Nonlinear gravitational wave interactions with plasmas

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We consider the interactions of a strong gravitational wave with electromagnetic fields using the 1+3 orthonormal tetrad formalism. A general system of equations is derived, describing the influence of a plane fronted parallel (pp) gravitational wave on a cold relativistic multicomponent plasma. We focus our attention on phenomena that are induced by terms that are higher order in the gravitational wave amplitude. In particular, it is shown that parametric excitations of plasma oscillations take place, due to higher order gravitational nonlinearities. The implications of the results are discussed.

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I. INTRODUCTION

There have been numerous investigations on the scattering of electromagnetic waves off gravitational fields (see Refs. [1,2]). Previous research has mostly directed its interest towards the effects on vacuum electromagnetic fields (although there are exceptions; see, e.g., Refs. [3,4], where the effects of plasmas have been taken into account). Similarly, much work concerning gravitational waves have considered the linearized theory, which is obviously the relevant regime for gravitational wave detectors or, in general, for distances far away from the gravitational wave source. Alternatively, there has been an interest in exact solutions, and thus a number of exact gravitational wave solutions (see, e.g., Ref. [5] and references therein) have been found. In the present paper we will choose an intermediate approach, starting with an exact gravitational wave solution, but focusing on a weak amplitude (but still nonlinear) approximation, and studying the effects induced in a plasma.

The question under study in this paper is whether nonlinear gravitational wave effects—which may be of significance close to the gravitational wave source—can give rise to qualitatively new phenomena in plasmas that are absent in linearized theory. Close to the source, additional effects apart from nonlinearities—due to, for example, the three-dimensional geometry and/or the nonradiative part of the gravitational field—are likely to be important for astrophysical applications. However, in order to focus on the processes directly induced by nonlinearities, a somewhat simpler model problem with a unidirectional gravitational wave will be studied: To facilitate the analysis of the nonlinear interaction between a plasma and a gravitational wave, we make use of the pp-wave solution of Einstein's field equations. Furthermore, we introduce a Lorentz tetrad in order to define physical variables in a straightforward manner. With this setup, the governing plasma equations can be written in a simple three-dimensional form. In Maxwell's equations, the gravitational effects are given by effective charge and current densities. Moreover, the fluid equations are given for a cold plasma.

Previously, the parametric excitation of a Langmuir wave and an electromagnetic wave by a linearized gravitational wave was considered [6]. Here we address the question of whether higher order terms in the gravitational wave amplitude can result in new effects, using the above mentioned equations for a cold plasma. In order to demonstrate the usefulness of our set of equations, we study the stability properties of a plasma in the presence of a pp wave. We show that including second order gravitational wave effects may give rise to new phenomena. In particular it is found that electrostatic waves can be excited at a resonant surface where the gravitational wave frequency is equal to the local plasma frequency. Our results are summarized and discussed in the last section of the paper.

II. PRELIMINARIES

A. Equations for a general space-time

We follow the approach presented in [3] for handling gravitational effects in Maxwell's equations. Suppose an observer moves with four-velocity u^a ($a=0,\dots,3$). This observer will measure the electric and magnetic (EM) fields [7]

$$E_a \equiv F_{ab}u^b, \quad B_a \equiv \tfrac{1}{2}\epsilon_{abc}F^{bc}, \tag{1}$$

respectively, where F_{ab} is the EM field tensor. Here ϵ_{abc} is the volume element on hypersurfaces orthogonal to u^a.

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We denote the fluid velocity $V^a \equiv (\gamma, \gamma \boldsymbol{v})$, where $\gamma \equiv (1 - \boldsymbol{v}^2)^{-1/2}$. Let q be the particle charge and n the proper number density. Furthermore, we introduce the orthonormal frame (ONF) $\{\boldsymbol{e}_a, a=0,\ldots,4\}$, each of which is a linear combination of the coordinate derivatives $\partial_i \equiv \partial/\partial x^i$, i.e., $\boldsymbol{e}_a = e_a^i \partial_i$. Using the split (1) together with $j^a = qnV^a$, Maxwell's equations $\nabla_b F^{ab} = j^a$, $\nabla_{[a}F_{bc]} = 0$ read

