First order phase transitions in a Bianchi type-I universe

Minu Joy* and V. C. Kuriakose†

Department of Physics, Cochin University of Science and Technology, Cochin, India 682022

(Received 6 June 2000; published 18 October 2000)

Considering the theory of induced gravity coupled to matter fields, taking the ϕ^6 interaction potential model we evaluate the one-loop effective potential in a (3+1)-dimensional Bianchi type-I spacetime. It is proved that the ϕ^6 theory can be regularized in (3+1)-dimensional curved spacetime. We evaluate the finite temperature effective potential and study the temperature dependence of phase transitions. The nature of phase transitions in the early universe is clarified to be of first order. The effects of spacetime curvature and arbitrary field coupling on the phase transitions in the early universe are also discussed.

PACS number(s): 04.62.+v, 11.10.Gh, 11.10.Wx, 11.15.Ex

I. INTRODUCTION

In the early universe, symmetries that are spontaneously broken today were restored and during the evolution of the universe there were phase transitions, perhaps many, associated with the spontaneous breakdown of symmetries (SSB) [1]. During such a phase transition it is possible for the field to acquire nonzero vacuum expectation values. In general, a symmetry breaking phase transition can be first or second order. For a first order phase transition the change in ϕ in going from one phase to the other must be discontinuous, while for a second order transition there is no barrier at the transition point and the transition occurs smoothly. Of particular interest for cosmology is the nature of phase transition, whether or not it is first order. If the phase transition is strongly first order, the Universe may be dominated by the vacuum energy and undergo a period of inflation. In this case, the transition proceeds by the nucleation of bubbles of the true vacuum. If the phase transition is higher order, or weakly first order, thermal fluctuations may drive the transition.

Quantum fields have profound influence on the dynamical behavior of the early universe [2–4]. Quantum field theory in an external classical gravitational field is usually regarded as a first step towards a more complete theory of quantum gravity [5]. At high energies the quantum matter fields are free from all the interactions except the conformal one with an external metric. The requirement of conformal invariance is especially important for the scalar field, as it fixes the value of the nonminimal parameter of the scalar curvature interaction to the conformal value. The effect of the quantum conformal factor leads to a first order phase transition induced by curvature where the scalar field plays the role of the order parameter [6–8].

The influence of quantum fields and the gravitational effects on the cosmological phase transitions have been investigated by many authors [9–12]. From an analysis based on the one loop renormalized effective potential it is concluded that the scalar gravitational coupling ξ and the magnitude of the scalar curvature R crucially determine the fate of symmetry. At the classical level the scalar curvature acts as an effective mass of the field and thus influence the phase transition of the system. The effect of anisotropy on the static spacetimes such as a mixmaster or Taub universe on the process of symmetry breaking and restoration has also been discussed [13,14].

In the present work we discuss the quantum field effects on phase transition and the temperature dependence of phase transition for a ϕ^6 theory in a Bianchi type-I universe. This is the most general model for a self-interacting scalar field exhibiting $\phi \to -\phi$ symmetry. Self-interactions up to ϕ^6 exhibit three lowest levels well separated from the rest [15]. Boyanovsky and Masperi have shown that the nature of phase transitions associated with such a field system may be of first order or second order depending on the relative depths of the wells and the strength of coupling.

One of the simplest models of an anisotropic universe that describes a homogeneous and a spatially flat universe is the Bianchi type-I cosmology. Unlike the Friedmann-Robertson-Walker (FRW) model which has the same scale factor for each of the three spatial directions, the Bianchi type-I cosmology has a different scale factor in each direction, which produces an anisotropy in expansion. Futamase has considered the effective potential in a Bianchi type-I universe [16] which reduces to the spatially flat Robertson-Walker model for zero anisotropy. Huang has discussed the fate of symmetry in a Bianchi type-I universe using an adiabatic approximation for a massless field with arbitrary coupling to gravity [17]. Berkin has also calculated the effective potential in a Bianchi type-I universe, for a scalar field having arbitrary mass and coupling to gravity [18].

ϕ^6 model is known to be nonrenormalizable in (3+1)-dimensional flat spacetime. Nonrenormalizability of the field theory does not mean that the theory is not interesting and it does not mean, of course, that finite renormalized prescription for the calculation of one-loop effective potential does not exist [19]. Using the present ϕ^6 model we have obtained a finite expression for the one-loop effective potential. The present calculations show that the ϕ^6 model can be regularized using the effective potential method in (3+1)-dimensional curved spacetime. This paper is organized in the following way. In Sec. II we evaluate the one-loop effective potential for ϕ^6 theory in a (3+1)-dimensional Bianchi type-I spacetime with small anisotropy and discuss the prop-

*Email address: minujoy@cusat.ac.in

†Email address: vck@cusat.ac.in

erties of quantum field corrections to the symmetry breaking or restoration. The finite temperature effects on the phase transitions of early universe are discussed in Sec. III and the nature of phase transitions is examined in Sec. IV. The crucial dependence of phase transitions of the early universe on spacetime curvature and the gravitational-scalar coupling is made clear in Sec. V. Section VI is devoted to discussions and conclusions of the present calculations.

