Geometrodynamics of variable-speed-of-light cosmologies

Bruce A. Bassett,[1,2,*] Stefano Liberati,[3,4,†] Carmen Molina-París,[5,‡] and Matt Visser[6,§]
[1]*Relativity and Cosmology Group, Division of Mathematics and Statistics, Portsmouth University, PO1 2EG, England*
[2]*Department of Theoretical Physics, University of Oxford, 1 Keble Road, OX1 3NP, England*
[3]*International School for Advanced Studies (SISSA), Via Beirut 2–4, 34014 Trieste, Italy*
[4]*Istituto Nazionale di Fisica Nucleare (INFN), sezione di Trieste, Italy*
[5]*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545*
[6]*Physics Department, Washington University, Saint Louis, Missouri 63130-4899*
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Variable-speed-of-light (VSL) cosmologies are currently attracting interest as an alternative to inflation. We investigate the fundamental geometrodynamic aspects of VSL cosmologies and provide several implementations which do not explicitly break Lorentz invariance (no ''hard'' breaking). These ''soft'' implementations of Lorentz symmetry breaking provide particularly clean answers to the question ''VSL with respect to what?.'' The class of VSL cosmologies *we consider* are compatible with both classical Einstein gravity and low-energy particle physics. These models solve the ''kinematic'' puzzles of cosmology as well as inflation does, but *cannot* by themselves solve the flatness problem, since in their purest form no violation of the strong energy condition occurs. We also consider a heterotic model (VSL plus inflation) which provides a number of observational implications for the low-redshift universe if χ contributes to the ''dark energy'' either as CDM or quintessence. These implications include modified gravitational lensing, birefringence, variation of fundamental constants and rotation of the plane of polarization of light from distant sources.

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I. INTRODUCTION

High-energy cosmology is flourishing into a subject of observational riches but theoretical poverty. Inflation stands as the only well-explored paradigm for solving the puzzles of the early universe. This monopoly is reason enough to explore alternative scenarios and new angles of attack. Variable-speed-of-light (VSL) cosmologies have recently generated considerable interest as alternatives to cosmological inflation which serve both to sharpen our ideas regarding falsifiability of the standard inflationary paradigm, and also to provide a contrasting scenario that is hopefully amenable to observational test.

The major variants of VSL cosmology under consideration are those of Moffat [1–3], Ellis, Mavromatos, and Nanopoulos [4], Clayton and Moffat [5,6], and Albrecht, Barrow, and Magueijo [7–11], plus more recent contributions by Avelino and Martins [12], Drummond [13], Kiritsis [14], and Alexander [15]. The last two are higher-dimensional, brane-inspired implementations. For completeness we also mention the earlier work by Levin and Freese [16] which discussed the inflationary-type cosmologies resulting from a dynamical Planck's constant.

The covariance of general relativity means that the set of cosmological models consistent with the existence of the apparently universal class of preferred rest frames defined by the cosmic microwave background (CMB) is very small and non-generic. Inflation alleviates this problem by making the flat Friedmann-Lemaitre-Robertson-Walker (FLRW) model an attractor within the set of almost-FLRW models, at the cost of violating the strong energy condition (SEC). Most of the above quoted VSL cosmologies, by contrast, sacrifice (or at the very least, grossly modify) Lorentz invariance at high energies, again making the flat FLRW model an attractor. In contrast, we will see that the *''soft breaking''* prescription we advocate cannot solve the flatness problem without additional external sources of energy condition violation, despite recent claims to the contrary (see Sec. V B for details).

In this paper we want to focus on some basic issues in VSL cosmology that are to our minds still less than clear. In particular, we wish to answer the question ''Can we have VSL without explicitly violating Lorentz invariance?'' As we will see, our approach is to split the degeneracy between the (effective) null cones of various species of particles. This means that in *our implementations* of VSL cosmology the Lorentz symmetry is broken in a ''soft'' manner, rather than in a ''hard'' manner. This ''soft'' breaking of Lorentz invariance, due to the nature of the ground state or initial conditions, is qualitatively similar to the notion of spontaneous symmetry breaking in particle physics, whereas ''hard'' breaking, implemented by brute force, is qualitatively similar to the notion of explicit symmetry breaking in particle physics.

We will have little specific to say about ''hard'' breaking, in the style of Albrecht-Barrow-Magueijo, other than to point out that ''hard'' breaking is a rather radical modification of standard physics. In comparison, ''soft'' breaking is rather benign and is easier to formulate in a geometrodynamic manner, as we discuss in Sec. II.

We specifically want to assess the geometric consistency

*Electronic mail: bruce.bassett@port.ac.uk
†Electronic mail: liberati@sissa.it
‡Also at Centro de Astrobiología, INTA, Carretera de Ajalvir Km. 4, 28850 Torrejón, Madrid, Spain. Electronic mail: carmen@t6-serv.lanl.gov
§Electronic mail: visser@kiwi.wustl.edu

of the VSL idea and ask to what extent it is compatible with Einstein gravity. This is not a trivial issue: Ordinary Einstein gravity has the constancy of the speed of light built into it at a fundamental level; c is the ''conversion constant'' that relates time to space. We need to use c to relate the zeroth coordinate to time: $dx^0 = c\,dt$. Thus, simply replacing the *constant c* by a position-dependent *variable* $c(t,\vec{x})$, and writing $dx^0 = c(t,\vec{x})dt$ is a suspect proposition. Indeed, even the choice $dx^0 = c(t,\vec{x})dt$ is a coordinate dependent statement. It depends on the way one slices up the spacetime with spacelike hypersurfaces. Different slicings would lead to different metrics, and so one has destroyed the coordinate invariance of the theory right at step one. This is not a good start for the VSL program, as one has performed an act of extreme violence to the mathematical and logical structure of general relativistic cosmology, moving well outside the confines of standard curved-spacetime Lorentzian geometry.

Another way of viewing this is to start with the ordinary FLRW metric

$$ds^2 = -c^2 dt^2 + a(t)^2 h_{ij} dx^i dx^j, \tag{1}$$

and compute the Einstein tensor. In the natural orthonormal basis one can write

$$G_{\hat{t}\hat{t}} = \frac{3}{a(t)^2}\left[\frac{\dot{a}(t)^2}{c^2} + K\right], \tag{2}$$

$$G_{\hat{i}\hat{j}} = -\frac{\delta_{\hat{i}\hat{j}}}{a(t)^2}\left[2\frac{a(t)\ddot{a}(t)}{c^2} + \frac{\dot{a}(t)^2}{c^2} + K\right], \tag{3}$$

with the spatial curvature $K = 0, \pm 1$. If one replaces $c \to c(t)$ *in the metric*, then the physics does not change since this particular ''variable speed of light'' can be undone by a coordinate transformation: $c\,dt_{\text{new}} = c(t)dt$. While a coordinate change of this type will affect the (coordinate) components of the metric and the (coordinate) components of the Einstein tensor, the orthonormal components and (by extension) all physical observables (which are coordinate invariants) will be unaffected.

An alternative, which does have observable consequences, is the possibility of replacing $c \to c(t)$ directly *in the Einstein tensor*. This is the route chosen by Barrow and Magueijo [8–10], and by Albrecht and Magueijo [7,11]. Avelino and Martins [12] adopt a slightly different viewpoint, making the change in the metric, but subject to a time-dependent redefinition of units. Then

$$G^{\text{modified}}_{\hat{t}\hat{t}} = \frac{3}{a(t)^2}\left[\frac{\dot{a}(t)^2}{c(t)^2} + K\right], \tag{4}$$

$$G^{\text{modified}}_{\hat{i}\hat{j}} = -\frac{\delta_{\hat{i}\hat{j}}}{a(t)^2}\left[2\frac{a(t)\ddot{a}(t)}{c(t)^2} + \frac{\dot{a}(t)^2}{c(t)^2} + K\right]. \tag{5}$$

Note that the replacement $c \to c(t)$ directly in the Einstein tensor is a specific implementation of the general prescription presented in [7]: ''take all time derivatives at fixed c and then replace $c \to c(t)$ in the result.''

Unfortunately, if one does so, the modified ''Einstein tensor'' so defined is *not* covariantly conserved (it does *not* satisfy the contracted Bianchi identities), and this modified ''Einstein tensor'' is not obtainable from the curvature tensor of *any* spacetime metric. Indeed, if we define a timelike vector $V^\mu = (\partial/\partial t)^\mu = (1,0,0,0)$ a brief computation yields

$$\nabla_\mu G^{\mu\nu}_{\text{modified}} \propto \dot{c}(t) V^\nu. \tag{6}$$

Thus, violations of the Bianchi identities for this modified ''Einstein tensor'' are part and parcel of this particular way of trying to make the speed of light variable. Indeed, as we will see later, in that VSL implementation these violations are the source of the solution of the flatness problem. Alternatively one can define *modified* Bianchi identities by moving the RHS above over to the LHS [10] and then speak of these modified Bianchi identities as being satisfied. Nevertheless the *usual* Bianchi identities are violated in their formalism. This may be interpreted as a statement that such an implementation of VSL is not based on pseudo-Riemannian geometry (Lorentzian geometry), but that instead one is dealing with some more complicated structure whose geometric interpretation is far more complex than usual.

If one couples this modified ''Einstein tensor'' to the stress-energy via the Einstein equation

$$G_{\mu\nu} = \frac{8\pi G_{\text{Newton}}}{c^4} T_{\mu\nu}, \tag{7}$$

then the stress-energy tensor divided by c^4 cannot be covariantly conserved either (here we do not need to specify just yet if we are talking about a variable c or a fixed c), and so $T^{\mu\nu}/c^4$ cannot be variationally obtained from *any* action. [The factor of c^4 is introduced to make sure all the components of the stress-energy tensor have the dimensions of energy density, ε (the same dimensions as pressure, p). When needed, mass density will be represented by ρ.] This non-conservation of stress-energy is a tremendous amount of physics to sacrifice and we do *not* wish to pursue this particular avenue any further.

Since this point can cause considerable confusion, let us be clear about what we are claiming: In VSL theories which violate the usual Bianchi identities [7,10], the stress-energy tensor cannot be obtained by variational differentiation of any local Lagrangian density based on a pseudo-Riemannian geometry. One can try to generalize the notion of pseudo-Riemannian geometry but this is an alien procedure from the standpoint of standard relativity and cosmology.

One of the earliest VSL formulations, and one which does satisfy the Bianchi identities, is that of Ellis *et al.* [4]. Inspired by non-critical string theory, the evolution of c was driven by non-trivial renormalization group dynamics associated with the Liouville mode which obeys a generalization of the Zamolodchikov C-theorem and therefore provides a natural cosmic arrow of time. The advantage of this formulation is that no extra (and arbitrary) scalar fields are required to generate the variations in c, the disadvantage, as they point out, is the possibility of making a coordinate transformation to nullify the VSL effects.

