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Null branes in string theory backgrounds

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We consider null bosonic *p*-branes moving in curved space-times and develop a method for solving their equations of motion and constraints, which is suitable for string theory backgrounds. As an application, we give an exact solution for such a background in ten dimensions.

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I. INTRODUCTION

The null *p*-branes are the zero tension limit $T_p \to 0$ of the usual *p*-branes, the one-brane being a string. This relationship between them generalizes the correspondence between massless and massive particles. Thus, the tensionless branes may be viewed as a high-energy limit of the tensile ones.

The *p*-branes are characterized by an energy scale $T_p^{1/(p+1)}$ and therefore by a length scale $T_p^{-1/(p+1)}$. The gravitational field provides another length scale, the curvature radius of the space-time R_c. For a *p*-brane moving in a gravitational field an appropriate parameter is the dimensionless constant $\mathcal{D} = R_c T_p^{1/(p+1)}$. Large values of $\mathcal{D}$ imply a weak gravitational field. One may reach such values of $\mathcal{D}$ by letting $T_p \to \infty$. In the opposite limit, small values of $\mathcal{D}$, one encounters strong gravitational fields and it is appropriate to consider $T_p \to 0$, i.e., null or tensionless branes.

A Lagrangian which could describe, under certain conditions, null bosonic branes in *D*-dimensional Minkowski space-time was first proposed in Ref. [1]. An action for a tensionless *p*-brane with space-time supersymmetry was first given in Ref. [2]. Since then, other types of actions and Hamiltonians (with and without supersymmetry) have been introduced and studied in the literature [3–10]. Owing to their zero tension, the worldvolume of the null *p*-branes is a lightlike, $(p+1)$-dimensional hypersurface, imbedded in the Minkowski space-time. Correspondingly, the determinant of the induced metric is zero. As in the tensile case, the null brane actions can be written in reparametrization and space-time conformally invariant form. However, their distinguishing feature is that at the classical level they may have any number of global space-time supersymmetries and be κ-invariant in all dimensions, which support Majorana (or Weyl) spinors. At the quantum level, they are anomaly free and do not exhibit any critical dimension, when appropriately chosen operator ordering is applied [11,3,4,7,12]. The only exception are the tensionless branes with manifest conformal invariance, with critical dimension $D=2$ for the bosonic case and $D=2-2N$ for the spinning case, N being the number of worldvolume supersymmetries [7].

Let us mention also the paper [13], which is devoted to the construction of field theory propagators of null strings and *p*-branes, as well as the corresponding spinning versions.

Almost all of the above investigations deal with *free* null branes moving in *flat* background (a qualitative consideration of null *p*-brane interacting with a scalar field has been done in [4]). The interaction of tensionless membranes $(p=2)$ with antisymmetric background tensor field in four-dimensional Minkowski space, described by means of Wess-Zumino-like action, is studied in Ref. [14]. The resulting equations of motion are successfully integrated exactly.

To our knowledge, the only papers until now devoted to the classical dynamics of *null* *p*-branes $(p>1)$ moving in *curved* space-times are Refs. [15–17]. In Ref. [15], the null *p*-branes living in *D*-dimensional Friedmann-Robertson-Walker spacetime with flat space-like section $(k=0)$ have been investigated. The corresponding equations of motion have been solved exactly. It was argued that an ideal fluid of null *p*-branes may be considered as a source of gravity for Friedmann-Robertson-Walker universes.

In Ref. [16], the classical mechanics of null branes in a gravity background was formulated. The Batalin-Fradkin-Vilkovisky approach in its Hamiltonian version was applied to the considered dynamical system. Some exact solutions of the equations of motion and of the constraints for the null membrane $(p=2)$ in general stationary axially symmetric four dimensional gravity background were found. The examples of Minkowski, (A)dS, Schwarzschild, Taub-NUT (Newman-Unti-Tamborino), and Kerr space-times were considered. Another exact solution, for the Demianski-Newman background, can be found in Ref. [17].