II. QUANTUM FIELD EFFECTS ON SYMMETRY BREAKING AND RESTORATION IN BIANCHI TYPE-I SPACETIME

The Lagrangian density £ describing a massive self-interacting scalar field ϕ coupled arbitrarily to the gravitational background is

$$\text{£} = \sqrt{-g}\left\{\frac{1}{2}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \xi R\phi^2] - \frac{1}{2}\lambda^2\phi^2(\phi^2 - m/\lambda)^2\right\}, \tag{1}$$

where the classical potential corresponding to this Lagrangian is

$$V(\phi) = \frac{1}{2}\xi R\phi^2 + \frac{1}{2}\lambda^2\phi^2(\phi^2 - m/\lambda)^2. \tag{2}$$

This Lagrangian exhibits $\phi \to -\phi$ symmetry. The equation of motion associated with the Lagrangian (1) is

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + (m^2 + \xi R)\phi - 4\kappa\phi^3 + 3\lambda^2\phi^5 = 0 \tag{3}$$

in which we put $m\lambda = \kappa$. We can write

$$\phi = \phi_c + \phi_q, \tag{4}$$

where ϕ_c is the classical field and ϕ_q is a quantum field with vanishing vacuum expectation value $\langle\phi_q\rangle = 0$. Introducing the renormalized parameters

$$m^2 = m_r^2 + \delta m^2, \quad \xi = \xi_r + \delta\xi,$$
$$\kappa = \kappa_r + \delta\kappa, \quad \lambda^2 = \lambda_r^2 + \delta\lambda^2, \tag{5}$$

the field equation for the classical field ϕ_c becomes

$$\begin{aligned} &g^{\mu\nu}\nabla_\mu\nabla_\nu\phi_c + [(m_r^2 + \delta m^2) + (\xi_r + \delta\xi)R]\phi_c, \\ &-4(\kappa_r + \delta\kappa)\phi_c^3 - 12(\kappa_r + \delta\kappa)\phi_c\langle\phi_q^2\rangle, \\ &+3(\lambda_r^2 + \delta\lambda^2)\phi_c^5 + 30(\lambda_r^2 + \delta\lambda^2)\phi_c^3\langle\phi_q^2\rangle \\ &+15(\lambda_r^2 + \delta\lambda^2)\phi_c\langle\phi_q^4\rangle = 0 \end{aligned} \tag{6}$$

and to the one loop quantum effect, the field equation for the quantum field ϕ_q is

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi_q + (m_r^2 + \xi R)\phi_q - 12\kappa_r\phi_c^2\phi_q + 15\lambda_r^2\phi_c^4\phi_q = 0. \tag{7}$$

The effective potential V_{eff} is given by

$$\begin{aligned} V_{\text{eff}} = &\frac{1}{2}[(m_r^2 + \delta m^2) + (\xi_r + \delta\xi)R][\phi_c^2 + \langle\phi_q^2\rangle] - (\kappa_r \\ &+ \delta\kappa)\phi_c^4 - 6(\kappa_r + \delta\kappa)\phi_c^2\langle\phi_q^2\rangle - (\kappa_r + \delta\kappa)\langle\phi_q^4\rangle \\ &+ \frac{1}{2}(\lambda_r^2 + \delta\lambda^2)\phi_c^6 + \frac{15}{2}(\lambda_r^2 + \delta\lambda^2)\phi_c^4\langle\phi_q^2\rangle \\ &+ \frac{15}{2}(\lambda_r^2 + \delta\lambda^2)\phi_c^2\langle\phi_q^4\rangle + \frac{1}{2}(\lambda_r^2 + \delta\lambda^2)\langle\phi_q^6\rangle. \end{aligned} \tag{8}$$

To make V_{eff} finite, the following renormalization conditions are used:

$$m_r^2 = \left(\frac{\partial^2 V_{\text{eff}}}{\partial\phi_c^2}\right)_{\phi_c = R = 0}, \tag{9}$$

$$\xi_r = \left(\frac{\partial^3 V_{\text{eff}}}{\partial R\,\partial\phi_c^2}\right)_{\phi_c = R = 0},$$

$$\kappa_r = \left(\frac{\partial^4 V_{\text{eff}}}{\partial\phi_c^4}\right)_{\phi_c = R = 0},$$

$$\lambda_r^2 = \left(\frac{\partial^6 V_{eff}}{\partial\phi_c^6}\right)_{\phi_c = R = 0}.$$

To evaluate $\langle\phi_q^2\rangle$, $\langle\phi_q^4\rangle$, and $\langle\phi_q^6\rangle$ we adopt the canonical quantization relations

$$[\phi_q(t,x), \phi_q(t,y)] = [\pi_q(t,x), \pi_q(t,y)] = 0,$$
$$[\phi_q(t,x), \pi_q(t,y)] = i\,\delta^3(x - y), \tag{10}$$

where the conjugate momentum π_q is defined by $\pi_q = \partial\text{£}/\partial(\partial_i\phi)$. Due to the space homogeneity we expand the quantum field ϕ_q by the summation or integration over modes in the form

$$\begin{aligned} \phi_q(t,x) = C^{-1/2}(t)\int d\mu(k)[&a_k\chi_k(t)y_k(x) \\ &+ a_k^+\chi_k^*(t)y_k^*(x)], \end{aligned} \tag{11}$$

where $y_k(x)$ is a normalized eigenfunction of the spatial part of field equation, while $\chi_k(t)$ is that of the time part. An explicit functional form of the mode solutions $\chi_k(t)$ and $y_k(x)$ can only be found after specifying the background spacetime.