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho_E + \rho, \tag{2a}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = \rho_B, \tag{2b}$$

$$\dot{\boldsymbol{E}} - \boldsymbol{\nabla} \times \boldsymbol{B} = -\boldsymbol{j}_E - \boldsymbol{j}, \tag{2c}$$

$$\dot{\boldsymbol{B}} + \boldsymbol{\nabla} \times \boldsymbol{E} = -\boldsymbol{j}_B, \tag{2d}$$

where the ''effective'' (gravity induced) charge densities and current densities are

$$\rho_E \equiv -\Gamma^\alpha_{\beta\alpha} E^\beta - \epsilon^{\alpha\beta\gamma}\Gamma^0_{\alpha\beta} B_\gamma, \tag{3a}$$

$$\rho_B \equiv -\Gamma^\alpha_{\beta\alpha} B^\beta + \epsilon^{\alpha\beta\gamma}\Gamma^0_{\alpha\beta} E_\gamma, \tag{3b}$$

$$j_E^\alpha \equiv -(\Gamma^\alpha_{0\beta} - \Gamma^\alpha_{\beta 0})E^\beta + \Gamma^\beta_{0\beta}E^\alpha - \epsilon^{\alpha\beta\gamma}(\Gamma^0_{\beta 0}B_\gamma + \Gamma^\delta_{\beta\gamma}B_\delta), \tag{3c}$$

$$j_B^\alpha \equiv -(\Gamma^\alpha_{0\beta} - \Gamma^\alpha_{\beta 0})B^\beta + \Gamma^\beta_{0\beta}B^\alpha + \epsilon^{\alpha\beta\gamma}(\Gamma^0_{\beta 0}E_\gamma + \Gamma^\delta_{\beta\gamma}E_\delta), \tag{3d}$$

while $\rho \equiv \Sigma_{\text{p.s.}} q\gamma n$ and $\boldsymbol{j} \equiv \Sigma_{\text{p.s.}} q\gamma n \boldsymbol{v}$ are the matter charge and current densities, respectively (the sums are over all particle species). Here Γ^a_{bc} are the Ricci rotation coefficients with respect to the ONF $\{\boldsymbol{e}_a\}$. We have introduced the notation $\boldsymbol{E} \equiv (E^\alpha) = (E^1, E^2, E^3)$ etc., $\boldsymbol{\nabla} \equiv (\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3)$, and the overdot stands for derivative in the direction of the timelike vector $\boldsymbol{e}_0$. The dot and cross products are defined in the usual Euclidean way.

The energy-momentum tensor for each particle species is assumed to take the form of pressure free matter (dust), $T^{ab} = mnV^aV^b$, where m is the rest mass of the particles. Then the conservation equations $\nabla_b T^{ab} = qnF^{ab}V_b$ give

$$\boldsymbol{e}_0(\gamma n) + \boldsymbol{\nabla} \cdot (\gamma n \boldsymbol{v}) = -\gamma n(\Gamma^\alpha_{0\alpha} + \Gamma^\alpha_{00}v_\alpha + \Gamma^\alpha_{\beta\alpha}v^\beta), \tag{4a}$$

$$(\boldsymbol{e}_0 + \mathbf{v}\cdot\boldsymbol{\nabla})\gamma \mathbf{v} = \frac{q}{m}(\boldsymbol{E} + \mathbf{v} \times \boldsymbol{B}) - \gamma[\Gamma^\alpha_{00} + (\Gamma^\alpha_{0\beta} + \Gamma^\alpha_{\beta 0}) \times v^\beta + \Gamma^\alpha_{\beta\gamma}v^\beta v^\gamma]\boldsymbol{e}_\alpha. \tag{4b}$$

B. Basic relations in the field of a *pp* wave

Previous examinations of interactions between gravitational radiation and EM waves have focused on linearized gravitation. On the other hand, one may suspect that there will be interesting effects in the nonlinear regime, not present to linear order. Below we will show that this is indeed the case.

In order to address the issue of how strong gravitational radiation may be involved in generation of EM waves, we look at the *plane fronted parallel* (pp) waves (for a discussion, see Ref. [8]), in the special case of a linearly polarized plane wave:

$$ds^2 = -dt^2 + a(u)^2\, dx^2 + b(u)^2\, dy^2 + dz^2, \tag{5}$$

where $u = z - t$, a and b satisfy $ab_{uu} + a_{uu}b = 0$, and the subscript u denotes a derivative with respect to retarded time. Note that we have chosen a vacuum geometry; i.e., we have omitted the influence of the plasma on the metric.

