

Geodesics and Newton's law in brane backgrounds

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(Received 25 May 2000; published 23 October 2000)

In brane world models our universe is considered as a brane embedded into a higher-dimensional space. We discuss the behavior of geodesics in the Randall-Sundrum background and point out that free massive particles cannot stably move along the brane. The brane is repulsive, and matter will be expelled from the brane into the extra dimension. This is undesirable, and hence we study a simple alternative model with a noncompact extra dimension, but with an attractive brane embedded into the higher-dimensional space. We study the linearized gravity equations and show that Newton's gravitational law is valid on the brane also in the alternative background.

PACS number(s): 11.10.Kk, 04.50.+h, 11.25.Mj

I. INTRODUCTION

It has recently been suggested by Randall and Sundrum [1] that four-dimensional gravity can arise at long distances on a brane embedded in a five-dimensional anti-de Sitter space. In their model the fifth dimension is noncompact. An effective dimensional reduction occurs, because the metric perturbations admit a bound state solution that looks similar to a four-dimensional graviton bound to the brane. Earlier work appeared in Refs. [2–4]. This interesting alternative to compactification has been discussed in a number of recent papers [5–29].

The metric of the Randall-Sundrum (RS) background has the form

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (1)$$

where $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, $\mu, \nu = 1, \dots, 4$ and $k > 0$. It was argued in Ref. [1] that the Kaluza-Klein excitations, as they are light, are suppressed near the brane and almost decouple from the matter fields. Moreover, it is assumed that matter fields are trapped to the brane by a certain mechanism.

If our spacetime is the ordinary Minkowski spacetime, then there should exist trajectories for free massive particle located on the brane only. Although this is true for the metric (1), these trajectories are not stable. In fact, the five-dimensional trajectory of the free particle in the metric (1) has the form (see Sec. II for details, here we take $y_0 \approx \dot{y}_0 = 0$)

$$x^\mu = x_0^\mu + v^\mu t, \quad |y| = \frac{1}{2k} \ln(1 - v^2 k^2 t^2). \quad (2)$$

For $y_0 = 0$ exactly, the trajectory is $y(t) = 0$, because the particle is pulled to each side of the brane with equal force. However, a perturbation will lead to expulsion from the

brane following a trajectory of the form (2). It seems this is undesirable, even if for a very small (Planck scale) k the time needed for the significant deviation in the fifth direction will be rather large. Thus, in the RS background, some other, nongravitational mechanism is needed in order to trap matter on the brane. The simplest way to obtain an attractive brane would be to change the sign of the brane tension, although one might argue that this would imply other undesirable features. This alternative was considered in Refs. [4, 30–32].

In the present paper, we shall study the RS background and the alternative possibility, whose background metrics are given by Eq. (1), with $k > 0$ for the RS background, and $k < 0$ for the alternative background. The metric (1) is a solution to Einstein's equation for the action

$$S = \int d^4x \int dy \sqrt{-g} (R - 2\Lambda) + \sigma \int_{y=0} d^4x \sqrt{-g_B}, \quad (3)$$

where the cosmological constant and brane tension are

$$\Lambda = -6k^2, \quad \sigma = -12k. \quad (4)$$

We shall now give a brief outline of the rest of the paper. First, in Sec. II we study the geodesics in the two backgrounds and find that only for the alternative background gravity provides a mechanism for the trapping of matter on the brane. Second, in Sec. III we consider the linearized gravity equations, which shall be used in Sec. IV to derive the Newtonian limit of gravity on the brane in both backgrounds. We find that in both cases the gravitational potential for a static point source will be $\sim -1/r$, and we find an exact formula for the corrections. In Sec. V, we give expressions for the graviton modes in both backgrounds. Spacelike modes are absent in the RS background, but our results are inconclusive for the alternative background.

II. GEODESICS

In this section, we explicitly solve the geodesic equation in the RS and alternative backgrounds. We shall find that, in the RS background, ordinary matter will be expelled from the brane, but in the alternative background, the brane is attractive.