In this article we consider the classical evolution of tensionless bosonic *p*-branes in a particular type of *D*-dimensional curved background. In Sec. II we develop a method for solving their equations of motion and constraints. In Sec. III, as an application of the method proposed, we give an explicit exact solution for the ten-dimensional solitonic five-brane gravity background. Section IV is devoted to our concluding remarks.

II. SOLVING THE EQUATIONS OF MOTION

We will use the following reparametrization invariant action for the null bosonic *p*-brane living in a *D*-dimensional curved space-time with metric tensor $g_{MN}(x)$:

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$$S=\int d^{p+1}\xi\mathcal{L},\quad \mathcal{L}=V^mV^n\partial_m x^M\partial_n x^N g_{MN}(x),$$

$$\partial_m=\partial/\partial\xi^m,\quad \xi^m=(\xi^0,\xi^a)=(\tau,\sigma^a),$$

$$m,n=0,1,\dots,p,\quad a,b=1,\dots,p,$$

$$M,N=0,1,\dots,D-1. \tag{1}$$

It is a natural generalization of the flat space-time action given in Refs. [5,6].

Let us rewrite the Lagrangian density from Eq. (1) into the form ($\partial_\tau=\partial/\partial\tau,\partial_a=\partial/\partial\sigma^a$):

$$\mathcal{L}=\frac{1}{4\lambda^0}g_{MN}(x)(\partial_\tau-\lambda^a\partial_a)x^M(\partial_\tau-\lambda^b\partial_b)x^N, \tag{2}$$

where the connection between V^m and (λ^0,λ^a) is given by

$$V^m=(V^0,V^a)=\left(-\frac{1}{2\sqrt{\lambda^0}},\frac{\lambda^a}{2\sqrt{\lambda^0}}\right).$$

The Euler-Lagrange equation for x^K are

$$\partial_\tau\left[\frac{1}{2\lambda^0}(\partial_\tau-\lambda^b\partial_b)x^K\right]-\partial_a\left[\frac{\lambda^a}{2\lambda^0}(\partial_\tau-\lambda^b\partial_b)x^K\right]$$
$$+\frac{1}{2\lambda^0}\Gamma^K_{MN}(\partial_\tau-\lambda^a\partial_a)x^M(\partial_\tau-\lambda^b\partial_b)x^N=0, \tag{3}$$

where Γ^K_{MN} is the connection compatible with the metric $g_{MN}(x)$:

$$\Gamma^K_{MN}=\tfrac{1}{2}g^{KL}(\partial_M g_{NL}+\partial_N g_{ML}-\partial_L g_{MN}).$$

The equations of motion for the Lagrange multipliers λ^0 and λ^a which follow from Eq. (2) give the constraints

$$g_{MN}(x)(\partial_\tau-\lambda^a\partial_a)x^M(\partial_\tau-\lambda^b\partial_b)x^N=0, \tag{4}$$

$$g_{MN}(x)(\partial_\tau-\lambda^b\partial_b)x^M\partial_a x^N=0. \tag{5}$$

From now on, we will work in the gauge $\lambda^0,\lambda^a=constants$, in which Eqs. (3) have the form

$$(\partial_\tau-\lambda^a\partial_a)(\partial_\tau-\lambda^b\partial_b)x^K$$
$$+\Gamma^K_{MN}(\partial_\tau-\lambda^a\partial_a)x^M(\partial_\tau-\lambda^b\partial_b)x^N=0. \tag{6}$$

We are going to look for solutions of the equations of motion (6) and constraints (4),(5) for the following type of gravity background:

$$ds^2=g_{MN}dx^Mdx^N$$
$$=g_{qq}(dx^q)^2+2g_{qs}dx^qdx^s+g_{ss}(dx^s)^2+g_{\alpha\beta}dx^\alpha dx^\beta$$
$$+\sum_i g_{ii}(dx^i)^2, \tag{7}$$

where $x^q\neq x^s$ are two arbitrary coordinates and it is supposed that g_{MN} does not depend on them.

