Nambu-Goldstone mechanism at finite temperature in the imaginary-time and real-time formalism

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In the imaginary-time formalism of thermal field theory, and also in the real-time formalism, but by means of some redefined physical propagators for scalar bound states by diagonalization of four-point function matrices, we reexamine the Nambu-Goldstone mechanism of electroweak symmetry breaking in a one-generation fermion condensate scheme, based on the Schwinger-Dyson equation in the fermion bubble diagram approximation, and compare the obtained results. We have reached the conclusion that in both formalisms the Goldstone theorem of spontaneous electroweak symmetry breaking is rigorously true for the case of mass-degenerate two flavors of fermions and only approximately valid at low energy scales for the mass-nondegenerate case, in spite of the existence of some differences between the two formalisms in the imaginary parts of the denominators of the propagators for scalar bound states. When the two flavors of fermions have unequal nonzero masses, the induced possible fluctuation effect for the Higgs particle is negligible if the momentum cutoff in the zero temperature loops is large enough. All the results show physical equivalence of the two formalisms in the present discussed problems.

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I. INTRODUCTION

In the research on spontaneous symmetry breaking at finite temperature [1–6], in addition to the central problem of the phase transition and critical behavior of a system, the theoretical exploration of the Nambu-Goldstone mechanism [7,8] at finite temperature is certainly quite significant for a deeper understanding of spontaneous symmetry breaking [9–11]. When symmetry breaking is induced dynamically by fermion condensates, the Nambu–Jona-Lasinio (NJL) model with four-fermion interactions may be a simple and physically clear laboratory for this research [8]. The key point of such research lies in verifying the existence of the Nambu-Goldstone bosons as products of spontaneous symmetry breaking, i.e., determining the masses of relevant scalar and pseudoscalar bound states consisting of fermions and antifermions. For the sake of examining the mass-difference effect of constituent fermions in a bound state, we prefer the Schwinger-Dyson equation approach of the Green functions to the auxiliary scalar field method which was extensively used in research on models of the NJL form [6].

Based on the above strategy, we have researched the Nambu-Goldstone mechanism at finite temperature in the real-time formalism of thermal field theory in two models of the NJL form [10,11]. It was shown that the Goldstone theorem is true rigorously if the constituent fermions of a bound state have the same masses, otherwise it is only valid approximately at low energy scales. The mass difference between the fermions and the antifermions in a bound state could lead to that the Higgs boson has doubled masses and some would-be Goldstone bosons will no longer be massless rigorously. However, it should be indicated that in obtaining the above results we have used a special definition of the propagators for relative scalar bound states. Although the effects induced by the mass difference of the fermions are negligible when the momentum cutoff is very large, we still want to know if the above results represent general conclusions of a thermal field theory, or if some of them are only due to the use of the special definition of the propagators for scalar bound states in the real-time formalism there.

To clarify this problem, in this paper, based on the same strategy as above, we will reexamine the Nambu-Goldstone mechanism at finite temperature first in the imaginary-time formalism of thermal field theory and then also in the real-time formalism but by means of some redefined physical propagators for scalar bound states [12]. We will again take the one-generation fermion condensate scheme of electroweak symmetry breaking and work by the Schwinger-Dyson equation in the fermion bubble diagram approximation.

The paper is arranged as follows. In Sec. II we give the Lagrangian of the model and the gap equation at finite temperature in the imaginary-time formalism. In Sec. III we will first calculate the Matsubara propagator for a scalar bound state, then continue analytically for the energy of the bound state from discrete frequency to physical values and determine the physical mass of the scalar bound state. In Secs. IV and V the same procedure will be applied to pseudoscalar and charged-scalar bound states. In Sec. VI we will derive the redefined physical propagators for scalar bound states in the real-time formalism and compare the results obtained in the two formalisms. Finally in Sec. VII our conclusions follow.

II. GAP EQUATION IN IMAGINARY-TIME FORMALISM

In the one-generation fermion condensate scheme of electroweak symmetry breaking, the one generation of Q fermi-

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ons form a $SU_L(2)\times U_Y(1)$ doublet (U,D) and are assigned in the representation R of the color group $SU_c(3)$ with the dimension $d_Q(R)$. The symmetry breaking is induced by the effective four-fermion Lagrangian among the Q fermions below some high momentum scale Λ [12]

$$\mathcal{L}_{4F}=\mathcal{L}_{4F}^{S}+\mathcal{L}_{4F}^{P}+\mathcal{L}_{4F}^{C}, \tag{2.1}$$

where the neutral scalar couplings

$$\mathcal{L}_{4F}^{S}=\frac{1}{4}\sum_{Q,Q'} g_{Q'Q}(\bar{Q}'Q')(\bar{Q}Q), \tag{2.2}$$

with

$$g_{Q'Q}=g_{Q'Q'}^{1/2}g_{QQ}^{1/2}, \quad Q,Q'=U,D, \tag{2.3}$$

the neutral pseudoscalar couplings

$$\mathcal{L}_{4F}^{P}=\frac{1}{4}\sum_{Q,Q'} g'_{Q'Q}(\bar{Q}'i\gamma_5 Q')\bar{Q}i\gamma_5 Q), \tag{2.4}$$

with

$$g'_{Q'Q}=(-1)^{I_{Q'}^3-I_Q^3}g_{Q'Q}, \quad Q,Q'=U,D, \tag{2.5}$$

and the denotation I_Q^3 being the third component of the weak isospin of the Q fermions, and the charged scalar couplings

$$\mathcal{L}_{4F}^{C}=\frac{G}{2}(\bar{D}\Gamma^{+}U)(\bar{U}\Gamma^{-}D), \tag{2.6}$$

with

$$\Gamma^{\pm}=\frac{1}{\sqrt{2}}[\cos\varphi-\sin\varphi\pm(\cos\varphi+\sin\varphi)\gamma_5],$$

$$G=g_{UU}+g_{DD}, \quad \cos^2\varphi=g_{UU}/G, \quad \sin^2\varphi=g_{DD}/G. \tag{2.7}$$

$\mathcal{L}_{4F}$ can also be expressed by [13]

$$\mathcal{L}_{4F}=\frac{G}{4}[(\phi_S^0)^2+(\phi_P^0)^2+2\phi^{+}\phi^{-}], \tag{2.8}$$

where

$$\phi_S^0=\cos\varphi(\bar{U}U)+\sin\varphi(\bar{D}D),$$

$$\phi_P^0=\cos\varphi(\bar{U}i\gamma_5 U)-\sin\varphi(\bar{D}i\gamma_5 D),$$

$$\phi^{-}=(\bar{U}\Gamma^{-}D), \quad \phi^{+}=(\bar{D}\Gamma^{+}U) \tag{2.9}$$

are, respectively, the configurations of the physical neutral scalar, neutral pseudoscalar, and charged scalar bound states. In the imaginary-time (Euclidean) field theory, we will use the conventional time-space coordinate $(\tau=it,\ \vec{x})$, the four-momentum

$$\bar{p}=(\bar{p}^0,\bar{p}^i)=(ip^0,p^i) \tag{2.10}$$

and the γ-matrices in spinor space

$$\bar{\gamma}^0=i\gamma^0, \quad \bar{\gamma}^i=\gamma^i, \quad \gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3=\bar{\gamma}^0\bar{\gamma}^1\bar{\gamma}^2\bar{\gamma}^3 \tag{2.11}$$

which submit to the anticommutation relations

$$\{\bar{\gamma}^{\mu},\bar{\gamma}^{\nu}\}=-2\delta^{\mu\nu}, \quad \{\bar{\gamma}^{\mu},\gamma_5\}=0, \tag{2.12}$$

where $\gamma^{\mu}(\mu=0,1,2,3)$ are the ordinary γ-matrices in the real-time (Minkowski) field theory. In this way, the propagator in the momentum space for the Q fermion with mass m_Q and chemical potential μ_Q will be expressed by

$$\frac{m_Q-\bar{\not{l}}_Q}{(\omega_n+i\mu_Q)^2+\vec{l}^2+m_Q^2}=S_Q(-i\omega_n+\mu_Q,\vec{l}),$$

$$\omega_n=\frac{(2n+1)\pi}{T},$$

$$\bar{\not{l}}_Q=\bar{\gamma}^{\mu}\bar{l}_Q^{\mu},$$

$$\bar{l}_Q^{\mu}=(\omega_n+i\mu_Q,\vec{l}) \tag{2.13}$$

and the Feynman rule, for example, corresponding to the four-fermion couplings in $\mathcal{L}_{4F}^{S}$, will be $g_{Q'Q}/2$.

The derivation of the gap equation is similar to that made in the real-time formalism [11], the main change is to replace the integral of the loop energy by the sum of Matsubara frequency. Therefore when assuming the thermal expectation value $\Sigma_{Q=U,D}g_{QQ}\langle\bar{Q}Q\rangle_T\neq 0$ we will obtain the mass of the Q fermion

$$m_Q(T,\mu)\equiv m_Q=-\frac{1}{2}g_{QQ}^{1/2}\sum_{Q'=U,D}g_{Q'Q'}^{1/2}\langle\bar{Q}'Q'\rangle_T, \tag{2.14}$$

which will lead to the relation

$$m_Q/g_{QQ}^{1/2}=m_{Q'}/g_{Q'Q'}^{1/2} \tag{2.15}$$

and the gap equation at finite temperature T,

$$1=\sum_{Q=U,D}g_{QQ}I_Q, \tag{2.16}$$

with

$$\begin{aligned}I_Q&=-\frac{1}{2m_Q}\langle\bar{Q}Q\rangle_T\\&=\frac{d_Q(R)}{2m_Q}\int\frac{d^3l}{(2\pi)^3}T\sum_{n=-\infty}^{\infty}\frac{\mathrm{tr}(m_Q-\bar{\not{l}}_Q)}{(\omega_n+i\mu_Q)^2+\vec{l}^2+m_Q^2}\\&=2d_Q(R)\int\frac{d^3l}{(2\pi)^3}T\sum_{n=-\infty}^{\infty}\frac{1}{(\omega_n+i\mu_Q)^2+\omega_{Ql}^2},\end{aligned} \tag{2.17}$$

$$\omega_{Ql}^2=\vec{l}^2+m_Q^2. \tag{2.18}$$

To find the frequency sum here and later, we define the Fourier transform [14]

$$\frac{1}{(\omega_n+i\mu_Q)^2+\omega_{Ql}^2}=\int_0^\beta d\tau e^{-i\omega_n\tau}\tilde{\Delta}(\tau,\omega_{Ql},\mu_Q), \tag{2.19}$$

with $\beta=1/T$ and the inverse formula

$$\tilde{\Delta}(\tau,\omega_{Ql},\mu_Q)=T\sum_{n=-\infty}^{\infty}e^{i\omega_n\tau}\frac{1}{(\omega_n+i\mu_Q)^2+\omega_{Ql}^2}. \tag{2.20}$$