We consider a (3+1)-dimensional Bianchi type-I spacetime with small anisotropy which has the line element

$$ds^2 = C(\eta)d\eta^2 - a_1^2(\eta)dx^2 - a_2^2(\eta)dy^2 - a_3^2(\eta)dz^2, \tag{12}$$

$$C = (a_1 a_2 a_3)^{2/3}.$$

In this model the mode function can be written in the separated form as $u_k = C^{-1/2}(2\pi)^{-3/2}\exp(i\kappa.x)\chi_k(\eta)$ and then

$$\langle \phi_q^2(\eta)\rangle = \frac{1}{8\pi^3 C(\eta)}\int d^3k \chi_k(\eta)\chi_k^*(\eta),$$

$$\langle \phi_q^4(\eta)\rangle = \frac{1}{64\pi^6 C^2(\eta)}\int d^3k [\chi_k(\eta)\chi_k^*(\eta)]^2, \tag{13}$$

and

$$\langle \phi_q^6(\eta)\rangle = \frac{1}{512\pi^9 C^3(\eta)}\int d^3k [\chi_k(\eta)\chi_k^*(\eta)]^3.$$

The wave equation Eq. (7) will then lead to

$$\ddot{\chi} + \left\{ C\left[m_r^2 + \left(\xi_r - \frac{1}{6}\right)R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \sum_i \frac{k_i^2}{a_i^2}\right] + Q\right\}\chi_k = 0, \tag{14}$$

where the spacetime curvature function R and the anisotropic function Q are

$$R = 6C^{-1}(\dot{H} + H^2 + Q), \quad H = \sum_i h_i, \tag{15}$$

$$h_i = \frac{\dot{a}_i}{a_i}, \quad Q = \frac{1}{36}\sum_{i<j}(h_i - h_j)^2.$$

When the metric is slowly varying Eq. (14) possesses WKB approximation solution

$$\chi_k = (2W_k)^{-1/2}\exp\left(-i\int d\eta W_k\right), \tag{16}$$

where

$$W_k = \left\{ C\left[m_r^2 + \left(\xi_r - \frac{1}{6}\right)R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \sum_i \frac{k_i^2}{a_i^2}\right] + Q\right\}^{1/2}.$$

Substituting the above solution in Eq. (13),

$$\langle \phi_q^2\rangle = \frac{1}{16\pi^3 C(\eta)}\int d^3k \left\{ C\left[m_r^2 + \left(\xi_r - \frac{1}{6}\right)R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \sum_i \frac{k_i^2}{a_i^2}\right] + Q\right\}^{-1/2}$$
$$= \frac{1}{16\pi}\left\{ \Lambda^2 + \frac{1}{2}\left[m_r^2 + \left(\xi_r - \frac{1}{6}\right)R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \frac{Q}{C}\right]\left[1 + \ln\left(\frac{m_r^2 + \left(\xi_r - \frac{1}{6}\right)R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \frac{Q}{C}}{4\Lambda^2}\right)\right]\right\} \tag{17}$$

and similarly,

$$\langle \phi_q^4\rangle = \frac{\Lambda}{128\pi^4 C^{3/2}}\left\{ 1 - \frac{\left[m_r^2 + \left(\xi_r - \frac{1}{6}\right)R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \frac{Q}{C}\right]^{1/2}}{\Lambda} \times \arctan\frac{\Lambda}{\left[m_r^2 + \left(\xi_r - \frac{1}{6}\right)R - 12\kappa_r\phi_c^2 + 15\lambda_r^2\phi_c^4 + \frac{Q}{C}\right]^{1/2}}\right\}, \tag{18}$$

where we have introduced a momentum cutoff Λ to regularize the k integration. From the renormalization conditions given by Eq. (9), the renormalization counterterms are evaluated as

$$\delta m^2=\frac{3(\kappa_r+\delta\kappa)}{4\pi}\left\{\Lambda^2+\frac{1}{2}\left(m_r^2+\frac{Q}{C}\right)\left(1+\ln\left[\frac{m_r^2+\frac{Q}{C}}{4\Lambda^2}\right]\right)\right\}-\frac{15(\lambda_r^2+\delta\lambda^2)}{128\pi^4C^{3/2}}\left[\Lambda-\left(m_r^2+\frac{Q}{C}\right)^{1/2}\arctan\left(\frac{\Lambda}{\left(m_r^2+\frac{Q}{C}\right)^{1/2}}\right)\right], \tag{19}$$

$$\delta\xi=\frac{3(\kappa_r+\delta\kappa)}{8\pi}\left(\xi_r-\frac{1}{6}\right)\left[2+\ln\left(\frac{m_r^2+\frac{Q}{C}}{4\Lambda^2}\right)\right]+\frac{15(\lambda_r^2+\delta\lambda^2)}{256\pi^4C^{3/2}}\left(\xi_r-\frac{1}{6}\right)\left[\frac{1}{\left(m_r^2+\frac{Q}{C}\right)^{1/2}}\arctan\left(\frac{\Lambda}{\left(m_r^2+\frac{Q}{C}\right)^{1/2}}\right)\right.$$
$$\left.-\frac{\Lambda}{\left(m_r^2+\frac{Q}{C}+\Lambda^2\right)}\right], \tag{20}$$