We feel therefore, that if one wants to uniquely specify that it is the speed of light that is varying, then the most ''natural'' thing to do is to seek a theory that contains two natural speed parameters, call them $c_{\rm photon}$ and $c_{\rm gravity}$, and then ask that the ratio of these two speeds is a time-dependent quantity. Naturally, once we go beyond idealized FLRW cosmologies, to include perturbations, we will let this ratio depend on space as well as time. Thus we would focus attention on the dimensionless ratio

$$\zeta \equiv \frac{c_{\rm photon}}{c_{\rm gravity}}. \tag{8}$$

An interesting alternative is to consider the ratio of $c_{\rm photon}$ at different *frequencies*. This ratio is non-trivial in D-brane and quantum gravity-inspired scenarios [17] which alter the photon dispersion relation at high energies.

With this idea in mind, we have found that it is simplest to take $c_{\rm gravity}$ to be fixed and position-independent and to set up the mathematical structure of differential geometry needed in implementing Einstein gravity: $dx^0 = c_{\rm gravity} dt$, the Einstein-Hilbert action, the Einstein tensor, etc. One can reserve $c_{\rm photon}$ for photons, and give an objective meaning to the VSL concept. Observationally, as recently emphasized by Carlip [18], direct experimental evidence tells us that in the current epoch $c_{\rm gravity} \approx c_{\rm photon}$ to within about one percent tolerance. This limit is perhaps a little more relaxed than one would have naively expected, but the looseness of this limit is a reflection of the fact that direct tests of general relativity are difficult due to the weakness of the gravitational coupling $G_{\rm Newton}$.

Although we will focus on models and systems of units in which $c_{\rm photon}$ varies while $c_{\rm gravity}$ is fixed, in the Appendix we consider the reverse. This is important for discussions of varying fine-structure constant α. Since $\alpha \propto c_{\rm photon}^{-1}$, the models we present in the following sections *do lead to variation of the fine-structure constant*. This issue will be important in model-building if the Webb *et al.* [19] results on time-varying α are confirmed.

The above approach naturally leads us into the realm of two-metric theories, and the next section will be devoted to discussing the origin of our proposal. In brief, we will advocate using at least *two* metrics: a spacetime metric $g_{\alpha\beta}$ describing gravity, and a second ''effective metric,'' $[g^{\rm em}]_{\alpha\beta}$ describing the propagation of photons. Other particle species could, depending on the specific details of the model we envisage, couple either to their own ''effective metric,'' to g, or to $g^{\rm em}$.

Specific early examples of a VSL model based on a two-metric theory are those of Moffat [1,2], with a more recent implementation being that of Drummond [13]. Moffat chooses to keep $c_{\rm photon}$ fixed and let $c_{\rm gravity}$ vary, which leads to some translation difficulties in comparing those papers with the current one; but it is clear that there are substantial areas of agreement. This paper can be viewed as an extension of those previous investigations.

To help set the background, we wish to emphasize that the basic idea of a quantum-induced effective metric, which affects only photons and differs from the gravitational metric, is actually far from radical. This concept has gained a central role in the discussion of the propagation of photons in non-linear electrodynamics. In particular, we stress that ''anomalous'' ($c_{\rm photon} > c_{\rm gravity}$) photon speeds have been calculated in relation with the propagation of light in the Casimir vacuum [20–22], as well as in gravitational fields [23–26].

These articles have shown that special quantum vacuum states (associated with ''polarization'' of the vacuum) can lead to a widening of lightcones (although possibly only in some directions and for special photon polarization). In recent papers [27,28] it has been stressed that such behavior can be described in a geometrical way by the introduction of an effective metric which is related to the spacetime metric and the renormalized stress-energy tensor by a relation such as

$$[g_{\rm em}^{-1}]^{\mu\nu} = A g^{\mu\nu} + B \langle \psi | T^{\mu\nu} | \psi \rangle, \tag{9}$$

where A and B depend on the detailed form of the effective (one-loop) Lagrangian for the electromagnetic field.

Warning. We will always raise and lower indices using the spacetime metric g. This has the side-effect that one can no longer use index placement to distinguish the matrix $[g_{\rm em}]$ from its matrix inverse $[g_{\rm em}^{-1}]$. (Since $[g_{\rm em}]^{\mu\nu} \equiv g^{\mu\sigma} g^{\nu\rho} [g_{\rm em}]_{\sigma\rho} \neq [g_{\rm em}^{-1}]^{\mu\nu}$.) Accordingly, whenever we deal with the EM metric, we will always explicitly distinguish $[g_{\rm em}]$ from its matrix inverse $[g_{\rm em}^{-1}]$.

It is important to note that such effects can safely be described without needing to take the gravitational back reaction into account. The spacetime metric g is only minimally affected by the vacuum polarization, because the formula determining $[g^{\rm em}]$ is governed by the fine structure constant, while backreaction on the geometry is regulated by Newton's constant. Although these deviations from standard propagation are extremely tiny for the above quoted cases (black holes and the Casimir vacuum) we can ask ourselves if a similar sort of physics could have been important in the early evolution of our universe.

Drummond and Hathrell [23] have, for example, computed one-loop vacuum polarization corrections to QED in the presence of a gravitational field. They show that at low momenta the effective Lagrangian is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4 m_e^2} (\beta_1 R F_{\mu\nu} F^{\mu\nu} + \beta_2 R_{\mu\nu} F^{\mu\alpha} F^{\nu}{}_{\alpha}) - \frac{\beta_3}{4 m_e^2} R_{\mu\nu\alpha\beta} F^{\mu\alpha} F^{\mu\beta}. \tag{10}$$

Drummond and Hathrell were able to compute the low momentum coefficients $\beta_i, i = 1 \ldots 3$, but their results are probably not applicable to the case $R/m_e \gg 1$ of primary interest here. It is the qualitative structure of their results that should be compared with our prescriptions as developed in the next section.

In the main body of this paper we sketch out a number of scenarios based on two-metric interpretations of the VSL idea. We present different models that are consistent (i.e., mathematically and logically consistent), and which satisfy zeroth-order compatibility with observations (i.e., at least reduce to ordinary special relativity in the here and now). We also indicate how the various puzzles of the standard cosmological model can be formulated in this language, and start a preliminary analysis of these issues.

Since doing anything to damage and violate Lorentz symmetry is at first glance a rather radical step, we also wish to add a few words regarding the various approaches to the breaking of Lorentz invariance that are well-established in the literature. Perhaps the most important observation is that quantum field theories that are not Lorentz invariant can nevertheless exhibit an approximate Lorentz invariance in the low energy limit. See, for instance, the work of Nielsen *et al.* [29–31], where they demonstrate that Lorentz invariance is often a stable infrared fixed point of the renormalization group flow of a quantum field theory. An alternative model for the breakdown of Lorentz invariance has also been discussed by Everett [32,33].

Additionally, there are physical systems (in no sense relativistic, and based on the flowing fluid analogy for Lorentzian spacetimes) that demonstrate that Lorentz invariance can arise as a low energy property [34–39]. In the flowing fluid analogy for Lorentzian spacetimes the fluid obeys the non-relativistic Euler and continuity equations, while sound waves propagating in the fluid behave as though they ''feel'' a Lorentzian metric (with appropriate symmetries) that is built algebraically out of the dynamical variables describing the fluid flow.

Furthermore, as yet another example of ''soft'' Lorentz symmetry breaking we mention the well-studied Scharnhorst effect [20–22], wherein quantum vacuum effects lead to an anomalous speed of light for photons propagating perpendicular to a pair of conducting metal plates. The relevant one-loop quantum physics is neatly summarized by the Euler-Heisenberg effective Lagrangian, which explicitly exhibits a symmetry under the full $(3+1)$-dimensional Lorentz group. However the ground state (field theoretic vacuum state) exhibits a *reduced* symmetry, being invariant only under boosts that are parallel to the plates. In this situation the boundary conditions have ''softly'' broken the symmetry from $(3+1)$-dimensional Lorentz invariance down to $(2+1)$-dimensional Lorentz invariance, even though the fundamental physics encoded in the bulk Lagrangian is still manifestly symmetric under the larger group.

These comments bolster the view that we should not be too worried by a gentle breaking of Lorentzian symmetry. In this vein, Coleman and Glashow, building on the formalism developed by Colladay and Kostelecky, have recently investigated the possibility of small, renormalizable perturbations to the standard model which break Lorentz invariance while preserving the anomaly cancellation [40]. These perturbations are important at high energies and may provide an explanation for the existence of ultra-high energy cosmic rays beyond the GZK cut-off [40].

Finally, we should again remind the reader that VSL implementations based on two-metric theories are certainly closer in spirit to the approaches of Moffat and Clayton [1–3,5,6] and Drummond [13], than to the early Albrecht-Barrow-Magueijo [7,10,11] and Avelino-Martins [12] prescriptions. We have so far been unable to develop any really clean geometrodynamic framework that more closely parallels the phenomenological approach of the Barrow *et al.* approach, though we hope to be able to return to that issue in the future.

In Table I we give a list of variables and symbols used in this paper together with a brief description and appropriate defining equation.

II. TWO-METRIC VSL COSMOLOGIES

Based on the preceding discussion, we think that the first step towards making a ''geometric'' VSL cosmology is to write a two-metric theory in the form

$$S_I = \int d^4x \sqrt{-g}\{R(g) + \mathcal{L}_{\rm matter}(g)\} + \int d^4x \sqrt{-g_{\rm em}}\{[g_{\rm em}^{-1}]^{\alpha\beta} F_{\beta\gamma} [g_{\rm em}^{-1}]^{\gamma\delta} F_{\delta\alpha}\}. \quad (11)$$

We have made the first of many *choices* here by choosing the volume element for the electromagnetic Lagrangian to be $\sqrt{-g_{\rm em}}$ rather than, say $\sqrt{-g}$. This has been done to do minimal damage to the electromagnetic sector of the theory. As long as we confine ourselves to making *only* electromagnetic measurements this theory is completely equivalent to ordinary curved space electromagnetism in the spacetime described by the metric $g_{\rm em}$. As long as we *only* look at the ''matter'' fields it is only the ''gravity metric'' g that is relevant.