The $\tilde{\Delta}(\tau,\omega_{Ql},\mu_Q)$ in Eq. (2.20) obeys the antiperiodicity condition

$$\tilde{\Delta}(\tau,\omega_{Ql},\mu_Q)=-\tilde{\Delta}(\tau-\beta,\omega_{Ql},\mu_Q) \tag{2.21}$$

and can be calculated by the formula

$$\tilde{\Delta}(\tau,\omega_{Ql},\mu_Q)=\frac{i}{4\pi}\int_{C_1\cup C_2}dz f(z)\tan\frac{\beta}{2}(z-i\mu_Q),$$

$$f(z)=e^{i(z-i\mu_Q)\tau}\frac{1}{z^2+\omega_{Ql}^2}, \tag{2.22}$$

where C_1 and C_2 represent the integral paths $-\infty+i(\mu_Q-\varepsilon)\rightarrow+\infty+i(\mu_Q-\varepsilon)$ and $+\infty+i(\mu_Q+\varepsilon)\rightarrow-\infty+i(\mu_Q+\varepsilon)$ respectively in complex z plane. The result is

$$\tilde{\Delta}(\tau,\omega_{Ql},\mu_Q)=\frac{1}{2\omega_{Ql}}\{[1-n(\omega_{Ql}-\mu_Q)]e^{-(\omega_{Ql}-\mu_Q)\tau}-n(\omega_{Ql}+\mu_Q)e^{(\omega_{Ql}+\mu_Q)\tau}\}, \tag{2.23}$$

where the denotations

$$n(\omega_{Ql}\pm\mu_Q)=1/[e^{\beta(\omega_{Ql}\pm\mu_Q)}+1] \tag{2.24}$$

has been used. Obviously, it can be obtained from Eqs. (2.20) and (2.23) that

$$\begin{aligned}T\sum_{n=-\infty}^{\infty}\frac{1}{(\omega_n+i\mu_Q)^2+\omega_{Ql}^2}&=\tilde{\Delta}(\tau=0,\omega_{Ql},\mu_Q)\\&=\frac{1}{2\omega_{Ql}}[1-n(\omega_{Ql}-\mu_Q)-n(\omega_{Ql}+\mu_Q)]\\&=\int_{-\infty}^{\infty}\frac{dl^0}{2\pi}\Bigg[\frac{i}{l^{02}-\omega_{Ql}^2+i\varepsilon}-2\pi\delta(l^{02}-\omega_{Ql}^2)\\&\quad\times\sin^2\theta(l^0,\mu_Q)\Bigg],\end{aligned} \tag{2.25}$$

with the definition

$$\sin^2\theta(l^0,\mu_Q)=\frac{\theta(l^0)}{\exp[\beta(l^0-\mu_Q)]+1}+\frac{\theta(-l^0)}{\exp[\beta(-l^0+\mu_Q)]+1}. \tag{2.26}$$

Substituting Eq. (2.25) into Eq. (2.17), we can express the gap equation (2.16) by

$$1=\sum_{Q=U,D}g_{QQ}2d_Q(R)\int\frac{d^4l}{(2\pi)^4}\times\left[\frac{i}{l^2-m_Q^2+i\varepsilon}-2\pi\delta(l^2-m_Q^2)\sin^2\theta(l^0,\mu_Q)\right], \tag{2.27}$$

which is precisely the expression of the gap equation in the real-time formalism [11]. It should be indicated that such identity of the gap equation in the two formalisms depends on the fact that the equation comes from the two-point Green function and is determined by the loops bounded by a single fermion propagator. Equation (2.16) or (2.27), as has been shown [15], could be satisfied only at the temperature T below the electroweak symmetry restoration temperature T_c. Therefore, in the following discussions we will always assume $T<T_c$ with the gap equation being obeyed.

III. SCALAR BOUND STATE IN THE IMAGINARY-TIME FORMALISM

Since a bound state is formed by the four-fermion interactions, its propagator must correspond to a four-point amputated Green function. Thus in the imaginary-time formalism, the propagator for a scalar bound state can be calculated by means of the four-point amputated functions $\Gamma_S^{Q'Q}(-i\Omega_m,\vec{p})$ for the transition from $(\bar{Q}Q)$ to $(\bar{Q}'Q')$, where

$$\Omega_m=\frac{2\pi m}{T},\quad m=0,\pm1,\pm2,\ldots \tag{3.1}$$

represent the Matsubara frequency of the scalar bound state and $\vec{p}$ is its three dimension momentum. Based on the scalar four-fermion couplings $\mathcal{L}_{4F}^S$ in Eq. (2.2), in the fermion bubble diagram approximation, $\Gamma_S^{Q'Q}(-i\Omega_m,\vec{p})$ must obey the algebraic equations

$$\Gamma_S^{Q'Q''}(-i\Omega_m,\vec{p})[\delta_{Q''Q}-N_{Q''}(-i\Omega_m,\vec{p})g_{Q''Q}]=g_{Q'Q}/2, \tag{3.2}$$

where $2N_Q(-i\Omega_m,\vec{p})$ is the contribution of the Q fermion loop with scalar coupling vertices and

$$N_Q(-i\Omega_m,\vec{p}) = -\frac{d_Q(R)}{2}\int\frac{d^3l}{(2\pi)^3}T \times\sum_{n=-\infty}^{\infty}\mathrm{tr}\,S_Q(-i\omega_n+\mu_Q,\vec{l}) \times S_Q(-i\omega_n-i\Omega_m+\mu_Q,\vec{l}+\vec{p}). \tag{3.3}$$

Equations (3.2) have the solutions

$$\Gamma_S^{Q'Q}(-i\Omega_m,\vec{p}) = \frac{g_{Q'Q}}{2\Delta(-i\Omega_m,\vec{p})}, \tag{3.4}$$

where

$$\Delta(-i\Omega_m,\vec{p}) = 1-\sum_{Q=U,D} g_{QQ}N_Q(-i\Omega_m,\vec{p}). \tag{3.5}$$

The propagator for the scalar bound state ϕ_S^0 shown in Eq. (2.9) becomes

$$\Gamma_I^{\phi_S^0}(-i\Omega_m,\vec{p}) = G/2\Delta(-i\Omega_m,\vec{p}). \tag{3.6}$$

By means of Eqs. (2.13), (2.12) and (3.3), we find

$$N_Q(-i\Omega_m,\vec{p}) = I_Q+2d_Q(R)\int\frac{d^3l}{(2\pi)^3}T\sum_n\frac{-(\Omega_m^2+\vec{p}^2)-2m_Q^2-(\omega_n+i\mu_Q)\Omega_m-\vec{l}\cdot\vec{p}}{[(\omega_n+i\mu_Q)^2+\vec{l}^2+m_Q^2][(\omega_n+\Omega_m+i\mu_Q)^2+(\vec{l}+\vec{p})^2+m_Q^2]}. \tag{3.7}$$

From Lorentz invariance, in the imaginary-time formalism, $\Gamma_I^{\phi_S^0}(-i\Omega_m,\ \vec{p})$ should be a function of $-(\Omega_m^2+\vec{p}^2)$, thus must obey the constraint

$$\Gamma_I^{\phi_S^0}(-i\Omega_m,\vec{p}) = \Gamma_I^{\phi_S^0}(+i\Omega_m,-\vec{p}) = \Gamma_I^{\phi_S^0}(-i\Omega_{-m},-\vec{p}). \tag{3.8}$$

From Eqs. (3.6) and (3.5), the same constraint on $N_Q(-i\Omega_m,\vec{p})$ is implied and this will lead to the equality

$$\int d^3lT\sum_n\frac{-(\omega_n+i\mu_Q)\Omega_m-\vec{l}\cdot\vec{p}}{[(\omega_n+i\mu_Q)^2+\vec{l}^2+m_Q^2][(\omega_n+\Omega_m+i\mu_Q)^2+(\vec{l}+\vec{p})^2+m_Q^2]} = \int d^3lT\sum_n\frac{(\Omega_m^2+\vec{p}^2)/2}{[(\omega_n+i\mu_Q)^2+\vec{l}^2+m_Q^2][(\omega_n+\Omega_m+i\mu_Q)^2+(\vec{l}+\vec{p})^2+m_Q^2]}. \tag{3.9}$$

As a result, we obtain

$$N_Q(-i\Omega_m,\vec{p}) = I_Q-\frac{1}{2}(\Omega_m^2+\vec{p}^2+4m_Q^2)K_Q^T(-i\Omega_m,\vec{p}), \tag{3.10}$$

where

$$K_Q^T(-i\Omega_m,\vec{p}) = 2d_Q(R)\int\frac{d^3l}{(2\pi)^3}A_Q(-i\Omega_m,\vec{p},\vec{l}), \tag{3.11}$$

$$A_Q(-i\Omega_m,\vec{p},\vec{l}) = T\sum_n\frac{1}{(\omega_n+i\mu_Q)^2+\omega_{Ql}^2}\,\frac{1}{(\omega_n+\Omega_m+i\mu_Q)^2+\omega_{Ql+p}^2}, \tag{3.12}$$

$$\omega_{Ql+p}^2 = (\vec{l}+\vec{p})^2+m_Q^2. \tag{3.13}$$

By means of Eqs. (2.19), (2.23) and the formula

$$T\sum_n e^{i\omega_n(\tau-\tau')} = \delta(\tau-\tau'), \tag{3.14}$$

we can find out the frequency sum (3.12) and obtain

$$A_Q(-i\Omega_m,\vec{p},\vec{l})=\int_0^\beta d\tau e^{-i\Omega_m\tau}\tilde{\Delta}(\tau,\omega_{Ql},-\mu_Q)\tilde{\Delta}(\tau,\omega_{Ql+p},\mu_Q)=\frac{1}{4\omega_{Ql}\omega_{Ql+p}}\left\{\frac{1-n(\omega_{Ql}+\mu_Q)-n(\omega_{Ql+p}-\mu_Q)}{i\Omega_m+\omega_{Ql}+\omega_{Ql+p}}\right.$$
$$\left.+\frac{n(\omega_{Ql}+\mu_Q)-n(\omega_{Ql+p}+\mu_Q)}{i\Omega_m+\omega_{Ql}-\omega_{Ql+p}}-\frac{n(\omega_{Ql}-\mu_Q)-n(\omega_{Ql+p}-\mu_Q)}{i\Omega_m-\omega_{Ql}+\omega_{Ql+p}}-\frac{1-n(\omega_{Ql}-\mu_Q)-n(\omega_{Ql+p}+\mu_Q)}{i\Omega_m-\omega_{Ql}-\omega_{Ql+p}}\right\}, \tag{3.15}$$

where $\exp(-i\Omega_m\beta)=1$ has been used. Substituting Eq. (3.10) into Eqs. (3.5) and (3.6) and considering the gap equation (2.16), we obtain the propagator for scalar bound state ϕ_S^0 in the imaginary-time formalism

$$\Gamma_I^{\phi_S^0}(-i\Omega_m,\vec{p})=-G\Bigg/\sum_Q g_{QQ}(-\Omega_m^2-\vec{p}^2-4m_Q^2)K_Q^T(-i\Omega_m,\vec{p}). \tag{3.16}$$