In order to make interpretations simple, we introduce the canonical Lorentz frame

$$\boldsymbol{e}_0 = \partial_t,\ \boldsymbol{e}_1 = a^{-1}\partial_x,\ \boldsymbol{e}_2 = b^{-1}\partial_y,\ \boldsymbol{e}_3 = \partial_z. \tag{6}$$

With this frame, the effective charge and current densities (3) read

$$\rho_E = -(\ln ab)_u E^3, \tag{7a}$$

$$\rho_B = -(\ln ab)_u B^3, \tag{7b}$$

$$\boldsymbol{j}_E = -(\ln b)_u (E^1 - B^2)\, \boldsymbol{e}_1 - (\ln a)_u (E^2 + B^1)\, \boldsymbol{e}_2 - (\ln ab)_u E^3\, \boldsymbol{e}_3, \tag{7c}$$

$$\boldsymbol{j}_B = -(\ln b)_u (E^2 + B^1)\, \boldsymbol{e}_1 + (\ln a)_u (E^1 - B^2)\, \boldsymbol{e}_2 - (\ln ab)_u B^3\, \boldsymbol{e}_3. \tag{7d}$$

Apart from Maxwell's equations (2a)–(2d) [together with the effective charge and current densities (7a)–(7d)] we also need the fluid equations. From the conservation equations (4) we obtain the fluid equations using the frame (6):

$$\frac{\partial}{\partial t}(\gamma n) + \boldsymbol{\nabla} \cdot (\gamma n \boldsymbol{v}) = \gamma n (\ln ab)_u (1 - v_\parallel), \tag{8a}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}\right)\gamma\mathbf{v} = \frac{q}{m}(\boldsymbol{E} + \mathbf{v}\times\boldsymbol{B}) + \gamma[(\ln a)_u v_1 \boldsymbol{e}_1 + (\ln b)_u v_2 \boldsymbol{e}_2](1 - v_\parallel) + \gamma[(\ln a)_u v_1^2 + (\ln b)_u v_2^2]\boldsymbol{e}_3, \tag{8b}$$

where $v_\parallel \equiv v_3$ is the velocity parallel to the gravitational wave propagation direction. These equations should be satisfied for each particle species. In the limit of small gravitational wave amplitudes and nonrelativistic velocities, Eqs. (7),(8), together with Maxwell's equations, were given in Ref. [3]. All terms with factors $(\ln ab)_u$ are, however, new, and—as we will demonstrate in the remainder of this article—they may induce new phenomena, compared to the linear regime.

III. EXAMPLE: PARAMETRIC EXCITATION OF PLASMA OSCILLATIONS

The longitudinal ''currents'' and ''charges'' are second order in the gravitational wave amplitude (see the Appendix for further details). These second order terms can give rise to qualitatively new phenomena compared to the linear regime, and we demonstrate this by considering a simple, but illustrative, example. In what follows, we will investigate longitudinal perturbations, i.e., $\boldsymbol{E}=(0,0,E)$, $\boldsymbol{v}=(0,0,v)$, etc., around a cold one-component equilibrium plasma. Compared to the case of weak gravitational waves [3], we now have $\rho_{E,B}$ different from zero, and we also have a longitudinal contribution to the effective currents. This means that longitudinal EM and plasma waves can be excited.

In the unperturbed plasma, $\partial n_0/\partial t=0$, $\boldsymbol{E}_0=\boldsymbol{0}$, and $\boldsymbol{B}_0=\boldsymbol{0}$ [9]. We denote the number density perturbation by $\overline{n}$, i.e., $n(z,t)=n_0(z)+\overline{n}(z,t)$, and assume that all perturbed quantities only depend on t and z. To first order, Maxwell's equation (2c) becomes

$$\frac{\partial E}{\partial t}=(\ln ab)_u E-\mu_0 q n_0 v, \tag{9}$$

where we have used $j_m=qn_0v$. Furthermore, the momentum equation (8b) becomes

$$\frac{\partial v}{\partial t}=\frac{q}{m}E. \tag{10}$$

Taking the time derivative of Eq. (9) and using Eq. (10), we obtain

$$\frac{\partial^2 E}{\partial t^2}+\omega_{\mathrm{p}}^2(z)E=\frac{\partial}{\partial t}[(\ln ab)_u E], \tag{11}$$

where $\omega_p(z)=[n_0(z)q^2\mu_0/m]^{1/2}$ is the local plasma frequency. Thus the left hand side is the usual equation for plasma oscillations in a cold inhomogeneous plasma, and the right hand side is the modification induced by the pp wave. We next focus on weak periodic deviations from flat spacetime (see the Appendix), where the periodicity is $2\pi/\omega$. At the resonant surface where $\omega_p(z_{\mathrm{res}})=\omega$, we can then have parametric excitation of plasma oscillations. We let $E(z_{\mathrm{res}},t)=\hat{E}(t)\exp[-\mathrm{i}\omega t]+\mathrm{c.c.}$, where c.c. denotes the complex conjugate, and assume that $|\partial\hat{E}(t)/\partial t|\ll\omega|\hat{E}(t)|$. At the resonant surface Eq. (11) then reduces to