Since the photons couple to a second, separate metric, distinct from the spacetime metric that describes the gravitational field, we can now give a precise physical meaning to VSL. If the two null-cones (defined by g and $g_{\rm em}$, respectively) do not coincide one has a VSL cosmology. Gravitons and all matter except for photons, couple to g. Photons couple to the electromagnetic metric $g_{\rm em}$. A more subtle model is provided by coupling all the gauge bosons to $g_{\rm em}$, but everything else to g.

$$S_{II} = \int d^4x \sqrt{-g}\{R(g) + \mathcal{L}_{\rm fermions}(g,\psi)\} + \int d^4x \sqrt{-g_{\rm em}} {\rm Tr}\{[g_{\rm em}^{-1}]^{\alpha\beta} F^{\rm gauge}_{\beta\gamma} [g_{\rm em}^{-1}]^{\gamma\delta} F^{\rm gauge}_{\delta\alpha}\}. \quad (12)$$

For yet a third possibility: couple *all* the matter fields to $g_{\rm em}$, keeping gravity as the only field coupled to g. That is

TABLE I. Symbols used in the paper with a brief description and an equation where it is first used, if applicable.

Symbol	Brief description	Eq.
$g_{\rm em}$	The electromagnetic metric	(14)
ϵ	Energy density	(27)
ρ	Mass density	(36)
p	Pressure	(27)
$c_{\rm gravity}$	Velocity of gravitons	(8)
$c_{\rm photon}$	Velocity of photons	(21)
c_{e^-}	Maximum velocity of electrons	
$\beta_{1,2,3}$	Coefficients of 1-loop QED corrections	(10)
ζ	The ratio of photon to graviton velocity	(8)
m_e	The electron mass	(10)
χ	The VSL-inducing field	(14)
ψ	A generic spinor field	(12)
M	The scale for χ non-renormalization effects	(14)
A	The coupling constant for the interaction between χ and $F_{\mu\nu}$	(14)
K	The tri-curvature constant: $K=0,\pm 1$	(3)
$G_{\rm Newton}$	Newton's gravitational constant	(7)
ρ_Λ	The energy density in Λ	(56)
Λ	The cosmological constant	(56)
$n_{\rm em}$	The effective refractive index of spacetime	(57)
γ	A generic photon	
$V(\chi)$	The χ potential	(11)
ω	Photon frequency	
τ	Time scale for the χ-field phase transition	

$$S_{III}=\int d^4x\sqrt{-g}R(g)+\int d^4x\sqrt{-g_{\rm em}}\{\mathcal{L}_{\rm fermions}(g_{\rm em},\psi)\}$$
$$+\int d^4x\sqrt{-g_{\rm em}}{\rm Tr}\{[g_{\rm em}^{-1}]^{\alpha\beta}F^{\rm gauge}_{\beta\gamma}[g_{\rm em}^{-1}]^{\gamma\delta}F^{\rm gauge}_{\delta\alpha}\}. \tag{13}$$

Note that we have used $dx^0=c\,dt$, with the c in question being $c_{\rm gravity}$. It is this $c_{\rm gravity}$ that should be considered fundamental, as it appears in the local Lorentz transformations that are the symmetry group of all the non-electromagnetic interactions. It is just that $c_{\rm gravity}$ is no longer the speed of ''light.''

Most of the following discussion will focus on the first model S_I, but it is important to realize that VSL cosmologies can be implemented in many different ways, of which the models I, II, and III are the cleanest exemplars. We will see later that there are good reasons to suspect that model III is more plausible than models I or II, but we concentrate on model I for its pedagogical clarity. If one wants a model with even more complexity, one could give a *different* effective metric to each particle species. A model of this type would be so unwieldy as to be almost useless.

If there is no relationship connecting the EM metric to the gravity metric, then the theory has too much freedom to be useful, and the equations of motion are under-determined. To have a useful theory we need to postulate some relationship between g and $g_{\rm em}$, which in the interest of simplicity we take to be algebraic. A particularly simple electromagnetic (EM) metric we have found useful to consider is[1]

$$[g_{\rm em}]_{\alpha\beta}=g_{\alpha\beta}-(AM^{-4})\nabla_\alpha\chi\nabla_\beta\chi, \tag{14}$$

with the inverse metric

$$[g_{\rm em}^{-1}]^{\alpha\beta}=g^{\alpha\beta}+(AM^{-4})\frac{\nabla^\alpha\chi\nabla^\beta\chi}{1+(AM^{-4})(\nabla^\alpha\chi)^2}. \tag{15}$$

Here we have introduced a dimensionless coupling A and taken $\hbar=c_{\rm gravity}=1$, in order to give the scalar field χ its canonical dimensions of mass-energy.[2] The normalization

[1]The form of this metric is similar to the Kerr-Schild-Trautmann ansatz for generating exact solutions: $g_{ab}=\eta_{ab}-2Vk_ak_b$, where k_a is null in both the flat and non-flat metrics. k_a is geodesic if and only if $T_{ab}k^ak^b=0$. This generates a family of vacuum and Einstein-Maxwell solutions [41].

[2]Remember that indices are always raised and/or lowered by using the gravity metric g. Similarly, contractions always use the gravity metric g. If we ever need to use the EM metric to contract indices we will exhibit it explicitly.

energy scale, M, is defined in terms of $\hbar$, $G_{\rm Newton}$, and $c_{\rm gravity}$. The EM light cones can be much wider than the standard (gravity) ones without inducing a large back reaction on the spacetime geometry from the scalar field χ, provided M satisfies $M_{\rm Electroweak} < M < M_{\rm Pl}$. The presence of this dimensionfull coupling constant implies that when viewed as a quantum field theory, χVSL cosmologies will be non-renormalizable. In this sense the energy scale M is the energy at which the non-renormalizability of the χ field becomes important. [This is analogous to the Fermi scale in the Fermi model for weak interactions, although in our case M could be as high as the grand unified theory (GUT) scale.] Thus, χVSL models should be viewed as ''effective field theories'' valid for sub-M energies. In this regard χVSL models are certainly no worse behaved than many of the models of cosmological inflation and/or particle physics currently extant.

In comparison, note that Moffat [5] introduces a somewhat similar vector-based model for an effective metric which in our notation would be written as

$$[g_{\rm em}]_{\alpha\beta} = g_{\alpha\beta} - (AM^{-2}) V_\alpha V_\beta, \tag{16}$$

with the inverse metric

$$[g_{\rm em}^{-1}]^{\alpha\beta} = g^{\alpha\beta} + (AM^{-2}) \frac{V^\alpha V^\beta}{1+(AM^{-2})(V^\alpha)^2}. \tag{17}$$

However there are many technical differences between that paper and this one, as will shortly become clear. In the more recent paper [6] a scalar-based scenario more similar to our own is discussed.

The evolution of the scalar field χ will be assumed to be governed by some VSL action

$$S_{\rm VSL} = \int d^4x \sqrt{-g}\, \mathcal{L}_{\rm VSL}(\chi). \tag{18}$$

We can then write the complete action for model I as

$$\begin{aligned} S_I = \int d^4x \sqrt{-g}\{R(g) + \mathcal{L}_{\rm matter}\} + \int d^4x \sqrt{-g_{\rm em}(\chi)} \\ \times \{[g_{\rm em}^{-1}]^{\alpha\beta}(\chi) F_{\beta\gamma} [g_{\rm em}^{-1}]^{\gamma\delta}(\chi) F_{\delta\alpha}\} \\ + \int d^4x \sqrt{-g}\, \mathcal{L}_{\rm VSL}(\chi) + \int d^4x \sqrt{-g}\, \mathcal{L}_{\rm NR}(\chi,\psi), \end{aligned} \tag{19}$$

where $\mathcal{L}_{\rm NR}(\chi,\psi)$ denotes the non-renormalizable interactions of χ with the standard model.

Let us suppose the potential in this VSL action has a global minimum, but the χ field is displaced from this minimum in the early universe: either trapped in a metastable state by high-temperature effects or displaced due to chaotic initial conditions. The transition to the global minimum may be either of first or second order and during it $\nabla_\alpha \chi \neq 0$, so that $g_{\rm em} \neq g$. Once the true global minimum is achieved, $g_{\rm em} = g$ again. Since one can arrange χ today to have settled to the true global minimum, current laboratory experiments would automatically give $g_{\rm em} = g$.

It is only via observational cosmology, with the possibility of observing the region where $g_{\rm em} \neq g$ that we would expect VSL effects to manifest themselves. We will assume the variation of the speed of light to be confined to very early times, of order of the GUT scale, and hence none of the low-redshift physics can be directly affected by this transition. We will see in Sec. VII how indirect tests for the presence of the χ field are indeed possible.

Note that in the metastable minimum $V(\chi) \geq 0$, thus the scalar field χ can mimic a cosmological constant, as long as the kinetic terms of the VSL action are negligible when compared to the potential contribution. If the lifetime of the metastable state is too long, a de Sitter phase of exponential expansion will ensue. Thus, the VSL scalar has the possibility of driving an inflationary phase in its own right, over and above anything it does to the causal structure of the spacetime (by modifying the speed of light). While this direct connection between VSL and inflation is certainly interesting in its own right, we prefer to stress the more interesting possibility that, by coupling an independent inflaton field ϕ to $g_{\rm em}$, χVSL models can be used to improve the inflationary framework by enhancing its ability to solve the cosmological puzzles. We will discuss this issue in detail in Sec. V C.

During the transition, (adopting FLRW coordinates on the spacetime), we see

$$[g_{\rm em}]_{tt} = -1 - (AM^{-4})(\partial_t \chi)^2 \leq -1. \tag{20}$$

This means that the speed of light for photons will be larger than the ''speed of light'' for everything else—the photon null cone will be wider than the null cone for all other forms of matter.[3] Actually one has

$$c_{\rm photon}^2 = c_{\rm gravity}^2 [1 + (AM^{-4})(\partial_t \chi)^2] \geq c_{\rm gravity}^2. \tag{21}$$

The fact that the photon null cone is wider implies that ''causal contact'' occurs over a larger region than one thought it did—and this is what helps smear out inhomogeneities and solve the horizon problem.

The most useful feature of this model is that it gives a precise *geometrical* meaning to VSL cosmologies: something that is difficult to discern in the extant literature.

Note that this model is by no means unique: (1) the VSL potential is freely specifiable; (2) one could try to do similar things to the Fermi fields and/or the non-Abelian gauge fields—use one metric for gravity and $g_{\rm em}$ for the other fields. We wish to emphasize some features and pitfalls of two-metric VSL cosmologies:

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[3]For other massless fields the situation depends on whether we use model I, II, or III. In model I it is *only* the photon that sees the anomalous light cones, and neutrinos for example are unaffected. In model II all gauge bosons (photons, $W^\pm$, Z^0, and gluons) see the anomalous light cones. Finally, in model III everything *except* gravity sees the anomalous light cones.

