Inflation in models with large extra dimension driven by a bulk scalar field

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(Received 26 July 2000; published 25 October 2000)

We discuss inflation in models with large extra dimensions, driven by a bulk scalar field. The brane inflaton is then a single effective field, obtained from the bulk scalar field by scaling. The self-interaction terms of the effective brane inflaton are then naturally suppressed. The picture is consistent with a fundamental string scale in the TeV range without the problem of a superlight inflaton. If hybrid inflation is considered, the right prediction for the density perturbations as observed by the Cosmic Background Explorer can be obtained without any fine-tuning. The bulk inflaton then decays preferentially into brane Higgs fields and reheating follows.

PACS number(s): 11.10.Kk, 12.90.+b, 98.80.Cq

I. INTRODUCTION

Theories with extra dimensions in which our four-dimensional world is a hypersurface (3-brane) embedded in a higher-dimensional space (the bulk) have been the focus of intense scrutiny during the last two years. It is generally assumed that in this picture the standard model particles are in the brane whereas gravity and perhaps other standard model singlets propagate in the bulk. The main motivation for these models comes from string theories where the Horava-Witten solution [1] of the nonperturbative regime of the $E_8 \times E_8$ string theory provided one of the first models of this kind (although from a phenomenological point of view the idea was discussed early on by several authors [2,3]). Additional interest arose from the observation [4] that the bulk size could be as large as a millimeter leading to new observable deviations from Newton's inverse square law [5] at the millimeter scale, where curiously enough Newton's law remains largely untested.

A key formula that relates the string scale to the radius of the large extra dimension in these models is

$$M^{2+n} R^n = M_{\mathrm{Pl}}^2, \tag{1}$$

where R is the common radius of the n extra dimensions, M is the string scale, and M_{Pl} is the Planck scale. For $R \sim$ millimeter, M can be as low as few TeV thereby providing another resolution of the long standing hierarchy problem. This has been another motivation for these theories.

While this picture leads to many interesting consequences for collider and other phenomenology [6], it seems to require drastic rethinking of the prevailing view of cosmology [7]. In particular, one runs into a great deal of difficulty in implementing the standard pictures of inflation [8]. For instance, if the inflaton is required to be a brane field, its mass becomes highly suppressed, making it difficult to understand the reheating process. Also, a wall inflaton makes it hard to understand the density perturbations observed by the Cosmic Background Explorer (COBE) [8].

As a way to solve these problems, Arkani-Hamed *et al.* [9] proposed a scenario where it was assumed that inflation occurs before the stabilization of the internal dimensions. With the dilaton field playing the role of the inflaton field, they argued that early inflation, when the internal dimensions are small, can successfully overcome the above complications. Another possible way out was proposed in Ref. [10], where it was suggested that the brane could be out of its stable point at early times, and inflation is induced on the brane by its movement through the extra space. Still other ideas can be found in Ref. [11].

The common point of the first two scenarios is that they share the same basic assumption of an unstable extra dimension. However, it is still possible that, due to some dynamical mechanism, the extra dimension gets stabilized long before the Universe exited from inflation, as in some scenarios in Kaluza-Klein (KK) theories, where the stabilization potential is generated by the Casimir force [12,13]. Other possible sources for this stabilizing potential could be present in brane-bulk theories; for instance, the formation of the brane at very early times may give rise to vacuum energy that plays a role in eventually stabilizing the extra dimension. It therefore appears to us that it would be of interest to seek inflationary scenarios where stabilization occurs before inflation ends. Clearly in this case, one cannot expect the dilaton field

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to play the role of the inflaton field and we need to find a new way to generate inflation, that can solve the problems faced by the brane inflaton.