To simplify the equations of motion (6) and constraints (4),(5), we introduce the ansatz

$$x^q(\tau,\sigma^a)=C^qF(z^a)+y^q(\tau),\quad x^s(\tau,\sigma^a)=C^sF(z^a)+y^s(\tau),$$

$$x^M(\tau,\sigma^a)=y^M(\tau)\ \text{for}\ M\neq q,s,\quad z^a=\lambda^a\tau+\sigma^a, \tag{8}$$

where $F(z^a)$ is an arbitrary function of z^a, and C^q, C^s are constants. Inserting Eq. (8) in Eqs. (4), (5), and (6), one obtains (the overdot is used for $d/d\tau$)

$$\ddot{y}^K+\Gamma^K_{MN}\dot{y}^M\dot{y}^N=0, \tag{9}$$

$$g_{MN}\dot{y}^M\dot{y}^N=0, \tag{10}$$

$$(C^qg_{qq}+C^sg_{qs})\dot{y}^q+(C^qg_{qs}+C^sg_{ss})\dot{y}^s=0. \tag{11}$$

It turns out that for the given metric (7), the equations for $\dot{y}^q$ and $\dot{y}^s$ in Eq. (9) become linear differential equations. On the other hand, with the help of Eq. (11), we can separate the variables in them. The corresponding solution, compatible with Eq. (11), is ($C=$const)

$$\dot{y}^q(\tau)=-C(C^qg_{qs}+C^sg_{ss})\exp(-\mathcal{H}),$$

$$\dot{y}^s(\tau)=+C(C^qg_{qq}+C^sg_{qs})\exp(-\mathcal{H}),$$

$$\mathcal{H}=\int(g^{qq}dg_{qq}+2g^{qs}dg_{qs}+g^{ss}dg_{ss}). \tag{12}$$

Now we observe that if we introduce the matrix

$$\mathbf{h}=\begin{pmatrix} g_{qq} & g_{qs}\\ g_{qs} & g_{ss}\end{pmatrix},$$

then the following equality holds:

$$\exp(\mathcal{H})=\exp\left(\int Tr\mathbf{h}^{-1}d\mathbf{h}\right)=\det\mathbf{h}\equiv h. \tag{13}$$

At the same time, the above equality is the compatibility condition for the solution (12) with the other equations of motion and constraint (10).

Using Eqs. (12) and (13), the equations for the other coordinates and the remaining constraint can be rewritten as ($K,M,N\neq q,s$)

$$\ddot{y}^K+\Gamma^K_{MN}\dot{y}^M\dot{y}^N+\frac{1}{2}g^{KM}\partial_M\left(C^2\frac{G}{h}\right)=0,$$

$$g_{MN}\dot{y}^M\dot{y}^N+\left(C^2\frac{G}{h}\right)=0, \tag{14}$$

where

$$G=(C^q)^2g_{qq}+2C^qC^sg_{qs}+(C^s)^2g_{ss}.$$

At this stage, taking into account the general structure of the string theory gravity backgrounds in D dimensions, we introduce an additional restriction on the metric (7). Namely, we suppose that the set of values of the index M is expressed by the subsets $M=(q,s,\alpha,i)$ such that g_{MN} does not depend on coordinates $x^\alpha(\tau,\sigma^a)=y^\alpha(\tau)$ in addition to x^q, x^s. Under this condition, one can reduce the order of the differential equations for y^α by one with the result

$$\dot{y}^\alpha=\exp\left(-\int g^{\alpha\beta}dg_{\beta\gamma}\right)C^\gamma,\quad C^\gamma=\text{const.} \tag{15}$$

The condition on Eq. (15) to be in accordance with the equations for y^i and the constraint (14) is

$$2\frac{dg_{\alpha\beta}}{d\tau}\dot{y}^\alpha\dot{y}^\beta+g_{\alpha\beta}\frac{d}{d\tau}(\dot{y}^\alpha\dot{y}^\beta)=0,$$

and it is identically satisfied.

