Frame dragging in the spacetime of a superconducting cosmic string

Marcus J. Thatcher* and Michael J. Morgan†
Department of Physics, Monash University, Victoria 3800, Australia
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In this paper we calculate particle geodesics in the spacetime of a supermassive superconducting cosmic string ($G\mu > 10^{-3}$). Numerical techniques are used to compare the spacetime of a string supporting a fermionic supercurrent with a string supporting a stationary spin-0 current condensate. In both cases it is found that frame dragging leads to exotic behavior of geodesics, in which particles are trapped by the string spacetime, without the string violating the dominant energy condition. In the case of a string with a fermionic current it is also found that the string core is completely isolated from nonrelativistic particles in the ''outside'' universe.

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I. INTRODUCTION

Grand unified theories (GUTs) predict topological defects which formed during phase transitions in the early Universe [1]. Of particular interest is cosmic string defects having unusual spacetime properties, capable of generating the cosmic microwave background (CMB) anisotropy [2–4] and large scale structure [5–7]. Present observational constraints restrict the mass per unit length of a cosmic string μ to satisfy $G\mu \leqslant 10^{-6}$ (where G is Newton's constant). It is possible that supermassive cosmic strings existed in the early Universe and became unstable at a subsequent phase transition.

In a previous paper [8] we considered the spacetime of a supermassive superconducting cosmic string ($G\mu > 10^{-3}$). Although the spacetime of superconducting cosmic strings has been examined previously (see, e.g., [9–13]), we were motivated to examine the spacetime near a ''vorton'' [14]. In this case it is appropriate to consider a frame where the momentum is nonzero. Analytic and numerical solutions to the exterior spacetime of a supermassive superconducting cosmic string suggest that such objects are associated with exotic spacetime properties. In particular, the solution exhibits oscillatory metric components, resulting in the isolation of the string from particles (geodesics) in the ''outside'' Universe.

It was subsequently pointed out by Gleiser and Tiglio [15] that for a string to have an oscillatory spacetime metric, the string must violate the dominant energy condition (DEC). However, this is not to say that realistic superconducting cosmic strings do not exhibit exotic spacetime behavior. In this paper we show that the spacetime of a (realistic) supermassive superconducting cosmic string does indeed prevent particles from escaping, or reaching, the string core. Furthermore, the rotation of timelike vectors and apparent causal paradoxes discussed in [8] are still exhibited by the string spacetime when the DEC is not violated. This is because the exotic effects are a consequence of frame dragging due to the spacetime, and are not dependent on the oscillatory behavior of the metric components. In particular, we consider the spacetime of a string with a fermionic supercurrent which cannot be boosted to a frame where the current condensate is stationary. The results are compared to the spacetime of a string with a spin-0 supercurrent in a frame where the current condensate is stationary.

The organization of this paper is as follows. In Sec. II we briefly review the oscillatory spacetime solution and its exotic properties. In Sec. III we numerically calculate the spacetime of a supermassive string with a fermionic supercurrent. The numerical solution is employed in Sec. IV to examine the behavior of particle geodesics in the string spacetime and to demonstrate exotic effects. In Sec. V these results are compared to a string with a spin-0 supercurrent. The main results and implications of this work are summarized in Sec. VI.

II. OSCILLATORY SPACETIME

In [8] we obtained a spacetime solution for a straight superconducting cosmic string (orientated along the z direction) of the form

$$ds^2 = \cos[4GM\ln(r)](dt^2 - dz^2) - 2\sin[4GM\ln(r)]dt\,dz - dr^2 - (1-4G\mu)^2 r^2 d\theta^2, \tag{1}$$

where M is the momentum per unit length, μ is the mass per unit length and $G = 6.72\times 10^{-39}$ GeV^{-2} is Newton's constant in natural units. However, as pointed out by Gleiser and Tiglio [15], Eq. (1) is not regular at $r=0$. The regular exterior (vacuum) solution is

$$ds^2 = r^{b/2}\cos[4GM\ln(r)](dt^2 - dz^2) - 2r^{b/2}\sin[4GM\ln(r)]dt\,dz - dr^2 - (1-4G\mu)^2 r^{2-b} d\theta^2, \tag{2}$$

where b is a constant determined by

$$b = \frac{4}{3}\{1 \pm [1 + 12(GM)^2]^{1/2}\}. \tag{3}$$

To accord with a conical spacetime in the limit $M\to 0$ we need only consider the negative solution for b in Eq. (2).