$$\delta\kappa=-\kappa_r-\frac{\lambda_r^2}{60\left\{\frac{45\lambda_r^2}{4\pi}\left(2+\ln\left[\frac{m_r^2+\frac{Q}{C}}{4\Lambda^2}\right]\right)+\frac{54\kappa_r^2}{\pi\left(m_r^2+\frac{Q}{C}\right)}\right\}}, \tag{21}$$

$$\delta\lambda^2=-\lambda_r^2+\frac{8\left\{-\lambda_r^2\pi+27\kappa_r\lambda_r^2\left(2+\ln\left[\frac{m_r^2+\frac{Q}{C}}{4\Lambda^2}\right]\right)+\frac{135\kappa_r^3}{\left(m_r^2+\frac{Q}{C}\right)}\right\}}{225\left\{\Lambda^2+\frac{1}{2}\left(m_r^2+\frac{Q}{C}\right)\left(1+\ln\frac{m_r^2+\frac{Q}{C}}{4\Lambda^2}\right)+\frac{3\kappa_r}{16\pi^3}\left(\frac{1}{\left(m_r^2+\frac{Q}{C}\right)^{\frac{1}{2}}}\arctan\left[\frac{\Lambda}{\left(m_r^2+\frac{Q}{C}\right)^{\frac{1}{2}}}\right]-\frac{\Lambda}{m_r^2+\frac{Q}{C}+\Lambda^2}\right)\right\}}$$
$$\times\frac{1}{\left[\frac{45\lambda_r^2}{4\pi}\left(2+\ln\left[\frac{m_r^2+\frac{Q}{C}}{4\Lambda^2}\right]\right)+\frac{54\kappa_r^2}{\pi\left(m_r^2+\frac{Q}{C}\right)}\right]}. \tag{22}$$

Substituting the renormalization counterterms, we find $\partial V_{\mathrm{eff}}/\partial\phi_c$ obtained from Eq. (8) as

$$
\begin{aligned}
\frac{\partial V_{\mathrm{eff}}}{\partial \phi_c} = {} & (m_r^2+\xi_r R)\phi_c + \frac{\kappa_r\left(\frac{n\pi}{2}\right)}{100\pi^3 C\frac{3}{2}\left[\left(m_r^2+\frac{Q}{C}\right)+\frac{3\kappa_r}{16\pi^3\left(m_r^2+\frac{Q}{C}\right)^{\frac{1}{2}}}\left(\frac{n\pi}{2}\right)\right]} \\
& \times\left\{\left(m_r^2+\frac{Q}{C}\right)^{\frac{1}{2}}-\left[m_r^2+\left(\xi_r-\frac{1}{6}\right)R-12\kappa_r\phi_c^2+15\lambda_r^2\phi_c^4+\frac{Q}{C}\right]\right\}\phi_c \\
& +\left[-\frac{\left(\xi_r-\frac{1}{6}\right)}{900}+\frac{\left(\xi_r-\frac{1}{6}\right)\kappa_r\left(\frac{n\pi}{2}\right)}{200\pi^3\left(m_r^2+\frac{Q}{C}\right)^{\frac{1}{2}}\left[\left(m_r^2+\frac{Q}{C}\right)+\frac{3\kappa_r}{16\pi^3\left(m_r^2+\frac{Q}{C}\right)^{\frac{1}{2}}}\left(\frac{n\pi}{2}\right)\right]}\right]R\phi_c \\
& +\frac{2\kappa_r\left[m_r^2+\left(\xi_r-\frac{1}{6}\right)R-12\kappa_r\phi_c^2+15\lambda_r^2\phi_c^4+\frac{Q}{C}\right]}{25\left[\left(m_r^2+\frac{Q}{C}\right)+\frac{3\kappa_r}{16\pi^3\left(m_r^2+\frac{Q}{C}\right)^{\frac{1}{2}}}\left(\frac{n\pi}{2}\right)\right]} \\
& \times\left[1+\ln\left(m_r^2+\left(\xi_r-\frac{1}{6}\right)R-12\kappa_r\phi_c^2+15\lambda_r^2\phi_c^4+\frac{Q}{C}\right)\right]\phi_c^3 \\
& +\frac{32\kappa_r\pi}{375\left[\left(m_r^2+\frac{Q}{C}\right)+\frac{3\kappa_r}{16\pi^3\left(m_r^2+\frac{Q}{C}\right)^{\frac{1}{2}}}\left(\frac{n\pi}{2}\right)\right]}\phi_c^5, \quad \text{where} \quad n=1,2,3\ldots. \qquad (23)
\end{aligned}
$$

The above equation shows that we can obtain finite expression for the one loop effective potential using this ϕ^6 model in (3+1)-dimensional Bianchi type I spacetime. Thus it is clear that the ϕ^6 theory in 3+1 dimensions can be regularized in a curved anisotropic spacetime using the effective potential method. It is to be noted that once we let the anisotropy in the above equation be zero, our result is consistent with that of the symmetric homogeneous case.

