Global monopoles in the Brans-Dicke theory

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A gravitating global monopole produces a repulsive gravitational field outside the core in addition to a solid angular deficit in the Brans-Dicke theory. As a new feature, the angular deficit is dependent on the values of ϕ_∞ and ω, where ϕ_∞ is the asymptotic value of the scalar field in spacelike infinity and ω is the Brans-Dicke parameter.

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The phase transitions in the early Universe can give rise to topological defects of various kinds [1]. The idea that a monopole ought to exist has proved to be remarkably durable. The first ones to study the effects of gravity on the global monopole were Barriola and Vilenkin [2]. When gravity is taken into account, the linearly divergent mass of a global monopole has an effect analogous to that of a tiny mass at the origin. Harari and Loustò [3], and Shi and Li [4] have shown that this small gravitational potential is actually repulsive. Recently, Li *et al.* [5–7] have described a new class of cold stars, which are called *D* stars (defect stars). Compared to *Q* stars, one further requires, as a new feature, that in the absence of matter field the theory has monopole solutions. This requirement is such that the characteristics of these objects, for instance a deficit angle, differ quite substantially from those of *Q* stars. On the other hand, there has been renewed interest in the Brans-Dicke (BD) theory [8], in which the usual metric gravitational field is augmented by a scalar field ϕ which couples to the curvature via a parameter ω. The modern studies of BD theory are motivated by the fact that they appear as the low energy limit of string theory [9,10]. Spherically symmetric charge distributions in BD theory have also been investigated before [11–14]. In this paper, we study global monopoles in the BD theory. We show that the monopole produces a repulsive gravitational field outside the core in addition to a solid angular deficit. As a new feature, the angular deficit is dependent on the values of ϕ_∞ and ω, where ϕ_∞ is the asymptotic value of scalar field in spacelike infinity.

To be specific, we shall work within a particular model in units $G=C=1$, where a global $O(3)$ symmetry is broken down to U(1) in the framework of BD theory. Its action is given by

$$\mathcal{S}=\frac{1}{16\pi}\int d^4x\sqrt{-g}\bigg(\phi R-\frac{\omega}{\phi}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}-8\pi g^{\mu\nu}\psi^a{}_{,\mu}\psi^a{}_{,\nu}-4\pi\lambda(\psi^a\psi^a-\eta^2)^2\bigg) \tag{1}$$

where $g=\det(g_{\mu\nu})$. R is the scalar curvature, ω is a constant, ϕ^a is a triplet of the Goldstone field, and $a=1,2,3$. The Goldstone field configuration describing a global monopole is

$$\psi^a=\eta\sigma(r)\frac{x^a}{r},\quad \text{with}\ \ x^ax^a=r^2 \tag{2}$$

so that we will actually have a monopole solution if $\sigma\to1$ at spatial infinity. The general static metric with spherical symmetry can be written as

$$ds^2=-B(\rho)+A(\rho)dr^2+\rho^2(d\theta^2+\sin^2\theta d\phi^2) \tag{3}$$

with the usual relation between the spherical coordinates ρ,θ,ϕ and the ''Cartesian'' coordinates x^a. Let us now introduce a dimensionless parameter $r\equiv\eta\rho$. From the action (1) and the definition for σ, the equation of motion is as follows:

$$\frac{1}{A}\sigma''+\left[\frac{2}{Ar}+\frac{1}{2B}\left(\frac{B}{A}\right)'\right]\sigma'-\frac{2}{r^2}\sigma-\lambda(\sigma^2-1)\sigma=0, \tag{4}$$

with the prime denoting differentiation with respect to r. Varying the action with respect to $g^{\mu\nu}$ and ϕ we obtain the field equations

$$R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=\frac{8\pi}{\phi}T_{\mu\nu}+\frac{\omega}{\phi^2}(\phi_{,\mu}\phi_{,\nu}-\tfrac{1}{2}g_{\mu\nu}\phi^{,\alpha}\phi_{,\alpha})+\frac{1}{\phi}(\phi_{,\mu;\nu}-g_{\mu\nu}\Box\phi) \tag{5}$$