*Email address: Marcus.Thatcher@sci.monash.edu.au
†Email address: Michael.Morgan@sci.monash.edu.au

In [8] we showed that the oscillatory spacetime is associated with exotic spacetime properties. In particular, timelike vectors were found to rotate through the directions t, $t-z$, z, $t+z$, and back to t with increasing radial distance from the string core. This indicates that the light cone is forced to rotate with radial distance, resulting in restrictions on particle trajectories. For example, it is found that all particle geodesics became undefined at a finite radial distance from the string. In this case particles are dragged in the string spacetime until they are moving at the local speed of light in the z direction and cannot continue to move radially outwards. In a similar way particles in the ''outside'' Universe are prevented from reaching the string. It is found that particles can move backwards in coordinate time and according to an observer at the string core particles return to the string core prior to departing. However, an object moving backwards in coordinate time results in the particle moving along the z direction. Hence causal paradoxes are avoided, since the returning particle is always spacelike separated from the outgoing particle.

Gleiser and Tiglio have shown that for the metric components (2) to be oscillatory requires a violation of the DEC. However, the exotic spacetime effects discussed in this paper are a consequence of frame dragging in the spacetime of a superconducting cosmic string. Although realistic cosmic strings do not possess an oscillatory spacetime, they still retain exotic properties as is shown in Sec. III.

III. STRINGS WITH FERMIONIC SUPERCURRENTS

To demonstrate exotic spacetime effects for a realistic string we consider a string with a fermionic supercurrent. We choose a fermionic supercurrent since it is moving at the speed of light and cannot be boosted to a frame where the momentum is zero. In this case the relationship between frame dragging and exotic effects (see Sec. IV) is emphasized.

We begin by writing the general form of the spacetime metric of a straight string with constant momentum

$$ds^2=\Lambda(r)dt^2-2\Phi(r)dtdz-dr^2-\Psi^2(r)d\theta^2-\Omega(r)dz^2, \tag{4}$$

where the string is orientated along the z axis and $g_{tz}=g_{zt}=\Phi(r)$. The Ricci tensor components and boundary conditions at $r=0$ have been discussed previously in [8]. To simplify the equations of motion we use the relation

$$2\Phi(r)=\Lambda(r)-\Omega(r), \tag{5}$$

for which the field equation for R_{tz} is automatically satisfied. Equation (5) is a consequence of invariance under Lorentz boosts in the z direction.

To describe a cosmic string we use the Abelian symmetry breaking model, with metric signature diag$(+,-,-,-)$:

$$\mathcal{L}=(D^\mu\phi)^\dagger D_\mu\phi+\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}-\tfrac{1}{2}\lambda(\phi^\dagger\phi-\eta^2)^2, \tag{6}$$

where ϕ is the Higgs field, $D_\mu=\nabla_\mu-iqA_\mu$ is the gauge covariant derivative, ∇_μ is the ''conventional'' covariant derivative, A_μ is the $U(1)$ vector boson field, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, q is the coupling strength between the vector and Higgs fields and λ and η determine the form of the Mexican hat (symmetry breaking) potential. We have chosen a local string to avoid singularities typically associated with the spacetime of a global string [16–18]. The fermionic current fields are coupled to the Higgs field in the usual fashion (see, e.g., [19]):

$$\mathcal{L}_F=i\psi_L^\dagger\gamma^0\gamma^\mu\nabla_\mu\psi_L+i\psi_R^\dagger\gamma^0\gamma^\mu\nabla_\mu\psi_R-g_f\psi_L^\dagger\gamma^0\phi\psi_R-g_f\psi_R^\dagger\gamma^0\phi^\dagger\psi_L, \tag{7}$$

where ψ_L and ψ_R are the left and right helicity lepton fields and g_f is the coupling between the lepton and Higgs field. For simplicity we have neglected gauge fields in the description of the fermionic supercurrent (i.e., we consider a neutral current). However, because of the coupling between the Higgs and lepton fields, the Lagrangian is only invariant under global gauge transformations of the Higgs field. Nevertheless, the results can be generalized to charged fermionic supercurrents and therefore the model provides a description of a general fermionic current.