With this background, we work in the framework of the Arkani-Hamed–Dimopoulos–Dvali (ADD) scenario where stabilization of the internal dimensions occurred long before the end of inflation. The main new ingredient of our work is that the inflaton is a bulk field rather than a field in the brane. We give qualitative arguments to show that this provides a different way out to the problems introduced with a brane inflaton. We should stress that while inflation proceeds, in principle, as in the former KK theories, the postinflationary era has a different behavior mainly due to the fact that reheating must take place on the boundaries where all matter resides. This raises interesting questions regarding the reheating process since, naively, one might expect a bulk inflaton to reheat the bulk instead of the brane by releasing all its energy into the internal space in the form of gravitons. However, as we will discuss, bulk heating is much less efficient than brane heating, thereby circumventing this possibility.

The paper is organized as follows. In Sec. II, we review the problems with brane inflation and comment on the possible solutions, leaving the analysis of the present proposal (a bulk inflaton with stable bulk) for Sec. III. In Sec. IV, we discuss density perturbation. In Sec. V, we address some details of the reheating era to explore the puzzles introduced by the possible production of gravitons. We will close our discussion with some remarks.

II. BRANE INFLATION

To see how letting the inflaton arise from the brane fields leads to problems [8], let us consider a typical chaotic inflation scenario [14]. If the highest scale in the theory is M, during inflation, the inflaton potential cannot be larger than M^4, regardless of the number of extra dimensions. Since successful inflation (the slow roll condition) requires that the inflaton mass be less than the Hubble parameter, which is given as

$$H\sim\sqrt{\frac{V(\phi)}{3M_{\rm Pl}^2}} \tag{2}$$

we have the inequality

$$m\leqslant H\leqslant M^2/M_P\,. \tag{3}$$

For $M\sim 1$ TeV, one then gets the bound $m\leqslant 10^{-3}$ eV, which is a severe fine-tuning constraint on the parameters of the theory. It further implies that inflation occurs on a time scale H^{-1} much greater than M^{-1}. As emphasized by Kaloper and Linde [8], this is conceptually very problematic since it requires that the Universe should be large and homogeneous enough from the very beginning so as to survive the large period of time from $t=M^{-1}$ to $t=H^{-1}$.

Moreover, for chaotic inflation [14] with $V(\phi)=\frac{1}{2}m^2\phi^2$, we get for the density perturbations

$$\frac{\delta\rho}{\rho}\sim 50\frac{m}{M_{\rm Pl}}\leqslant 10^{-31}. \tag{4}$$

For the case where $\lambda\phi^4$ term dominates the density one gets the same old fine-tuning condition $\delta\rho/\rho\sim\lambda^{1/2}$. Assuming hybrid inflation [15], with the potential $V(\phi,\sigma)=(1/4\lambda)(M^2-\lambda\sigma^2)^2+\frac{1}{2}m^2\phi^2+g^2\phi^2\sigma^2$, does not improve those results [8], since it needs either a value of m six orders of magnitude smaller or a strong fine tuning on the parameters, to match the COBE result $\delta\rho/\rho\sim 10^{-5}$.

There are two possible ways to overcome this theorem. First, as emphasized in Ref. [9], we can imagine that during the inflation era the extra dimensions were as small as M^{-1}. Thus, instead of Eq. (2) we will get

$$H^2=\frac{V(\phi)}{3M^2}, \tag{5}$$

which naturally removes the suppression (3). However, one cannot allow the extra dimension to grow considerably during inflation since large changes on the internal size will significantly affect the scale invariance of the density perturbations. Therefore the radius of the extra dimension must remain essentially static while the Universe expands. After inflation ends, the extra dimension should grow to its final size, which may, however, produce a contraction period on the brane [9]. In this scenario, the radion should slow roll during inflation and could be identified as the inflaton. Nevertheless, it also poses some complications for the understanding of reheating since the radion is long lived, and its mass could be very small (its mass is lower bounded by 10^{-3} eV).

Here, we consider the second possibility where the dynamics of the radion stabilizes the radius of the extra dimension before inflation ends ($\tau_{\rm stab}\ll\tau_{\rm inf}$). There are examples of some KK inspired theories [12,13] where this happens. In such a theory, a different way around the above problems is needed. Notice that, in this case, the scale invariance of density perturbations requires that stabilization occurs long before the last 80 e foldings or so. Clearly, in this case the radion cannot play the roll of the inflaton since it will not slow roll. As we will discuss in the next section, if a bulk scalar field plays the role of the inflaton, a simple solution to the above problems may be given. One must, however, investigate the question of reheating carefully in this model. Another important question in this model is the origin of stabilization of the extra dimension. We do not address this difficult question here but simply assume the condition that $\tau_{\rm stab}\ll\tau_{\rm inf}$.