The imaginary-time propagator $\Gamma_I^{\phi_S^0}(-i\Omega_m,\vec{p})$ is defined at discrete values $-i\Omega_m(m=0,\pm1,\pm2,\dots)$ in the imaginary axis on the complex energy p^0 plane. The analytic continuation to physical real values of energy can be made by the replacement [14]

$$-i\Omega_m\to p^0+i\varepsilon p^0,\quad \varepsilon=0_+. \tag{3.17}$$

This means that one can rotate the integral path from the imaginary axis on the complex p^0 plane clockwise to the real axis without meeting any singularities. As will be seen later, all the results derived from Eq. (3.17) will at least automatically reproduce the expressions of the causal propagators obtained in usual zero temperature field theory when $T=0$, and this fact justifies the continuation. Under the analytic continuation (3.17), we will have the substitutions

$$-(\Omega_m^2+\vec{p}^2)\to p^{02}-\vec{p}^2+i\varepsilon=p^2+i\varepsilon, \tag{3.18}$$

$$A_Q(-i\Omega_m,\vec{p},\vec{l})\to A_Q(p^0,\vec{p},\vec{l})=A_Q(p,\vec{l}), \tag{3.19}$$

where

$$A_Q(p,\vec{l})=\frac{1}{4\omega_{Ql}\omega_{Ql+p}}\left\{\frac{1-n(\omega_{Ql}+\mu_Q)-n(\omega_{Ql+p}-\mu_Q)}{-p^0+\omega_{Ql}+\omega_{Ql+p}-i\varepsilon}+\frac{n(\omega_{Ql}+\mu_Q)-n(\omega_{Ql+p}+\mu_Q)}{-p^0+\omega_{Ql}-\omega_{Ql+p}-i\varepsilon\,\eta(\omega_{Ql}-\omega_{Ql+p})}\right.$$
$$\left.-\frac{n(\omega_{Ql}-\mu_Q)-n(\omega_{Ql+p}-\mu_Q)}{-p^0-\omega_{Ql}+\omega_{Ql+p}+i\varepsilon\,\eta(\omega_{Ql}-\omega_{Ql+p})}-\frac{1-n(\omega_{Ql}-\mu_Q)-n(\omega_{Ql+p}+\mu_Q)}{-p^0-\omega_{Ql}-\omega_{Ql+p}+i\varepsilon}\right\}, \tag{3.20}$$

with the definition

$$\eta(\omega_{Ql}-\omega_{Ql+p})=\begin{cases}1 & \text{if}\quad \omega_{Ql}>\omega_{Ql+p},\\ -1 & \text{if}\quad \omega_{Ql}<\omega_{Ql+p},\end{cases} \tag{3.21}$$

and, from Eq. (3.11),

$$K_Q^T(-i\Omega_m,\vec{p})\to K_Q^T(p)=2d_Q(R)\int\frac{d^3l}{(2\pi)^3}A_Q(p,\vec{l}). \tag{3.22}$$

For making a comparison between the results obtained in the imaginary-time and in the real-time formalism, we may change $A_Q(p,\vec{l})$ into an integral representation. In fact, by the formula

$$\frac{1}{X+i\varepsilon}=\frac{X}{X^2+\varepsilon^2}-i\pi\delta(X) \tag{3.23}$$

and the definition (2.26), we can write

$$A_Q(p,\vec{l}) = \int \frac{dl^0}{2\pi} \frac{-i}{[l^{0^2} - \omega_{Ql}^2 + i\varepsilon][(l^0+p^0)^2 - \omega_{Ql+p}^2 + i\varepsilon]}$$
$$+ \int dl^0 \left\{ \frac{\delta(l^{0^2} - \omega_{Ql}^2)}{(l^0+p^0)^2 - \omega_{Ql+p}^2 + i\varepsilon} \sin^2\theta(l^0,\mu_Q) + \frac{\delta[(l^0+p^0)^2 - \omega_{Ql+p}^2]}{l^{0^2} - \omega_{Ql}^2 + i\varepsilon} \sin^2\theta(l^0+p^0,\mu_Q) \right\}$$
$$+ i2\pi \int dl^0 [\theta(l^0)\theta(l^0+p^0) + \theta(-l^0)\theta(-l^0-p^0)] \delta(l^{02} - \omega_{Ql}^2) \delta[(l^0+p^0)^2 - \omega_{Ql+p}^2] \sin^2\theta(l^0,\mu_Q). \quad (3.24)$$

Applying Eqs. (3.18), (3.22) and (3.24) to Eq. (3.16) and considering the existence of a factor i in a four-point function after the analytic continuation we obtain the physical propagator for the scalar bound state ϕ_S^0:

$$\Gamma_I^{\phi_S^0}(p) \equiv i\Gamma_I^{\phi_S^0}(-i\Omega_m \to p^0 + i\varepsilon p^0, \vec{p})$$
$$= -iG \Big/ \sum_Q g_{QQ}(p^2 - 4m_Q^2 + i\varepsilon) K_Q^T(p), \quad (3.25)$$

where

$$K_Q^T(p) = K_Q(p) + H_Q(p) - iS_Q^I(p). \quad (3.26)$$

In Eq. (3.26) the functions

$$K_Q(p) = -2d_Q(R) \times \int \frac{id^4l}{(2\pi)^4} \frac{1}{(l^2 - m_Q^2 + i\varepsilon)[(l+p)^2 - m_Q^2 + i\varepsilon]}$$
$$= \frac{d_Q(R)}{8\pi^2} \int_0^1 dx \left(\ln\frac{\Lambda^2 + M_Q^2}{M_Q^2} - \frac{\Lambda^2}{\Lambda^2 + M_Q^2} \right),$$
$$M_Q^2 = m_Q^2 - p^2 x(1-x), \quad (3.27)$$

with the four-dimension Euclidean momentum cutoff Λ,

$$H_Q(p) = 4\pi d_Q(R) \int \frac{d^4l}{(2\pi)^4} \times \left\{ \frac{(l+p)^2 - m_Q^2}{[(l+p)^2 - m_Q^2]^2 + \varepsilon^2} + (p \to -p) \right\} \times \delta(l^2 - m_Q^2) \sin^2\theta(l^0,\mu_Q), \quad (3.28)$$

and

$$S_Q^I(p) = 4\pi^2 d_Q(R) \int \frac{d^4l}{(2\pi)^4} \delta(l^2 - m_Q^2) \delta[(l+p)^2 - m_Q^2]$$
$$\times [1 - \theta(l^0)\theta(l^0+p^0) - \theta(-l^0)\theta(-l^0-p^0)]$$
$$\times [\sin^2\theta(l^0,\mu_Q) + \sin^2\theta(l^0+p^0,\mu_Q)]. \quad (3.29)$$

When deriving the expression (3.29), we have used the equality $S_Q^I(p) = S_Q^I(-p)$ coming from the equality $\Gamma_I^{\phi_S^0}(p) = \Gamma_I^{\phi_S^0}(-p)$. It should be indicated that the expressions (3.26) –(3.29) for $K_Q^T(p)$ are true for both $\omega_{Ql} > \omega_{Ql+p}$ and $\omega_{Ql} < \omega_{Ql+p}$ in Eq. (3.20) of $A_Q(p,\vec{l})$. By means of the relation

$$g_{QQ}/G = m_Q^2 \Big/ \sum_Q m_Q^2 \quad (3.30)$$

derived from Eq. (2.15), we obtain

$$\Gamma_I^{\phi_S^0}(p) = -i\sum_Q m_Q^2 \Big/ \sum_Q (p^2 - 4m_Q^2 + i\varepsilon) m_Q^2 K_Q^T(p)$$
$$= -i\sum_Q m_Q^2 \Big/ \sum_Q (p^2 - 4m_Q^2 + i\varepsilon) m_Q^2 \times [K_Q(p) + H_Q(p) - iS_Q^I(p)]. \quad (3.31)$$

It is indicated that the term containing $S_Q^I(p)$ in Eq. (3.31) will make the pole of $\Gamma^{\phi_S^0}(p)$ possibly become complex. In addition, $K_Q(p)$ in Eq. (3.27) can also be complex when $p^2 > 4m_Q^2$. Denote $K_Q(p) = K_{Qr}(p) - iK_{Qi}$ with $K_{Qi} > 0$ and let

$$\sum_Q m_Q^2 K_{Qr}(p) = k_r, \quad \sum_Q m_Q^2 K_{Qi}(p) = k_i,$$
$$\sum_Q m_Q^2 H_Q(p) = h, \quad \sum_Q m_Q^2 S_Q^I(p) = s^I,$$
$$\sum_Q m_Q^4 K_{Qr}(p) = \tilde{k}_r, \quad \sum_Q m_Q^4 K_{Qi}(p) = \tilde{k}_i,$$
$$\sum_Q m_Q^4 H_Q(p) = \tilde{h}, \quad \sum_Q m_Q^4 S_Q^I(p) = \tilde{s}^I. \quad (3.32)$$

We can obtain from Eq. (3.31) the equation to determine the mass of ϕ_S^0

$$m_{\phi_S^0}^2 = p^2 = 4\frac{\tilde{k}_r + \tilde{h} - i(\tilde{k}_i + \tilde{s}^I)}{k_r + h - i(k_i + s^I)}. \quad (3.33)$$