Using the zeroth order terms of the connections given in

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the Appendix, the geodesic equation takes the form

$$\frac{d^2 x^\mu}{d\theta^2} - 2k \operatorname{sgn} y \frac{dx^\mu}{d\theta} \frac{dy}{d\theta} = 0, \quad (5)$$

$$\frac{d^2 y}{d\theta^2} + k \operatorname{sgn} y e^{-2ky} \eta_{\mu\nu} \frac{dx^\mu}{d\theta} \frac{dx^\nu}{d\theta} = 0. \quad (6)$$

We start by integrating Eq. (5), which yields

$$\frac{dx^\mu}{d\theta} = v^\mu e^{2k|y|}, \quad (7)$$

where v^μ is a constant four-vector. Equation (6) is explicitly solved by the first integral of the geodesic equation

$$\left(\frac{dy}{d\theta}\right)^2 + e^{-2ky} \eta_{\mu\nu} \frac{dx^\mu}{d\theta} \frac{dx^\nu}{d\theta} = C,$$

where C is a constant. Hence, inserting Eq. (7), we find

$$\left(\frac{dy}{d\theta}\right)^2 + e^{2k|y|} v^2 = C, \quad (8)$$

where $v^2 = v^\mu v_\mu$.

It is convenient to change the parameterization to a non-affine parameter t such that

$$\frac{dt}{d\theta} = e^{2k|y|}.$$

Then, Eq. (7) becomes (notation $\dot{x} = dx/dt$)

$$\dot{x}^\mu = v^\mu \Rightarrow x^\mu = x_0^\mu + v^\mu t, \quad (9)$$

which shows that we can choose a reference frame such that $t = x^0$, i.e., t is the time on the brane. Moreover, Eq. (8) becomes

$$\dot{y}^2 e^{4k|y|} + e^{2k|y|} v^2 = C, \quad (10)$$

and we can determine C from the initial data

$$C = \dot{y}_0^2 e^{4k|y_0|} + v^2 e^{2k|y_0|}. \quad (11)$$

Before integrating Eq. (10), we note that it depends only on $|y|$, as long as we do not pass through $y=0$. Therefore, it is sufficient to consider $y>0$; replacing y with $|y|$ at the end will take care of the case $y<0$. For $y>0$, we can integrate Eq. (10) and find

$$\sqrt{C - v^2 e^{2ky}} + \pm v^2 k t + \sqrt{C - v^2 e^{2ky_0}},$$

where we have again expressed the integration constant by the initial data. The $\pm$ sign in front of the term containing t on the right-hand side stems from the ambiguity of taking a square root. After some simple steps involving the substitution of C from Eq. (11) we obtain

$$e^{2ky_0} - e^{2ky} = v^2 k^2 t^2 \pm |y_0| 2k e^{2ky_0} t,$$

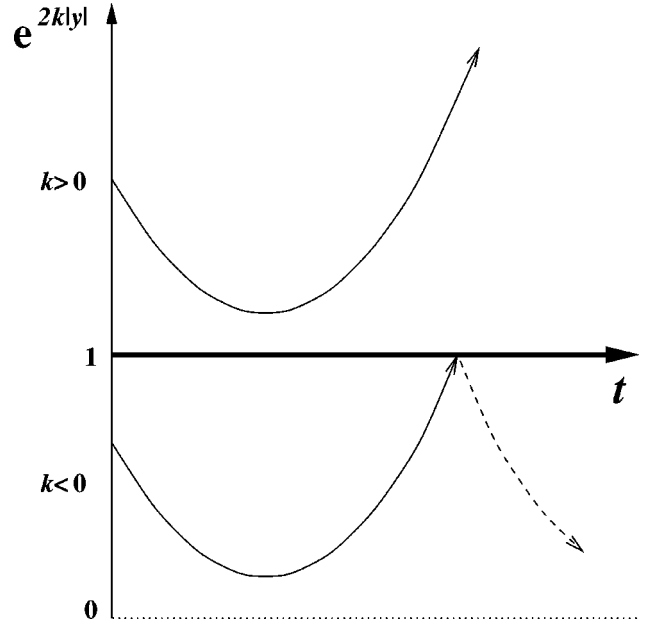


FIG. 1. The solution (12) for ordinary matter, $v^2 < 0$. The region $k>0$ corresponds to the RS background, the region $k<0$ to the alternative background. In both cases, the brane sits at $e^{2k|y|}=1$.

and we can deduce from the initial data that we have to replace $\pm |\dot{y}_0|$ by $-\dot{y}_0$. Thus, the final result is

$$e^{2k|y|} = e^{2k|y_0|} + 2k\dot{y}_0 e^{2k|y_0|} t - v^2 k^2 t^2. \quad (12)$$

The solution (12) is valid, as long as $|y| \neq 0$.