To reduce the number of independent parameters we rescale according to $t\rightarrow\delta t$, $\mathbf{x}\rightarrow\delta\mathbf{x}$, $\phi\rightarrow\eta\phi$, $A_\mu\rightarrow\eta A_\mu$, $\psi_{L,R}\rightarrow\eta^{3/2}(\lambda/2)^{1/4}\psi_{L,R}$, $q\rightarrow(\lambda/2)^{1/2}q$, and $g_f\rightarrow(\lambda/2)^{1/2}g_f$, where $\delta=(\lambda\eta^2/2)^{1/2}$ is the Compton width of the Higgs field. We write the equations of motion for the Higgs, vector boson and fermion fields for an arbitrary spacetime (in the Lorentz gauge):

$$D^\mu D_\mu\phi+\phi(|\phi|^2-1)-g_f\psi_R^\dagger\gamma^0\psi_L=0 \tag{8a}$$

$$\nabla^\mu\nabla_\mu A_\nu+2qIm[\phi^\dagger D_\nu\phi]=0 \tag{8b}$$

$$i\gamma^\mu\nabla_\mu\psi_L-g_f\phi\psi_R=0 \tag{8c}$$

$$i\gamma^\mu\nabla_\mu\psi_R-g_f\phi^\dagger\psi_L=0. \tag{8d}$$

To solve for the superconducting cosmic string in a curved spacetime we use the *Ansätze*

$$\phi(r,\theta)=f(r)e^{i\theta} \tag{9a}$$

$$A_\theta=\frac{a_\theta(r)}{q\Psi(r)} \tag{9b}$$

$$A_t=A_r=A_z=0 \tag{9c}$$

$$\psi_L(t,r,z)=\chi s(r)e^{ik(z-t)} \tag{9d}$$

$$\psi_R(t,r,z)=-i\gamma^1\psi_L(t,r,z), \tag{9e}$$

where k is a constant and χ is a spinor, defined by $\chi=-\gamma_5\chi$ and $\chi=i\gamma_1\gamma_2\chi$. This form of χ results in a zero contribution from the fermionic fields in Eq. (8a) (i.e., no backreaction). The *Ansätze* (9) represent the conventional description of a cosmic string supporting a fermionic current condensate (i.e., a zero mode [19]). From an index theorem due to Weinberg [20], we know that this is the only zero mode solution for a cosmic string with winding number $n=1$. Substituting the superconducting string *Ansätze* (9) into the equations of motion (8) gives

$$\nabla_r^2 f(r) - \frac{f(r)}{\Psi^2(r)}(a_\theta(r)-1)^2 - f(r)(f^2(r)-1) = 0 \tag{10a}$$

$$\frac{\Omega(r)}{\Phi^2(r)+\Lambda(r)\Omega(r)}\left(\frac{d^2a_\theta(r)}{dr^2}\right)+\frac{\Psi^2(r)}{\sqrt{-g(r)}}\left[\frac{d}{dr}\left(\frac{\Omega(r)}{\sqrt{-g(r)}}\right)\right] \times\left(\frac{da_\theta(r)}{dr}\right) - 2q^2 f^2(r)(a_\theta(r)-1) = 0 \tag{10b}$$

$$\frac{ds(r)}{dr} + g_f f(r) s(r) = 0 \tag{10c}$$

$$[2\Phi(r) - \Lambda(r) + \Omega(r)]s(r)e^{ik(z-t)} = 0, \tag{10d}$$

where ∇_r^2 is

$$\nabla_r^2 = \frac{d^2}{dr^2} + \frac{1}{\sqrt{-g(r)}}\left(\frac{d\sqrt{-g(r)}}{dr}\right)\frac{d}{dr}, \tag{11}$$

and $g(r)$ is the determinant of the metric:

