Brane world: Disappearing massive matter

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In a brane (domain wall) scenario with infinite extra dimensions and localized gravity, bulk fermions and scalars often have bound states with a zero four-dimensional mass. In this way, massless matter residing on the brane may be obtained. We consider what happens when one tries to introduce a small, but nonvanishing mass to these matter fields. We find that the discrete zero modes turn into quasilocalized states with a finite four-dimensional mass and finite width. The latter is due to tunneling of massive matter into extra dimensions. We argue that this phenomenon is generic to fields that can have bulk modes. We also point out that, in theories meant to describe massive scalars, the four-dimensional scalar potential has, in fact, a power-law behavior at large distances.

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I. INTRODUCTION AND SUMMARY

It was suggested a long time ago [1,2] that localization of particles on a defect in a higher-dimensional space may serve as an alternative to standard Kaluza-Klein compactification. The simplest example of such a defect is a domain wall in $4+1$ dimensions. In the domain wall scenario, the extra dimension is infinite, with the observed four-dimensional fields being zero modes of bulk fields in the domain wall background. These zero modes are localized around the domain wall and thus behave, at low energies, as four-dimensional massless fields. An explicit field theoretic realization of the localization scenario in theories without gravity was straightforward in the case of scalars and fermions [1]; localization of gauge bosons is much more difficult [3].

An interesting recent development concerns the gravitational sector [4] (see Refs. [5–7] for extension to six dimensions). With fine-tuning between a (negative) bulk cosmological constant and (positive) brane tension, a thin-brane solution to the five-dimensional Einstein equations exists which has flat four-dimensional hypersurfaces

$$ds^2 = a^2(z)\eta_{\mu\nu}dx^\mu dx^\nu - dz^2. \qquad (1)$$

Here

$$a(z) = \exp(-k|z|),$$

and the parameter k is determined by the five-dimensional Planck mass and bulk cosmological constant. It has been found that the gravitational field perturbations about the background (1) have a localized zero mode, a four-dimensional graviton. Although continuum modes are arbitrarily light in this case, their interactions with matter on the brane are suppressed. As a result, gravity experienced by matter residing on the brane is effectively four dimensional at distances $r \gg k^{-1}$ [4,8,9].

It has also been shown [10,9] that massless bulk scalars in the Randall-Sundrum background (1) have similar properties

as gravitons: there exist a localized zero mode, and a Kaluza-Klein continuum of arbitrarily light states weakly interacting with matter residing on the brane. Finally, the original mechanism of the localization of massless fermions on a domain wall [1,11] works, in a range of parameters, in curved space (1) as well [10].

In order to serve as a prototype for any realistic model, the brane construction has to be supplemented with a mechanism of mass generation for the four-dimensional fields. In the usual Kaluza-Klein scenario this can be done merely by adding a small mass term to the higher-dimensional action. As we will see shortly, this apparently innocent step has nontrivial consequences in the domain wall case. The point is that even in the presence of an explicit bulk mass term, the operator which determines the modes (and corresponding eigenvalues, i.e., masses of four-dimensional particles) always has a continuous spectrum starting from zero. Indeed, consider the case of a free five-dimensional massive scalar field described by the action

$$S = \int dz d^4x \sqrt{-g}\left(\frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi - \frac{1}{2}\mu^2\phi^2\right), \qquad (2)$$

where the metric g^{ab} is given by Eq. (1). The field equation in this background reads

$$[-\partial_z^2 + 4k\,\mathrm{sgn}(z)\partial_z + \mu^2 - m^2 e^{2k|z|}]\phi(z,p) = 0, \qquad (3)$$

where $m^2 = p^\mu p_\mu$ is the four-dimensional mass. Clearly, at large $|z|$ the bulk mass term μ^2 is negligible as compared to the term $m^2 e^{2k|z|}$, so Eq. (3) reduces to one with $\mu = 0$. Since the continuum eigenvalues are determined by the large-$|z|$ asymptotics which is not affected by the bulk mass term μ^2, Eq. (3) has the same continuum spectrum as the equation with $\mu = 0$, i.e., the continuum spectrum starts from zero m.