If we hit the brane at $y=0$, we have to match a solution for $y>0$ with a solution for $y<0$. However, from Eq. (6) we see that the velocity $\dot{y}$ must be continuous at $y=0$, since the second term in that equation is finite. Thus, the brane will not deflect particles gravitationally, but we might expect that nongravitational interactions with matter on the brane do so.

The interpretation of the solution (12) is rather simple: In the RS background ($k>0$) ordinary matter ($v^2 < 0$) is repelled by the brane. A special case is a particle initially moving along the brane, $y_0 = \dot{y}_0 = 0$. Equation (12) predicts expulsion, but the particle would not know in which direction it is to go, so it will keep moving along the brane in an unstable equilibrium. On the other hand, tachyonic particles ($v^2 > 0$) are attracted to the brane, whereas massless particles are not affected by its presence. For $k<0$, the brane attracts ordinary matter. This is sketched in Fig. 1.

Another interesting fact is that, for the alternative background, $e^{2k|y|}=0$ corresponds to $|y|=\infty$. Thus, tachyonic particles will be expelled to $|y|=\infty$ in finite brane time, as will massless particles with the right initial conditions. Moreover, there exist initial conditions for ordinary particles, which will yield $|y|=\infty$ in finite brane time.

III. LINEARIZED GRAVITY

In this section, we shall study the linearized gravity equations with two applications in mind: The derivation of Newton's law on the brane and the study of graviton modes, which will be carried out in Secs. IV and V, respectively.

For our purpose, we introduce a matter perturbation on the brane

$$\delta T_{00} = \delta(y) t_{00}(x), \quad (13)$$

and solve the linearized gravity equations for this source.

The form (1) of the background metric suggests to use the time slicing formalism [33] for calculating the metric perturbations, although we do not slice with respect to time, but with respect to the transverse coordinate y . Let us first give some useful formulas. In the time slicing formalism, we split up the metric tensor as

$$g_{ab} = \begin{pmatrix} g_{\mu\nu} & n_\nu \\ n_\mu & n_\mu n^\mu + n^2 \end{pmatrix},$$

$$g^{ab} = \frac{1}{n^2} \begin{pmatrix} n^2 g^{\mu\nu} + n^\mu n^\nu & -n^\nu \\ -n^\mu & 1 \end{pmatrix}, \quad (14)$$

where $a, b = 1, \dots, 5$, $x^5 = y$, and $g_{\mu\nu}(x, y)$ are the induced metrics in the hypersurfaces with internal coordinates x^μ . The quantities n and n^μ are the lapse function and shift vector, respectively, and are fixed to their respective background values in axial gauge

$$n^\mu = 0, \quad n = 1. \quad (15)$$

Thus, we consider a metric of the form

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + dy^2. \quad (16)$$

Then, the second fundamental form measuring the extrinsic curvature on the hypersurfaces is given by

$$H_{\mu\nu} = \frac{1}{2n} \left(\frac{\partial}{\partial y} g_{\mu\nu} - \nabla_\mu n_\nu - \nabla_\nu n_\mu \right) = \frac{1}{2} \frac{\partial}{\partial y} g_{\mu\nu}, \quad (17)$$

where ∇_μ is the covariant derivative on the hypersurfaces, and the second equality holds in axial gauge.

Einstein's equation is

$$R_{ab} - \frac{1}{2} g_{ab} R = -g_{ab} \Lambda + 8\pi T_{ab}, \quad (18)$$

where $T_{ab} = \bar{T}_{ab} + \delta T_{ab}$, and $\bar{T}_{ab}$ is the background from the brane, whose nonzero components are found from Eq. (3) as

$$\bar{T}_{\mu\nu} = -\frac{3k}{4\pi} \delta(y) g_{\mu\nu}. \quad (19)$$

One can observe from Eqs. (13) and (19) that $T_{a5} = 0$. Therefore, by virtue of the Gauss-Codazzi equations [33], the normal and mixed components of Eq. (18) become

$$\hat{R} + H^\mu_\nu H^\nu_\mu - H^2 = 2\Lambda, \quad (20)$$

$$\nabla_\mu H - \nabla_\nu H^\nu_\mu = 0, \quad (21)$$

respectively, where $H = H^\mu_\mu$, and $\hat{R}$ is the intrinsic scalar curvature of the hypersurfaces. The tangential components of Eq. (18) simply read

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -g_{\mu\nu} \Lambda + 8\pi T_{\mu\nu}. \quad (22)$$

Equation (22) is the equation of motion for $g_{\mu\nu}$, whereas Eqs. (20) and (21) are constraints.