$$g(r) = -\Psi^2(r)[\Phi^2(r) + \Lambda(r)\Omega(r)]. \tag{12}$$

The equation of motion for $s(r)$ in Eq. (10c) is identical to the Minkowski spacetime form and has the exact solution

$$s(r) = \exp\left(-g_f\int_0^r f(\rho)d\rho\right). \tag{13}$$

The equation of motion (10d) governing the current condensate is also readily solved using the relationship between metric components in Eq. (5). Therefore, we only need to numerically solve for $f(r)$ and $a_\theta(r)$ in curved spacetime. The boundary conditions imposed on the Higgs field and vector boson field must reconcile an undefined phase at the center of the defect and finite energy far from the defect, i.e.,

$$f(0) = 0, \quad f(r\to\infty) = 1, \tag{14}$$

$$a_\theta(0) = 0, \quad a_\theta(r\to\infty) = 1. \tag{15}$$

To determine the field equations we first calculate the components of the energy-momentum tensor for the superconducting cosmic string:

$$T_{\mu\nu} = 2\frac{\delta\mathcal{L}}{\delta g^{\mu\nu}} - g_{\mu\nu}\mathcal{L}. \tag{16}$$

The nonzero components are

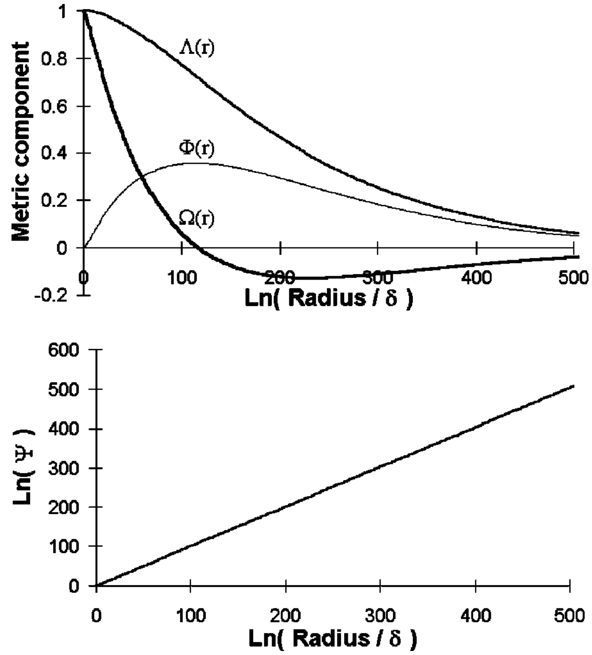

FIG. 1. The metric components for a cosmic string with a fermionic supercurrent ($k=5\times10^{-4}\delta$ and $g_f\delta\eta=1$). Some aspects of the spacetime behavior observed in the analytic solution (2) are retained in our numerical model. In particular it is noted that $\Omega(r)$ becomes negative at some distance from the string core.

$$T_{tt} = \frac{\eta^2}{\delta^2}[4ks^2(r) + \Lambda(r)L(r)] \tag{17a}$$

$$T_{rr} = \frac{\eta_\phi^2}{\delta^2}\left[2\left(\frac{df(r)}{dr}\right)^2 + \frac{1}{q^2\Psi^2(r)}\left(\frac{da_\theta(r)}{dr}\right)^2 - L(r)\right] \tag{17b}$$

$$T_{\theta\theta} = \frac{\eta^2}{\delta^2}\left[2f^2(r)[a_\theta(r)-1]^2 + \frac{1}{q^2}\left(\frac{da_\theta(r)}{dr}\right)^2 - \Psi^2(r)L(r)\right] \tag{17c}$$

$$T_{zz} = \frac{\eta^2}{\delta^2}[4ks^2(r) - \Omega(r)L(r)] \tag{17d}$$

$$T_{tz} = T_{zt} = \frac{\eta^2}{\delta^2}[-4ks^2(r) - \Phi(r)L(r)], \tag{17e}$$

where

$$L(r) = \left(\frac{df(r)}{dr}\right)^2 + \frac{f^2(r)(a_\theta(r)-1)^2}{\Psi^2(r)} + \frac{1}{2q^2\Psi^2(r)}\left(\frac{da_\theta(r)}{dr}\right)^2 + \frac{1}{2}[f^2(r)-1]^2. \tag{18}$$

The derivation of $L(r)$ in Eq. (18) has been simplified by using Eq. (5).