Let us turn to the equations of motion for the remaining coordinates y^i. These are

$$\ddot{y}^i+\Gamma^i_{jk}\dot{y}^j\dot{y}^k+\frac{1}{2}g^{ij}\left[\partial_j\left(C^2\frac{G}{h}\right)-(\partial_j g_{\alpha\beta})\dot{y}^\alpha\dot{y}^\beta\right]=0, \tag{16}$$

where $\dot{y}^\alpha$ are given by Eq. (15). The following step is to use the equality:

$$(\partial_j g_{\alpha\beta})\dot{y}^\alpha\dot{y}^\beta=-\partial_j(g_{\alpha\beta}\dot{y}^\alpha\dot{y}^\beta),$$

which is an identity on the solutions (15). This allows us to transform Eqs. (16) into the form

$$2\frac{d}{d\tau}(g_{ij}\dot{y}^j)-(\partial_i g_{jk})\dot{y}^j\dot{y}^k+\partial_i\left(C^2\frac{G}{h}+V\right)=0,$$

where

$$V=\left[C\exp\left(\int gdg^{-1}\right)\right]^\alpha g_{\alpha\beta}\left[\exp\left(-\int g^{-1}dg\right)C\right]^\beta.$$

Taking into account that the matrix g_{ij} is a diagonal one, we can further transform the equations for $y^i(\tau)$ to obtain (there is no summation over i):

$$\frac{d}{d\tau}(g_{ii}\dot{y}^i)^2+\dot{y}^i\partial_i\left[g_{ii}\left(C^2\frac{G}{h}+V\right)\right]+\dot{y}^i\sum_{j\neq i}\partial_i\left(\frac{g_{ii}}{g_{jj}}\right)(g_{jj}\dot{y}^j)^2=0. \tag{17}$$

In receiving Eq. (17), the constraint (14) rewritten in the form

$$g_{ii}(\dot{y}^i)^2+\sum_{j\neq i}g_{jj}(\dot{y}^j)^2+V+C^2\frac{G}{h}=0 \tag{18}$$

is also used.

Now it is evident from Eq. (17) that we can reduce the order of these differential equations by one, if

$$\partial_i\left(\frac{g_{ii}}{g_{jj}}\right)=0\quad\text{for}\quad i\neq j \tag{19}$$

or

$$\partial_i(g_{jj}\dot{y}^j)^2=0\quad\text{for}\quad i\neq j. \tag{20}$$

Keeping in mind the aim to apply our results to the string theory gravity backgrounds, we choose the following combination of the two existing possibilities: for all coordinates y^i except one, which we call y^r, the equalities (19) are satisfied; for $i=r$, the equalities (20) hold. Then the result of integration, compatible with Eq. (18), is

$$\begin{aligned}(g_{kk}\dot{y}^k)^2&=C_k(y^r,\dots,y^{k-1},y^{k+1},\dots)-g_{kk}\left(C^2\frac{G}{h}+V\right)\\&=E_k(\dots,y^{k-1},y^k,y^{k+1},\dots),\end{aligned} \tag{21}$$

$$(g_{rr}\dot{y}^r)^2=g_{rr}\left\{\left(\sum_k -1\right)\left(C^2\frac{G}{h}+V\right)-\sum_k\frac{C_k}{g_{kk}}\right\}=E_r(y^r), \tag{22}$$

$$\partial_k\left(\frac{g_{kk}}{g_{ii}}\right)=0,\quad k\neq q,s,\alpha,r. \tag{23}$$

Here C_k, E_k, and E_r are arbitrary functions. C_k depend on all coordinates on which depends the metric, but y^k (for every fixed value of k). E_k do not depend on y^r, but depend on y^k. Obviously, the right hand sides of Eqs. (21) and (22) have to be nonnegative.