Obviously, there is no zero mode at $\mu \neq 0$. The above argument shows that there are no true localized modes with nonzero four-dimensional mass m either (there are no true

bound states embedded in the continuum). This property is generic to fields that can have bulk modes: by using a scaling argument, bulk mass terms are suppressed by a factor $a^2(z)$ as compared to p^2, hence they become irrelevant at large distances from the wall. We will see this explicitly in the case of fermions as well. Let us note in passing that the phenomenon we are discussing persists also when the mass terms are introduced on the brane itself (say, when the effective action contains an additional term $\int d^4x dz \, \delta(z)[-(1/2)\sqrt{(-\bar{g})}\mu'^2\phi^2]$ due to some dynamics on the brane).

The question of whether the domain wall scenario is at all capable of incorporating objects which, to a certain approximation, behave as four-dimensional particles of small, but nonvanishing mass arises. In this paper we give an affirmative answer in both scalar and fermion cases. We will see, however, that these four-dimensional particles are metastable. In other words, we show that at small enough μ, quasilocalized modes exist whose width Γ is much smaller than their four-dimensional mass m. These quasilocalized modes are metastable states that decay into the continuum modes. From the point of view of four-dimensional observer, the quasilocalized modes correspond to massive particles that propagate in three spatial dimensions for some time, and then literally disappear (into the fifth dimension).

Quasilocalized scalars and fermions are similar to quasilocalized gravitons [12–14] that emerge in a class of models [15,12] with flat large-z asymptotics of the five-dimensional space-time. Unlike the latter, the models we consider need not contain potentially dangerous [14,16–18] dynamical branes of negative tension.

The suppression of the width Γ depends on the mechanism of the localization of particles on the wall. We find that in the scalar model (2), the width is suppressed with respect to the mass m by a factor $(m/k)^2$ at small m/k. In the case of fermions the suppression factor has more complicated form and is exponential in a range of parameters.

Yet another manifestation of the continuum starting from zero m is a power-law behavior of the four-dimensional propagator in the infrared. In the scalar case this corresponds to a power-law potential between static sources at large distances (in a model meant to describe massive four-dimensional particles). We will explicitly calculate this potential in Sec. II.

II. SCALAR FIELD

There are several ways to see that an effective four-dimensional theory contains a massive metastable particle. The easiest way is to directly find a complex eigenvalue from the equation which determines the mass spectrum [Eq. (3) in the case of scalars]. As in ordinary quantum mechanics, this complex eigenvalue appears when one imposes the radiation (outgoing wave) boundary conditions[1] at $z \rightarrow \pm \infty$.

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[1]In the brane-world context, this approach was used in Ref. [14] for calculating the lifetime of quasilocalized gravitons in models of the type of Refs. [15,12].

Alternatively, one calculates the Feynman propagator between two points on the brane: if there exists a metastable state, this propagator has a pole at a complex value of the mass. The two ways should of course lead to consistent results.

Let us begin with applying the first method to scalars. We wish to show that the mode equation, Eq. (3), has a complex eigenvalue when the radiation boundary conditions are imposed (see Ref. [9]) at $z \rightarrow \pm \infty$. To the left and to the right of the brane, the solutions to Eq. (3) which satisfy the radiation boundary conditions are

$$f(z<0)=c_1 e^{-2kz}H_\nu^{(1)}\left(\frac{m}{k}e^{-kz}\right),$$

$$f(z>0)=c_2 e^{2kz}H_\nu^{(1)}\left(\frac{m}{k}e^{kz}\right), \quad (4)$$

where $H_\nu^{(1)}(x)$ is the Hankel function and

$$\nu=\sqrt{4+\frac{\mu^2}{k^2}}.$$

The eigenvalues are determined by matching these solutions at $z=0$. The continuity requires that $c_1=c_2$. The first derivative should also be continuous, as is clear from Eq. (3),

$$\partial_z f(+0)-\partial_z f(-0)=0.$$

The latter condition implies the equation for the eigenvalue m,

$$\frac{mH_{\nu-1}^{(1)}(m/k)}{kH_\nu^{(1)}(m/k)}+2-\nu=0. \quad (5)$$

Let us consider the case $\mu \ll k$, and search for solutions with $m \ll k$. In this regime one writes