Now we are in a position to investigate the gravitational and quantum field effects on the cosmological phase transitions. This can be done by considering the case $\phi_c \rightarrow 0$. In the case of conformal coupling ($\xi_r=\frac{1}{6}$) or vanishing scalar curvature ($R=0$) we have

$$\left(\frac{\partial V_{\mathrm{eff}}}{\partial \phi_c}\right)_{\phi_c\rightarrow 0} \sim m_r^2\phi_c \qquad (24)$$

which shows that in such situations, the one-loop quantum correction does not change the fate of symmetry. For the other cases, we can find from the above equations that only for some suitable values of scalar gravitational coupling could the symmetry be radiatively broken or restored.

The perturbative method of calculating the effective potential can be improved by using renormalization group (RG) approach [20]. Such RG improved effective potential can be calculated in curved spacetime too [21]. The condition expressing the independence of the effective potential from the renormalization point leads to renormalization group equation (RGE) [6]. This property in renormalizable theories may be used for construction of famous RG improved effective potential, which is much more exact than one loop-effective potential, because it takes into account all orders of the perturbation theory. However, unlike such multiplicatively renormalizable theories RG improved potential will not give leading log approximation in the present ϕ^6 model as the theory is not multiplicatively renormalizable.

III. FINITE TEMPERATURE BEHAVIOR

The evolution of particles in vacuum and in a thermal bath are very different. Similarly, the nature of evolution of field changes when coupled to a thermal bath. Under certain conditions, the changes may be absorbed in a temperature-dependent potential, the finite temperature effective potential. The temperature dependence of finite temperature effective potential in quantum field theory leads to phase transitions in the early universe [22]. In this case the vacuum expectation value is replaced by the thermal average $\langle \phi \rangle_T = \sigma_T$ taken with respect to a Gibbs ensemble [1].

In this section we evaluate the effective potential at finite temperature and show that the symmetry breaking present in the model can be removed if the temperature is raised above a certain value called the critical temperature. Considering the same Lagrangian density as above, the zero loop effective potential is temperature independent,

$$V_0(\sigma)=\frac{1}{2}\xi R\sigma^2+\frac{1}{2}\lambda^2\sigma^2(\sigma^2-m/\lambda)^2. \tag{25}$$

The one loop approximation to finite temperature effective potential has been computed by many authors [23–26] and is given by

$$\begin{aligned} V_1^\beta(\sigma) &= \frac{1}{2\beta}\sum_n \int \frac{d^3k}{(2\pi)^3}\ln(k^2-M^2) \\ &= \frac{1}{2\beta}\sum_n \int \frac{d^3k}{(2\pi)^3}\ln\left(\frac{-4\pi^2n^2}{\beta^2}-E_M^2\right), \end{aligned} \tag{26}$$

where

$$E_M^2=k^2+M^2, \tag{27}$$

$$M^2=m^2+\xi R-12\lambda m\sigma^2+15\lambda^2\sigma^4.$$

The sum over n diverges; it may be evaluated by the method of Dolan and Jackiw [23] and we get

$$\begin{aligned} V_1^\beta(\sigma) &= \int \frac{d^3k}{(2\pi)^3}\left[\frac{E_M}{2}+\frac{1}{\beta}\ln(1-e^{-\beta E})\right] \\ &= V_1^0(\sigma)+\overline{V_1^\beta}(\sigma), \end{aligned} \tag{28}$$

where

$$V_1^0(\sigma)=\int \frac{d^3k}{(2\pi)^3}\frac{E_M}{2} \tag{29}$$

and

$$\begin{aligned} \overline{V_1^\beta}(\sigma) &= \int \frac{d^3k}{(2\pi)^3}\frac{1}{\beta}\ln(1-e^{-\beta E}) \\ &= \frac{1}{4\pi\beta^3}\int_0^\infty x\,dx\,\ln[1-\exp-(x^2+\beta^2M^2)^{1/2}], \end{aligned} \tag{30}$$

where we put $x^2/\beta^2=E_M^2-M^2$. The integral may be evaluated by expanding $\overline{V_1^\beta}(\sigma)$ as a Taylor series and in the high-temperature limit we find that

$$V_1^\beta(\sigma)=\frac{-\pi^2}{90\beta^4}+\frac{M^2}{24\beta^2}-\frac{M^3}{12\pi\beta}-\frac{M^4}{64\pi^2}\ln M^2\beta^2. \tag{31}$$

The critical temperature in the present case is

$$T_c=\left[\frac{(m^2+\xi R)}{\lambda m}\right]^{1/2}. \tag{32}$$

The symmetry breaking present in the model can be removed if the temperature is raised above a certain value called the critical temperature. The order parameter of the theory is temperature dependent.