Now, we are interested in finding exact solutions of the above equations. It turns out that it is preferable to use a slightly different approach for multidimensional and for lower-dimensional space-times. This will allow us to obtain solutions in more general class of metrics in the lower dimensional case. At first, we will try to find solutions appropriate for application to higher-dimensional backgrounds.

A simple analysis shows that we can integrate Eqs. (21) and (22) completely, if we fix the coordinates on which the background depends, except y^r. We prefer to consider just this possibility in connection with further applications in mind. Because $g_{MN}=g_{MN}(y^r,y^k)$, $(k\neq q,s,\alpha,r)$, we fix the coordinates y^k: $y^k=y^k_0=\text{const}$. Then the *exact* solution of the equations of motion and constraints for a null p-brane in this background is given by (8), where y^k are constants and

$$y^q = y_0^q \mp C\int_{y_0^r}^{y^r} du\,\frac{(C^q g_{qs}^0 + C^s g_{ss}^0)}{h^0 W_0^{1/2}}, \quad y_0^q, y_0^r = \text{const},$$

$$y^s = y_0^s \pm C\int_{y_0^r}^{y^r} du\,\frac{(C^q g_{qq}^0 + C^s g_{qs}^0)}{h^0 W_0^{1/2}}, \quad y_0^s = \text{const},$$

$$y^\alpha = y_0^\alpha \pm C^\gamma \int_{y_0^r}^{y^r}\frac{du}{W_0^{1/2}}\exp\left(-\int dg_{\gamma\beta}^0 g^{0\beta\alpha}\right), \quad y_0^\alpha = \text{const},$$

$$\tau = \tau_0 \pm \int_{y_0^r}^{y^r}\frac{du}{W_0^{1/2}}, \quad \tau_0 = \text{const},$$

$$W_0 = -\frac{C_k^0}{g_{rr}^0 g_{kk}^0}, \quad C_k^0 = g_{kk}^0\left(C^2\frac{G^0}{h^0} + V^0\right). \tag{24}$$

In the above equalities $g_{MN}^0 = g_{MN}^0(y^r) = g_{MN}(y^r, y_0^k)$ and analogously for G^0, h^0, V^0, and C_k^0.

Let us turn to the lower dimensional case. This separate consideration is necessary, because in obtaining the solution (24) we have restricted the metric to be independent on too many variables. In four dimensions, for instance, it is preferable to have a metric, which depends at least on two of the coordinates. This gives us the possibility to consider different types of black hole backgrounds, for example. Taking this into account, now we would like to find exact solutions of the differential equations (21),(22) for background metric, which does not depend only on y^q and y^s. To this end, we set $\alpha = \{\varnothing\}$ and choose the coordinates y^k, $(k \neq q,s,r)$ to be constant. Then for the remaining coordinates one obtains

$$y^q = y_0^q \mp \int_{y_0^r}^{y^r} du (C^q g_{qs}^0 + C^s g_{ss}^0)\left[-\frac{g_{rr}^0}{G^0 h^0}\right]^{1/2},$$

$$y^s = y_0^s \pm \int_{y_0^r}^{y^r} du (C^q g_{qq}^0 + C^s g_{qs}^0)\left[-\frac{g_{rr}^0}{G^0 h^0}\right]^{1/2},$$

$$\tau = \tau_0 \pm \int_{y_0^r}^{y^r} du\left[-\frac{g_{rr}^0 h^0}{C^2 G^0}\right]^{1/2}. \tag{25}$$

The corresponding *exact* solution for the tensionless *p*-brane in the chosen *D*-dimensional gravity background is given again by Eq. (8) with Eq. (25) inserted in there. Finally, we note that for obtaining exact solutions in cosmological type backgrounds, one can identify x^0 with y^r in Eq. (24) or in Eq. (25).