In the special case when only single-flavor Q fermions exist (e.g. in the top-quark condensate scheme [16]) or all the Q fermions are mass degenerate, we will have the real $m^2_{\phi^0_S} = p^2 = 4m^2_Q$. In the other cases, the solution of p^2 (or say p^0) will be complex. Denote $p^0 = p^0_r + ip^0_i$, then Eq. (3.33) will become

$$(p^0_r + ip^0_i)^2 - \vec{p}^2 = [a(p) + ib(p)]|_{p^2 = m^2_{\Phi^0_S}}, \quad (3.34)$$

with

$$a(p) = 4\frac{(k_r+h)(\tilde{k}_r+\tilde{h}) + (k_i+s^I)(\tilde{k}_i+\tilde{s}^I)}{(k_r+h)^2 + (k_i+s^I)^2},$$

$$b(p) = \frac{(\tilde{k}_r+\tilde{h})(k_i+s^I) - (k_r+h)(\tilde{k}_i+\tilde{s}^I)}{(k_r+h)^2 + (k_i+s^I)^2}. \quad (3.35)$$

It is easy to find from Eq. (3.35) that $b(p) \equiv 0$ for the cases with both single-flavor and mass-degenerate Q fermions and $b(p)$ could be very small if the momentum cutoff Λ in the $K_Q(p)$ is very large. Thus we can define the squared mass of ϕ^0_S by the solution of the real part of Eq. (3.34), i.e.

$$m^2_{\phi^0_S} = p^2_r = a(p_r)$$
$$= 4\frac{(k_r+h)(\tilde{k}_r+\tilde{h}) + (k_i+s^I)(\tilde{k}_i+\tilde{s}^I)}{(k_r+h)^2 + (k_i+s^I)^2}\Bigg|_{p=p_r}. \quad (3.36)$$

Since $K_{Qr}(p)$ and $H_Q(p)$ is real and positive [11], and based on the expression (3.29), the same is true to $S^I_Q(p)$; we can deduce the mass inequalities from Eq. (3.36)

$$2(m_Q)_{\min} \leq m_{\phi^0_S} \leq 2(m_Q)_{\max}, \quad (3.37)$$

where $(m_Q)_{\min}$ and $(m_Q)_{\max}$ are respectively the minimal and the maximal mass of the Q fermions. When $0 \neq m_U \neq m_D \neq 0$, only the signs of inequality are left in Eq. (3.37). In this case, if we set $m_D = \alpha m_U$ $(\alpha > 0)$, then the numerator of $b(p)$ in Eq. (3.35) will become $(\alpha^2 - \alpha^4)m^6_U[(K_{Ur} + H_U)(K_{Di} + S^I_D) - (K_{Dr} + H_D)(K_{Ui} + S^I_U)]$. Considering the inequalities in Eq. (3.37) and that the imaginary parts $K_{Qi}(p) = 0$ and $S^I_Q(p) = 0$ when $p^2 < 4m^2_Q$, we can obtain that whether $\alpha < 1$ $(m_D < m_U)$ or $\alpha > 1$ $(m_D > m_U)$ always have $b(p) > 0$. This means that p^0 will contain a positive imaginary part

$$p^0_i \simeq \frac{b(p_r)}{2p^0_r}, \quad (3.38)$$

hence the amplitude associated with the scalar bound state ϕ^0_S will get a growth factor $\exp(p^0_i t)$. In other words, owing to the mass difference between massive U and D fermions, the scalar bound state ϕ^0_S will encounter some effect of fluctuation. It should be mentioned that such amplitude growth factor of ϕ^0_S always exists in the case with unequal nonzero masses of the two flavors of fermions in a one-generation fermion condensate model even if we let the temperature $T \to 0$. At finite temperature, it is only modified by thermal effect and displays itself more plainly. However, such fluctuation effect of ϕ^0_S is physically completely negligible considering that $b(p_r)$ will be extremely small if the momentum cutoff Λ in the zero temperature loops is large enough and that ϕ^0_S generally has a finite decay life in a real model.

IV. NEUTRAL PSEUDOSCALAR BOUND STATE IN THE IMAGINARY-TIME FORMALISM

The propagator for a neutral pseudoscalar bound state can be calculated by the four-point amputated Green functions $\Gamma^{Q'Q}_P(-i\Omega_m, \vec{p})$ for the transition from $(\overline{Q} i\gamma_5 Q)$ to $(\overline{Q'} i\gamma_5 Q')$. Based on the pseudoscalar four-fermion coupling $\mathcal{L}^P_{4F}$ [Eq. (2.4)], in the fermion bubble diagram approximation, $\Gamma^{Q'Q}_P(-i\Omega_m, \vec{p})$ will obey the algebraic equations

$$\Gamma^{Q'Q''}_P(-i\Omega_m, \vec{p})[\delta_{Q''Q} - N_{Q''5}(-i\Omega_m, \vec{p})g'_{Q''Q}] = g'_{Q'Q}/2, \quad (4.1)$$

where $2N_{Q5}(-i\Omega_m, \vec{p})$ is the contribution of the Q fermion loop with pseudoscalar coupling vertices and

$$N_{Q5}(-i\Omega_m, \vec{p}) = -\frac{d_Q(R)}{2}\int \frac{d^3l}{(2\pi)^3} T \times \sum_{n=-\infty}^{\infty} \mathrm{tr}[i\gamma_5 S_Q(-i\omega_n + \mu_Q, \vec{l}) \times i\gamma_5 S_Q(-i\omega_n - i\Omega_m + \mu_Q, \vec{l} + \vec{p})]. \quad (4.2)$$

Similar to Eqs. (3.2), Eqs. (4.1) have the solutions

$$\Gamma^{Q'Q}_P(-i\Omega_m, \vec{p}) = \frac{g'_{Q'Q}}{2\Delta'(-i\Omega_m, \vec{p})}, \quad (4.3)$$

with

$$\Delta'(-i\Omega_m, \vec{p}) = 1 - \sum_{Q=U,D} g_{QQ} N_{Q5}(-i\Omega_m, \vec{p}). \quad (4.4)$$

The propagator in the imaginary-time formalism for the pseudoscalar bound state ϕ^0_P shown in Eq. (2.9) is

$$\Gamma^{\phi^0_P}_I(-i\Omega_m, \vec{p}) = G/2\Delta'(-i\Omega_m, \vec{p}). \quad (4.5)$$

By means of Eqs. (2.13), (2.12) and (4.2), we find

$$N_{Q5}(-i\Omega_m,\vec{p})=I_Q+2d_Q(R)\int\frac{d^3l}{(2\pi)^3}T\sum_n\frac{-(\Omega_m^2+\vec{p}^2)-(\omega_n+i\mu_Q)\Omega_m-\vec{l}\cdot\vec{p}}{[(\omega_n+i\mu_Q)^2+\vec{l}^2+m_Q^2][(\omega_n+\Omega_m+i\mu_Q)^2+(\vec{l}+\vec{p})^2+m_Q^2]}$$
$$=I_Q-\frac{1}{2}(\Omega_m^2+\vec{p}^2)K_Q^T(-i\Omega_m,\vec{p}), \tag{4.6}$$

where Eq. (3.9) has been used. Substituting Eq. (4.6) into Eqs. (4.4) and (4.5) and considering the gap equation (2.16), we obtain

$$\Gamma_I^{\phi_P^0}(-i\Omega_m,\vec{p})=-G/(-\Omega_m^2-\vec{p}^2)\sum_Q g_{QQ}K_Q^T(-i\Omega_m,\vec{p}). \tag{4.7}$$

Then, by the analytic continuation to physical real energy p^0 similar to that made for $\Gamma_I^{\phi_S^0}(-i\Omega_m,\vec{p})$, we obtain the physical propagator for pseudoscalar bound state ϕ_P^0:

$$\Gamma_I^{\phi_P^0}(p)=-iG/(p^2+i\varepsilon)\sum_Q g_{QQ}K_Q^T(p)$$
$$=-i\sum_Q m_Q^2/(p^2+i\varepsilon)\sum_Q m_Q^2K_Q^T(p), \tag{4.8}$$

where Eq. (3.30) has been used and $K_Q^T(p)$ is still given by Eqs. (3.26)–(3.29). The expression (4.8) indicates that $\Gamma_I^{\phi_P^0}(p)$ has a single pole at $p^2=0$; thus ϕ_P^0 is a massless neutral pseudoscalar Goldstone boson. In addition, since when $p^2=0$, $H_Q(p)=0$ and $S_Q^I(p)=0$ in $K_Q^T(p)$, we also have

$$\Gamma_I^{\phi_P^0}(p)=-i\sum_Q m_Q^2/(p^2+i\varepsilon)\sum_Q m_Q^2K_Q(p),\quad \text{if}\quad p^2\to 0, \tag{4.9}$$

which has the same form as the propagator for ϕ_P^0 at $T=0$, except that m_Q now implies the dynamical mass of the Q fermions at temperature T [13]. These results are independent of the mass difference between the two flavors of fermions.

V. CHARGED SCALAR BOUND STATES IN THE IMAGINARY-TIME FORMALISM

By Eq. (2.9), the configuration of the charged scalar bound state is $\phi^-=(\bar{U}\Gamma^-D)$; thus the four-point amputated function for the transition from $(\bar{U}\Gamma^-D)$ to $(\bar{D}\Gamma^+U)$ will simply correspond to the propagator for ϕ^- (as well as its hermitian conjugate ϕ^+). Based on $\mathcal{L}_{4F}^C$ in Eq. (2.6), the imaginary-time propagator $\Gamma_I^{\phi^-}(-i\Omega_m,\vec{p})$ for ϕ^- obeys the algebraic equation

$$\Gamma_I^{\phi^-}(-i\Omega_m,\vec{p})=\frac{G}{2}+GL(-i\Omega_m,\vec{p})\Gamma_I^{\phi^-}(-i\Omega_m,\vec{p}), \tag{5.1}$$

from which we find

$$\Gamma_I^{\phi^-}(-i\Omega_m,\vec{p})=G/2[1-GL(-i\Omega_m,\vec{p})], \tag{5.2}$$

where $2L(-i\Omega_m,\ \vec{p})$ represents the contribution of the fermion loops bounded by a U fermion and a D fermion propagator with a Γ^+ and a Γ^- coupling vertex, i.e. we have

$$GL(-i\Omega_m,\vec{p})=G\frac{d_Q(R)}{2}\int\frac{d^3l}{(2\pi)^3}T\sum_n\frac{-\mathrm{tr}[\Gamma^-(m_U-\not{\bar{l}}_U)\Gamma^+(m_D-\not{\bar{l}}_D-\not{\bar{p}})]}{[(\omega_n+i\mu_U)^2+\vec{l}^2+m_U^2][(\omega_n+\Omega_m+i\mu_D)^2+(\vec{l}+\vec{p})^2+m_D^2]}, \tag{5.3}$$