IV. NATURE OF PHASE TRANSITION

The characteristic of a first order phase transition is the existence of a barrier between the symmetric and the broken phase [27]. The temperature dependence of $V_{\rm eff}$ for a first order phase transition obtained using the present ϕ^6 model is shown in Figs. 1(a)–1(e). When $T \gg T_c$, the effective potential attains a minimum at $\sigma=0$, which corresponds to the completely symmetric case. When the temperature decreases, a global minimum appears at $\sigma=0$ and two local minima at $\sigma\neq 0$, which shows the existence of a barrier between the global and local minima. At $T=T_c$, all the minima are degenerate, that means the symmetry is broken. For $T<T_c$ the minima at $\sigma\neq 0$ becomes the global one. If for $T\leqslant T_c$ the extremum at $\sigma=0$ remains a local minimum, there must be a barrier between the minimum at $\sigma=0$ and at $\sigma\neq 0$. Therefore the change in σ in going from one phase to the other must be discontinuous, indicating a first order phase transition. The phase transition starts at T_c by tunneling, however, if the barrier is high enough the tunneling effect is very small and the phase transition effectively starts at a temperature $T\ll T_c$ [28]. This shows that the present model can describe first order phase transitions which might have taken place during the evolution of the early universe.

V. DEPENDENCE ON SCALAR CURVATURE *R* AND SCALAR-GRAVITATIONAL COUPLING ξ

Using this ϕ^6 model, it is proved that the curvature can restore broken symmetries for a wide range of parameters from conformal to near minimal couplings, even if the temperature is below critical temperature. Figure 2 clearly shows that the first order phase transition takes place as R changes.

The scalar-gravitational coupling constant ξ is found to play a crucial role in symmetry breaking phase transitions. Classically, a positive ξ restores symmetry, while the opposite effects are found for negative coupling [18]. Quantum effects depend on the value of ξ relative to the conformal value $\frac{1}{6}$. The present calculations show that the symmetry is restored as the scalar coupling constant ξ is increased. This phase transition, induced by the coupling constant ξ is also