We linearize Eqs. (20)–(22) around the background (1), for which purpose we use an induced metric of the form

$$g_{\mu\nu} = e^{-2k|y|} (\eta_{\mu\nu} + \gamma_{\mu\nu}), \quad (23)$$

where γ is a small perturbation compared to η . The indices of γ shall be raised and lowered using the Lorentz metric η . Some useful expressions for the connections and curvatures are given in the Appendix. Equations (20)–(22) take the forms

$$e^{2k|y|} (\gamma^{\mu\nu}_{,\mu\nu} - \square \gamma) + 3k \operatorname{sgn} y \gamma_{,y} = 0, \quad (24)$$

$$\frac{1}{2} \partial_y (\gamma_{,\mu} - \gamma^\nu_{,\mu,\nu}) = 0, \quad (25)$$

$$\frac{1}{2} (\gamma^\mu_{\rho,\mu\nu} + \gamma^\mu_{\nu,\mu\rho} - \square \gamma_{\nu\rho} - \gamma_{,\nu\rho})$$

$$- \frac{1}{2} \eta_{\nu\rho} (\gamma^{\mu\lambda}_{,\mu\lambda} - \square \gamma) + e^{-2k|y|} \left[-\frac{1}{2} \gamma_{\nu\rho,y} + \frac{1}{2} \eta_{\nu\rho} \gamma_{,\gamma\gamma} + 2k \operatorname{sgn} y (\gamma_{\nu\rho,y} - \eta_{\nu\rho} \gamma_{,y}) \right] = 8\pi \delta T_{\nu\rho}, \quad (26)$$

where the background has been cancelled using Eqs. (4) and (19).

Let us start by solving the constraints. First, from Eq. (25) we find

$$\gamma^\nu_{\mu,\nu} = \gamma_{,\mu} + \xi_\mu(x), \quad (27)$$

where ξ_μ are functions of the brane coordinates x^μ only. Second, after substituting Eq. (27), Eq. (24) leads to

$$e^{2k|y|} \xi^\mu_{,\mu} + 3k \operatorname{sgn} y \gamma_{,y} = 0. \quad (28)$$

Thus, integrating Eq. (28) yields the trace γ as

$$\gamma = -\frac{1}{6k^2} \xi^\mu_{,\mu} (e^{2k|y|} - 1), \quad (29)$$

where we have used the residual gauge freedom to impose $\gamma = 0$ on the brane. We see from Eq. (29) that γ is unbounded for the RS background ($k > 0$), if $\xi^\mu_{,\mu} \neq 0$. Moreover, this observation is independent of whether we use the residual gauge freedom as indicated or not. We shall see later that we do not have the choice of setting $\xi^\mu = 0$, if a matter perturbation is present on the brane. This indicates that, in the RS background, the linear approximation is not consis-

tent. However, it might be an artifact of the particular choice of gauge. For further discussion of this problem, see Ref. [15,28].

Finally, substituting Eqs. (13), (27), and (29) into the equation of motion (26), we obtain the equation

$$\begin{aligned} \square \gamma_{\nu\rho} + \partial_y (e^{-2k|y|} \gamma_{\nu\rho,y}) - 2k \operatorname{sgn} y e^{-2k|y|} \gamma_{\nu\rho,y} \\ - \xi_{\nu,\rho} - \xi_{\rho,\nu} + \frac{1}{3} \eta_{\nu\rho} \xi^\mu{}_{,\mu} + \frac{1}{6k^2} (e^{2k|y|} - 1) \xi^\mu{}_{,\mu\nu\rho} \\ + \frac{2}{3k} \delta(y) \eta_{\nu\rho} \xi^\mu{}_{,\mu} = -16\pi \delta(y) t_{\nu\rho}. \end{aligned} \quad (30)$$

We can take the trace of Eq. (30) and find

$$\xi^\mu{}_{,\mu} = -8\pi k t. \quad (31)$$

Thus, as indicated earlier, the four-divergence $\xi^\mu{}_{,\mu}$ is fixed by the content of matter perturbation on the brane.