and

$$\Box\phi=\frac{8\pi T}{2\omega+3}, \tag{6}$$

where the energy-momentum tensor is

$$T_{\mu\nu}=\partial_\nu\psi^a\partial_\nu\psi^a-\tfrac{1}{2}g_{\mu\nu}[g^{\alpha\beta}\partial_\alpha\psi^a\partial_\beta\psi^a+\tfrac{1}{2}\lambda(\psi^a\psi^a-\eta^2)^2] \tag{7}$$

and T is tr $T_{\mu\nu}$. Using Eqs. (2) and (3), we have

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$$A' = Ar\left[\frac{4\pi\eta^2\sigma'^2(2\omega+1)}{\phi(2\omega+3)} + \frac{8\pi\eta^2 A(2\omega-1)}{\phi(2\omega+3)}\right.$$
$$\left.\times\left(\frac{\sigma^2}{r^2}+\frac{\lambda}{4}(\sigma^2-1)^2\right) - \frac{B'\phi'}{2B\phi} - \frac{\omega\phi'^2}{2\phi} + \frac{1}{r^2}(1-A)\right], \tag{8}$$

$$B' = \frac{rB}{2\phi+r\phi'}\left[8\pi\eta^2\sigma'^2 - 8\pi\eta^2 A\left(\frac{\sigma^2}{r^2}+\frac{\lambda}{4}(\sigma^2-1)^2\right)\right.$$
$$\left. - \frac{4}{r}\phi' + \frac{\omega}{\phi}\phi'^2 + \frac{2\phi}{r^2}(A-1)\right], \tag{9}$$

$$\phi'' = \phi'\left(\frac{A'}{2A} - \frac{B'}{2B} - \frac{2}{r}\right) - \frac{2A\eta^2}{2\omega+3}\left[\frac{2\sigma^2}{r^2} + \frac{\sigma'^2}{A}\right.$$
$$\left. + \lambda(\sigma^2-1)^2\right]. \tag{10}$$

Equations (8)–(10) can be reduced to the general relativistic (GR) ones [2–4] when $\omega\to\infty$ and $\phi\to 1$. We introduce the ''scalar charge'' of BD theory

$$S = \lim_{r\to\infty}\left(r^2\frac{d\phi}{dr}\right). \tag{11}$$

Note that this is not, in general, a conserved quantity. We define it here since it is included in the expression for the mass of the monopole which we will give below. The equation for ϕ can be formally intergrated, we have an alternative expression for the scalar charge

$$S = \int_0^\infty dr\, r^2\left\{\phi'\left(\frac{A'}{2A} - \frac{B'}{2B}\right) - \frac{2A\eta^2}{2\omega+3}\right.$$
$$\left.\times\left(\frac{2\sigma^2}{r^2} + \frac{\sigma'^2}{A} + \lambda(\sigma^2-1)^2\right)\right\}. \tag{12}$$

Expanding the metric and the scalar field equation in powers of r^{-1} about $r=\infty$ one can write the field equations in a linearized form. Using the boundary condition of the asymptotically flat

$$A \sim A_\infty + O\left(\frac{1}{r^n}\right),$$
$$B \sim B_\infty + O\left(\frac{1}{r^n}\right),$$
$$\phi \sim \phi_\infty + O\left(\frac{1}{r^n}\right), \tag{13}$$

where $n\geqslant 1$ and the subscript ''∞'' denotes values at space-like ∞. We also require $\sigma\to 1$ at least as fast as r^{-1}. In fact, as we will show below, the conditions above imply that $\sigma\to 1$ exponentially as $r\to\infty$.