$$\frac{H_{\nu-1}^{(1)}(m/k)}{H_\nu^{(1)}(m/k)}=\frac{N_{\nu-1}(m/k)}{N_\nu(m/k)}\left\{1-i\frac{J_{\nu-1}(m/k)}{N_{\nu-1}(m/k)}+\cdots\right\},$$

where dots denote terms suppressed by at least one power of m/k, and we keep the contribution that is imaginary at real m. Plugging this expression into Eq. (5) and expanding the Bessel functions at small argument one finds

$$m=m_0-i\Gamma$$

with

$$m_0^2=\frac{\mu^2}{2}, \quad (6)$$

$$\frac{\Gamma}{m_0}=\frac{\pi}{16}\left(\frac{m_0}{k}\right)^2. \quad (7)$$

Thus, there exists a quasidiscrete level with the width suppressed by $(m/k)^2$.

It is instructive to reproduce this result in terms of the scalar propagator $\Delta(z,z',p^2)$ (by four-dimensional Lorentz invariance, the latter depends only on $p^2=p^\mu p_\mu$). The pole of the propagator at certain $p^2=m^2$ corresponds to a particle with the four-dimensional mass m. In the case when one of the arguments is located on the brane, the propagator is straightforward to find from Eq. (4)

$$\Delta(z,0,p^2)=c(p)e^{2k|z|}H_\nu^{(1)}\left(\frac{p}{k}e^{k|z|}\right),$$

where the radiation boundary conditions are imposed, and $c(p)$ is determined by the normalization condition

$$\partial_z\Delta(z,0,p^2)|_{z=0}=1.$$

The propagator has a particularly simple form when both arguments are on the brane

$$\Delta(0,0,p^2)=\left[\frac{pH_{\nu-1}^{(1)}(p/k)}{kH_\nu^{(1)}(p/k)}+2-\nu\right]^{-1}. \qquad (8)$$

Comparing Eq. (8) with Eq. (5) one finds that the propagator has the pole at the complex value of p^2 which corresponds to the unstable massive particle with the mass and width given by Eqs. (6) and (7).

Finally, let us consider the static potential between two sources on the brane, which is induced by the scalar exchange. The potential receives contributions from all modes and is given by the following integral:

$$V(r)=q_1q_2\int\frac{e^{-mr}}{r}\phi_m^2(0)dm, \qquad (9)$$

where q_1 and q_2 are the charges of the sources and $\phi_m(z)$ are the eigenmodes of Eq. (3) which are even under the reflection $z\rightarrow-z$. These eigenmodes are normalized with the measure $\exp(-2k|z|)$ [10,9],

$$\int dze^{-2k|z|}\phi_m(z)\phi_{m'}(z)=\delta(m-m').$$

One finds

$$\phi_m(z)=\sqrt{\frac{m}{2k}}e^{2k|z|}\left[a_mJ_\nu\left(\frac{m}{k}e^{k|z|}\right)+b_mN_\nu\left(\frac{m}{k}e^{k|z|}\right)\right],$$

where the coefficients a_m and b_m are determined by the normalization condition

$$a_m^2+b_m^2=1$$

and the boundary condition on the brane

$$\partial_z\phi_m(z)|_{z=0}=0.$$

The solution to these equations can be written in the form

$$a_m=-\frac{A_m}{\sqrt{1+A_m^2}}, \qquad b_m=\frac{1}{\sqrt{1+A_m^2}},$$

where

$$A_m=\frac{N_{\nu-1}(m/k)-(\nu-2)(k/m)N_\nu(m/k)}{J_{\nu-1}(m/k)-(\nu-2)(k/m)J_\nu(m/k)}.$$