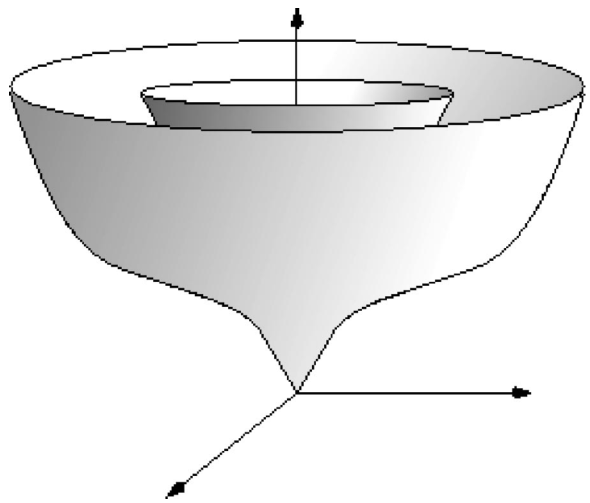

FIG. 1. A schematic illustration of the two future null cones $C^+_{\rm gravity}$ and $C^+_{\rm photon}$. Initially they coincide, followed by a transition after which $c_{\rm photon} \gg c_{\rm gravity}$ and then by another transition in which $c_{\rm photon} \simeq c_{\rm gravity}$.

The causal structure of spacetime is now ''divorced'' from the null geodesics of the metric g. Signals (in the form of photons) can travel at a speed $c_{\rm photon} \gg c_{\rm gravity}$.

We must be extremely careful whenever we need to assign a specific meaning to the symbol c. We are working with a *variable* $c_{\rm photon}$, which has a larger value than the standard one, and a *constant* $c_{\rm gravity}$ which describes the speed of propagation of all the other massless particles. In considering the cosmological puzzles and other features of our theory (including the ''standard'' physics) we will always have to specify if the quantities we are dealing with depend on $c_{\rm photon}$ or $c_{\rm gravity}$ (see Fig. 1).

Stable causality: If the gravity metric g is causally stable, if the coupling $A \geq 0$, and if $\partial_\mu \chi$ is a timelike vector with respect to the gravity metric, then the photon metric is also causally stable. This eliminates the risk of nasty causal problems like closed timelike loops. This observation is important since with two metrics (and two sets of null cones), one must be careful to not introduce causality violations—and if the two sets of null cones are completely free to tip over with respect to each other it is very easy to generate causality paradoxes in the theory.

If χ is displaced from its global minimum we expect it to oscillate around this minimum, causing $c_{\rm photon}$ to have periodic oscillations. This would lead to dynamics very similar to that of preheating in inflationary scenarios [42].

During the phase in which $c_{\rm photon} \gg c_{\rm gravity}$ one would expect photons to emit gravitons in an analogue of the Cherenkov radiation. We will call this effect *Gravitational Cherenkov Radiation*. This will cause the frequency of photons to decrease and will give rise to an additional stochastic background of gravitons.

Other particles moving faster than $c_{\rm gravity}$ (i.e., models II and III) would slow down and become subluminal relative to $c_{\rm gravity}$ on a characteristic time-scale associated to the emission rate of gravitons. There will therefore be a natural mechanism for slowing down massive particles to below $c_{\rm gravity}$.

In analogy to photon Cherenkov emission [43], longitudinal graviton modes may be excited due to the non-vacuum background [44].

III. STRESS-ENERGY TENSOR, EQUATION OF STATE, AND EQUATIONS OF MOTION

A. The two stress-energy tensors

The definition of the stress-energy tensor in a VSL cosmology is somewhat subtle since there are two distinct ways in which one could think of constructing it. If one takes gravity as being the primary interaction, it is natural to define

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}}, \tag{22}$$

where the metric variation has been defined with respect to the gravity metric. This stress-energy tensor is the one that most naturally shows up in the Einstein equation. One could also think of defining a different stress-energy tensor for the photon field (or in fact any form of matter that couples to the photon metric) by varying with respect to the photon metric, that is

$$\tilde{T}^{\mu\nu} = \frac{2}{\sqrt{-g_{\rm em}}} \frac{\delta S}{\delta g^{\rm em}_{\mu\nu}}. \tag{23}$$

This definition is most natural when one is interested in non-gravitational features of the physics.

In the formalism we have set up, by using the chain rule and the relationship that we have assumed between $g_{\rm em}$ and g, it is easy to see that

$$T^{\mu\nu}_{\rm em} = \sqrt{\frac{g_{\rm em}}{g}} \tilde{T}^{\mu\nu}_{\rm em} = \sqrt{1-(AM^{-4})[(\nabla^\alpha \chi)^2]} \tilde{T}^{\mu\nu}_{\rm em}. \tag{24}$$

Thus, these two stress-energy tensors are very closely related. When considering the way the photons couple to gravity, the use of $T^{\mu\nu}_{\rm em}$ is strongly recommended. Note that $T^{\mu\nu}_{\rm em}$ is covariantly conserved with respect to ∇_g, whereas $\tilde{T}^{\mu\nu}_{\rm em}$ is conserved with respect to $\nabla_{g_{\rm em}}$. It should be noted that $\tilde{T}^{\mu\nu}_{\rm em}$ is most useful when discussing the non-gravitational behavior of matter that couple to $g_{\rm em}$ rather than g. (Thus in type I models this means we should only use it for photons.) For matter that couples to g (rather than to $g_{\rm em}$), we have not found it to be indispensable, or even useful, and wish to discourage its use on the grounds that it is dangerously confusing.

An explicit calculation, assuming for definiteness a type I model and restricting attention to the electromagnetic field, yields

$$T^{\mu\nu}_{\text{em}} = \sqrt{1-(AM^{-4})[(\nabla^\alpha\chi)^2]} \times \left\{ [g^{-1}_{\text{em}}]^{\mu\sigma} F_{\sigma\rho} [g^{-1}_{\text{em}}]^{\rho\lambda} F_{\lambda\pi} [g^{-1}_{\text{em}}]^{\pi\nu} - \frac{1}{4}[g^{-1}_{\text{em}}]^{\mu\nu}(F^2) \right\}, \tag{25}$$

with

$$(F^2) = [g^{-1}_{\text{em}}]^{\alpha\beta} F_{\beta\gamma} [g^{-1}_{\text{em}}]^{\gamma\delta} F_{\delta\alpha}. \tag{26}$$

(In particular, note that both $\tilde{T}^{\mu\nu}_{\text{em}}$ and $T^{\mu\nu}_{\text{em}}$ are traceless with respect to g_{em}, not with respect to g. This observation will prove to be very useful.)

B. Energy density and pressure: The photon equation of state

In an FLRW universe the high degree of symmetry implies that the stress-energy tensor is completely defined in terms of energy density and pressure. We will define *the* physical energy density and pressure as the appropriate components of the stress-energy tensor when referred to an orthonormal basis *of the metric that enters the Einstein equation* (from here on denoted by single-hatted indices)

$$\varepsilon = T^{\hat{t}\hat{t}} = T^{tt}/|g^{tt}| = |g_{tt}|T^{tt}, \tag{27}$$

$$p = \frac{1}{3}\delta_{\hat{i}\hat{j}} T^{\hat{i}\hat{j}} = \frac{1}{3} g_{ij} T^{ij}. \tag{28}$$

It is this ε and this p that will enter the Friedmann equations governing the expansion and evolution of the universe.

On the other hand, if one defines the stress-energy tensor in terms of a variational derivative with respect to the electromagnetic metric, then when viewed from an orthonormal frame adapted to the *electromagnetic* metric (denoted by double hats), one will naturally define *different* quantities for the energy density $\tilde{\varepsilon}$ and pressure $\tilde{p}$. We can then write

$$\tilde{\varepsilon} = \tilde{T}^{\hat{\hat{t}}\hat{\hat{t}}} = \tilde{T}^{tt}/|g^{tt}_{\text{em}}| = |g^{\text{em}}_{tt}|\tilde{T}^{tt}, \tag{29}$$

$$\tilde{p} = \frac{1}{3}\delta_{\hat{\hat{i}}\hat{\hat{j}}} \tilde{T}^{\hat{\hat{i}}\hat{\hat{j}}} = \frac{1}{3} g^{\text{em}}_{ij} \tilde{T}^{ij}. \tag{30}$$

From our previous discussion [Eq. (24)] we know that the two definitions of stress-energy are related, and using the symmetry of the FLRW geometry we can write

$$T^{\mu\nu} = \frac{c_{\text{photon}}}{c_{\text{gravity}}} \tilde{T}^{\mu\nu}. \tag{31}$$

If we combine this equation with the previous definitions, we have

$$\varepsilon = \frac{c_{\text{gravity}}}{c_{\text{photon}}} \tilde{\varepsilon}, \tag{32}$$

$$p = \frac{c_{\text{photon}}}{c_{\text{gravity}}} \tilde{p}. \tag{33}$$

(Note that the prefactors are *reciprocals* of each other.) From a gravitational point of view any matter that couples to the photon metric has its energy density depressed and its pressure enhanced by a factor of $c_{\text{gravity}}/c_{\text{photon}}$ relative to the energy density and pressure determined by ''electromagnetic means.'' This ''leverage'' will subsequently be seen to have implications for strong energy condition (SEC) violations and inflation.

In order to investigate the equation of state for the photon field, our starting point will be the standard result that the stress-energy tensor of photons is traceless. By making use of the tracelessness and symmetry arguments one can (in one-metric theories) deduce the relationship between the energy density and the pressure $\varepsilon = 3p$. However, in two-metric theories (of the type presented here) the photon stress-energy tensor is traceless with respect to g_{em}, but not with respect to g. Thus in this bi-metric theory we have

$$\tilde{\varepsilon} = 3\tilde{p}. \tag{34}$$

When translated into ε and p, (quantities that will enter the Friedmann equations governing the expansion and evolution of the universe), this implies

$$p_{\text{photons}} = \frac{1}{3}\varepsilon_{\text{photons}} \frac{c^2_{\text{photon}}}{c^2_{\text{gravity}}}. \tag{35}$$

As a final remark it is interesting to consider the speed of sound encoded in the photon equation of state. If we use the relationship $\rho_{\text{photons}} = \varepsilon_{\text{photons}}/c^2_{\text{gravity}}$, we can write

$$\rho_{\text{photons}} = \frac{3p_{\text{photons}}}{c^2_{\text{photon}}}. \tag{36}$$

And therefore

$$(c_{\text{sound}})_{\text{photons}} = \sqrt{\frac{\partial p_{\text{photons}}}{\partial \rho_{\text{photons}}}} = \frac{c_{\text{photon}}}{\sqrt{3}}. \tag{37}$$

That is, oscillations in the density of the photon fluid propagate at a relativistic speed of sound which is $1/\sqrt{3}$ times the speed of ''light'' *as seen by the photons*.