The equation for the metric coefficient $A(r)$ reads

$$\left(\frac{r}{A}\right)' + \Omega(r)\left(\frac{r}{A}\right) = 1 - \Delta(r)\left[\sigma^2 + \frac{\lambda}{4}r^2(\sigma^2-1)^2\right], \tag{14}$$

where

$$\Omega(r) = \frac{4\pi\sigma'^2\eta^2(2\omega+1)r}{(2\omega+3)} - \frac{B'\phi'}{2B\phi} + \frac{\omega\phi'^2}{2\phi} \tag{15}$$

and

$$\Delta(r) = \frac{8\pi\eta^2(2\omega-1)}{\phi(2\omega+3)}. \tag{16}$$

We define

$$\Delta \equiv \Delta(\infty) = \frac{8\pi\eta^2(2\omega-1)}{\phi_\infty(2\omega+3)} \tag{17}$$

and we will show below, Δ describes a solid angular deficit in the BD theory. Integrating Eq. (14), $A(r)$ can be written as

$$A^{-1} = 1 - \Delta - \frac{2M(r)}{r}, \tag{18}$$

where

$$M(r) = \frac{1}{2}[\Delta(r)-\Delta]r + \frac{1}{2}\exp\left[-\int_0^r \Omega(y)dy\right]$$
$$\times\int_0^r dy\left\{\Delta(y)\left[(\sigma^2-1) + \frac{\lambda y^2}{4}(\sigma^2-1)^2\right.\right.$$
$$\left.\left. + [1-\Delta(y)]y\Omega(y)\right] + 8\pi\frac{\phi'}{\phi}\Delta(y)\right\}\left[\int_0^y \Omega(z)dz\right]. \tag{19}$$

One can show that $\lim_{r\to\infty}M(r) = M_{ADM}$, which is the Arnowitt-Deser-Misner mass [15,16] of the monopole. Let us first discuss the GR case. When $\omega\to\infty$ and $\phi\to 1$, we have $\Delta = 8\pi G\eta^2$ and

$$M(r) = \frac{\Delta}{2}\exp\left[-\frac{\Delta}{2}\int_0^r dy\,\sigma'^2(y)y\right]$$
$$\times\int_0^r dy\left[(\sigma^2-1) + \frac{y^4}{4}(\sigma^2-1)^2 + (1-\Delta)\frac{y^2}{2}\sigma'^2\right]$$
$$\times\exp\left[\frac{\Delta}{2}\int_0^y dz\,\sigma'^2 z\right], \tag{20}$$

which is known as a formula in Ref. [3]. Analogously,

$$B(r) = 1 - \Delta - \frac{2M_B(r)}{r} \tag{21}$$

with

$$M_B(r)=M(r)\exp\left[\Delta\int_\infty^r dy\,\sigma'^2 y\right]+(1-\Delta)\frac{r}{2}\left[1-\exp\left(\Delta\int_\infty^r dy\,\sigma'^2 y\right)\right]. \tag{22}$$

One finds the asympototic expansions

$$f(r)=1-\frac{1}{r^2}-\frac{3/2-\Delta}{r^4}+O(r^{-6}),$$

$$M(r)=M_{ADM}+\frac{\Delta}{2r}+O(r^{-3}),$$

$$M_B(r)=M_{ADM}\left[1-\frac{\Delta}{r^4}\right]+\frac{(1-\Delta)}{2}\frac{\Delta}{r^3}+O(r^{-7}). \tag{23}$$

Solving Eqs. (8)–(10) in a linearized form, we find the scalar field has the following asymptotic form:

$$\phi=\phi_\infty-\frac{S}{r}+O\left(\frac{1}{r^2}\right), \tag{24}$$

while the line element in this limit is

$$ds^2=-\left[1-\Delta-\frac{2M_K}{r}+O\left(\frac{1}{r^2}\right)\right]dt^2+\left[1-\Delta-\frac{2M_{ADM}}{r}+O\left(\frac{1}{r^2}\right)\right]^{-1}dr^2+r^2(d\theta^2+\sin^2\theta d\phi), \tag{25}$$

where

$$M_K=M_{ADM}-\frac{S}{\phi_\infty} \tag{26}$$

is the Keplerian mass of the monopole. The M_K is the active gravitational mass measured by a non-self-gravitating test particle in a circular orbit at spacelike infinity about the monopole. Substituting the metric components appearing in the asymptotic form of the line element in Eq. (4), we have

$$(1-\sigma)''+\frac{2}{r}(1-\sigma)'-2\lambda\left(1+\Delta+\frac{2M_{ADM}}{r}\right)(1-\sigma)+O\left(\frac{1}{r^2}\right)=0. \tag{27}$$

We obtain the asymptotic solution

$$\sigma=1-r^{-b}e^{-kr}\left[1+O\left(\frac{1}{r^2}\right)\right], \tag{28}$$

where

$$k=[2\lambda(1+\Delta)]^{1/2},$$

$$b=1+\left(\frac{2\lambda}{(1+\Delta)}\right)^{1/2}M_{ADM}. \tag{29}$$

Therefore, σ field tends to one exponentially with r in the asymptotic region.