$$\frac{d\hat{E}}{dt}=-\frac{1}{2}\,\mathrm{i}\exp(2\mathrm{i}\omega z_{\mathrm{res}})\omega\hat{h}^2\hat{E}^*, \tag{12}$$

where the asterisk denotes the complex conjugate (see the Appendix). Taking the time derivative of Eq. (12) and using the complex conjugate of the same equation, we find $\hat{E}\propto\exp(\Gamma t)$ where the growth rate is

$$\Gamma=\tfrac{1}{2}\omega|\hat{h}^2|. \tag{13}$$

Note that the threshold value for excitation is zero, since we have not included any dissipation mechanism of the plasma oscillations. Adding an electron-ion collision in Eq. (8b), the threshold value $\hat{h}_{\mathrm{thr}}$ of this instability is of the order of $(\nu_{e\text{-}i}/\omega_p)^{1/2}$, where $\nu_{e\text{-}i}$ is the electron–ion collision frequency.

Clearly, our instability does not occur unless higher order gravitational perturbations are included, in contrast to the results in Ref. [6]. Thus the corresponding growth rate is smaller in our case for a given source of gravitational radiation. There are still two interesting properties of the above instability as compared to the process in Ref. [6], where parametric excitation of a Langmuir wave and an electromagnetic wave was considered.

(i) The frequency matching condition in our case is $\omega=\omega_p$, which requires a rather high gravitational frequency [10], but is less severe than the condition in Ref. [6], where $\omega\geqslant 2\omega_p$.

(ii) In contrast with most parametric instabilities in plasmas we have no wave vector matching condition, but instead the process takes place at a localized resonance surface $z=z_{\mathrm{res}}$ where $\omega=\omega_p(z_{\mathrm{res}})$. This means that there is no threshold value for the instability introduced by plasma inhomogeneities. Normally the threshold value is inversely proportional to the inhomogeneity scale length [11], and close to a binary system, where the effects of gravitational radiation are likely to be most important, such a condition for parametric excitation may thus be rather severe. Unfortunately, the result of the ''no inhomogeneity threshold'' depends on the cold plasma approximation, and a finite temperature is likely to change the picture.

IV. SUMMARY AND DISCUSSION

In the present paper we have investigated a higher order effect of gravitational waves on a plasma. For this purpose we have developed a Lorentz tetrad formalism for a cold plasma in the presence of a strong gravitational wave. The Lorentz tetrad approach has of course been widely used before, perhaps most notably in the membrane paradigm approach to black hole spacetimes [2]. The obvious advantage of using a Lorentz tetrad is its direct connection to measurements. It is possible to formulate Maxwell's equations such that the gravitational contributions take the form of ''charge'' and ''current'' densities. Similarly, the fluid equations are modified by effective particle sources and gravitational forces. Of course, this is not the physical picture behind the equations, but it still provides a useful tool for predicting the consequences of the gravitational influence.

The main purposes of this study have been to (i) provide a framework for investigating strong gravitational pulse effects in cold multicomponent plasmas and (ii) to show that higher order gravitational wave effects may be of importance, since they introduce effective charges and longitudinal currents, as well as effective ''particle sources'' and gravitational forces. As demonstrated, this in turn makes new processes—such as parametric generation of electrostatic waves—possible. Since the effect under discussion is of order $\hat{h}^2$, we do not believe that it will be of significance concerning *direct* Earth based observations of gravitational waves. It is possible, however, that there exist favorable circumstances, e.g., close to a binary merger, for which the higher order gravitational effective charge and current densities can play an important role. Close to such sources, the gravitational wave amplitudes can reach considerable strength, implying observational possibilities for the induced phenomena.

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APPENDIX: PERTURBATIVE EXPANSION OF THE *pp* WAVE

In many situations of interest the gravitational wave amplitude is small, i.e., $|a-1|\ll 1$, $|b-1|\ll 1$, and it is appropriate to make approximations for the factors $(\ln a)_u$, $(\ln b)_u$, and $(\ln ab)_u$ that appear in the gravitational source terms in Eqs. (7) and (8). We will concentrate on approximately periodic gravitational waves, such as those generated by binary systems, in order to get definite results. Let $a(u)=\Sigma_{n=-\infty}^{\infty}\hat{a}_n\exp(in\omega u)$, $b(u)=\Sigma_{n=-\infty}^{\infty}\hat{b}_n\exp(in\omega u)$, where $\hat{a}_n,\hat{b}_n\ll 1$, $|\hat{a}_n|\sim|\hat{b}_n|\sim|\hat{a}_1|^{|n|}$, $\forall n$. Furthermore $\hat{a}_n^*=\hat{a}_{-n}$ and similarly for b. Then $a_{uu}b+ab_{uu}=0$ becomes

$$\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}(n^2+m^2)\hat{a}_n\hat{b}_m\exp[\mathrm{i}\omega(n+m)]=0. \quad \text{(A1)}$$