with

$$\bar{p}=(\Omega_m,\vec{p}). \tag{5.4}$$

The trace in Eq. (5.3) can be expressed by

$$a=-\mathrm{tr}[\Gamma^-(m_U-\not{\bar{l}}_U)\Gamma^+(m_D-\not{\bar{l}}_D-\not{\bar{p}})]=4\bar{l}_U\cdot(\bar{l}_D+\bar{p})+8\frac{m_U^2m_D^2}{m_U^2+m_D^2},$$

where we have used Eqs. (2.7) and (3.30). It can be further written by either

$$a\equiv a_U=4\{(\omega_n+\Omega_m+i\mu_D)^2+(\vec{l}+\vec{p})^2+m_D^2$$
$$-[\omega_n+i\mu_U+\Omega_m+i(\mu_D-\mu_U)][\Omega_m+i(\mu_D-\mu_U)]-(\vec{l}+\vec{p})\cdot\vec{p}+m_D^2(m_U^2-m_D^2)/(m_U^2+m_D^2)\} \tag{5.5}$$

or

$$a\equiv a_D=4\{(\omega_n+i\mu_U)^2+\vec{l}^2+m_U^2+(\omega_n+i\mu_U)[\Omega_m+i(\mu_D-\mu_U)]+\vec{l}\cdot\vec{p}+m_U^2(m_D^2-m_U^2)/(m_U^2+m_D^2)\}. \tag{5.6}$$

From these expressions we obtain

$$GL(-i\Omega_m,\vec{p})=\frac{d_Q(R)}{2}\int\frac{d^3l}{(2\pi)^3}T\sum_n\frac{g_{UU}a_U+g_{DD}a_D}{[(\omega_n+i\mu_U)^2+\vec{l}^2+m_U^2][(\omega_n+\Omega_m+i\mu_D)^2+(\vec{l}+\vec{p})^2+m_D^2]} \tag{5.7}$$

$$=\sum_{Q=U,D}g_{QQ}I_Q+2d_Q(R)\int\frac{d^3l}{(2\pi)^3}T \times\sum_n\frac{(g_{DD}-g_{UU})\{(\omega_n+i\mu_U)[\Omega_m+i(\mu_D-\mu_U)]\}+\vec{l}\cdot\vec{p}-g_{UU}\{[\Omega_m+i(\mu_D-\mu_U)]^2+\vec{p}^2\}}{[(\omega_n+i\mu_U)^2+\vec{l}^2+m_U^2][(\omega_n+\Omega_m+i\mu_D)^2+(\vec{l}+\vec{p})^2+m_D^2]}, \tag{5.8}$$

where the result coming from Eq. (3.30)

$$g_{UU}m_D^2-g_{DD}m_U^2=0 \tag{5.9}$$

has been used. Substituting Eq. (5.8) into Eq. (5.2) and considering the gap equation (2.16), we obtain

$$\Gamma_I^{\phi^-}(-i\Omega_m,\vec{p})=-G/4d_Q(R)\int\frac{d^3l}{(2\pi)^3}(\{(g_{DD}-g_{UU})i\mu_U(\Omega_m+i\mu_D-i\mu_U+\vec{l}\cdot\vec{p})-g_{UU}[(\Omega_m+i\mu_D-i\mu_U)^2+\vec{p}^2]\} \times A_c(-i\Omega_m,\vec{p},\vec{l})+(g_{DD}-g_{UU})(\Omega_m+i\mu_D-i\mu_U)B_c(-i\Omega_m,\vec{p},\vec{l})), \tag{5.10}$$

where the Matsubara frequency sums

$$A_c(-i\Omega_m,\vec{p},\vec{l})=T\sum_n\frac{1}{[(\omega_n+i\mu_U)^2+\omega_{Ul}^2][(\omega_n+\Omega_m+i\mu_D)^2+\omega_{Dl+p}^2]}, \tag{5.11}$$

$$B_c(-i\Omega_m,\vec{p},\vec{l})=T\sum_n\frac{\omega_n}{[(\omega_n+i\mu_U)^2+\omega_{Ul}^2][(\omega_n+\Omega_m+i\mu_D)^2+\omega_{Dl+p}^2]}. \tag{5.12}$$

By means of a similar method to calculate $A_Q(-i\Omega_m,\ \vec{p},\vec{l})$ in Eq. (3.12), we can express

$$A_c(-i\Omega_m,\vec{p},\vec{l})=\int_0^\beta d\tau e^{-i\Omega_m\tau}\tilde{\Delta}(\tau,\omega_{Ul},-\mu_U)\tilde{\Delta}(\tau,\omega_{Dl+p},\mu_D), \tag{5.13}$$

$$B_c(-i\Omega_m,\vec{p},\vec{l})=i\int_0^\beta d\tau e^{-i\Omega_m\tau}\left[\frac{\partial}{\partial\tau}\tilde{\Delta}(\tau,\omega_{Ul},-\mu_U)\right]\tilde{\Delta}(\tau,\omega_{Dl+p},\mu_D) =-i\mu_UA_c(-i\Omega_m,\vec{p},\vec{l})+D_c(-i\Omega_m,\vec{p},\vec{l}), \tag{5.14}$$

with the results

$$\left.\begin{array}{l}A_c(-i\Omega_m,\vec{p},\vec{l})\\ D_c(-i\Omega_m,\vec{p},\vec{l})\end{array}\right\}=\left.\begin{array}{l}-i\omega_{Ul}\\ 1\end{array}\right\}\frac{1}{4\omega_{Ul}\omega_{Dl+p}}\left[\frac{1-n(\omega_{Ul}+\mu_U)-n(\omega_{Dl+p}-\mu_D)}{i\Omega_m-(\mu_D-\mu_U)+\omega_{Ul}+\omega_{Dl+p}}+\frac{n(\omega_{Ul}+\mu_U)-n(\omega_{Dl+p}+\mu_D)}{i\Omega_m-(\mu_D-\mu_U)+\omega_{Ul}-\omega_{Dl+p}}\right. \left.\mp\frac{n(\omega_{Ul}-\mu_U)-n(\omega_{Dl+p}-\mu_D)}{i\Omega_m-(\mu_D-\mu_U)-\omega_{Ul}+\omega_{Dl+p}}\mp\frac{1-n(\omega_{Ul}-\mu_U)-n(\omega_{Dl+p}+\mu_D)}{i\Omega_m-(\mu_D-\mu_U)-\omega_{Ul}-\omega_{Dl+p}}\right]. \tag{5.15}$$

In the derivation of Eq. (5.14) we have used the antiperiodicity condition (2.21) of $\tilde{\Delta}(\tau,\omega_{Ql},\mu_Q)$.

For analytic continuation of the definition region of the propagator for ϕ^- from $-i\Omega_m$ to the real axis of p^0, we will make the replacement

$$-i\Omega_m+\mu_D-\mu_U\rightarrow p^0+i\varepsilon p^0,\quad \varepsilon=0_+, \tag{5.16}$$

considering that the charged scalar bound state $\phi^- = (\bar{U}\Gamma^- D)$ composed of the U antifermions and the D fermions may have the chemical potential $\mu_D - \mu_U$. Correspondingly, we will have the following substitutions:

$$A_c(-i\Omega_m,\vec{p},\vec{l}) \to A_c(p,\vec{l}), \quad D_c(-i\Omega_m,\vec{p},\vec{l}) \to D_c(p,\vec{l}). \tag{5.17}$$

For the purpose of making a comparison between the results in the imaginary-time formalism and in the real-time formalism, we will express $A_c(p,\vec{l})$ and $D_c(p,\vec{l})$ respectively by an integral over the real l^0. In this way, it is found that

$$A_c(p,\vec{l}) = \int \frac{dl^0}{2\pi} \tilde{A}_c(p,l^0,\vec{l}), \tag{5.18}$$

$$D_c(p,\vec{l}) = \int \frac{dl^0}{2\pi} il^0 \tilde{A}_c(p,l^0,\vec{l}), \tag{5.19}$$

where

$$\begin{aligned}
\tilde{A}_c(p,l^0,\vec{l}) = {} & \frac{-i}{(l^2-m_U^2+i\varepsilon)[(l+p)^2-m_D^2+i\varepsilon]} + 2\pi \frac{\delta(l^2-m_U^2)}{(l+p)^2-m_D^2+i\varepsilon}\sin^2\theta(l^0,\mu_U) \\
& + 2\pi \frac{\delta[(l+p)^2-m_D^2]}{l^2-m_U^2+i\varepsilon}\sin^2\theta(l^0+p^0,\mu_D) + i4\pi^2\delta(l^2-m_U^2)\delta[(l+p)^2-m_D^2] \\
& \times[\theta(l^0)\theta(l^0+p^0)+\theta(-l^0)\theta(-l^0-p^0)][\theta(\omega_{Ul}-\omega_{Dl+p})\sin^2\theta(l^0,\mu_U)+\theta(\omega_{Dl+p}-\omega_{Ul})\sin^2\theta(l^0+p^0,\mu_D)].
\end{aligned} \tag{5.20}$$