III. THE EXPLICIT SOLUTION

In this section we are going to apply the method proposed in the previous one for finding an explicit exact solution. We start by considering a null *p*-brane moving in the solitonic $(\tilde{d}-1)$-brane background [18]

$$\begin{aligned} ds^2 &= g_{MN}dx^M dx^N \\ &= \exp(2A)\,\eta_{\mu\nu}dx^\mu dx^\nu + \exp(2B) \\ &\quad \times (dr^2 + r^2 d\Omega^2_{D-\tilde{d}-1}), \end{aligned}$$

$$\exp(2A) = \left(1 + \frac{k_{\tilde{d}}}{r^d}\right)^{-d/(d+\tilde{d})},$$

$$\exp(2B) = \left(1 + \frac{k_{\tilde{d}}}{r^d}\right)^{+\tilde{d}/(d+\tilde{d})},$$

$$k_{\tilde{d}} = \text{const}, \quad +\tilde{d} = D-2,$$

$$\eta_{\mu\nu} = \text{diag}(-,+,\dots,+), \quad \mu,\nu = 0,1,\dots,\tilde{d}-1.$$

The $(D-\tilde{d}-1)$-dimensional sphere $\mathbf{S}^{D-\tilde{d}-1}$ is supposed to be parametrized so that

$$g_{kk} = \exp(2B)r^2\prod_{n=1}^{D-k-1}\sin^2\theta_n,$$

$$D-k-1 = 1,2,\dots,D-\tilde{d}-2,$$

$$g_{D-1,D-1} = \exp(2B)r^2.$$

If we now set $q=0$, $\alpha = 1,2,\dots,\tilde{d}-2, D-1$, $s = \tilde{d}-1$, $r = \tilde{d}$, $k = \tilde{d}+1,\dots,D-2$, the conditions (23) are fulfilled and we can use the general formula (24). The result is

$$y^\mu = y_0^\mu \pm E^\mu \int_{r_0}^{r} du\left(1 + \frac{k_{\tilde{d}}}{u^d}\right)\left(\mathcal{E} - \frac{(C^{D-1})^2}{u^2} + \frac{\mathcal{E}k_{\tilde{d}}}{u^d}\right)^{-1/2},$$

$$r \equiv y^{\tilde{d}},$$

$$E^\mu = (E^0, E^1, \dots, E^{\tilde{d}-1}) = (CC^{\tilde{d}-1}, C^1, \dots, CC^0),$$

$$\varphi = \varphi_0 \pm C^{D-1}\int_{r_0}^{r}\frac{du}{u^2}\left(\mathcal{E} - \frac{(C^{D-1})^2}{u^2} + \frac{\mathcal{E}k_{\tilde{d}}}{u^d}\right)^{-1/2},$$

$$\varphi \equiv y^{D-1},$$

$$\tau = \tau_0 \pm \int_{r_0}^{r} du\left(1 + \frac{k_{\tilde{d}}}{u^d}\right)^{\frac{\tilde{d}}{d+\tilde{d}}}\left(\mathcal{E} - \frac{(C^{D-1})^2}{u^2} + \frac{\mathcal{E}k_{\tilde{d}}}{u^d}\right)^{-1/2},$$

$$\mathcal{E} \equiv -E^\mu E^\nu \eta_{\mu\nu} = (CC^{\tilde{d}-1})^2 - (CC^0)^2 - \sum_{\alpha=1}^{\tilde{d}-2}(C^\alpha)^2 \geqslant 0. \tag{26}$$