Neglecting the mass term in Eqs. (18) and (21), and rescaling variables r and t at a large distance off the core, the monopole metric can be rewritten as

$$ds^2=-dt^2+dr^2-(1-\Delta)r^2(d\theta^2+\sin^2\theta d\phi^2). \tag{30}$$

This metric describes a space with a deficit solid angle: the area of a sphere with radius r is not $4\pi r^2$, but a little smaller. In the BD theory, angular deficit is dependent on the values of ϕ_∞ and ω.

Note that the integrand in Eq. (19) contains a nonpositive definite term, so $M(r)$ may be negative. The strongest constrains on BD theories are usually assumed to come from the solar system weak field tests [15]. As it is well known, observations constrain the BD parameter to have a value of $\omega>500$. Using numerical integration, $M(r)$ is indeed negative all the way from the origin and quickly approaches an asymptotic value of order $M\approx-6\pi\eta^2(2\omega-1)/[\phi_\infty(2\omega+3)]$. However, one has taken $\omega=-1$ in the simplest string effective action [10], which is relevant to the study of global monopole in very early Universe, although in this case $\phi_\infty=1$ is physically unrealistic: to approximate a monopole embedded in the string dominated Universe, we should choose a value $\phi_\infty<1$. In this case, we have $\Delta=-24\eta^2/\phi_\infty$. This means that there is not angular deficit, but angular surplus in the string dominated cosmology.

Finally, the general scalar-tensor gravitational theories arise from dimension reduction of higher dimensional theories [17] and string theory [10]. The general theories can satisfy the rather strict constraints determined at the current epoch, while still differing considerably from Einstein's theory in the past [18]. Details of the monopole in more general scalar-tensor theories will be considered elsewhere.

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[1] A. Vilenkin, Phys. Rep. **121**, 263 (1985).
[2] M. Barriola and A. Vilenkin, Phys. Rev. Lett. **63**, 341 (1989).
[3] D. Harari and C. Loustó, Phys. Rev. D **42**, 2626 (1990).
[4] X. Shi and X.Z. Li, Class. Quantum Grav. **8**, 37 (1991).
[5] X.Z. Li, X.H. Zhai, and G. Chen, Astropart. Phys. **13**, 245 (2000).
[6] X.Z. Li and X.H. Zhai, Phys. Lett. B **364**, 212 (1995).
[7] J.M. Li and X.Z. Li, Chin. Phys. Lett. **15**, 3 (1998).
[8] C. Brans and R.H. Dicke, Phys. Rev. **124**, 925 (1961).
[9] E.S. Fradkin and A.A. Tseytlin, Nucl. Phys. **B261**, 1 (1985).
[10] C.G. Callan, D. Friedan, E.J. Martinec, and M.J. Perry, Nucl. Phys. **B262**, 593 (1985).

[11] R.T. Singh and J.K. Usham, Astrophys. Space Sci. **155**, 225 (1989).
[12] D.R.K. Reddy, J. Math. Phys. **20**, 23 (1979).
[13] D.R.K. Reddy and V.U.M. Rao, J. Phys. A **14**, 1973 (1981).
[14] A.W. Whinnett and D.F. Torres, Phys. Rev. D **60**, 104050 (1999).
[15] R. Arnowitt, S. Deser, and C.W. Misner, Phys. Rev. **122**, 997 (1961).
[16] C.M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1993).
[17] M. Rainer and A. Zhuk, Phys. Rev. D **54**, 6186 (1996).
[18] J.D. Barrow, Phys. Rev. D **51**, 2729 (1995).