By means of Eqs. (5.10), (5.14) and (5.16)–(5.19), we can analytically continue the imaginary-time propagator for ϕ^- to the physical propagator, i.e.,

$$i\Gamma_I^{\phi^-}(-i\Omega_m,\vec{p}) \to \Gamma^{\phi^-}(p), \tag{5.21}$$

with

$$\begin{aligned}
\Gamma_I^{\phi^-}(p) &= -iG/4d_Q(R)\int \frac{d^3l}{(2\pi)^3}\{[(g_{DD}-g_{UU})\vec{l}\cdot\vec{p}+g_{UU}(p^2+i\varepsilon)]A_c(p,\vec{l})+(g_{DD}-g_{UU})ip^0D_c(p,\vec{l})\} \\
&= -iG/4d_Q(R)\int \frac{d^4l}{(2\pi)^4}[(g_{UU}-g_{DD})l\cdot p+g_{UU}(p^2+i\varepsilon)]\tilde{A}_c(p,l^0,\vec{l}) \\
&= -i\sum_Q m_Q^2/4d_Q(R)\int \frac{d^4l}{(2\pi)^4}[(m_U^2-m_D^2)l\cdot p+m_U^2(p^2+i\varepsilon)]\tilde{A}_c(p,l^0,\vec{l}),
\end{aligned} \tag{5.22}$$

where Eq. (3.30) has been used once again. Substituting Eq. (5.20) into Eq. (5.22) and using the formula (3.23), we finally obtain the physical propagator for the charged scalar bound state ϕ^-

$$\Gamma_I^{\phi^-}(p) = -i/\{(p^2+i\varepsilon)[K_{UD}(p)+H_{UD}(p)]+E_{UD}(p)-i(p^2-\bar{M}^2)S_{UD}^I(p)\}, \tag{5.23}$$

where

$$\bar{M}^2 = (m_U^2-m_D^2)^2/(m_U^2+m_D^2), \tag{5.24}$$

$$\begin{aligned}
K_{UD}(p) &= \frac{d_Q(R)}{4\pi^2}\int_0^1 dx \frac{m_U^2(1-x)+m_D^2x}{m_U^2+m_D^2}\left[\ln\frac{\Lambda^2+M_{UD}^2(p)}{M_{UD}^2(p)} - \frac{\Lambda^2}{\Lambda^2+M_{UD}^2(p)}\right], \\
M_{UD}^2(p) &= m_U^2(1-x)+m_D^2x-p^2x(1-x),
\end{aligned} \tag{5.25}$$

$$H_{UD}(p)=4\pi d_Q(R)\int\frac{d^4l}{(2\pi)^4}\left\{\frac{(l+p)^2-m_D^2}{[(l+p)^2-m_D^2]^2+\varepsilon^2}\delta(l^2-m_U^2)\sin^2\theta(l^0,\mu_U)+(p\rightarrow -p,m_U\leftrightarrow m_D,\mu_U\leftrightarrow\mu_D)\right\}, \tag{5.26}$$

$$E_{UD}(p)=4\pi d_Q(R)\frac{m_U^2-m_D^2}{m_U^2+m_D^2}\int\frac{d^4l}{(2\pi)^4}\times\left\{\frac{[(l+p)^2-m_U^2][(l+p)^2-m_D^2]}{[(l+p)^2-m_D^2]^2+\varepsilon^2}\delta(l^2-m_U^2)\sin^2\theta(l^0,\mu_U)-(p\rightarrow -p,m_U\leftrightarrow m_D,\mu_U\leftrightarrow\mu_D)\right\}, \tag{5.27}$$

$$S^I_{UD}(p)=4\pi^2 d_Q(R)\int\frac{d^4l}{(2\pi)^4}\delta(l^2-m_U^2)\delta[(l+p)^2-m_D^2]\{\sin^2\theta(l^0,\mu_U)+\sin^2\theta(l^0+p^0,\mu_D) -2[\theta(l^0)\theta(l^0+p^0)+\theta(-l^0)\theta(-l^0-p^0)][\theta(\omega_{Ul}-\omega_{Dl+p})\sin^2\theta(l^0,\mu_U)+\theta(\omega_{Dl+p}-\omega_{Ul})\sin^2\theta(l^0+p^0,\mu_D)]\}. \tag{5.28}$$

As for the key question under what condition $\phi^{\mp}$ could be massless bound states, we can answer it in two cases.

(1) $m_U=m_D=m_Q$. For the mass-degenerate U and D fermions, we will have $K_{UD}(p)=K_Q(p)$, $H_{UD}(p)=H_Q(p)$, $S^I_{UD}(p)=S^I_Q(p)$, $E_{UD}(p)=0$, and $\overline{M}^2=0$, thus

$$\Gamma_I^{\phi^-}(p)=-i/(p^2+i\varepsilon)[K_Q(p)+H_Q(p)-iS^I_Q(p)]. \tag{5.29}$$

Equation (5.29) implies that $p^2=0$ is the single pole of $\Gamma^{\phi^-}(p)$ hence ϕ^- and ϕ^+ are both massless scalar bound states and can be identified with the charged Nambu-Goldstone bosons. As was indicated in Sec. III, when $p\rightarrow 0$, no pinch singularity could emerge from $S^I_Q(p)$.

(2) $m_U\neq m_D$. For the mass-nondegenerate U and D fermions, we will have $E_{UD}\neq 0$ and $\overline{M}^2\neq 0$, thus the pole of $\Gamma_I^{\phi^-}(p)$ will be determined by the equation

$$p^2=-\frac{E_{UD}(p)+i\overline{M}^2S^I_{UD}(p)}{K_{UD}(p)+H_{UD}(p)-iS^I_{UD}(p)}. \tag{5.30}$$

Hence, the masses of ϕ^- and ϕ^+ at finite temperature are not equal to zeros. However, as long as the momentum cutoff Λ in $K_{UD}(p)$ is large enough, the single pole of $\Gamma_I^{\phi^-}(p)$ could still be approximately at $p^2=0$. On the other hand, when $p^2=0$ and $p^0=|\vec{p}|\rightarrow 0$, both $E_{UD}(p)$ and $S^I_{UD}(p)$ in the numerator of the right-hand side of Eq. (5.30) approach zero. Therefore, at low energy scales it is still possible that ϕ^- and ϕ^+ are considered as approximate massless bound states and identified with the charged Nambu-Goldstone bosons.

VI. PHYSICAL PROPAGATORS FOR SCALAR BOUND STATES IN THE REAL-TIME FORMALISM

The propagators for scalar bound states in the real-time formalism were discussed in Ref. [11]. However, the propagators are defined there by $\Gamma_B^{Q'1Q1}(p)$ $(B=S,P)$ and $\Gamma^{11}_{\phi^-}(p)$ respectively for neutral scalar, pseudoscalar and charged scalar bound states which are the four-point amputated functions for the transition between the physical field configurations, e.g., $(\overline{Q}Q)^{(1)}$ and $(\overline{Q'}Q')^{(1)}$ etc. These definitions can give the main physical features of the bound states including their masses but also lead some extra result, e.g. a neutral scalar bound state ϕ_S^0 could have double masses due thermal fluctuation. In addition, the resulting expressions of the propagators for ϕ_S^0, ϕ_P^0 and $\phi^{\mp}$ have quite explicit difference with Eqs. (3.31), (4.8) and (5.23) obtained here in the imaginary-time formalism. For seeking a closer correspondence between the physical propagators for the above scalar bound states in the real-time and the imaginary-time formalism, following Ref. [12], we will redefine the physical propagators for ϕ_S^0, ϕ_P^0 and $\phi^{\mp}$ by diagonalization of corresponding matrix propagators. These matrix propagators have in fact been obtained in Ref. [11].

First let us deal with the case of the neutral scalar bound state. By Eqs. (3.3) and (3.7) in Ref. [11], the matrix of the four-point amputated functions for the transition from $(\overline{Q}Q)^{(a)}$ and $(\overline{Q'}Q')^{(b)}$ can be expressed by

$$\Gamma_S^{Q'bQa}(p)=\frac{g_{Q'Q}}{G}\Gamma^{\phi_S^0 ba}(p), \quad Q',Q=U,D, \quad b,a=1,2, \tag{6.1}$$

where $\Gamma^{\phi_S^0 ba}(p)$ $(b,a=1,2)$ is the matrix propagator for the neutral scalar bound state ϕ_S^0 given in Eq. (2.9), its explicit form is

$$\begin{pmatrix} \Gamma^{\phi_S^0 11}(p) & \Gamma^{\phi_S^0 12}(p) \\ \Gamma^{\phi_S^0 21}(p) & \Gamma^{\phi_S^0 22}(p) \end{pmatrix} = \frac{-i\Sigma_Q m_Q^2}{[p^2(k+h)-4(\tilde{k}+\tilde{h})]^2+(p^2 s-4\tilde{s})^2-(p^2 r-4\tilde{r})^2} \times \begin{pmatrix} (p^2-i\varepsilon)(k+h+is)-4(\tilde{k}+\tilde{h}+i\tilde{s}) & -i(p^2 r-4\tilde{r}) \\ -i(p^2 r-4\tilde{r}) & -(p^2+i\varepsilon)(k+h-is)+4(\tilde{k}+\tilde{h}-i\tilde{s}) \end{pmatrix}. \quad (6.2)$$

In Eq. (6.2) the denotations k, h, $\tilde{k}$ and $\tilde{h}$ have been given by Eq. (3.32) and

$$s=\sum_Q m_Q^2 S_Q(p), \quad r=\sum_Q m_Q^2 R_Q(p),$$

$$\tilde{s}=\sum_Q m_Q^4 S_Q(p), \quad \tilde{r}=\sum_Q m_Q^4 R_Q(p), \quad (6.3)$$

where $K_Q(p)$ and $H_Q(p)$ are expressed by Eqs. (3.27) and (3.28) and

$$S_Q(p)=4\pi^2 d_Q(R)\int \frac{d^4 l}{(2\pi)^4}\,\delta(l^2-m_Q^2)\,\delta[(l+p)^2-m_Q^2] \times[\sin^2\theta(l^0+p^0,\mu_Q)\cos^2\theta(l^0,\mu_Q) + \cos^2\theta(l^0+p^0,\mu_Q)\sin^2\theta(l^0,\mu_Q)], \quad (6.4)$$

and

$$R_Q(p)=2\pi^2 d_Q(R)\int \frac{d^4 l}{(2\pi)^4}\,\delta(l^2-m_Q^2) \times\delta[(l+p)^2-m_Q^2]\sin 2\theta(l^0,\mu_Q) \times\sin 2\theta(l^0+p^0,\mu_Q), \quad (6.5)$$

with $\sin^2\theta(l^0,\mu_Q)$ given by Eq. (2.26). In deriving Eq. (6.2), the gap equation (2.27) in the real-time formalism has been used. The matrix $\Gamma^{\phi_S^0 ba}(p)$ $(b,a=1,2)$ can be diagonalized by a thermal matrix M_S, i.e.,

$$\begin{pmatrix} \Gamma^{\phi_S^0 11}(p) & \Gamma^{\phi_S^0 12}(p) \\ \Gamma^{\phi_S^0 21}(p) & \Gamma^{\phi_S^0 22}(p) \end{pmatrix} = M_S^{-1}\begin{pmatrix} \Gamma_R^{\phi_S^0}(p) & 0 \\ 0 & \Gamma_R^{\phi_S^{0*}}(p) \end{pmatrix} M_S^{-1}, \quad (6.6)$$

where

$$M_S=\begin{pmatrix} \cosh\theta_S & \sinh\theta_S \\ \sinh\theta_S & \cosh\theta_S \end{pmatrix}, \quad (6.7)$$

with

$$\cosh\theta_S=\frac{1}{\sqrt{2}}\left(\frac{S}{\sqrt{S^2-R^2}}+1\right)^{1/2},$$

$$\sinh\theta_S=\frac{1}{\sqrt{2}}\left(\frac{S}{\sqrt{S^2-R^2}}-1\right)^{1/2}, \quad (6.8)$$

and

$$S=p^2 s-4\tilde{s}, \quad R=p^2 r-4\tilde{r}. \quad (6.9)$$

We indicate that since $S_Q(p)\pm R_Q(p)\geqslant 0$ [11] and $S_Q(p)=R_Q(p)=0$ when $p^2<4m_Q^2$, it can be deduced that $S^2-R^2=\Sigma_Q m_Q^2(p^2-4m_Q^2)(S_Q+R_Q)\,\Sigma_{Q'} m_{Q'}^2(p^2-4m_{Q'}^2)\times(S_{Q'}-R_{Q'})\geqslant 0$ no matter what value p^2 could take. $\Gamma_R^{\phi_S^0}(p)$ in Eq. (6.6) will be identified with the physical propagator for ϕ_S^0 in the real-time formalism and has the expression

$$\Gamma_R^{\phi_S^0}(p)=-i\sum_Q m_Q^2/\{[(p^2+i\varepsilon)[k+h-is\sqrt{1-R^2/S^2}] -4[\tilde{k}+\tilde{h}-i\tilde{s}\sqrt{1-R^2/S^2}]\}. \quad (6.10)$$