To highlight the exotic spacetime properties we choose the energy scale of the string to be supermassive, whence $G\eta^2=3\times10^{-2}$ ($\eta\sim10^{18}$ GeV). The momentum per unit length of the string is calculated from the T_{tz} component (17d), and depends on the parameter k according to

$$M=-2\pi\int_0^\infty dr T_{tz}\Psi(r)=2\pi\eta^2\int_0^\infty dr[4ks^2(r)+\Phi(r)L(r)]\Psi(r). \tag{19}$$

In the limit $k\rightarrow0$, the contributions to the energy-momentum tensor from the fermionic current vanish. In this case we obtain $\Phi=0$ and $T_{tz}=0$ as a solution to the Einstein field equations (i.e., zero momentum). In this situation, we find $T_{tt}=-T_{zz}$ (i.e., equivalent to a nonsuperconducting cosmic string) which results in a conical metric [21]. To calculate the spacetime for a cosmic string with a fermionic supercurrent we first numerically solve for the equations of motion of the particle fields (10) using a relaxation technique. The spacetime curvature is calculated from the Einstein field equations (see [8]) and the energy-momentum tensor (17) using a fourth-order Runge-Kutta scheme. The coupled vortex and Einstein field equations are solved iteratively until the solution converges.

An example of a numerical solution to the spacetime of a string with a fermionic supercurrent is shown in Fig. 1. It is important to note that the numerical spacetime solution retains aspects of the analytic metric (2), with the metric component $\Omega(r)$ becoming negative. This is indicative of a rotation of timelike vectors which restricts the ability of particles to escape from the string (Sec. IV). However, $\Lambda(r)$ does not become negative for any parameter choice and hence the spacetime never exhibits oscillatory behavior. This is consistent with Gleiser and Tiglio [15] who argue that the oscillatory behavior of the metric components, g_{tt}, g_{tz}, and g_{zz}, necessitates a violation of the DEC.

Using Eq. (5), the determinant (12) becomes

$$g(r)=-\frac{1}{4}\Psi^2(r)[\Lambda(r)+\Omega(r)]^2. \tag{20}$$

If $\Lambda(r)$ decreases sufficiently, so that $\Lambda(r)=-\Omega(r)$ [i.e., for $\Omega(r)<0$], then the determinant vanishes. Consequently, $\Lambda(r)$ is prevented by the Einstein field equations from adopting the value $-\Omega(r)$, and hence $\Lambda(r)$ is bounded and never becomes negative. Nevertheless, as we show in Sec. IV, the spacetime of a superconducting string can exhibit exotic properties.

IV. PARTICLE GEODESICS

In [8] we showed that a particle moving outwards from the string core is prevented from escaping from the string due to frame dragging in the z direction (i.e., along the current). Consider the geodesic equation for $\dot{r}$ (where an overdot denotes differentiation with respect to the affine parameter):

$$\dot{r}^2=\frac{\Lambda(r)[\Omega(r)-B\Phi(r)]^2+2\Phi(r)[\Omega(r)-B\Phi(r)][\Phi(r)+B\Lambda(r)]-\Omega(r)[\Phi(r)+B\Lambda(r)]^2}{[\Phi^2(r)+\Lambda(r)\Omega(r)]^2}. \tag{21}$$

In Eq. (21) coordinate time t is identified with proper time at $r=0$ and $B=-\dot{z}$ at $r=0$. For $B=0$, we have shown in [8] that the geodesic equation for $\dot{r}$ becomes undefined if $\Omega(r)<0$. This behavior is observed in Fig. 1, where photons corresponding to $B=0$ are trapped by the string spacetime despite the absence of oscillatory spacetime metric components. However, since the spacetime is not oscillatory, there exists a choice of B for which photons can escape to the ''outside'' Universe. This can be seen by defining a function $\beta(r)$, which corresponds to the value of B at a given distance r, for which $\dot{r}=0$, i.e.,

$$\dot{r}(B=\beta(r),r)=0. \tag{22}$$

After some straightforward algebra we can write the function $\beta(r)$ in terms of the metric components

$$\beta(r)=\frac{-\Phi(r)\pm[\Phi^2(r)+\Lambda(r)\Omega(r)]^{1/2}}{\Lambda(r)}. \tag{23}$$