More generally, for highly relativistic particles we expect

$$\varepsilon_i = 3p_i \frac{c^2_{\text{gravity}}}{c^2_i}, \tag{38}$$

and

$$(c_{\text{sound}})_i = \frac{c_i}{\sqrt{3}}. \tag{39}$$

Note that we could define the mass density (as measured by electromagnetic means) in terms of $\tilde{\rho}_{\text{photons}} = \tilde{\varepsilon}_{\text{photons}}/c^2_{\text{photon}}$. This definition yields the following identity:

$$\rho_{\text{photons}} = \frac{c_{\text{photon}}}{c_{\text{gravity}}} \tilde{\rho}_{\text{photons}}. \tag{40}$$

If the speed of sound is now calculated in terms of $\tilde{p}_{\text{photons}}$ and $\tilde{\rho}_{\text{photons}}$ we get the same result as above.

C. Equations of motion

The general equations of motion based on model I can be written as

$$G_{\mu\nu} = \frac{8\pi G_{\text{Newton}}}{c^4_{\text{gravity}}} (T^{\text{VSL}}_{\mu\nu} + T^{\text{em}}_{\mu\nu} + T^{\text{matter}}_{\mu\nu}). \tag{41}$$

All of these stress-energy tensors have been defined with the ''gravity prescription''

$$T^{\mu\nu}_i = \frac{2}{\sqrt{-g}} \frac{\delta S_i}{\delta g_{\mu\nu}}. \tag{42}$$

In a FLRW spacetime the Friedmann equations (summing over all particles present) for a χVSL cosmology read as follows:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2_{\text{gravity}}} \sum_i \varepsilon_i - \frac{K c^2_{\text{gravity}}}{a^2}, \tag{43}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2_{\text{gravity}}} \sum_i (\varepsilon_i + 3p_i), \tag{44}$$

where, as usual, $K = 0, \pm 1$.

The constant ''geometric'' speed of light implies that we get from the Friedmann equation separate conservation equations valid for each species individually (provided, as is usually assumed for at least certain portions of the universe's history, that there is no significant energy exchange between species)

$$\dot{\varepsilon}_i + 3\frac{\dot{a}}{a}(\varepsilon_i + p_i) = 0. \tag{45}$$

In the relativistic limit we have already seen, from Eq. (35), that $p_i = \frac{1}{3}\varepsilon_i (c_i^2/c^2_{\text{gravity}})$. [We are generalizing slightly to allow each particle species to possess its own ''speed-of-light.''] So we can conclude that

$$\dot{\varepsilon}_i + \left(3 + \frac{c_i^2}{c^2_{\text{gravity}}}\right)\frac{\dot{a}}{a}\varepsilon_i = 0. \tag{46}$$

Provided c_i is slowly changing with respect to the expansion of the universe (and it is not at all clear whether such an epoch ever exists), we can write for each relativistic species

$$\varepsilon_i a^{3 + (c_i^2/c^2_{\text{gravity}})} \approx \text{const}. \tag{47}$$

This is the generalization of the usual equation ($\varepsilon_i a^4 \approx \text{const}$) for relativistic particles in a constant-speed-of-light model. This implies that energy densities will fall much more rapidly than naively expected in this bi-metric VSL formalism, provided $c_i > c_{\text{gravity}}$.

IV. COSMOLOGICAL PUZZLES AND PRIMORDIAL SEEDS

In the following we will discuss the main cosmological puzzles showing how they are mitigated (if not completely solved) by the χVSL models. Given its complexity, the peculiar case of the flatness problem will be treated in a separate section.

A. The isotropy and horizon problems

One of the major puzzles of the standard cosmological model is that the isotropy of the CMB seems in conflict with the best estimates of the size of causal contact at last scattering. The formula for the (coordinate) size of the particle horizon at the time of last scattering t_* is

$$R_{\text{particle-horizon}}(t_*) = \int_0^{t_*} \frac{c_{\text{gravity}} dt}{a(t)}. \tag{48}$$

For photons this should now be modified to

$$R_{\text{photon-horizon}}(t_*) = \int_0^{t_*} \frac{c_{\text{photon}} dt}{a(t)} \tag{49}$$

$$\geq R_{\text{particle-horizon}}(t_*). \tag{50}$$

The quantity $R_{\text{photon-horizon}}$ sets the distance scale over which photons can transport energy and thermalize the primordial fireball. On the other hand, the coordinate distance to the surface of last scattering is

$$R_{\text{last-scattering}}(t_*, t_0) = \int_{t_*}^{t_0} \frac{c_{\text{photon}} dt}{a(t)}. \tag{51}$$

(Here t_0 denotes the present epoch.) The observed large-scale homogeneity of the CMB implies (in order to have the CMB coming from opposite points on the sky)

$$R_{\text{photon-horizon}}(t_*) \geq 2R_{\text{last-scattering}}(t_*, t_0), \tag{52}$$

which can be achieved by having $c_{\text{photon}} \gg c_{\text{gravity}}$ early in the expansion. (In order not to change late-time cosmology too much it is reasonable to expect $c_{\text{photon}} \approx c_{\text{gravity}}$ between last scattering and the present epoch.) Instead of viewing our observable universe as an inflated small portion of the early universe (standard inflationary cosmology), we can say that in a VSL framework the region of early causal contact is underestimated by a factor that is roughly approximated by the ratio of the maximum photon speed to the speed with which gravitational perturbations propagate.

We can rephrase the horizon problem as a constraint on the ratio between the photon horizon at last scattering and the photon horizon at the present day. Indeed if we add $2R_{\text{photon-horizon}}(t_*)$ to both sides of the previous equation, then

$$3R_{\text{photon-horizon}}(t_*) \geqslant 2R_{\text{photon-horizon}}(t_0). \tag{53}$$

In terms of the *physical distance* to the photon horizon $[l(t)=a(t)R(r)]$, this implies

$$l_{\text{photon-horizon}}(t_*) \geqslant \frac{2}{3}\frac{a(t_*)}{a(t_0)} l_{\text{photon-horizon}}(t_0). \tag{54}$$

This formulation of the observed ''horizon constraint'' is as model-independent as we can make it—this constraint is a purely kinematical statement of the observational data and is not yet a ''problem;'' even in standard cosmology it will not become a problem until one uses *dynamics* to deduce a specific model for $a(t)$. In the present VSL context we will need to choose or deduce dynamics for both $a(t)$ and $c(t)$ before this constraint can be used to discriminate between acceptable and unacceptable cosmologies. More on this point below.

B. Monopoles and relics

The Kibble mechanism predicts topological defect densities that are inversely proportional to powers of the correlation length of the Higgs fields. These are generally bounded above by the particle horizon at the time of defect formation.

To simplify the analysis it is useful to use the related concept of Hubble distance

$$R_{\text{Hubble}} = \frac{c_{\text{photon}}}{H}. \tag{55}$$

The above quantity (often known as the Hubble radius or, speaking loosely, ''the horizon'') is often mistakenly *identified* with the particle horizon [45]. The two concepts, though related, are distinct. In particular the Hubble scale evolves in the same way as the particle horizon in simple FLRW models and hence measures the domain of future influence of an event in these models [46]. If fields interact only through gravity, then the Hubble scale *is* useful as a measure of the minimum spatial wavelength of those modes that are effectively ''frozen in'' by the expansion of the universe. A mode is said to be ''frozen in'' if its frequency is smaller than the Hubble parameter, since then there is not enough time for it to oscillate before the universe changes substantially, the evolution of that mode is governed by the expansion of the universe. Therefore, for modes travelling at the speed c_{photon}, if the ''freeze out'' occurs at $\omega < H$, this implies that $\lambda > c_{\text{photon}}/H$, as claimed above. Note that this discussion crucially assumes that only gravity is operating. As soon as interactions between fields are allowed, such as occurs in inflationary reheating, the Hubble scale is irrelevant for determining the evolution of modes and modes with $k/aH \ll 1$ can evolve extremely rapidly without violating causality, as indeed typically occurs in preheating [47].

If we suppose a good thermal coupling between the photons and the Higgs field to justify using the photon horizon scale in the Kibble freeze-out argument then we can argue as follows: Inflation solves the relics puzzle by diluting the density of defects to an acceptable degree, χVSL models deal with it by varying c in such a way as to make sure that the photon horizon scale is large when the defects form. Thus, we need the transition in the speed of light to happen *after* the spontaneous symmetry breaking (SSB) that leads to monopole production.

Alternatively, we could arrange a model where both photons and the Higgs field couple directly to g_{em}, along the lines of S_{III} above; this obviates the need for postulating good thermal coupling since the Higgs field, and its dynamics, is now directly controlled by the variable speed of light.

So far the discussion assumes thermal equilibrium, but one should develop a formalism which takes into account the non-equilibrium effects and the characteristic time scales (quench and critical slowing down scales). As a first remark one can note that the larger the Higgs correlation length ξ_Φ is, the lower the density of defects (with respect to the standard estimates) will be. This correlation length characterizes the period *before* the variation of the speed of light, when we suppose that the creation of topological defects has taken place. Remember that in the Zurek mechanism $\rho_{\text{defects}} \sim \xi_\Phi^{-n}$ with $n=1,2$, and 3, for domain walls, strings, and monopoles, respectively [48].

We could also consider the possibility that the change in c is driven by a symmetry breaking (Higgs-like) mechanism, and try to relate changes in c to symmetry breaking at the GUT or electro-weak scale. Unfortunately such considerations require a much more specific model than the one considered here, and we want to keep the discussion as general as possible.

C. Λ and the Planck problem

In this χVSL approach we are not affecting the cosmological constant Λ, except indirectly via $\mathcal{L}_{\text{VSL}}$. The vacuum energy density is given by

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G_{\text{Newton}}}. \tag{56}$$

But which is the c appearing here? The speed of light c_{photon}? Or the speed of gravitons c_{gravity}? In our two-metric approach it is clear that for any fundamental cosmological constant one should use c_{gravity}. On the other hand, for any contribution to the total cosmological constant from quantum zero-point fluctuations (ZPF) the situation is more complex. If the quantum field in question couples to the metric g_{em}, one would expect c_{photon} in the previous equation, not least in the relationship between ρ_{zpf} and p_{zpf}.

While we do nothing to mitigate the cosmological constant problem we also do not encounter the ''Planck problem'' considered by Coule [49]. He stressed the fact that in earlier VSL formulations [3,7,10] a varying speed of light also affects the definition of the Planck scale. In fact, in the standard VSL one gets two different Planck scales (determined by the values of c before and after the transition). The number of Planck times separating the two Planck scales turns out to be larger than the number of Planck times separating us from the standard Planck era. So, in principle, the standard fine-tuning problems are even worse in these models.