III. INFLATION WITH A BULK SCALAR FIELD

Let us now discuss the picture of inflation, when the inflaton is a higher-dimensional scalar field. To keep things simple, we will assume only a single extra dimension. However, we stress that our results hold for any number of extra dimensions as long as the inflaton propagates in all of them.

Let us start by assuming that the extra dimensions are already stabilized by some (yet unknown) dynamical mechanism [16]. As the inflaton is now a bulk field, we will further assume that it is homogeneous along the extra dimensions, just as in the former KK theories. This is another way to state the perfect fluid assumption for the Φ field in five dimensions (i.e., $T_{05}=0$ where T_{05} is one of the components of the energy momentum tensor). Obviously this will make our theory of inflation similar to those models. However, what will make our theory different from the usual KK theories is the fact that matter is attached to the branes and that will affect the inflaton decay in an essential manner. Notice that the condition $T_{05}=0$ makes the inflaton a zero mode, which is also a necessary condition if we want to reproduce the ADD scenario at late times (a flat and factorizable geometry). Therefore, in the effective four-dimensional theory, the inflaton field $\tilde{\phi}$ and the bulk field Φ are related by $\tilde{\phi} = \sqrt{R}\Phi_0$. Notice that these assumptions are consistent as long as the brane densities remain smaller than M^4. If brane densities were large, one would have to consider the branes as sources for the metric in the Einstein equations and one will depart from the ADD picture towards a Randall-Sundrum type nonfactorizable geometry. We will not consider such scenarios here. Some ideas on this regard can be found in Ref. [17].

Once the extra dimension is stable, one gets, in the effective four dimensional theory, the usual form of the Hubble parameter as

$$H^2=\frac{V_{\rm eff}}{3M_{\rm Pl}^2}, \tag{6}$$

where the effective four-dimensional potential is defined by

$$V_{\rm eff}=RV_{\rm 5D}=\left(\frac{M_{\rm P\,l}^2}{M^3}\right)V_{\rm 5D}. \tag{7}$$

As we will see, this condition translates into the scaling of the inflaton as mentioned above. Now we focus on the implications of this formula.

First, since the inflaton is now a bulk field, the upper bound on the five-dimensional potential is M^5 (instead of M^4 for the case of the brane inflaton) and the effective potential has the upper bound $RM^5=M^2M_{\rm P\,l}^2$. Therefore, one gets $m \leqslant H \leqslant M$, which does not require a superlight inflaton. This also keeps the explanation of the flatness and horizon problems as usual, since now, the time for inflation could be as short as in the standard theory. Thus, our bulk field naturally overcomes the problem noted by Lyth and Kaloper and Linde.

To proceed further, let us assume the following five-dimensional potential for the bulk field

$$V_{\rm 5D}(\Phi)=\frac{1}{2}m^2\Phi^2+\frac{\lambda}{4M}\Phi^4. \tag{8}$$

The effective potential that drives inflation on the brane can be derived from the above equation to be

$$V_{\rm eff}(\tilde{\phi})=\frac{1}{2}m^2(\tilde{\phi})^2+\frac{\tilde{\lambda}}{4}(\tilde{\phi})^4, \tag{9}$$

where $\tilde{\lambda}:=(M^2/M_{\rm P\,l}^2)\lambda$ is a naturally suppressed coupling. Now, as in the old fashioned chaotic scenario, inflation will start in those small patches of size H^{-1} where the effective inflaton reaches an homogeneous value $\tilde{\phi}_c \geqslant M_{\rm P\,l}$. However, because of scaling, this requires that for the bulk field, we must have $\Phi_c(0)\geqslant M^{3/2}$, which is a natural value in our picture.