At relatively large distances, $r\gg k^{-1}$, only modes with $m\ll k$ are relevant. Assuming again that $\mu\ll k$, we find

$$A_m\approx\frac{2\Gamma(\nu+1)\Gamma(\nu-1)}{\pi(\nu+2)}\left(\frac{m}{2k}\right)^{2-2\nu}$$

$$\times\left[1-2(\nu-2)(\nu-1)\left(\frac{k}{m}\right)^2\right],$$

$$\phi_m^2(0)\approx\frac{1}{\pi^2}\left(\frac{m}{k}\right)^{-3}\frac{8}{1+A_m^2},$$

so the scalar potential (9) takes the following form:

$$V(r)=\frac{8q_1q_2}{\pi^2}\int\frac{e^{-mr}}{r}\left(\frac{m}{k}\right)^{-3}\frac{1}{1+A_m^2}dm. \qquad (10)$$

There are two competing contributions to this integral. The first one comes from the region where A_m are small, i.e., the last factor in the integrand of Eq. (10) is peaked. It is straightforward to check that this region corresponds exactly to the resonance (6), (7) described above. The resonance contribution to the potential is equal to

$$V_{\text{res}}(r)=\frac{8q_1q_2}{\pi^2}\frac{e^{-m_0r}}{r}\left(\frac{m_0}{k}\right)^{-3}2\pi\Gamma=q_1q_2k\frac{e^{-m_0r}}{r}.$$

As one might have expected, this is the usual Yukawa potential with the mass m_0 (extra factor k accounts for the difference in the mass dimensions of charges in five and four dimensions).

The second contribution comes from the light modes with $m\ll\mu$. It is suppressed by the large factor $(1+A_m^2)\propto(k\mu)^4/m^8$. Explicitly,

$$V_{\text{light}}(r)=\frac{q_1q_2}{2}\int\frac{e^{-mr}}{r}\frac{m^5}{km_0^4}dm=60q_1q_2\frac{1}{km_0^4}\frac{1}{r^7}.$$

We see that almost massless modes lead to power-law behavior at large r. The resulting potential

$$V(r)=V_{\text{res}}(r)+V_{\text{light}}(r)$$

is dominated by the power-like contribution at distances $r\gtrsim2m_0^{-1}\ln(k/m_0)$.

III. FERMIONS

Fermion fields are not localized on the positive tension brane by gravitational interactions only [10]. Hence, one invokes the localization mechanism of Refs. [11,1]. The simplest setup is as follows. One considers a domain wall formed by some scalar field χ. This scalar field has a double-well potential with two degenerate vacua at $\chi=\pm v$; the domain wall separates the region $\chi=-v$ at $z<0$ from the re-

gion $\chi=v$ at $z>0$. A fermion field which has a Yukawa coupling to the scalar $g\chi\bar{\psi}\psi$ has an exact zero mode in the domain wall background. This zero mode is topological and its existence does not depend on the details of the profile of the scalar filed across the wall. Therefore, it also exists for the infinitely thin wall, which is the case we consider in what follows.

For a given sign of the Yukawa coupling g, the zero mode has a certain chirality. Since the four-dimensional fermion mass term requires both chiralities, it can only be introduced in models with two bulk fermion fields which have opposite couplings to the scalar χ. It is convenient to organize these spinors into one field

$$\Psi=\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix},$$

where ψ_1 and ψ_2 are four-component spinors living in five dimensions.

In the presence of the Yukawa interaction $g\chi\bar{\Psi}\tau_3\Psi$, the fields ψ_1 and ψ_2 have left and right zero modes, respectively. Mixing between these two modes that eventually gives rise to four-dimensional mass, is introduced by adding a term $\mu\bar{\Psi}\tau_1\Psi$. It is convenient to bring both these terms to the off-diagonal form by a global $SU(2)$ rotation. The resulting fermion action reads

$$S=\int dzd^4x\sqrt{g}\,\bar{\Psi}(i\gamma^a\nabla_a+g\chi\tau_1+\mu\tau_2)\Psi,$$

where ∇_a is the spinor covariant derivative with respect to the five-dimensional metric g_{ab}. The Dirac equation which follows from this action in the background (1) has the form

$$\left[\frac{1}{a}\gamma^\mu p_\mu+\gamma_5\partial_z-g\chi(z)\tau_1-\mu\tau_2\right]\Psi=0. \quad (11)$$

In the thin-wall limit one has $g\chi(z)=gv\,\text{sgn}\,z$.