As the next step, we consider the discontinuity of Eq. (30) at $y=0$. One easily finds

$$-16\pi t_{\nu\rho} = \gamma_{\nu\rho,y}|_{y=+0} - \gamma_{\nu\rho,y}|_{y=-0} + \frac{2}{3k} \eta_{\nu\rho} \xi^\mu{}_{,\mu}. \quad (32)$$

However, as the perturbation is symmetric around the brane, we need only look for solutions that are even in y . Therefore, we can consider Eq. (30) in the region $y>0$, and Eqs. (32) and (31) provide the Neumann boundary condition

$$\gamma_{\nu\rho,y}|_{y=+0} = -8\pi \left(t_{\nu\rho} - \frac{1}{3} \eta_{\nu\rho} t \right). \quad (33)$$

Consider now Eq. (30) for $y>0$. First, let us choose the vector ξ^μ as

$$\xi_\mu = -8\pi k \frac{1}{\square} \partial_\mu t, \quad (34)$$

which is consistent with the condition (31). Then, we shall write

$$\gamma_{\nu\rho} = \frac{4\pi}{3k} \left[\frac{1}{\square} t_{,\nu\rho} (e^{2ky} - 1) + 2k^2 \left(\eta_{\nu\rho} \frac{1}{\square} t - \frac{4}{\square^2} t_{,\nu\rho} \right) \right] + \bar{\gamma}_{\nu\rho} \quad (35)$$

in order to obtain from Eq. (30) the following homogeneous equation for $\bar{\gamma}_{\nu\rho}$:

$$\square \bar{\gamma}_{\nu\rho} + \partial_y (e^{-2ky} \bar{\gamma}_{\nu\rho,y}) - 2k e^{-2ky} \bar{\gamma}_{\nu\rho,y} = 0. \quad (36)$$

Moreover, from Eq. (29) we find that the trace $\bar{\gamma} \equiv 0$, and the Neumann boundary condition (33) yields

$$\bar{\gamma}_{\nu\rho,y}|_{y=+0} = -8\pi \left(t_{\nu\rho} - \frac{1}{3} \eta_{\nu\rho} t + \frac{1}{3\square} t_{,\nu\rho} \right). \quad (37)$$

Notice that a trivial (zero) solution to the homogeneous equation (36) is not consistent with this boundary condition.

In order to solve Eq. (36), let us Fourier transform with respect to the brane coordinates and change variables to $z = e^{2ky}$. Then, Eq. (36) becomes

$$\left(z^2 \partial_z^2 - z \partial_z - \frac{p^2}{4k^2} z \right) \bar{\gamma}_{\nu\rho} = 0, \quad (38)$$

whose solution can be expressed in terms of Bessel functions [34].

IV. NEWTON'S LAW

In order to derive Newton's law on the brane, we have to look for a unique solution to the linearized Einstein equations in the presence of a static point source on the brane. In the last section, we presented the general formalism of linearized gravity. Let us now continue the solution for static point source. In order to obtain the Newtonian limit, we have to calculate $\gamma_{00}(x,0)$, since the gravitational potential is given by

$$V = -\frac{1}{2} \gamma_{00}. \quad (39)$$

We need a second boundary condition for Eq. (36) in order to obtain a unique solution. We shall use

$$\bar{\gamma}_{\nu\rho}|_{y=\infty} = 0, \quad (40)$$

because it is applicable in both for $k>0$ and $k<0$. The more physical condition $\gamma_{\nu\rho}|_{y=\infty} = 0$ can be imposed only for $k<0$.

For static potentials, we have $p_0=0$, and therefore $p^2 \geq 0$ in Eq. (38). In fact, we need only consider $p^2>0$, as the solution for $p^2=0$ can be reconstructed as the limit $p^2 \rightarrow 0$. The solution of Eq. (38) for $p^2>0$ is

$$\bar{\gamma}_{\nu\rho}(p,y) = c_{\nu\rho}(p) e^{2ky} \begin{cases} K_2(e^{ky}|p|/|k|), \\ I_2(e^{ky}|p|/|k|), \end{cases} \quad (41)$$

where the choice between the two possible solutions is dictated by Eq. (40). We easily see that this amounts to choosing the solution with the K function for the RS background ($k>0$), and the solution with the I function for the alternative background.