On the other hand, by means of Eq. (3.32), $\Gamma_I^{\phi_S^0}(p)$ in Eq. (3.31) can be written as

$$\Gamma_I^{\phi_S^0}(p)=-i\sum_Q m_Q^2/[(p^2+i\varepsilon)(k+h-is^I)-4(\tilde{k}+\tilde{h}-i\tilde{s}^I)]. \quad (6.11)$$

Comparing Eq. (6.10) with Eq. (6.11), we find that the physical propagators for ϕ_S^0 in the two formalisms now have quite similar form, except that the replacements $s\sqrt{1-R^2/S^2}\to s^I$ and $\tilde{s}\sqrt{1-R^2/S^2}\to\tilde{s}^I$ in the imaginary parts of their denominators must be made when transiting from the real-time formalism to the imaginary-time formalism. It has been known for some time that for an amputated Green function, the results calculated in the two formalisms of thermal field theory have generally some difference [17]. In present case, we note that the difference appears only in imaginary parts of the denominators of the propagators which does not affect the main physical conclusions from the propagators. This means that all the conclusions of the mass of ϕ_S^0 deduced from $\Gamma_I^{\phi_S^0}(p)$ in Sec. III remain valid for $\Gamma_R^{\phi_S^0}(p)$, including that the possible fluctuation effect of the amplitude of ϕ_S^0 is also kept qualitatively. In particular, by redefining the physical propagator $\Gamma_R^{\phi_S^0}(p)$ for ϕ_S^0 by diagonalization of the corresponding matrix propagator, we no longer meet the problems of doubling of $m_{\phi_S^0}$ originated from the definition of the propagator for ϕ_S^0 by $\Gamma^{\phi_S^0 11}(p)$. We also point out that the imaginary parts of the denominators of both $\Gamma_R^{\phi_S^0}(p)$ and $\Gamma_I^{\phi_S^0}(p)$ may be identical and equal to zero if $0 \leqslant p^2 < 4(m_Q)^2_{\min}$, where $(m_Q)_{\min}$ is the smallest mass of the Q fermions. In that case, we have real k and $\tilde{k}$, and $s=\tilde{s}=r=\tilde{r}=0$ thus $S=R=0$ and $s^I=\tilde{s}^I=0$ because the δ-function product factor $\delta(l^2-m_Q^2)\delta[(l+p)^2-m_Q^2]$ in the integrand of each $S_Q(p)$ and $R_Q(p)$ will become zero if $0 \leqslant p^2 < 4m_Q^2$. In the other case the imaginary parts of the denominators of $\Gamma_R^{\phi_S^0}(p)$ and $\Gamma_I^{\phi_S^0}(p)$ will generally differ. We note that, different from the case of the gap equation, the appearance of such a difference here is related to the fact that we are dealing with the four-point amputated functions which are determined by the loops bounded by two fermion propagators. The difference could be technically attributed to a different order of analytic continuation of a Green function for discrete Matsubara frequencies in the two formalisms [12]. But it is more possible [18] that appearance of the difference is due to that we are calculating different functions e.g. $\Gamma_I^{\phi_S^0}(p)$ and $\Gamma_R^{\phi_S^0}(p)$ in the two formalisms. However, it should be pointed out that at present the two formalisms of thermal field theory are not used *as usual*, by the terminology in Ref. [18]. First, in the imaginary-time formalism, owing to the analytic continuation (3.17) used, $\Gamma_I^{\phi_S^0}(p)$ is now a time-ordered, i.e., physical propagator, not a retarded one. Thus it is not difficult to understand why $\Gamma_I^{\phi_S^0}(p)$ and $\Gamma_R^{\phi_S^0}(p)$ may have almost identical expressions except the imaginary parts in their denominators. Next, we note that in the fermion bubble approximation, the amputated four-point functions induced by the four-fermion interactions are essentially determined by one fermion loops, and the calculations of them may formally be reduced the ones of usual two-point functions, but the former is rather different from the latter. This is because we are dealing with problem of the scalar bound states whose propagators correspond to only pure amputated functions without external particle legs. In this case, the thermal matrix M_S [Eqs. (6.7) and (6.8)] which diagonalizes the amputated four-point functions matrix (6.6) in the real-time formalism is different from the thermal matrix of a free scalar particle and can only be considered as an ''effective'' one in the case of bound states. Obviously, some more work is needed before any definite relation between $\Gamma_I^{\phi_S^0}(p)$ and $\Gamma_R^{\phi_S^0}(p)$ is found, for instance, by some more formal calculations. At present, it seems to be premature for us to be able to make a judgment on which circumstances of proper function should be considered. Hence an alternative and more realistic way to treat such difference between the two formalisms is to compare the results obtained by respectively using the propagators in the two formalisms in the same physical problem and examine whether the difference indeed gives some physically unequal description or not.

Next we turn to the case of pseudoscalar bound state mode and make similar discussions. By means of Eqs. (4.3) and (4.4), the matrix of the four-point amputated functions for the transition from $(\bar{Q}i\gamma_5 Q)^{(a)}$ to $(\bar{Q}'i\gamma_5 Q')^{(b)}$ can be expressed by [11]

$$\Gamma_P^{Q'bQa}(p)=\frac{g'_{Q'Q}}{G}\Gamma^{\phi_P^0 ba}(p), \quad Q',Q=U,D, \quad b,a=1,2, \tag{6.12}$$

where $\Gamma^{\phi_P^0 ba}(p)$ $(b,a=1,2)$ is the matrix propagator for the neutral pseudoscalar bound state ϕ_P^0 in Eq. (2.9) and has the following explicit expression:

$$\begin{pmatrix} \Gamma^{\phi_P^0 11}(p) & \Gamma^{\phi_P^0 12}(p) \\ \Gamma^{\phi_P^0 21}(p) & \Gamma^{\phi_P^0 22}(p)\end{pmatrix}=\frac{-i\Sigma_Q m_Q^2}{[(p^2)^2+\varepsilon^2][(k+h)^2+s^2-r^2]}\begin{pmatrix}(p^2-i\varepsilon)(k+h+is) & -ip^2r \\ -ip^2r & -(p^2+i\varepsilon)(k+h-is)\end{pmatrix}. \tag{6.13}$$

The matrix $\Gamma^{\phi_P^0 ba}(p)$ $(b,a=1,2)$ can be diagonalized by a thermal matrix M_P, i.e.,

$$\begin{pmatrix} \Gamma^{\phi_P^0 11}(p) & \Gamma^{\phi_P^0 12}(p) \\ \Gamma^{\phi_P^0 21}(p) & \Gamma^{\phi_P^0 22}(p)\end{pmatrix}=M_P^{-1}\begin{pmatrix}\Gamma_R^{\phi_P^0}(p) & 0 \\ 0 & \Gamma_R^{\phi_P^0 *}(p)\end{pmatrix}M_P^{-1}, \tag{6.14}$$

where

$$M_P=\begin{pmatrix}\cosh\theta_P & \sinh\theta_P \\ \sinh\theta_P & \cosh\theta_P\end{pmatrix}, \tag{6.15}$$

with

$$\cosh\theta_P=\frac{1}{\sqrt{2}}\left(\frac{s}{\sqrt{s^2-r^2}}+1\right)^{1/2},\quad \sinh\theta_s=\frac{1}{\sqrt{2}}\left(\frac{s}{\sqrt{s^2-r^2}}-1\right)^{1/2}. \tag{6.16}$$

Here we again have $s^2-r^2\geqslant 0$ due to $S_Q(p)\pm R_Q(p)\geqslant 0$ [11]. The physical propagator for ϕ_P^0 in the real-time formalism will become

$$\Gamma_R^{\phi_P^0}(p)=-i\sum_Q m_Q^2/(p^2+i\varepsilon)[k+h-is\sqrt{1-r^2/s^2}]. \tag{6.17}$$

It can be compared with $\Gamma_I^{\phi_P^0}(p)$, the propagator for ϕ_P^0 in the imaginary-time formalism which, by Eqs. (4.8), (3.26) and (3.23), will have the expression

$$\Gamma_I^{\phi_P^0}(p)=-i\sum_Q m_Q^2/(p^2+i\varepsilon)(k+h-is^I). \tag{6.18}$$

We see that $\Gamma_R^{\phi_P^0}(p)$ and $\Gamma_I^{\phi_P^0}(p)$ have very similar expressions except that in the imaginary parts of their denominators the replacement $s\sqrt{1-r^2/s^2}\to s^I$ must be made when transiting from the real-time formalism to the imaginary-time formalism. Such difference does not change the main physical conclusion reached from $\Gamma_R^{\phi_P^0}(p)$ or $\Gamma_I^{\phi_P^0}(p)$: ϕ_P^0 is a massless neutral pseudoscalar particle and can be identified with a Nambu-Goldstone boson of electroweak symmetry breaking. We also note that when $0\leqslant p^2<4(m_Q)^2_{\min}$, $\Gamma_R^{\phi_P^0}(p)$ and $\Gamma_I^{\phi_P^0}(p)$ will be real and identical since $s=r=s^I=0$ in this case. In particular, when $p^2\to 0$, both have the same form as the propagator for ϕ_P^0 at $T=0$.