We wish to note parenthetically, that if the Φ^4 term dominates the energy density (i.e., $m \ll M$), then $\tilde{\phi}$ could develop a vacuum expectation value in which case an interesting connection between the scales M, m, and v_{ew} can emerge as follows. We can have the inflaton couple to matter fields in the brane (which is needed to reheat the brane Universe), via the following term:

$$hM^{1/2}\Phi\chi^2\delta(y), \tag{10}$$

where h is a dimensionless coupling constant and χ is the brane Higgs field. When Φ develops a vacuum expectation value, the last term will contribute to the mass term, μ_0^2, in the Higgs potential, which should be of order of the weak scale. From the potential given in Eq. (8), we get then the constraint

$$\mu_0^2=\frac{h}{\lambda^{1/2}}mM\approx v_{ew}^2. \tag{11}$$

Now assuming that $h,\lambda\sim 1$, and $m\sim 10$ GeV$\ll M$, we get $M\sim 10^2$ TeV, which is consistent with the strongest experimental limit [6].

IV. DENSITY PERTURBATION

The calculation of the density perturbation proceeds as in the usual four-dimensional theories. Let us write it down in terms of the five-dimensional potential

$$\frac{\delta\rho}{\rho}\sim\frac{[V_{\rm eff}(\tilde{\phi}_c)]^{3/2}}{M_{\rm P\,l}^3\left(\frac{\partial V_{\rm eff}}{\partial\tilde{\phi}_c}\right)}=\left(\frac{1}{M_{\rm P\,l}M^3}\right)\frac{[V_{\rm 5D}(\Phi_c)]^{3/2}}{\left(\frac{\partial V_{5D}}{\partial\Phi_c}\right)}. \tag{12}$$

Because the quartic term is suppressed for small values of M, we first assume that the mass term drives the inflation. Nevertheless, as expected we get $\delta\rho/\rho\sim m/M_{\rm P\,l}$, which is again very small for $m \ll M$. This result is similar to what one obtains in the brane inflaton models [8]. On the other hand, if the quartic term dominates the density, then

$$\frac{\delta\rho}{\rho}\sim\tilde{\lambda}^{1/2}=\left(\frac{M}{M_{\rm P\,l}}\right)\lambda^{1/2}.$$

and models with only large values of M would be satisfactory. Since our interest here is in models with large extra dimensions, we consider M in the multi-TeV range and therefore we must seek ways to solve this problem. In any case it is gratifying that a single bulk scalar field seems to solve two of the major problems faced by the brane inflaton models.

In order to improve the situation with respect to $\delta\rho/\rho$ in this model, we extend it to include an extra scalar field σ and considering bulk potential to have the same form as is used in implementing the hybrid inflation [14] picture:

$$V(\phi,\sigma)=\frac{M}{4\lambda}\left(M^2-\frac{\lambda}{M}\sigma^2\right)^2+\frac{m_0^2}{2}\phi^2+\frac{g^2}{M}\phi^2\sigma^2. \quad (13)$$

It is easy to check that inflation will require $\phi_c^2 \geqslant M^3/2g^2$. Therefore, our effective inflaton should be $\tilde{\phi}_c \geqslant M_{\mathrm{P1}}/\sqrt{2}g$, just as expected. One then uses Eq. (12) to get

$$\frac{\delta\rho}{\rho}\sim\left(\frac{g}{2\lambda^{3/2}}\right)\frac{M^3}{m_0^2 M_{\mathrm{P1}}}. \quad (14)$$

If for instance, we set in the last equation the values $M \sim 10^2$ TeV and $m_0 \sim m \sim 10$ GeV we find

$$\frac{\delta\rho}{\rho}\sim\left(\frac{g}{2\lambda^{3/2}}\right)\times 10^{-5}, \quad (15)$$

which is the right COBE result.