Moreover, from Eq. (41) we find the first derivative

$$\bar{\gamma}_{\nu\rho,y}(p,y) = |p| c_{\nu\rho}(p) e^{3ky} \begin{cases} -K_1(e^{ky}|p|/|k|), \\ I_1(e^{ky}|p|/|k|), \end{cases} \quad (42)$$

which, combined with the boundary condition (37), yields the coefficients

$$\begin{aligned} c_{\nu\rho} = \frac{8\pi}{|p|} \left[t_{\nu\rho} - \frac{1}{3} t \left(\eta_{\nu\rho} - \frac{p_\nu p_\rho}{p^2} \right) \right] \\ \times \begin{cases} [K_1(|p|/|k|)]^{-1} & \text{for } k>0, \\ [I_1(|p|/|k|)]^{-1} & \text{for } k<0. \end{cases} \end{aligned} \quad (43)$$

Thus, inserting Eqs. (41) and (43) into Eq. (35), we obtain the solution for the metric perturbation

$$\begin{aligned} \gamma_{\nu\rho}(p,y) = & \frac{4\pi}{3k} \left[\frac{p_\nu p_\rho}{p^2} (e^{2ky} - 1) - 2k^2 \left(\frac{\eta_{\nu\rho}}{p^2} - \frac{4p_\nu p_\rho}{p^4} \right) \right] t \\ & + \frac{8\pi}{|p|} \left[t_{\nu\rho} - \frac{1}{3} t \left(\eta_{\nu\rho} - \frac{p_\nu p_\rho}{p^2} \right) \right] e^{2ky} \\ & \times \begin{cases} \frac{K_2(e^{ky}|p|/k)}{K_1(|p|/k)} & \text{for } k > 0. \\ \frac{I_2(e^{ky}|p|/|k|)}{I_1(|p|/|k|)} & \text{for } k < 0. \end{cases} \end{aligned} \quad (44)$$

Let us now use a static point source, $t_{00}(p) = 2\pi\delta(p_0)a$, where $a = M/M_{Pl(5)}^3$, and solve for $\gamma_{00}(x,0)$. Consider first the RS background. We can use the recursion formula for modified Bessel functions

$$K_2(z) = \frac{2}{z} K_1(z) + K_0(z),$$

in order to separate the divergent term for $|p| \rightarrow 0$ in the Fourier integral. Then, from Eq. (44) we find

$$\begin{aligned} \gamma_{00}(x,0) = & -\frac{2ka}{3r} + \frac{8ka}{3r} + \lim_{y \rightarrow 0} \int \frac{d^3p}{(2\pi)^3} \\ & \times e^{-ip \cdot x} \frac{16\pi a}{3|p|} \frac{K_0(e^{ky}|p|/k)}{K_1(|p|/k)} \\ = & \frac{2ka}{r} + \frac{8a}{3\pi r^2} \lim_{y \rightarrow 0} \int_0^\infty ds \sin s \frac{K_0(e^{ky}s/kr)}{K_1(s/kr)}. \end{aligned} \quad (45)$$

It is interesting to note that the inhomogeneous terms of the solution would yield Newton's law with a wrong sign, but the homogeneous part, whose presence is necessary because of the Neuman boundary condition, takes care of this. In fact, the importance of the boundary conditions for obtaining the gravity on the brane has already been pointed out in Ref. [35]. Moreover, we observe that the integral in Eq. (45) is defined for any $y > 0$. It is for this reason that we take the limit $y \rightarrow 0$ after the integration. The nonzero y acts as a regulator of the integral for large s . The second term in Eq. (45) is a correction to Newton's law, which we shall demonstrate in a moment.

Let us consider now the alternative background. From Eq. (44) we obtain simply

$$\begin{aligned} \gamma_{00}(x,0) = & \frac{2|k|a}{3r} + \lim_{y \rightarrow 0} \int \frac{d^3p}{(2\pi)^3} e^{-ip \cdot x} \\ & \times \frac{16\pi a}{3|p|} \frac{I_2(e^{ky}|p|/|k|)}{I_1(|p|/|k|)} \\ = & \frac{2|k|a}{3r} + \frac{8a}{3\pi r^2} \lim_{y \rightarrow 0} \int_0^\infty ds \sin s \frac{I_2(e^{ky}s/|k|r)}{I_1(s/|k|r)}. \end{aligned} \quad (46)$$

The first term in Eq. (46) represents Newton's law, and the second term corrections, as we shall demonstrate now.