Let us restrict ourselves to the particular case of ten dimensional solitonic five-brane background. The corresponding values of the parameters D, $\tilde{d}$ and d are $D=10$, $\tilde{d}=6$, $d=2$. Taking this into account and performing the integration in Eq. (26), one obtains the following explicit exact solution of the equations of motion and constraints for a tensionless p-brane living in such curved space-time:

$$x^0(\tau,\sigma^a)=C^0F(z^a)+y^0(\tau),$$

$$x^5(\tau,\sigma^a)=C^5F(z^a)+y^5(\tau),$$

$$x^M(\tau,\sigma^a)=y^M(\tau)\quad\text{for}\quad M\neq 0,5,7,8,$$

$$x^{7,8}(\tau,\sigma^a)=y_0^{7,8}=\text{const},$$

where for $\mathcal{C}=k_6-(C^9)^2/\mathcal{E}>0$

$$y^\mu=y_0^\mu\mp\frac{k_6E^\mu}{(\mathcal{C}\mathcal{E})^{1/2}}\ln\left(\frac{\mathcal{C}^{1/2}/r+(1+\mathcal{C}/r^2)^{1/2}}{\mathcal{C}^{1/2}/r_0+(1+\mathcal{C}/r_0^2)^{1/2}}\right)\pm\frac{E^\mu}{\mathcal{E}^{1/2}}[(\mathcal{C}+r^2)^{1/2}-(\mathcal{C}+r_0^2)^{1/2}],$$

$$\varphi=\varphi_0\mp\frac{C^9}{(\mathcal{C}\mathcal{E})^{1/2}}\ln\left(\frac{\dfrac{\mathcal{C}^{1/2}}{r}+(1+\mathcal{C}/r^2)^{1/2}}{\dfrac{\mathcal{C}^{1/2}}{r_0}+(1+\mathcal{C}/r_0^2)^{1/2}}\right),$$

$$\begin{aligned}\tau=\tau_0&\mp\left(\frac{k_6^3}{\mathcal{C}^4\mathcal{E}^2}\right)^{1/4}r^{3/2}\left(1+\frac{\mathcal{C}}{r^2}\right)^{1/2}\\&\times F_2\left(3/4,1,-3/4;3/2,3/4;1+\frac{r^2}{\mathcal{C}},-\frac{r^2}{k_6}\right)\\&\mp 2\left(\frac{k_6^3r_0^2}{\mathcal{C}^2\mathcal{E}^2}\right)^{1/4}F_1\left(1/4,1/2,-3/4;5/4;-\frac{r_0^2}{\mathcal{C}},-\frac{r_0^2}{k_6}\right)\\&\pm\frac{\Gamma(1/4)k_6}{4\Gamma(3/4)}\sqrt{\frac{\pi}{\mathcal{C}}}\mathcal{E}2F_1\left(1/4,1/2;-1/4;1-\frac{k_6}{\mathcal{C}}\right)\\&\mp\frac{\Gamma(1/4)k_6}{2\Gamma(3/4)}\sqrt{\frac{\pi}{\mathcal{C}\mathcal{E}}}\left(1-\frac{k_6}{\mathcal{C}}\right)^{-1/4}\\&\times{}_2F_1\left[1/4,3/2;3/4;\left(1-\frac{k_6}{\mathcal{C}}\right)^{-1}\right].\end{aligned}\tag{27}$$

In the above expressions, $\Gamma(z)$ is the Euler's Γ function and $2F_1(a,b;c;z)$ is the Gauss' hypergeometric function. The functions $F_1(a,b,b';c;w,z)$ and $F_2(a,b,b';c,c';w,z)$ in Eq. (27) are two of the hypergeometric functions of two variables. The defining equalities for F_1 and F_2 are [19,20]

$$F_1(a,b,b';c;w,z)=\sum_{k,l=0}^{\infty}\frac{(a)_{k+l}(b)_k(b')_l}{k!l!(c)_{k+l}}w^kz^l\quad(|w|,|z|<1),$$

$$F_2(a,b,b';c,c';w,z)=\sum_{k,l=0}^{\infty}\frac{(a)_{k+l}(b)_k(b')_l}{k!l!(c)_k(c')_l}w^kz^l\quad(|w|+|z|<1),$$

where

$$(a)_k=\frac{\Gamma(a+k)}{\Gamma(a)}.$$

IV. CONCLUDING REMARKS

In this paper we performed some investigation on the classical dynamics of the null bosonic branes in a curved space-time. In the Sec. II, we found *exact* solutions of the equations of motion and constraints for a null p-brane in two particular types of D-dimensional curved backgrounds. However, the latter are general enough to include in itself many interesting cases of string theory gravity backgrounds in different dimensions (such as black branes, intersecting branes, and cosmological type backgrounds). In the Sec. III, we gave an explicit example of exact solution for the solitonic five-brane curved background in ten dimensions.