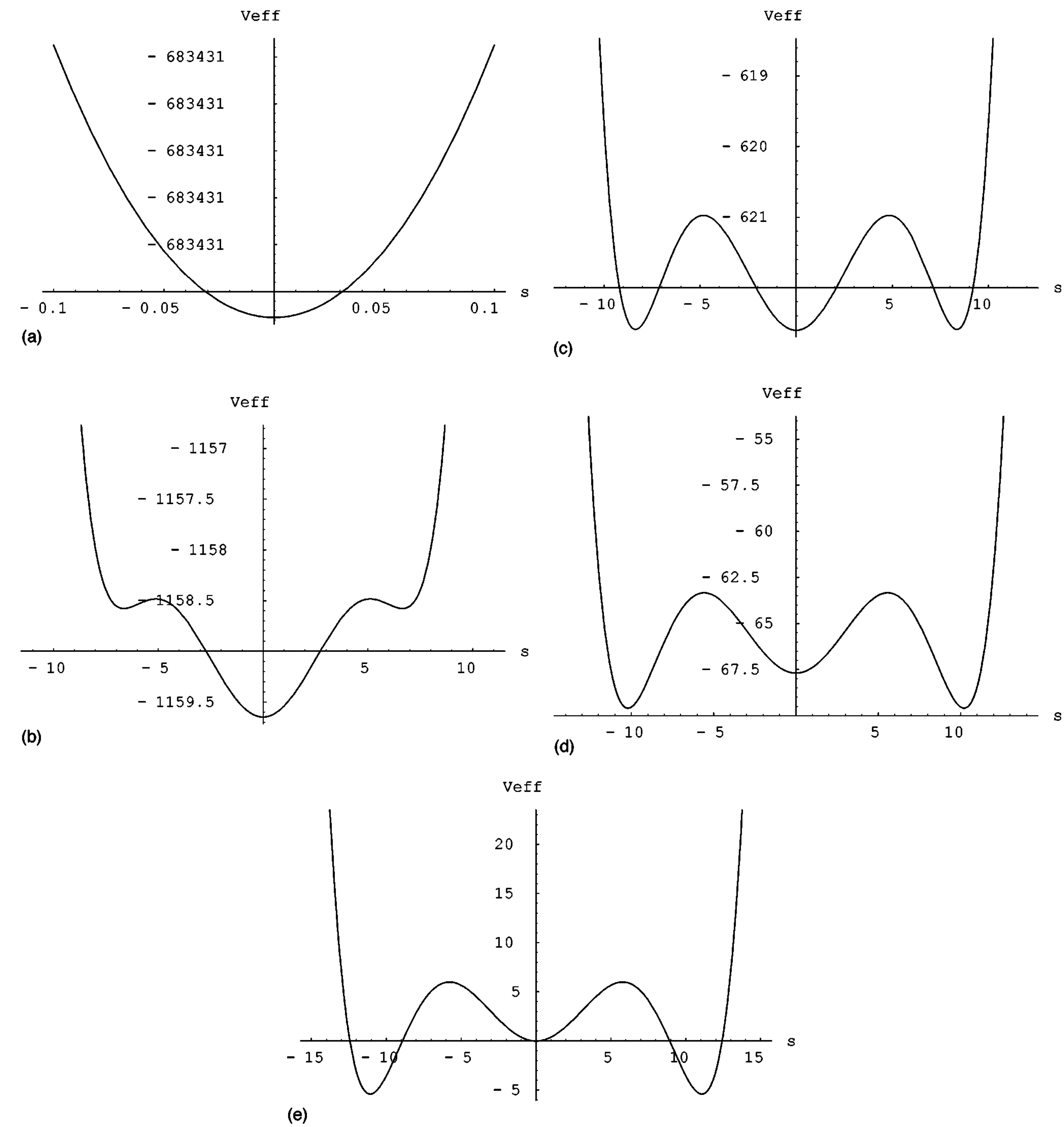

FIG. 1. (a) The behavior of finite temperature effective potential as a function of σ for fixed $m=0.9371$, $\lambda=0.008$, $R=11.2, \xi=1.6$ and $T=50$ such that $\mathbf{T} \gg \mathbf{T}_c$, for which the symmetry is completely restored. (b) The behavior of finite temperature effective potential as a function of σ for fixed $m=0.9371$, $\lambda=0.008$, $R=0.8, \xi=0.145$, and $T=10.15$ such that $\mathbf{T} > \mathbf{T}_c$. (c) The behavior of finite temperature effective potential as a function of σ for fixed $m=0.9371$, $\lambda=0.008$, $R=0.35, \xi=0.004$, and $T=8.69$ such that $\mathbf{T}=\mathbf{T}_c$. (d) The behavior of finite temperature effective potential as a function of σ for fixed $m=0.9371$, $\lambda=0.008$, $R=0.31, \xi=-0.22$, and $T=5$ such that $\mathbf{T} < \mathbf{T}_c$. (e) The behavior of finite temperature effective potential as a function of σ for fixed $m=0.9371$, $\lambda=0.008$, $R=0.3, \xi=-0.3$ and $\mathbf{T}=\mathbf{0}$.

found to be of first order. It is clear from Fig. 3 that there is a barrier between the symmetric and broken phases.

VI. DISCUSSIONS AND CONCLUSIONS

According to renormalizability considerations, degree of the interaction potential cannot be higher than four in 3+1 dimensions [19]. The present calculations show that the ϕ^6 theory in 3+1 dimensions can be regularized in curved spacetime and one can obtain finite expression for the one loop effective potential. In this paper we closely examine and verify the temperature dependence of phase transitions in the early universe and verify their nature to be of first

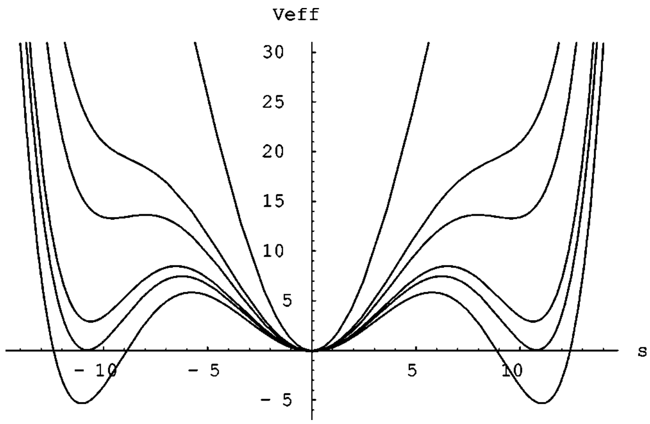

FIG. 2. The behavior of finite temperature effective potential as a function of σ for fixed $m=0.9371$, $\lambda=0.008$, $\xi=0.1$, and $T=1$. Starting from top the curves corresponds to the following values of the curvature: $R=15,4,2.5,0.5,0.001,-0.9$.

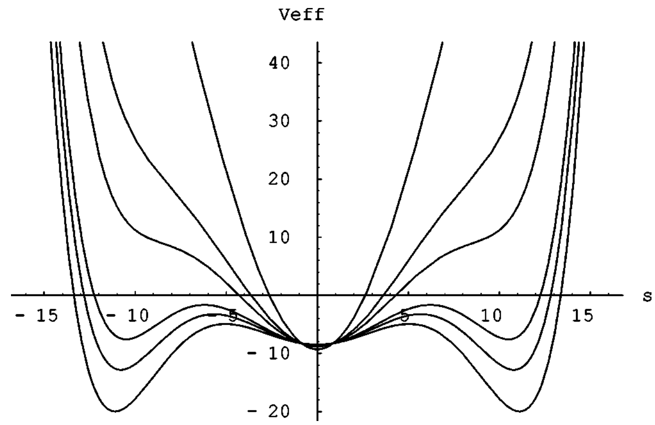

FIG. 3. The behavior of finite temperature effective potential as a function of σ for fixed $m=0.9371$, $\lambda=0.008$, $R=0.3$ and $T=3$. Starting from top the curves corresponds to the following values of the curvature: $\xi=6.5,2.3,1.25,0.01,-0.3,-0.7$.

order as the transition is found to be discontinuous.