Equation (11) determines the fermion modes. At $\mu=0$ and $gv>k/2$ [10], there exist two fermion zero modes of opposite chirality and continuous spectrum starting from zero. It is straightfroward to see that at $\mu>0$ the zero modes disappear, whereas the continuous spectrum still starts from zero. This is precisely the same situation as in the scalar case.

In order to see that there is a metastable massive state, let us find the complex eigenvalue at which there exists a solution to Eq. (11) with the radiation boundary conditions imposed at $z\rightarrow\pm\infty$. It is convenient to separate the spinor Ψ into the left and right components

$$\gamma_5\Psi_{L,R}=\pm\Psi_{L,R}.$$

In terms of $\Psi_{L,R}$ Eq. (11) translates into a set of coupled equations

$$\frac{(\gamma p)}{a}\Psi_R+\partial_z\Psi_L-(g\chi\tau_1+\mu\tau_2)\Psi_L=0, \quad (12)$$

$$\frac{(\gamma p)}{a}\Psi_L-\partial_z\Psi_R-(g\chi\tau_1+\mu\tau_2)\Psi_R=0. \quad (13)$$

After eliminating Ψ_R one obtains a second order equation for Ψ_L,

$$\left[\frac{m^2}{a^2}+\partial_z^2+\frac{a'}{a}\partial_z-\frac{a'}{a}(g\chi\tau_1+\mu\tau_2)-g\chi'\tau_1\right.$$
$$\left.-(g^2\chi^2+\mu^2)\right]\Psi_L=0. \quad (14)$$

Again, the explicit mass terms (the terms involving $g\chi$ and μ) are negligible at large $|z|$ as compared to m^2/a^2, and continuum indeed starts at zero m.

We solve Eq. (14) separately to the left and to the right of the brane, and then match the solutions. To the right of the brane one has $a=\exp(-kz)$, so Eq. (14) reads

$$[m^2e^{2kz}+\partial_z^2-k\partial_z+k(gv\tau_1+\mu\tau_2)-(g^2v^2+\mu^2)]\Psi_L=0. \quad (15)$$

It is convenient to introduce eigenvectors of the matrix $(gv\tau_1+\mu\tau_2)$. Let us define M and α in such a way that

$$gv+i\mu=Me^{i\alpha}.$$

Then these eigenvectors are

$$\Psi_\pm^{(>)}=\begin{pmatrix}e^{-i\alpha/2}\\\pm e^{i\alpha/2}\end{pmatrix}$$

with the eigenvalues $\pm M$.

It is now straightforward to obtain a general solution to Eq. (15) that obeys the radiation boundary conditions at $z\rightarrow+\infty$,

$$\Psi_L(z>0)=e^{kz/2}\left[c^{(>)}H_{\nu_+}^{(1)}\left(\frac{m}{k}e^{kz}\right)\Psi_+^{(>)}\right.$$

$$\left.+d^{(>)}H_{\nu_-}^{(1)}\left(\frac{m}{k}e^{kz}\right)\Psi_-^{(>)}\right]\psi_p,$$

where

$$\nu_\pm=\frac{M}{k}\mp\frac{1}{2},$$

$c^{(>)}$ and $d^{(>)}$ are two yet undetermined coefficients, and ψ_p is a z-independent left spinor.

The solution to the left of the brane is obtained in a similar way,

$$\Psi_L(z<0)=e^{-kz/2}\left[c^{(<)}H_{\nu_+}^{(1)}\left(\frac{m}{k}e^{-kz}\right)\Psi_+^{(<)}\right.$$

$$\left.+d^{(<)}H_{\nu_-}^{(1)}\left(\frac{m}{k}e^{-kz}\right)\Psi_-^{(<)}\right]\psi_p,$$

where

$$\Psi_{\pm}^{(<)} = \begin{pmatrix} e^{i\alpha/2} \\ \pm e^{-i\alpha/2} \end{pmatrix}$$

are eigenvectors of the matrix $(-gv\tau_1 + \tau_2)$.