Let us briefly comment on the area of applicability of the obtained results. Considering this, we have to take into account the following properties of the tensionless extended objects: (1) the null p-branes can be considered as a high energy limit of the tensile ones, when the role played by the string tension may be ignored; (2) in the presence of strong gravitational fields, it is appropriate to consider the null tension limit of a brane; (3) the large tension limit of a q-brane is related to the zero tension limit of the dual p-brane.

For example, if the null branes are viewed as space-time probes, the obtained exact solutions may have relevance to the singularity structure of branes. On the other hand, these solutions may have cosmological implications especially in the early universe. It is worth checking if this type of solutions leads to self-consistent brane cosmology. Another appropriate field of application of our results is the investigation of the solution properties near black hole horizons.

Outside the framework of the exact solutions, one can try to find an approximate solution for a tensile p-brane by perturbative expansion in powers of the brane tension. Then the exact null brane solution will be the zero approximation. However, it is more interesting to answer the question: can we calculate all the terms in such an expansion? In other words, does our method work in the tensile brane case? It turns out that the answer is positive at least for the tensile one-branes (strings). The appropriate ansatz is

$$x^{q,s}(\tau,\sigma^1)=C^{q,s}(z^1\pm 2\lambda^0T_1\tau)+y^{q,s}(\tau),$$

$$x^M(\tau,\sigma^1)=y^M(\tau)\quad\text{for}\quad M\neq q,s.$$

The corresponding solutions for y^M are

$$y^q=y_0^q\mp C\int_{y_0^r}^{y^r}du\frac{[C^qg_{qs}^0+C^sg_{ss}^0\pm 2\lambda^0T_1(C^qh^0/C)]}{h^0U_0^{1/2}},$$

$$y^s=y_0^s\pm C\int_{y_0^r}^{y^r}du\frac{[C^qg_{qq}^0+C^sg_{qs}^0\mp 2\lambda^0T_1(C^sh^0/C)]}{h^0U_0^{1/2}},$$

$$y^{\alpha}=y_0^{\alpha}\pm C^{\gamma}\int_{y_0^r}^{y^r}\frac{du}{U_0^{1/2}}\exp\left(-\int dg_{\gamma\beta}^0 g^{0\beta\alpha}\right),$$

$$y^k=\text{const},\quad \tau=\tau_0\pm\int_{y_0^r}^{y^r}\frac{du}{U_0^{1/2}},$$

$$U_0=-\frac{D_k^0}{g_{rr}^0 g_{kk}^0},\quad D_k^0=g_{kk}^0\left\{\left[\frac{C^2}{h^0}+(2\lambda^0 T_1)^2\right]G^0+V^0\right\}.$$

It is evident that taking the limit $T_1\rightarrow 0$ in the above expressions, we obtain our null string solution with $F(z^1)=z^1$.

Let us finally note that there exists another ansatz which leads to the same type of exact solutions and it is

$$x^q(\tau,\sigma^a)=C^qF(z^a)+y^q(\sigma),$$

$$x^s(\tau,\sigma^a)=C^sF(z^a)+y^s(\sigma),$$

$$x^M(\tau,\sigma^a)=y^M(\sigma)\quad\text{for}\quad M\neq q,s,$$

where σ is one of the world-volume coordinates $\sigma^1,\ldots,\sigma^p$. The corresponding tensile string ansatz is obvious.

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