In most of the works on cosmological phase transitions, the coupling to the background gravitational field is ignored. One deals with the quantum field theory in flat spacetime at finite temperature and the expansion of the Universe serves only to decrease the temperature. However, at sufficiently early times the spacetime curvature can be expected to be important. Many authors have argued that such effects may be important in the context of cosmological phase transitions in grand unified models [19,29–32]. Vilenkin and Ford have shown that spacetime curvature can drastically change the behavior of the system [33]. O'Connor and co-workers have confirmed the effect of spacetime curvature and arbitrary field coupling on the phase transitions of the early universe [34]. Janson [35], Grib and Mosteparenko [36], and Madsen [37] have independently shown that the interaction with the external gravitational field may lead to SSB. The present work proves that the phase transition taking place during such a SSB is first order. It is found that for $\xi=0$ or $R=0$ the system remains in the symmetry broken state for all values of $T\leq T_c$. As the temperature is increased above T_c, the symmetry is restored depending on the values of ξ and R also. It is also found that symmetry can be restored either by increasing the value of ξ or by increasing the value of R keeping the temperature constant. This shows that the scalar-gravitational coupling and the scalar curvature do play a crucial role in determining the nature of phase transitions which took place in the early universe.

These results may be useful for the study of quantum thermal processes in the early universe. To examine the symmetry behavior of the early universe closely one should take into consideration the effects of spacetime curvature and finite temperature corrections in their full rights.

ACKNOWLEDGMENTS

We thank Professor T. Padmanabhan and Professor Maria Lombardo for valuable discussions. One of us (M.J.) would like to thank UGC, N. Delhi for the financial support. V.C.K. acknowledges Associateship of IUCAA, Pune.

[1] A. D. Linde, Rep. Prog. Phys. **42**, 389 (1979).
[2] N. D. Birrel and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
[3] D. G. C. McKeon and G. Tsoupros, Class. Quantum Grav. **11**, 73 (1994).
[4] S. A. Ramsey and B. L. Hu, Phys. Rev. D **56**, 661 (1997).
[5] E. Elizalde, S. Leseduarte, S. D. Odintsov, and Yu I. Shil'nov, Phys. Rev. D **53**, 1917 (1996).
[6] I. L. Buchbinder and S. O. Odinstov, Class. Quantum Grav. **2**, 721 (1985).
[7] G. Cognola and I. L. Shapiro, Class. Quantum Grav. **15**, 787 (1998).
[8] T. Inagaki, T. Muta, and S. D. Odintsov, Mod. Phys. Lett. A **8**, 2117 (1993).
[9] W. H. Huang, Class. Quantum Grav. **10**, 2021 (1993).
[10] H. Ford and D. J. Toms, Phys. Rev. D **25**, 1510 (1982).
[11] B. L. Hu and Y. Zhang, Phys. Rev. D **37**, 2151 (1988).
[12] A. Ringwald, Phys. Rev. D **36**, 2598 (1987).
[13] R. Critchley and J. S. Dowker, J. Phys. A **15**, 157 (1982).
[14] T. C. Shen, B. L. Hu, and D. J. O'Connor, Phys. Rev. D **31**, 2401 (1985).
[15] D. Boyanovsky and L. Masperi, Phys. Rev. D **21**, 1550 (1980).
[16] T. Futamase, Phys. Rev. D **29**, 2783 (1984).
[17] W. H. Huang, Phys. Rev. D **42**, 1287 (1990).
[18] A. L. Berkin, Phys. Rev. D **46**, 1551 (1992).
[19] I. L. Buchbinder, S. D. Odinstov, and I. L. Shapiro, *Effective Action in Quantum Gravity* (IOP Publishing, London, 1992).
[20] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).
[21] E. Elizalde and S. D. Odinstov, Z. Phys. C **64**, 699 (1994).
[22] R. Brandenberger, in *Proceedings of the 1991 Summer School on High Energy Physics and Cosmology, ICTP, Trieste* (World Scientific, Singapore, 1991).
[23] L. Dolan and R. Jackiw, Phys. Rev. D **9**, 2904 (1974).

[24] K. Babu Joseph and V. C. Kuriakose, J. Phys. A **15**, 2231 (1982).
[25] I. Moss, D. Toms, and A. Wright, Phys. Rev. D **46**, 1671 (1992).
[26] S. Coleman and E. Weinberg, Phys. Rev. D **9**, 3320 (1974).
[27] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, New York, 1990).
[28] R. Dominguez-Tenreiro and Mariano Quiros, *An Introduction to Cosmology and Particle Physics* (World Scientific, Singapore, 1988), Chap. 7.
[29] G. Micle and P. Vitale, Nucl. Phys. **B494**, 365 (1997).
[30] Klaus Kirsten, Class. Quantum Grav. **10**, 1461 (1993).
[31] E. Elizalde, Yu I. Shil'nov, and V. V. Chitov, Class. Quantum Grav. **15**, 735 (1998).
[32] T. Inagaki, T. Muta, and S. D. Odintsov, Prog. Theor. Phys. Suppl. **127**, 93 (1997).
[33] A. Vilenkin and L. H. Ford, Phys. Rev. D **26**, 1231 (1982).
[34] D. J. O'Connor, B. L. Hu, and T. C. Shen, Phys. Lett. **130B**, 31 (1983).
[35] M. M. Janson, Lett. Nuovo Cimento Soc. Ital. Fis. **15**, 231 (1976).
[36] A. A. Grib and V. M. Mosteparenko, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 302 (1977) [JETP Lett. **25**, 277 (1977)].
[37] M. S. Madsen, Class. Quantum Grav. **5**, 627 (1988).