Consider an integral of the form $\int_0^\infty ds \sin sf(s/z,y)$, where f is a differentiable and integrable function. Obviously, the integrands in Eqs. (45) and (46) satisfy this property for any $y > 0$. Given the integrability of f , we can rewrite the integral as

$$\begin{aligned} & \int_0^\infty ds \sin sf(s/z,y) \\ & = \sum_{k=0}^\infty \int_{-\pi}^\pi ds (-\sin s) f\left(\frac{\pi(2k+1)+s}{z}, y\right). \end{aligned} \quad (47)$$

At this point, we can take the limit $y \rightarrow 0$, and we shall write $f(x,0) = f(x)$. For large z , the argument of f will change little in one period of the sin function, and we can write

$$\begin{aligned} \int_0^\infty ds \sin sf(s/z) \approx & - \sum_{k=0}^\infty \int_{-\pi}^\pi ds \sin s \left[f\left(\frac{\pi(2k+1)}{z}\right) \right. \\ & \left. + \frac{s}{z} f'\left(\frac{\pi(2k+1)}{z}\right) \right] \\ \approx & - \sum_{k=0}^\infty \frac{2\pi}{z} f'\left(\frac{\pi(2k+1)}{z}\right). \end{aligned}$$

Here, we observe that the $z \rightarrow \infty$ limit exists, namely it is just the integral

$$- \int_0^\infty dx f'(x) = f(0) - f(\infty).$$

For both integrands under consideration we have $f(0) = 0$ and $f(\infty) = 1$ [for $y > 0$ we would have the stronger $f(\infty) = 0$]. Thus, we find that the second terms in Eqs. (45) and (46) go to zero at least as fast as $1/r^2$ for large r and are in fact corrections to Newton's law.

Finally, from Eq. (39) we obtain the gravitational potential

$$V(r) = -\frac{|k|mM}{M_{Pl(5)}^3} \begin{cases} 1/r + \mathcal{O}(1/r^2) & \text{for } k > 0, \\ 1/(3r) + \mathcal{O}(1/r^2) & \text{for } k < 0. \end{cases} \quad (48)$$

Thus, we can read off the four-dimensional Planck masses as

$$M_{Pl(4)}^2 = \begin{cases} M_{Pl(5)}^3/k & \text{for } k > 0, \\ 3M_{Pl(5)}^3/|k| & \text{for } k < 0. \end{cases} \quad (49)$$

Our result for the RS background is in agreement with the result given in Ref. [36].

We would like to point out that the result (48) should be taken with a grain of salt. On the one hand, for the RS background, the linearization is inconsistent, because of the exponentially growing trace γ . Still, we believe the result (48) to be correct. On the other hand, for the alternative background one could also use, instead of Eq. (40), the more

physical boundary condition $\gamma_{\nu\rho}|_{y=\infty}=0$. Then, one would not obtain a leading $1/r$ term in the potential. Using Eq. (40) is not entirely unmotivated: The result for the RS background is supposedly correct, and a unified treatment of both cases was possible.

V. GRAVITON MODES

It is our objective in this section to study graviton modes. The equation of motion for gravitons in the gauge $\delta g_{a5}=0$ is the homogeneous equation (36) with the boundary condition

$$\gamma_{\nu\rho,y}|_{y=0}=0. \quad (50)$$

This boundary condition stems from Eq. (33) and implies that we consider only modes which are even in y . This would be natural for the orbifold S^1/Z_2 , but in general one would have to match a solution for $y>0$ with a solution for $y<0$.

Let us now give the graviton solutions. As we consider only modes which are even in y , we restrict ourselves to $y \geq 0$. Using Eq. (38), the solutions to Eq. (36) satisfying the boundary condition (50) are

$$\gamma_{\mu\nu}(p,y)=c_{\mu\nu}(p) \begin{cases} e^{2ky}[I_2(\alpha e^{ky})K_1(\alpha)+K_2(\alpha e^{ky})I_1(\alpha)] & \text{for } p^2>0, \\ 1 & \text{for } p^2=0, \\ e^{2ky}[J_2(\alpha e^{ky})N_1(\alpha)-N_2(\alpha e^{ky})J_1(\alpha)] & \text{for } p^2<0. \end{cases} \quad (51)$$

Here, we have set $\alpha=\sqrt{|p^2|}/|k|$.

