu}+(F^*k)^{\nu}k^{\mu}]\right.$$

$$\times\int_0^1 dv(1-v^2)u\int_0^{\infty}dz\exp\left(-i\left(zu+\frac{z^3}{3}\right)\right)$$

$$+\frac{4}{(kF^2k)}(F^*k)^{\mu}(F^2k)^{\nu}\left\{\int_0^1 dv\,u\int_0^{\infty}dz\right.$$

$$\times\exp\left(-i\left(zu+\frac{z^3}{3}\right)\right)+i\left.\right\}\right\}, \tag{15}$$

where

$$u=\left(\frac{e^2(kF^2k)}{16m^6}\right)^{-1/3}(1-v^2)^{-2/3}. \tag{16}$$

Earlier the amplitudes in crossed fields were considered in [10,26]. We can compare only parts of Eq. (15) with those existing in the literature, because all previous results were presented in contracted form and with photons on the mass shell. Taking this into account we can say that our result contradicts that of [10] and confirms that of [26].

III. CONCLUSIONS

We have presented the $\nu\nu\gamma$ amplitude in the presence of an external homogeneous constant electromagnetic field. The result is gauge invariant and reproduces the known results for an external magnetic field. We further identified the crossed-field limit. Let us also emphasize that the general case of electric and magnetic fields is in need when considering (low-frequency) multiphoton one-loop processes with and without external fields. The situation is similar to processes where the Heisenberg-Euler Lagrangian turns out to be useful as, e.g., in the study of photon-splitting [27] or in photon-neutrino processes as discussed in the recent article by Dicus and Repko [28]. Hence we conclude that our results allow for a variety of applications that will also be discussed in a future publication.

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