In contrast, in our two-metric formulation one has to decide from the start which c is referred to in the definition of the Planck length. The definition of the Planck epoch is the scale at which the gravitational action becomes of the order of $\hbar$. This process involves gravity and does not refer to photons. Therefore, the c appearing there is the speed of propagation of gravitons, which is unaffected in our model. Hence we have a VSL cosmology without a ''Planck problem,'' simply because we have not made any alterations to the gravity part of the theory.

D. Primordial fluctuations

The inflationary scenario owes its popularity not just to its ability to solve the main problems of the background cosmology. It is also important because it provides a plausible, causal, micro-physics explanation for the origin of the primordial perturbations which may have seeded large-scale structure. The phase of quasi–de Sitter expansion excites the quantum vacuum and leads to particle creation in squeezed states. As the expansion is almost exactly exponential, these particles have an (almost exactly) scale-invariant spectrum with amplitude given by the Hawking ''temperature'' $H/2\pi$ [50].

In the case of χVSL the creation of primordial fluctuations is again generic. The basic mechanism can be understood by modelling the change in the speed of light as a changing ''effective refractive index of the EM vacuum.'' In an FLRW background

$$n_{\rm em}=\frac{c_{\rm gravity}}{c_{\rm photon}}=\frac{1}{\sqrt{[1+(AM^{-4})(\partial_t\chi)^2]}}. \qquad (57)$$

Particle creation from a time-varying refractive index is a well-known effect [51–54][4] and shares many of the features calculated for its inflationary counterpart (e.g., the particles are also produced as squeezed couples). We point out at this stage that these mechanisms are not identical. In particular, in χVSL cosmologies it is only the fields coupled to the EM metric that will primarily be excited. Of course, it is conceivable, and even likely, that perturbations in these fields will spread to the others whenever some coupling exists. Gravitational perturbations could be efficiently excited if the χ field is non-minimally coupled to gravity.

A second, and perhaps more fundamental, point is that a scale-invariant spectrum of metric fluctuations on large scales is by no means guaranteed. The spectrum may have a nearly thermal distribution over those modes for which the adiabatic limit holds ($\tau\omega>1$, where τ is the typical time scale of the transition in the refractive index) [53]. If we assume that τ is approximately constant in time during the phase transition, then it is reasonable to expect an approximately Harrison-Zel'dovich spectrum over the frequencies for which the adiabatic approximation holds. Extremely short values of τ, or very rapid changes of τ during the transition, would be hard to make compatible with the present observations. Since a detailed discussion of the final spectrum of perturbations in χVSL cosmologies would force us to take into account the precise form of the χ-potential $V(\chi)$, (being very model dependent), we will not discuss these issues further here.

As final remarks we want to mention a couple of generic features of the creation of primordial fluctuations in χVSL cosmologies. Since we require inflation to solve the flatness problem, the χVSL spectrum must be folded into the inflationary spectrum as occurs in standard inflation with phase transitions (see, e.g., [55]). In addition to this also a preheating phase is conceivable in χVSL models if χ oscillates coherently. This would lead to production of primordial magnetic fields due to the breaking of the conformal invariance of the Maxwell equations.

V. FLATNESS

The flatness problem is related to the fact that in FLRW cosmologies the $\Omega=1$ solution appears as an unstable point in the evolution of the universe. Nevertheless observations seem to be in favor of such a value. In this section we will show that any two-metric implementation of the kind given in Eq. (14) does not by itself solve the flatness problem, let alone the quasi-flatness problem [8]. We will also explain how this statement is only apparently in contradiction with the claims made by Clayton and Moffat in their implementations of two-metric VSL theories. Finally we will show that χVSL can nevertheless enhance any mild SEC violation originated by an inflaton field coupled to $g_{\rm em}$.

A. Flatness in ''pure'' χVSL cosmologies

The question ''Which c are we dealing with?'' arises once more when we address the flatness problem. From the Friedmann equation we can write

$$\epsilon\equiv\Omega-1=\frac{Kc^2}{H^2a^2}=\frac{Kc^2}{\dot a^2}, \qquad (58)$$

where $K=0,\pm1$. We already know that one cannot simply replace $c\to c_{\rm photon}$ in the above equation. The Friedmann equation is obtained by varying the Einstein-Hilbert action. Therefore, the c appearing here must be the fixed $c_{\rm gravity}$, otherwise the Bianchi identities are violated and Einstein gravity loses its geometrical interpretation. Thus, we have

[4]It is important to stress that in the quoted papers the change of refractive index happens in a flat static spacetime. It is conceivable and natural that in an FLRW spacetime the expansion rate could play an important additional role. The results of [51–54] should then be considered as precise in the limit of a rapid ($\dot n/n\gg\dot a/a$) transition in the speed of light.

$$\epsilon = \frac{K c_{\mathrm{gravity}}^2}{\dot{a}^2}. \tag{59}$$

If we differentiate the above equation, we see that purely on *kinematic* grounds

$$\dot{\epsilon} = -2 K c_{\mathrm{gravity}}^2 \left(\frac{\ddot{a}}{\dot{a}^3} \right) = -2 \epsilon \left(\frac{\ddot{a}}{\dot{a}} \right). \tag{60}$$

From the way we have implemented VSL cosmology (two-metric model), it is easy to see that this equation is independent of the photon sector; it is unaffected if $c_{\mathrm{photon}} \neq c_{\mathrm{gravity}}$. The only way that VSL effects could enter this discussion is indirectly. When $c_{\mathrm{photon}} \neq c_{\mathrm{gravity}}$ the photon contribution to ρ and p is altered.

In particular, if we want to solve the flatness problem by making $\epsilon = 0$ a stable fixed point of the evolution (at least for some portion in the history of the universe), then we must have $\ddot{a} > 0$, and the expansion of the universe must be accelerating (for the same portion in the history of the universe).

It is well known that the condition $\ddot{a} > 0$ leads to violations of the SEC [56]. Namely, violations of the SEC are directly linked to solving the flatness problem. (It is for this reason that a positive cosmological constant, which violates the SEC, is so useful in mitigating the flatness problem.) By making use of the Friedmann equations (43) and (44), this can be rephrased as

$$\dot{\epsilon} = 2 \epsilon \left[\frac{4 \pi G_{\mathrm{Newton}} \sum_i (\varepsilon_i + 3 p_i)}{3 H c_{\mathrm{gravity}}^2} \right]. \tag{61}$$

In our bi-metric formalism the photon energy density ε and photon pressure p are both positive, and from Eq. (35) it is then clear that also $\varepsilon + 3p$ will be positive. This is enough to guarantee no violations of the SEC. This means that bi-metric VSL theories are no better at solving the flatness problem than standard cosmological (non-inflationary) FLRW models. To ''solve'' the flatness problem by making $\epsilon = 0$ a stable fixed point will require some SEC violations and cosmological inflation from other non-photon sectors of the theory.

B. Flatness in the Clayton-Moffat scenarios

In relation to the preceding discussion, we now wish to take some time to distinguish our approach from that of Moffat [1–3] and Clayton-Moffat [5,6]. The two clearest descriptions (of two separate VSL implementations, a vector-based approach and a scalar-based approach) appear in the recent papers [5,6].

1. The vector scenario

Let us first consider Clayton and Moffat's *vector* scenario as discussed in [5]. In this paper Clayton and Moffat claim to be able to solve the flatness problem directly from their VSL implementation (equivalent to asserting that they can induce SEC violations), an assertion we believe to be premature. The key observation is that from their Eq. (6), and retaining (as much as possible) their notation for now, it is easy to see that

$$\rho_{\mathrm{eff}} = \frac{\tilde{\rho}_{\mathrm{matter}}}{\sqrt{1+\beta \psi_0^2}} + \frac{1}{2} m^2 \frac{\psi_0^2}{c^2}, \tag{62}$$

$$p_{\mathrm{eff}} = \sqrt{1+\beta \psi_0^2}\, \tilde{p}_{\mathrm{matter}} + \frac{1}{2} m^2 \frac{\psi_0^2}{c^2}, \tag{63}$$

$$(\rho + 3p)_{\mathrm{eff}} = \left(\frac{\tilde{\rho}_{\mathrm{matter}}}{\sqrt{1+\beta \psi_0^2}} + 3 \tilde{p}_{\mathrm{matter}} \sqrt{1+\beta \psi_0^2} \right) + 2 m^2 \frac{\psi_0^2}{c^2}. \tag{64}$$

[Compare also with Eqs. (68) and (72) below.] Note that because the presentation in [5] is set up in a language where c_{photon} is kept fixed and c_{gravity} is allowed to vary, there are potential translation pitfalls in comparing that presentation to out own approach. Here $\tilde{\rho}_{\mathrm{matter}}$ and $\tilde{p}_{\mathrm{matter}}$ are the matter energy density and pressure as measured in an orthonormal frame adapted to the electromagnetic metric; they are simply called ρ and p in the Clayton–Moffat paper.[5]

The key observation is now that the contribution to the SEC arising from the VSL vector field is positive, and if the ordinary matter has positive pressure and energy density, then there is no possibility of violating the SEC. This is perhaps a little easier to see if (as is usual in the rest of the current paper) we go to an orthonormal frame adapted to the gravity metric, in that case

$$\rho_{\mathrm{eff}} = \rho_{\mathrm{matter}} + \frac{1}{2} m^2 \frac{\psi_0^2}{c^2}, \tag{65}$$

$$p_{\mathrm{eff}} = p_{\mathrm{matter}} + \frac{1}{2} m^2 \frac{\psi_0^2}{c^2}, \tag{66}$$

$$(\rho + 3p)_{\mathrm{eff}} = (\rho_{\mathrm{matter}} + 3 p_{\mathrm{matter}}) + 2 m^2 \frac{\psi_0^2}{c^2}. \tag{67}$$

The contribution to the SEC arising from the VSL vector field is manifestly positive, and because of the form of the stress-energy tensor, it is clear that the VSL vector field does not mimic a cosmological constant. Again, if the ordinary matter has positive pressure and energy density, then there is no possibility of violating the SEC.

[5]We wish to thank M. A. Clayton and J. W. Moffat for helpful comments on these translation issues.