Let us now compare our model with the case where one has hybrid inflation in the wall. In order to explain the density pertubation in the wall hybrid inflation models , an unpleasant fine tuning of either the mass of the inflaton field (to the level of $m_0 = 10^{-10}$ eV), or of the coupling constant (to the level of $\lambda = 10^{-8}$ [8]) is essential. On the other hand, as we just showed, if the inflaton is a bulk field, no such fine-tuning is required. We find this to be perhaps an interesting advantage of models with large extra dimensions over the conventional four-dimensional inflation models.

Let us also remark that in the case of more than one extra dimension, the only change on our above results come from the substitution of R for the volume of the extra space V_n in all the analysis. This does not affect the results of our analysis, since the effective theory is still given in terms of the same effective coupling constants, although the effective inflaton will be changed into $\sqrt{V_n}\Phi$. This rescaling does not affect our main expression in Eq. (12) nor (14).

V. REHEATING

The epoch of the Universe soon after inflation is called reheating. During this era, the inflaton is supposed to decay into matter populating the Universe and reheating it to a temperature T_R, called the reheating temperature. Since many important phenomena of cosmology, e.g., baryogenesis, depend on the Universe being very hot, the value of T_R is important. One also has to watch out for any unwanted particles that may be produced during the reheating, since they may create problems for the subsequent evolution of the Universe (as, for instance, is familiar from the study of gravitino production in supergravity theories). Reheating is followed by thermalization of particles produced, so that conventional Friedman expansion can begin subsequently. Again, thermalization is also dependent on T_R. Clearly, therefore reheating is a very important aspect of any model of inflation. In this section, we discuss how it works in our model.

In our discussion, we will use the elementary theory of reheating, which is based on perturbation theory [18] and where the reheating temperature T_R can be expressed in terms of the total decay rate of the inflaton, Γ and the Hubble parameter as follows: $T_R \sim 0.1\sqrt{\Gamma M_{\mathrm{P1}}}$ [19]. It has been pointed out that this approach faces some limitations in terms of efficiency [19] and possible improvements have been suggested using a first stage of heating (called preheating [20]) based on parametric resonance. We will not be concerned here with these extra subtleties and use perturbative reheating to get a crude idea of the postinflationary phase of the Universe and discuss how the Friedman Universe emerges following the end of the reheating period.

An obvious problem that could possibly arise is that the strong coupling of the KK modes of graviton, could induce a faster decay channel for the inflaton than matter. If this happens, the bulk would reheat while the brane would remain empty of matter giving rise to a nonstandard, undesirable Universe. As we show below, luckily this is not the case for our model.

When hybrid inflation ends, the field σ quickly goes to one of its minima $\sigma_\pm = \pm M^{3/2}/\sqrt{\lambda}$. As a result the mass term of the inflaton field receives a contribution $\sim g^2M^2/\lambda$ which dominates over m. This leads us to conclude that the inflaton field, which will oscillate around its minimum and generate reheating, has a mass about an order of magnitude below M (using the set of parameter chosen to explain density perturbations). Therefore, the decay $\phi \to \chi\chi$ is allowed, where χ is the Higgs field, for typical Higgs field masses in the 100–200 GeV range. Let us stress that this process will take place only on the boundaries of the extra dimension (i.e., in the brane), making it physically different from the former KK theories where the production of matter through inflaton decays occurs everywhere.

Following the steps of perturbative reheating theory, we can estimate the reheating temperature by calculating the decay rate of the inflaton field into two Higgs fields. That is,

$$\Gamma_{\phi\to\chi\chi}\sim\frac{M^4}{32\pi M_{\mathrm{P1}}^2 m_\phi}. \quad (16)$$

With an inflaton mass m_ϕ around $0.1M$ we estimate the reheating temperature to be $T_R > 100$ MeV. As would be desirable, this temperature is above that required for successful big bang nucleosynthesis.