To zeroth order, we assume that $\hat{a}_0=1=\hat{b}_0$. To first order, the solution to Eq. (A1) is $\hat{a}_1=-\hat{b}_1\equiv\hat{h}$. Clearly, quadratic nonlinear terms will generate second harmonic terms proportional to $\exp(2\mathrm{i}\omega u)$. Separating the frequencies in Eq. (A1) and concentrating on the second harmonic part, we obtain

$$2\hat{b}_2+2\hat{a}_2-\hat{h}^2=0. \quad \text{(A2)}$$

The canonical choice is $\hat{a}_2=\hat{b}_2=(1/4)\hat{h}^2$. Physically this means that we minimize the (pseudo)energy density at the second harmonic frequency. Thus—for this choice—all the oscillations at 2ω are strictly due to the nonlinearity of Einstein's equations, and no harmonics are assumed to be initially present, i.e., generated by a varying octopole moment of the binary source. For astrophysical applications this is not necessarily the most accurate choice (since binary systems may indeed have finite octopole moments), but it has the advantage of clearly isolating the effects due to nonlinearities. Furthermore, it turns out that including the effect of higher moments of the gravitational source (i.e., octupole moments and higher) do not influence our calculations in Sec. III. The reason is that an alternative solution to Eq. (A2) —$\hat{a}_2=(1/4)\hat{h}^2+\delta\hat{a}_2$, $\hat{b}_2=(1/4)\hat{h}^2-\delta\hat{a}_2$, instead of $\hat{b}_2=\hat{a}_2=(1/4)\hat{h}^2$—does not significantly affect the factor $\ln(ab)_u$, since it is independent of $\delta\hat{a}_2$ to second order in the gravitational amplitude, provided $\delta\hat{a}_2\sim\hat{h}^2$.

Continuing to third order, a similar calculation shows that we can make the natural choice $\hat{a}_3=\hat{b}_3=0$. For the fourth order terms, Eq. (A1) gives

$$32(\hat{a}_4+\hat{b}_4)-\hat{h}^4=0 \quad \text{(A3)}$$

where the canonical choice $\hat{a}_4=\hat{b}_4=(1/64)\hat{h}^4$ is made. Continuing this procedure, it turns out that all terms odd in $n\neq 1$ disappear, while the terms even in n satisfy $\hat{a}_n=\hat{b}_n\forall n$.

Using the above results, the logarithmic factors in Eqs. (7) and (8) become

$$(\ln a)_u=\mathrm{i}\omega\hat{h}\exp(\mathrm{i}\omega u)-\tfrac{1}{2}\mathrm{i}\omega\hat{h}^2\exp(2\mathrm{i}\omega u)+\tfrac{1}{4}\mathrm{i}\omega\hat{h}^3\times\exp(3\mathrm{i}\omega u)-\tfrac{1}{16}\omega\hat{h}^4\exp(4\mathrm{i}\omega u)+\text{c.c.}, \quad \text{(A4a)}$$

$$(\ln b)_u=-\mathrm{i}\omega\hat{h}\exp(\mathrm{i}\omega u)-\tfrac{1}{2}\mathrm{i}\omega\hat{h}^2\exp(2\mathrm{i}\omega u)-\tfrac{1}{4}\mathrm{i}\omega\hat{h}^3\times\exp(3\mathrm{i}\omega u)-\tfrac{1}{16}\omega\hat{h}^4\exp(4\mathrm{i}\omega u)+\text{c.c.}, \quad \text{(A4b)}$$

$$(\ln ab)_u=-\mathrm{i}\omega\hat{h}^2\exp(2\mathrm{i}\omega u)-\tfrac{1}{8}\omega\hat{h}^4\exp(4\mathrm{i}\omega u)+\text{c.c.} \quad \text{(A4c)}$$

to fourth order in the gravitational amplitude. This procedure may of course be continued to arbitrary order, noting that this in general will result in an asymptotic series; i.e., it does not *necessarily* converge towards a solution of $a_{uu}b+ab_{uu}=0$.

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[9] Note that as long as the magnetic field is parallel to the direction of propagation of the gravitational wave it will decouple from the rest of the equations, and our following results are therefore unaffected by the addition of such a field.

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