2. The scalar scenario

In Clayton and Moffat's *scalar* scenario [6] the discussion of the relationship between SEC violations is more nuanced, and we find ourselves largely in agreement with the point of view presented in that paper. Indeed, subtract Eq. (30) of that paper from Eq. (31) and divide by two to obtain (following the notation of that paper)

$$\frac{\ddot{R}}{R}=\frac{1}{3}c^2\Lambda+\frac{1}{3}c^2V(\phi)-\frac{1}{6}\dot{\phi}^2-\frac{\kappa c^2}{6}\left(\frac{\rho_M}{\sqrt{I}}+3p_M\sqrt{I}\right). \tag{68}$$

The quantity I is defined in Eq. (15) of that paper and satisfies $I>1$, so that the square root is well defined, ($\sqrt{I}\mapsto c_{\text{photon}}/c_{\text{gravity}}$ when mapped to our notation.) If the ''ordinary'' matter (ρ_M, p_M) is indeed ''ordinary'' ($\rho_M>0$, $p_M>0$), the only possible source of SEC violations (and inflation) is from the explicit cosmological constant or from letting the VSL field (ϕ in their notation, which becomes χ in ours) act as an inflaton field. Alternatively, if p_M is slightly negative and I is large, the effect of this negative pressure is greatly enhanced, possibly leading to SEC violations.

We conclude from the previous discussion that two-metric VSL cosmologies do not automatically solve the flatness problem—to solve the flatness problem one needs to make the universe expand rapidly, which means that there are SEC violations (with respect to the *gravity* metric).

Though we disagree with Clayton and Moffat on the technical issue of whether two-metric VSL cosmologies can automatically solve the flatness problem, we do wish to emphasise that we are largely in agreement with those papers on other issues—in particular, we strongly support the two-metric approach to VSL cosmologies. Furthermore, as we will now discuss, we agree that two-metric VSL cosmologies naturally lead to an amplification of any inflationary tendencies that might be present in those fields that couple to the photon metric.

C. Flatness in heterotic (inflaton+χVSL) models

To conclude this section we will show how two-metric VSL cosmologies *enhance* any inflationary tendencies in the matter sector. Let us suppose that we have an inflaton field coupled to the *electromagnetic* metric. We know that during the inflationary phase we can approximately write

$$T^{\mu\nu}_{\text{inflaton}}\propto g^{\mu\nu}_{\text{em}}. \tag{69}$$

We have repeatedly emphasized that it is important to define *the* physical energy density and pressure (ε,p) as the appropriate components of the stress-energy tensor when referred to an orthonormal basis *of the metric that enters the Einstein equation*. The condition $T^{\mu\nu}_{\text{inflaton}}\propto g^{\mu\nu}_{\text{em}}$, when expressed in terms of an orthonormal basis of the metric g asserts

$$p_{\text{inflaton}}=-\frac{c^2_{\text{photon}}}{c^2_{\text{gravity}}}\varepsilon_{\text{inflaton}}. \tag{70}$$

That is

$$(\varepsilon+3p)_{\text{inflaton}}=\left(1-3\frac{c^2_{\text{photon}}}{c^2_{\text{gravity}}}\right)\varepsilon_{\text{inflaton}}. \tag{71}$$

Thus, any ''normal'' inflation will be amplified during a VSL epoch. It is in this sense that VSL cosmologies heterotically improve standard inflationary models.

We can generalize this argument. Suppose the ''normal'' matter, when viewed from an orthonormal frame adapted to the *electromagnetic* metric, has energy density $\tilde{\varepsilon}$ and pressure $\tilde{p}$. From our previous discussion [Eqs. (31)–(33)] we deduce

$$\varepsilon+3p=\frac{c_{\text{gravity}}}{c_{\text{photon}}}\tilde{\varepsilon}+3\frac{c_{\text{photon}}}{c_{\text{gravity}}}\tilde{p}. \tag{72}$$

[Compare with Eqs. (64) and (68) above.] In particular, if $\tilde{p}$ is slightly negative, VSL effects can magnify this to the point of violating the SEC (defined with respect to the gravity metric). It is in this sense that two-metric VSL cosmologies provide a natural enhancing effect for negative pressures (possibly leading to SEC violations), even if they do not provide the seed for a negative pressure.

We point out that this same effect makes it easy to violate *all* the energy conditions. If $(\tilde{\varepsilon},\tilde{p})$ satisfy all the energy conditions with respect to the photon metric, and provided $\tilde{p}$ is only slightly negative, then VSL effects make it easy for (ε,p) to violate all the energy conditions with respect to the gravity metric—and it is the energy conditions with respect to the gravity metric that are relevant to the singularity theorems, positive mass theorem, and topological censorship theorem.

VI. THE ENTROPY PROBLEM

It is interesting to note that (at least in the usual framework) the two major cosmological puzzles described above (isotropy/horizon and flatness) can be reduced to a single problem related to the huge total amount of entropy that our universe appears to have today [57–59,16]. If we define $s\propto T^3$ the entropy density associated with relativistic particles and $S=a^3(t)s$ the total entropy per comoving volume, then it is easy to see from the Friedmann equation (43) that

$$a^2=\frac{Kc^2_{\text{gravity}}}{H^2(\Omega-1)}, \tag{73}$$

and so

$$S=\left[\frac{Kc^2_{\text{gravity}}}{H^2(\Omega-1)}\right]^{3/2}s. \tag{74}$$

The value of the total entropy can be evaluated at the present time and comes out to be $S>10^{87}$. One can then see that explaining why $\Omega\approx1$ (the flatness problem) is equivalent to explaining why the entropy of our universe is so huge.

In a similar way one can argue (at least in the usual framework) that the horizon problem can be related to the entropy problem [57–59]. In order to see how large the causally connected region of the universe was at the time of decoupling with respect to our present horizon, we can compare the particle horizon at time t for a signal emitted at $t=0$, $l_{\mathrm{h}}(t)$, with the radius at same time, $L(t)$, of the region which now corresponds to our observed universe of radius L_{present}. The fact that (assuming insignificant entropy production between decoupling and the present epoch) $(l_{\mathrm{h}}/L)^3|_{t_{\mathrm{decoupling}}} \ll 1$ is argued to be equivalent to the horizon problem. Once again, a mechanism able to greatly increase S via a non-adiabatic evolution would also automatically lead to the resolution of the puzzle.

χVSL cosmologies evade this connection between the horizon and flatness puzzles: We have just seen that although the horizon problem is straightforwardly solved, it is impossible to solve the flatness dilemma (at least in pure χVSL models). To understand how this may happen is indeed very instructive.

First of all, we can try to understand what happens to the entropy per comoving volume $S=a^3(t)s$. In the case of inflation we saw that the non-adiabatic evolution $\dot{S}\neq 0$ was due to the fact that although the entropy densities do not significantly change, $s_{\mathrm{before}} \approx s_{\mathrm{after}}$ thanks to reheating, nevertheless the enormous change in scale factor $a(t_{\mathrm{after}})=\exp[H(t_{\mathrm{after}}-t_{\mathrm{before}})]\cdot a(t_{\mathrm{before}})$ drives an enormous increase in total entropy per comoving volume. (Here ''before'' and ''after'' are intended with respect to the inflationary phase.)

In our case (bimetric VSL models) the scale factor is unaffected by the transition in the speed of light if the χ field is not the dominant energy component of the universe. Instead what changes is the entropy density s. As we have seen, a sudden phase transition affecting the speed of light induces particle creation and raises both the number and the average temperature of relativistic particles. Therefore one should expect that s grows as $c_{\mathrm{photon}} \rightarrow c_{\mathrm{gravity}}$.

From Eq. (54) it is clear that the increased speed of light is enough to ensure a resolution of the horizon problem, regardless of what happens to the entropy. At the same time one can instead see that the flatness problem is not solved at all. Equation (74) tells us that it is the *ratio* $SH^3/s \approx \dot{a}^3$ which determines the possibility of stretching the universe. Unfortunately this is not a growing quantity in the standard model as well as in *pure* bi-metric VSL theory. Once again only violations of the SEC ($\ddot{a}>0$) can lead to a resolution of the flatness problem.

VII. OBSERVATIONAL TESTS AND THE LOW-REDSHIFT χVSL UNIVERSE

At this point, it is important to note that due to the nature of the interaction (11), the χ field appears unable to decay completely. Decay of the χ field proceeds via $2\chi \rightarrow 2\gamma$ and hence, once the density of χ bosons drops considerably, ''freeze-out'' will occur and the χ field will stop decaying. This implies that the χ field *may* be dynamically important at low-redshift *if* its potential is such that its energy density drops less rapidly than that of radiation.

However, the χ correction to g_{em} corresponds to a dimension twelve operator, which is highly non-renormalizable. The vector model of Moffat [5] is a dimension eight operator. Nevertheless, for energies below M it is difficult to argue why either of these operators will not be negligibly small relative to dimension five operators, which would cause single body decays of the χ field. While it is possible that these dimension five operators are absent through a global symmetry [60], or the lifetime of the χ bosons is extremely long, we will see later that such non-renormalizable interactions with the standard model give rise to serious constraints. For the time being we neglect single-body decays, and we can imagine two natural dark-matter candidates, with the added advantage that they are distinguishable and detectable, at least in principle.

(i) If $V(\chi)$ has a quadratic minimum, the χ field will oscillate about this minimum and its average equation of state will be that of dust. This implies that the χ field will behave like axions or cold dark matter. Similarly if the potential is quartic, the average equation of state will be that of radiation.

(ii) If $V(\chi)$ has quintessence form, with no local minimum but a global minimum at $\chi \rightarrow \infty$. A typical candidate is a potential which decays to zero at large χ (less rapidly than an exponential) with $V(\chi) > Ae^{-\lambda\chi}$ for $\lambda>0$.

These two potentials lead to interesting observational implications for the low-redshift universe which we now proceed to analyze and constrain.

A. Clustering and gravitational lensing

It is interesting to note that the effective refractive index we introduced in Eq. (57) may depend, not just on time, but also on space and have an anisotropic structure. In particular the dispersion relation of photons in an anisotropic medium reads

$$\omega^2=[n^{-2}]^{ij}k_i k_j, \tag{75}$$

and from the above expression it is easy to see that the generalization of Eq. (57) then takes the form

$$[n^{-2}]^{ij}=g^{ij}_{\mathrm{em}}/|g^{tt}_{\mathrm{em}}|. \tag{76}$$

Scalar fields do not support small scale density inhomogeneities (largely irrespective of the potential). This implies that the transfer function tends to unity on small scales and the scalar field is locally identical to a cosmological constant.

However, on scales larger than 100 Mpc, the scalar field can cluster [61]. During such evolution both $\dot{\chi}\neq 0$ and $\partial_i \chi \neq 0$ will hold. This would lead to deviations from Eq. (57), as the ratio between the two speeds of light will not be only a function of time.

For instance, let us suppose we are in a regime where time derivatives of χ can be neglected with respect to spatial derivatives. Under these conditions the EM metric reduces to

$$g^{\mathrm{em}}_{tt}=g_{tt}=-|g_{tt}|, \tag{77}$$

$$g^{\mathrm{em}}_{ij}=g_{ij}-(AM^{-4})\partial_i\chi\partial_j\chi. \tag{78}$$

From Eq. (76) this is equivalent to a tensor refractive index n_{ij}, with

$$[n^2]_{ij}=\frac{g_{ij}-(AM^{-4})\partial_i\chi\partial_j\chi}{|g_{tt}|}. \tag{79}$$

This tensor refractive index may lead to additional lensing by large-scale structure, over and above the usual contribution from gravitational lensing [62].