The next point that needs to be investigated is the generation of gravitons by the excited modes of ϕ and σ fields. Because, if excessive graviton production drained away the energy stored in the inflaton field, it would lead to lower matter density compared to graviton density and matter may not reach a state of equilibrium. In the five-dimensional language, this would lead to an expansion of the bulk rather than the brane. Notice that such processes are not dangerous in KK theories where the compactification scale is very small and the excited modes decay preferentially into matter (on the bulk). Now that the radius of the extra space is large, a large number of KK gravitons h could be produced by a KK inflaton mode decaying into another excited mode plus a graviton $\phi_n \to \phi_l h_{n-l}$, where n and l are the KK numbers, which are conserved. To estimate the amount of gravitons produced, we have to first estimate the production rate for ϕ_n, the excited modes of the inflaton, since only ϕ_n decay can produce gravitons via the decay process just mentioned. The ϕ_n modes are produced via collision processes $\phi_0\phi_0 \to \phi_n\phi_n$ and the rate for this process is given by

$$\sigma_{\phi_0\phi_0 \to \phi_n\phi_n} \sim \lambda^2 \frac{M^2}{M_{\rm Pl}^4} \tag{17}$$

for $\sqrt{s} \sim M$. This has to be compared with $\Gamma_{\phi \to \chi\chi}$ above Eq. (16). Due to the very different Planck mass dependence, it is easy to see that the ϕ_n production is highly suppressed compared to the ϕ_0 decay to Higgs bosons. Once an excited mode has been produced, it will preferably decay into gravitons. Although the rate for this process is very small,

$$\Gamma_{\phi_n \to \phi_l h_{n-l}} \sim \frac{m_n m_l^2}{12\pi M_{\rm Pl}^2}, \tag{18}$$

where $m_n^2 = m_\phi^2 + n^2/R^2$ is the mass of the excited mode, the presence of a large number of accessible modes in the final state will enhance this value up to

$$\Gamma_{\phi_n,\rm total} = \sum_l \Gamma_{\phi_n \to \phi_l h_{n-l}} \sim \frac{m_n^3}{12\pi M^2} \tag{19}$$

making the excited mode very short lived. We should stress, however, that the final products of the shower induced by a KK mode decaying on lighter modes will always include ϕ_0's, which as we stated already only decay on the brane to Higgs fields and hence to matter. As a result, the KK excitations of the ϕ_0 will not be around to overclose the Universe and the associated graviton production is also unlikely to be significant.

Thus, the final scenario that emerges is as follows: after exiting inflation the inflaton will start moving relativistically, eventually producing both Higgs fields as well as its excited modes due to the possible (suppressed) four points self-interactions. As the rate estimates above show, most of the reheat energy will pass to the brane in the form of matter and a very small part will pass to the bulk in the form of gravitons produced through the fast decaying KK inflaton modes. As the Universe is still rapidly moving at that stage, we could imagine that the density of those gravitons will be substantially diluted. The Higgs bosons produced by the inflaton decay will quickly decay to quarks and leptons, which will attain equilibrium, via their strong and weak interactions. Friedmann expansion will resume, albeit starting with a lower temperature ($T \sim 0.1$ GeV) compared to the conventional grand unified theories. We also point out that this scenario, naively, will not be affected by the presence of the σ field of the hybrid inflation model.

VI. REMARKS AND DISCUSSION

We now conclude with a brief summary of our main results. Choosing the bulk scalar field as the source of the brane inflaton field leads to several advantages: first the fundamental scale M can be in the multi-TeV range, which turns out to be the natural bound for inflaton mass m_ϕ and Hubble constant H, in contrast with the brane inflaton models where m_ϕ and H are oversuppressed. Assuming hybrid inflation and M just above the current experimental limits, the COBE observation of $\delta\rho/\rho$ is also successfully explained without any fine-tuning. Finally, we also note that even though the bulk inflaton has KK modes, the reheating process leads to a Universe not dominated by their mass but rather by the standard model particles in equilibrium. Finally, we mention that all the results of this paper remain unchanged when more extra dimensions are involved, provided that the inflaton propagates in all the bulk, and that the Friedmann equation [Eq. (2)] holds.

ACKNOWLEDGMENTS

The work of R.N.M. was supported by a grant from the National Science Foundation under Grant No. PHY-9802551. The work of A.P.L. was supported in part by CONACyT (México). The work of C.P. was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP). We would like to thank S. Nussinov for several stimulating discussions on the reheating process.

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