The fermion wave function has to obey matching conditions at $z = 0$. These are the requirements of continuity of Ψ_L and Ψ_R across the brane,

$$\Psi_{L,R}(-0) = \Psi_{L,R}(+0). \tag{16}$$

Continuity of the left components requires

$$\gamma c^{(>)} + d^{(>)} = \exp(i\alpha)(\gamma c^{(<)} + d^{(<)}), \tag{17}$$

$$\gamma c^{(>)} - d^{(>)} = \exp(-i\alpha)(\gamma c^{(<)} - d^{(<)}), \tag{18}$$

where we have introduced the notation

$$\gamma \equiv \frac{H_{\nu_+}(m/k)}{H_{\nu_-}(m/k)}.$$

To obtain the second set of relations between c's and d's, one notices that, because of Eq. (12), continuity of Ψ_R across the brane is equivalent to continuity of

$$\partial_z \Psi_L - (g\chi\tau_1 + \mu\tau_2)\Psi_L.$$

Making use of the properties of Hankel functions, we obtain

$$c^{(>)} - \gamma d^{(>)} = \exp(i\alpha)(-c^{(<)} + \gamma d^{(<)}), \tag{19}$$

$$c^{(>)} + \gamma d^{(>)} = \exp(-i\alpha)(-c^{(<)} - \gamma d^{(<)}). \tag{20}$$

The determinant of the system (17)–(20) vanishes provided that γ obeys either of the four equations

$$\gamma = \pm\tan(\alpha/2), \quad \gamma = \pm\cot(\alpha/2).$$

At small μ/gv (i.e., small α), the relevant solution is $\gamma = \tan(\alpha/2)$. This equation determines the complex eigenvalue m. Explicitly, the eigenvalue equation at $\mu \ll gv$ reads

$$\frac{H_{\nu_+}^{(1)}(m/k)}{H_{\nu_-}^{(1)}(m/k)} = \frac{\mu}{2gv}.$$

The simplest case to consider is when μ is the smallest parameter, i.e., $\mu \ll k$. In this case one expands the Bessel functions at small values of the argument and obtains

$$m = m_0 - i\Gamma$$

with

$$m_0 = \left(1 - \frac{k}{2gv}\right)\mu,$$

$$\frac{\Gamma}{m_0} = \left(\frac{m_0}{2k}\right)^{2gv/k-1} \frac{\pi}{[\Gamma(gv/k + 1/2)]^2}.$$

Hence, the suppression of the width depends nontrivially on all parameters and may become very strong.

In the opposite case $\mu \gg k$ (but still $\mu \ll gv$, which implies also $gv \gg k$), one makes use of the approximation of the Bessel function by means of tangents, and obtains

$$m_0 = \mu,$$

$$\frac{\Gamma}{m_0} = \frac{1}{2}\left(\frac{m_0}{2M}\right)^{2M/k-1} e^{2M/k},$$

where $M = \sqrt{(gv)^2 + \mu^2}$. In this case the suppression of the width is always exponentially strong. One can show that at $\mu \gg k$, the width is exponentially suppressed also for $\mu \sim gv$,

$$\frac{\Gamma}{m_0} \propto e^{-2(M/k)(\beta - \tanh\beta)}, \quad \cosh\beta = \frac{M}{\mu} = \frac{M}{m_0}.$$

It is clear why the time fermions spend on the brane is large at small k. At $gv \gg k$, continuum modes with $p^2 \sim m_0^2$ barely penetrate the potential barrier extending from the brane to the large-z region. The would-be localized mode, on the other hand, is narrow in z direction $[\Delta z \sim (gv)^{-1}]$. Hence, the overlap between continuum modes and would-be localized mode is small, and the lifetime of the metastable state is large. This feature is absent in the scalar case considered in Sec. II, where both the potential barrier and the spatial extent of the would-be localized mode are governed by one and the same parameter k.

Peculiar features of massive matter in brane world have been found in this paper in field theory framework. It remains to be understood whether similar phenomena are present in D-brane theory. In particular one may wonder whether massive matter carrying gauge charges may disappear into extra dimensions. One may worry that this would contradict three-dimensional Gauss' law; however, the issue becomes not so obvious if one recalls that the gravitational analog of Gauss' law does not prevent massive particles to escape from the brane [19]. We hope to return to this and other related issues in future.

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