Lastly we apply the parallel discussions to the charged scalar bound state mode. By Eqs. (5.3) and (5.4) in Ref. [11], the matrix of the four-point amputated functions for the transition from $(\bar{U}\Gamma^-D)^{(a)}$ to $(\bar{D}\Gamma^+U)^{(b)}$ is just the matrix propagator $\Gamma_{\phi^-}^{ba}$ $(b,a=1,2)$ for $\phi^-=(\bar{U}\Gamma^-D)$ and its elements can be expressed as follows:

$$\begin{aligned}\Gamma_{\phi^-}^{11}(p)&=-i\frac{(p^2-i\varepsilon)[K_{UD}(p)+H_{UD}(p)+iS_{UD}(p)]+E_{UD}(p)-i\bar{M}^2S_{UD}(p)}{\{p^2[K_{UD}(p)+H_{UD}(p)]+E_{UD}(p)\}^2+(p^2-\bar{M}^2)^2[S_{UD}^2(p)-R_{UD}^2(p)]}=[\Gamma_{\phi^-}^{22}(p)]^*,\\ \Gamma_{\phi^-}^{12}(p)&=\frac{-e^{\beta(\mu_U-\mu_D)/2}(p^2-\bar{M}^2)R_{UD}(p)}{\{p^2[K_{UD}(p)+H_{UD}(p)]+E_{UD}(p)\}^2+(p^2-\bar{M}^2)^2[S_{UD}^2(p)-R_{UD}^2(p)]}=e^{\beta(\mu_U-\mu_D)}\Gamma_{\phi^-}^{21}(p),\end{aligned} \tag{6.19}$$

where $\bar{M}^2$, $K_{UD}(p)$, $H_{UD}(p)$ and $E_{UD}(p)$ have been given by Eqs. (5.24)–(5.27), respectively, and by Eqs. (5.11) and (5.12) in Ref. [11], we have

$$\begin{aligned}S_{UD}(p)=4\pi^2d_Q(R)\int\frac{d^4l}{(2\pi)^4}\,\delta(l^2-m_U^2)\,\delta[(l+p)^2-m_D^2]\\ \times\{\sin^2\theta(l^0,\mu_U)\cos^2\theta(l^0+p^0,\mu_D)+\cos^2\theta(l^0,\mu_U)\sin^2\theta(l^0+p^0,\mu_D)\},\end{aligned} \tag{6.20}$$

$$R_{UD}(p)=2\pi^2d_Q(R)\int\frac{d^4l}{(2\pi)^4}\,\delta(l^2-m_U^2)\,\delta[(l+p)^2-m_D^2]\sin 2\theta(l^0,\mu_U)\sin 2\theta(l^0+p^0,\mu_D). \tag{6.21}$$

By means of a thermal matrix M_C, the matrix $\Gamma_{\phi^-}^{ba}$ $(b,a=1,2)$ can be diagonalized, i.e.,

$$\begin{pmatrix}\Gamma_{\phi^-}^{11} & \Gamma_{\phi^-}^{12}\\ \Gamma_{\phi^-}^{21} & \Gamma_{\phi^-}^{22}\end{pmatrix}=M_C^{-1}\begin{pmatrix}\Gamma_R^{\phi^-}(p) & 0\\ 0 & \Gamma_R^{\phi^-*}(p)\end{pmatrix}M_C^{-1}, \tag{6.22}$$

where

$$M_C=\begin{pmatrix}\cosh\theta_C & e^{\alpha}\sinh\theta_C\\ e^{-\alpha}\sinh\theta_C & \cosh\theta_C\end{pmatrix}, \tag{6.23}$$

with

$$\alpha=\beta(\mu_U-\mu_D)/2, \quad \cosh\theta_C=\frac{1}{\sqrt{2}}\left[\frac{S_{UD}(p)}{\sqrt{S_{UD}^2(p)-R_{UD}^2(p)}}+1\right]^{1/2}, \quad \sinh\theta_C=\frac{1}{\sqrt{2}}\left[\frac{S_{UD}}{\sqrt{S_{UD}^2(p)-R_{UD}^2(p)}}-1\right]^{1/2}. \quad (6.24)$$

It is easy from Eqs. (6.20) and (6.21) to verify that

$$S_{UD}(p)\pm R_{UD}(p)=4\pi^2 d_Q(R) \times\int\frac{d^4l}{(2\pi)^4}\delta(l^2-m_U^2)\delta[(l+p)^2-m_D^2] \times\sin^2[\theta(l^0,\mu_U)\pm\theta(l^0+p^0,\mu_D)]\geqslant 0. \quad (6.25)$$

Thus we must have $S_{UD}^2(p)-R_{UD}^2(p)\geqslant 0$. $\Gamma_R^{\phi^-}(p)$ in Eq. (6.22) is now defined as the physical propagator for ϕ^- in the real-time formalism and can be expressed by

$$\Gamma_R^{\phi^-}(p)=-i/\{(p^2+i\varepsilon)[K_{UD}(p)+H_{UD}(p)]+E_{UD}(p) -i(p^2-\bar{M}^2)\sqrt{S_{UD}^2(p)-R_{UD}^2(p)}\}. \quad (6.26)$$

It has very similar form to $\Gamma_I^{\phi^-}(p)$ in Eq. (5.23), the propagator for ϕ^- in the imaginary-time formalism, but for the transition $\Gamma_R^{\phi^-}(p)\to\Gamma_I^{\phi^-}(p)$ we must make the replacement $\sqrt{S_{UD}^2(p)-R_{UD}^2(p)}\to S_{UD}^I(p)$ in the imaginary parts of their denominators. Obviously, all the conclusions coming from $\Gamma_R^{\phi^-}(p)$ will be the same as those from $\Gamma_I^{\phi^-}(p)$ in Sec. V. When $m_U=m_D=m_Q$ and $\mu_U=\mu_D=\mu_Q$, we will have $E_{UD}=\bar{M}^2=0$, $K_{UD}=K_Q$, $H_{UD}=H_Q$, $S_{UD}=S_Q$, and $R_{UD}=R_Q$, thus we get

$$\Gamma_R^{\phi^-}(p)=-i/(p^2+i\varepsilon)[K_Q(p)+H_Q(p) -i\sqrt{S_Q^2(p)-R_Q^2(p)}], \quad (6.27)$$

which has an identical form to $\Gamma_R^{\phi_P^0}(p)$ in this case. The thermal matrix element

$$\cosh\theta_C=\frac{1}{\sqrt{2}}[S_Q(p)/\sqrt{S_Q^2(p)-R_Q^2(p)}+1]^{1/2}$$

will also coincide with $\cosh\theta_P$. In addition, it can be proven that

$$\delta(l^2-m_U^2)\delta[(l+p)^2-m_D^2]=0,$$

when

$$(m_U-m_D)^2\leqslant p^2<(m_U+m_D)^2. \quad (6.28)$$

Thus $S_{UD}(p)$, $R_{UD}(p)$ and $S_{UD}^I(p)$ are all equal to zero and these will make $\Gamma_R^{\phi^-}(p)$ and $\Gamma_I^{\phi^-}(p)$ become identical in this case.

VII. CONCLUSIONS

In the imaginary-time formalism of thermal field theory we have reexamined the Nambu-Goldstone mechanism of electroweak symmetry breaking at finite temperature in a one-generation fermion condensate scheme and compared the results with those obtained in the real-time formalism through some redefined physical propagators for scalar bound states. By means of the Schwinger-Dyson equation in the fermion bubble diagram approximation, it is obtained that the propagators for scalar bound states have very similar forms in the two formalisms, except that the imaginary parts of the denominators of the propagators have some differences when the momenta squared of the bound states are within some given ranges. However, these differences do not change the common essential conclusions reached in the two formalisms. These conclusions are as follows.

(1) When the two flavors of the one generation of fermions are mass-degenerate, at the temperature T below the symmetry restoration temperature T_c, one may obtain a composite Higgs boson ϕ_S^0 with mass $2m_Q$, a composite neutral pseudoscalar Nambu-Goldstone boson ϕ_P^0 and two composite charged Nambu-Goldstone bosons ϕ^- and ϕ^+. Thus the Goldstone theorem representing the spontaneous breaking of electroweak group $SU_L(2)\times U_Y(1)\to U_Q(1)$ is valid rigorously in this case.

(2) When one of the two flavors of fermions is massless, we can obtain the same ϕ_S^0 and ϕ_P^0 as those in (1), but the charged scalar bound states ϕ^- and ϕ^+ will no longer be rigorously massless and they could be considered approximately massless only if the momentum cutoff Λ is very large and the considered energy scales are low. This means that the Goldstone theorem at finite temperature is only approximately valid at low energy scales.

(3) When the two flavors of fermions have unequal nonzero masses, besides the conclusions in (2) remaining valid, it seems that we will meet the possible fluctuation effect of the Higgs boson's amplitude originated from the imaginary part of its propagator's pole. However, it has been argued that such effect is compeletely negligible, if the momentum cutoff Λ is sufficiently large.

The above conclusions imply that, as far as the discussed Nambu-Goldstone mechanism in this model is concerned, the two formalisms of thermal field theory give a physically equivalent description. In particular, by the redefinition of physical propagators in the real-time formalism, the extra result of the Higgs boson mass being doubled which originates from the definition of physical propagator of ϕ_S^0 by 11 element of its matrix propagator has automatically disappeared.

Our calculations also show that the gap equation, which comes from two-point amputated functions and is determined by the loops bounded by a single fermion propagator, is identical in the two formalisms without the need for analytic continuation. However, the propagator for scalar bound states, when defined by four-point amputated functions and calculated in the fermion bubble diagram approximation, is determined by the loops bounded by two fermion propagators, which could have some differences in the imaginary parts of their denominators in the two formalisms. Such differences have close relation to the introduction of the ghost fields to cancel the pinch singularities in the real-time formalism and it seems that they could technically be attributed to a different order of analytic continuation for the discrete Matsubara frequencies in the two formalisms [12]. However, a greater possibility is that such differences reflect that we are calculating different functions in the two formalisms. This problem needs to be researched further, especially with the bound state propagators being involved. Before any definite relation between the propagators in the two formalisms is found, we may still respectively use the propagators in the two formalisms in the same physical problem and compare the results obtained, as has been done in this paper. Although the differences between the propagators do not predict any actual different physical effect in the model discussed here, whether, where and how they could do that is an interesting problem deserving of further research.

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