Let us discuss the normalizability of these modes. For the RS background, one finds that spacelike modes ($p^2>0$) are not normalizable, since I_2 diverges as e^{e^y} for large y . On the other hand, zerolike and timelike modes are normalizable.

On the other hand, for the alternative background the y integral diverges in all three cases just as the volume integral. Thus, just as in flat space, we might assume that we can form wave packets which describe normalizable wave functions. Alternatively, one might regularize integrals such as the norm of wave functions or the effective action by dividing by the total volume of space. We would like to leave these points open to further research.

VI. CONCLUSIONS

In this paper, we have studied the geodesics and the Newtonian limit of linearized gravity for the Randall-Sundrum and an alternative brane background. We found that matter will be repelled by the brane in the RS background. A similar behavior was observed by Rubakov *et al.* [37] in a different context. Therefore, the RS background is classically unstable, and it must be supplemented with a mechanism for confinement of matter. On the other hand, gravity provides a natural mechanism for the trapping of matter in the alternative background. It would be interesting to study a realization of the alternative background in supergravity. The RS background in supergravity has been discussed in Refs. [8,23–26].

Our derivation of gravity on the brane revealed the validity of Newton's law to leading order in both backgrounds, but the corresponding Planck masses on the brane are different. Moreover, we found exact formulas, Eqs. (45) and (46), for the corrections.

In the RS background there are no normalizable spacelike modes, which certainly is in favor of this background. For the alternative background, all modes are nonintegrable, but a thorough discussion of the confinement or nonconfinement of gravity remains an open problem. It seems that the confinement of gravity cannot be discussed separately from the confinement of matter.

ACKNOWLEDGMENTS

This research was partly supported by NSERC. I.V. is grateful to the Physics Department of Simon Fraser University for its kind hospitality. Moreover, we are grateful to I. Ya. Aref'eva and M. G. Ivanov for their help in the early stage of this work and to A. Linde and V. A. Rubakov for their critical remarks.

APPENDIX

We state here some expressions for the connections and curvature up to first order in the perturbations $\gamma_{\mu\nu}$. The only nonzero connections for the metric g_{ab} are

$$\Gamma^\mu_{\nu\lambda}=\frac{1}{2}(\gamma^\mu_{\nu,\lambda}+\gamma^\mu_{\lambda,\nu}-\gamma_{\nu\lambda}{}^{\mu}), \quad (A1)$$

$$\Gamma^y_{\nu\lambda}=k \operatorname{sgn} y g_{\nu\lambda}-\frac{1}{2}e^{-2k|y|}\gamma_{\nu\lambda,y}, \quad (A2)$$

$$\Gamma^\nu_{\lambda y}=\Gamma^\nu_{y\lambda}=-k \operatorname{sgn} y \delta^\nu_\lambda+\frac{1}{2}\gamma^\mu_{\lambda,y}. \quad (A3)$$

Moreover, we find from Eq. (17)

$$H^\mu_\nu=-k \operatorname{sgn} y \delta^\mu_\nu+\frac{1}{2}\gamma^\mu_{\nu,y}, \quad (A4)$$

and some expressions for the curvatures are ($\square=\eta^{\mu\nu}\partial_\mu\partial_\nu$)

$$\hat{R}_{\nu\rho}=\frac{1}{2}(\gamma^\mu_{\nu,\rho\mu}+\gamma^\mu_{\rho,\nu\mu}-\gamma_{\nu\rho}{}^\mu-\square\gamma_{\nu\rho}), \quad (A5)$$

$$\hat{R} = e^{2k|y|}(\gamma^{\mu\nu}_{,\mu\nu} - \square \gamma), \quad (\text{A6})$$

$$R^y_{\nu y \rho} = -k^2 g_{\nu \rho} + 2k \delta(y) g_{\nu \rho} + e^{-2k|y|} \left(k \operatorname{sgn} y \gamma_{\nu \rho, y} - \frac{1}{2} \gamma_{\nu \rho, yy} \right), \quad (\text{A7})$$

$$R_{\mu \rho} = \hat{R}_{\mu \rho} + H_{\mu}^{\nu} H_{\nu \rho} - H H_{\mu \rho} + R_{\mu y \rho}^y, \quad (\text{A8})$$

$$R = \hat{R} - 20k^2 + 16k \delta(y) + 5k \operatorname{sgn} y \gamma_{,y} - \gamma_{,yy}. \quad (\text{A9})$$

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