B. Quintessence and long-range forces

Another natural application is to attempt to use the χ field as the source of the ''dark energy'' of the universe, the putative source of cosmic acceleration. This is attractive for its potential to unify a large number of disparate ideas, but is severely constrained as well.

1. Constraints arising from variation of the fine-structure constant

As noted in the Introduction, a change of $c_{\rm photon}$ will cause a variation in the fine-structure constant. Such variation is very constrained. We point out two particularly interesting constraints. The first, arising from nucleosynthesis [63], is powerful due to the extreme sensitivity of nucleosynthesis to variations in the proton-neutron mass difference, which in turn is sensitive to α. This places the tight constraint that $|\dot{\alpha}/\alpha|\leqslant 10^{-14}\ {\rm yr}^{-1}$. However, this is only a constraint on $\dot{c}_{\rm photon}/c_{\rm photon}$ if no other constants appearing in α are allowed to vary. Further we have assumed $\dot{\alpha}$ was constant through nucleosynthesis.

A similar caveat applies to other constraints one derives for variations of $c_{\rm photon}$ through variations of α. Other tests are only sensitive to integrated changes in α over long time scales. At redshifts $z\leqslant 1$ constraints exist that $|\Delta\alpha/\alpha|<3\times 10^{-6}$ (quasar absorption spectra [64]) and $|\Delta\alpha/\alpha|<10^{-7}$ (Oklo natural reactor [65]).

2. Binary pulsar constraints

Unless we choose the unattractive solution that χ lies at the minimum of its potential but has non-zero energy (i.e., an explicit Λ term), we are forced to suggest that $\dot{\chi}\neq 0$ today and $V(\chi)$ is of the form $e^{-\lambda\chi}$ or χ^{-n} [66]. In this case, gravitons and photons do not travel at the same speed today. The difference in the two velocities is rather constrained by binary pulsar data to be less than 1% [18]; i.e., $|n_{\rm em}-1|<0.01$.

3. High-energy tests of VSL

Constraints on our various actions S_I-S_{III} also come from high energy experiments. In model *I*, photons travel faster than any other fields. This would lead to perturbations in the spectrum of nuclear energy levels [67].

Similarly, high energy phenomena will be sensitive to such speed differences. For example, if $c_{\rm photon}>c_{e^-}$, the process $\gamma\rightarrow e^-+e^+$ becomes kinematically possible for sufficiently energetic photons. The observation of primary cosmic ray photons with energies up to 20 TeV implies that today $c_{\rm photon}-c_{e^-}<10^{-15}$ [40]. The reverse possibility—which is impossible in our model I if $A>0$ in Eq. (21)—is less constrained, but the absence of vacuum Cherenkov radiation with electrons up to 500 GeV implies that $c_{e^-}-c_{\rm photon}<5\times 10^{-13}$. Similar constraints exist which place upper limits on the differences in speeds between other charged leptons and hadrons [40,68]. These will generally allow one to constrain models I–III, but we will not consider such constraints further.

4. Non-renormalizable interactions with the standard model

Our χVSL model is non-renormalizable and hence one expects an infinite number of *M*-scale suppressed, dimension five and higher, interactions of the form

$$\beta_i\frac{\chi^n}{M^n}\mathcal{L}_i, \tag{80}$$

where β_i are dimensionless couplings of order unity and $\mathcal{L}_i$ is any dimension-four operator such as $F^{\mu\nu}F_{\mu\nu}$.

For sub-Planckian χ-field values, the tightest constraints typically come from $n=1$ (dimension five operators) and we focus on this case. The non-renormalizable couplings will cause time variation of fundamental constants and rotation of the plane of polarization of distant sources [69]. For example, with $\mathcal{L}_{QCD}={\rm Tr}(G_{\mu\nu}G^{\mu\nu})$, where $G_{\mu\nu}$ is the QCD field strength, one finds the strict limit [70]

$$|\beta_{G^2}|\leqslant 10^{-4}(M/M_{\rm Planck}) \tag{81}$$

which, importantly, is χ independent.

If one expects that $|\beta_i|=O(1)$ on general grounds, then this already provides as strong a constraint on our model as it does on general quintessence models. This constraint is not a problem if there exist exact or approximate global symmetries [60]. Nevertheless, without good reason for adopting such symmetries this option seems unappealing.

Another dimension five coupling is given by Eq. (80) with $\mathcal{L}_{F^2}=F_{\mu\nu}F^{\mu\nu}$ which causes time-variation in α. Although there is some evidence for this [19], other tests have been negative as discussed earlier. These yield the constraint [60]

$$|\beta_{F^2}|\leqslant 10^{-6}(MH/\langle\dot{\chi}\rangle). \tag{82}$$

Clearly this does not provide a constraint on χVSL unless we envisage that $\dot{\chi}\neq 0$ today as required for quintessence. If χ has been at the minimum of its effective potential since around $z<5$, then neither this, nor the binary pulsar, constrain χVSL models. The CMB provides a more powerful probe of variation of fundamental constants and hence provides a test of χVSL if χ did not reach its minimum before $z\simeq 1100$ [71].

Another interesting coupling is $\mathcal{L}_{F*F}=F_{\mu\nu}{}^*F^{\mu\nu}$, where ${}^*F$ is the dual of F. As has been noted [60], this term is not suppressed by the exact global symmetry $\chi\rightarrow\chi+{\rm constant}$, since it is proportional to $(\nabla_\mu\chi)A_\nu{}^*F^{\mu\nu}$. A non-zero $\dot{\chi}$ leads to a polarization-dependent $(\pm)$ deformation of the dispersion relation for light

$$\omega^2 = k^2 \pm \beta_{F*F}(\dot{\chi} k/M). \tag{83}$$

If $\dot{\chi} \neq 0$ today, the resulting rotation of the plane of polarization of light traveling over cosmological distances is potentially observable. Indeed claims of such detection exist [72]. However, more recent data is consistent with no rotation [73,74]. Ruling out of this effect by high-resolution observations of large numbers of sources would be rather damning for quintessence but would simply restrict the χ field to lie at its minimum, i.e., $\Delta\chi \simeq 0$ for $z < 2$.

On the other hand, a similar and very interesting effect arises not from $\dot{\chi}$ but from spatial gradients of χ at low-redshifts due to the tensor effective refractive index of spacetime.

VIII. DISCUSSION

In this paper we have tried to set out a geometrically consistent and physically coherent formalism for discussing variable speed of light (VSL) cosmologies. An important observation is that taking the usual theory and simply replacing $c \to c(t)$ is *more radical a step than strictly necessary*. One either ends up with a coordinate change which does not affect the physics, or one is forced to move well outside the usual mathematical framework of Lorentzian differential geometry. In particular, replacing $c \to c(t)$ in the Einstein tensor of an FLRW universe violates the Bianchi identities and energy conservation and destroys the usual geometrical interpretation of Einstein gravity as arising from spacetime curvature. We do not claim that such a procedure is necessarily wrong, but point out that it is a serious and fundamental modification of our usual ideas.

In contrast, in the class of χVSL cosmologies presented in this article, where the Lorentz symmetry is ''softly broken,'' the ''geometrical interpretation'' is preserved, and the Bianchi identities are fulfilled. In particular, these ''soft breaking'' VSL scenarios are based on straightforward extensions of known physics, such as the Scharnhorst effect and anomalous electromagnetic propagation in gravitational fields, and so represent ''minimalist'' implementations of VSL theories. Indeed, these non-renormalizable VSL-inducing couplings should exist in supergravity theories, though they would be expected to be negligible at low energies.

In this article, we have argued for the usefulness of a two-metric approach. We have sketched a number of two-metric scenarios that are compatible with laboratory particle physics, and have indicated how they relate to the cosmological puzzles. We emphasize that there is considerable freedom in these models, and that a detailed confrontation with experimental data will require the development of an equally detailed VSL model. In this regard VSL cosmologies are no different from inflationary cosmologies. Since the models we discuss are non-renormalizable however, there may be interesting implications for the low-redshift universe through gravitational lensing and birefringence.

VSL cosmologies should be seen as a general scheme for attacking cosmological problems. This scheme has some points in common with inflationary scenarios, but also has some very strange peculiarities of its own. In particular, once $c_{\text{photon}} \neq c_{\text{gravity}}$ complications may appear in rather unexpected places.

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APPENDIX: VARYING c_{gravity}, KEEPING c_{photon} FIXED

In contrast with the main thrust of this paper, we will now ask what happens if we keep c_{photon} fixed, while letting c_{gravity} vary. This means that we are still dealing with a two-metric theory, and so it still makes sense to define VSL in terms of the ratio $c_{\text{photon}}/c_{\text{gravity}}$. Keeping c_{photon} fixed has the advantage that the photon sector (or more generally the entire matter sector) has the usual behavior. However a variable c_{gravity} has the potential for making life in the gravity sector rather difficult.

To make this model concrete, consider a relationship between the photon metric and the gravity metric of the form

$$[g_{\text{gravity}}]_{\alpha\beta} = [g_{\text{em}}]_{\alpha\beta} + (AM^{-4})\nabla_\alpha\chi\nabla_\beta\chi, \tag{A1}$$

where we now take g_{em} as fundamental, and g_{gravity} as the derived quantity. We postulate an action of the form

$$S_{IV} = \int d^4x \sqrt{-g_{\text{gravity}}}\, R(g_{\text{gravity}}) + \int d^4x \sqrt{-g_{\text{em}}}\, \mathcal{L}_{\text{matter}}(g_{\text{em}}, \psi, \chi), \tag{A2}$$

where the matter Lagrangian now includes *everything* non-gravitational and the χ field. The matter equations of motion are the usual ones and it makes most sense to define the stress-energy tensor with respect to the photon metric. (That is, use $\tilde{T}^{\mu\nu}$ as the primary quantity.) The Einstein equation is modified to read

$$\sqrt{\frac{g_{\rm gravity}}{g_{\rm em}}} G^{\mu\nu}\Big|_{g_{\rm gravity}=g_{\rm em}+AM^{-4}\nabla\chi\otimes\nabla\chi} = \tilde{T}^{\mu\nu}. \qquad (A3)$$

Though minor technical details differ from the approach adopted in this paper ($c_{\rm gravity}$ fixed, $c_{\rm photon}$ variable), the results are qualitatively similar to our present approach. We will for the time being defer further discussion of this possibility.

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