In the case of fermionic supercurrents the expression for $\beta(r)$ can be simplified further by using Eq. (5), i.e.,

$$\beta(r)=\frac{\Omega(r)}{\Lambda(r)} \text{ or } -1. \tag{24}$$

In Fig. 2, we have plotted the function $\beta(r)$ for the numerical spacetime solution in Fig. 1. As we move out radially from the string core we find that the two solutions for $\beta(r)$ approach each other. This means that the range of B values, describing geodesics which extend to some distance r, decreases as r increases.

There is a relationship between imaginary photon geodesics at a finite distance from the string core and the rotation of a timelike vector. To see this we make the coordinate transformation

$$t=\kappa\sin\alpha(r)+\zeta\cos\alpha(r) \tag{25a}$$

$$z=\kappa\cos\alpha(r)-\zeta\sin\alpha(r), \tag{25b}$$

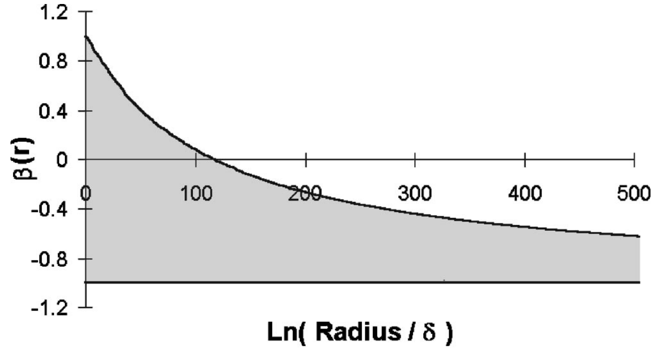

FIG. 2. The function $\beta(r)$ is plotted for the numerical solution to the metric in Fig. 1. The shaded region represents the range of B values corresponding to geodesics which are defined at some distance r. As we move out radially from the string core the range of B, which describes possible geodesics, becomes increasingly restricted. For incoming photons originating at large radial distances all geodesics reach the string core.

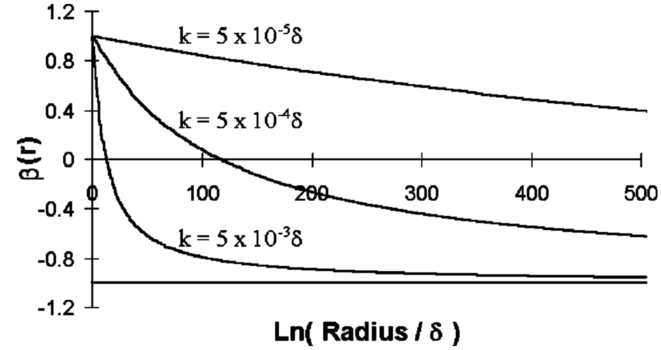

FIG. 3. Dependence of $\beta(r)$ on the parameter k. By increasing k, the range of B values, which describes particle geodesics extending to a distance r, is reduced.

where $\alpha(r)$ is an angle which varies with distance from the string core. In terms of the components $\Phi(r)$, $\Psi(r)$, $\Lambda(r)$, and $\Omega(r)$, the metric in the ζ-κ frame becomes

$$\begin{aligned} ds^2 = &[\Lambda(r)\cos^2\alpha(r) + 2\Phi(r)\cos\alpha(r)\sin\alpha(r) \\ &-\Omega(r)\sin^2\alpha(r)]d\zeta^2 + 2\{[\Lambda(r) \\ &+\Omega(r)]\cos\alpha(r)\sin\alpha(r) - 2\Phi(r)[\cos^2\alpha(r) \\ &-\sin^2\alpha(r)]\}d\zeta d\kappa + [\Lambda(r)\sin^2\alpha(r) \\ &-2\Phi(r)\cos\alpha(r)\sin\alpha(r) - \Omega(r)\cos^2\alpha(r)]d\kappa^2 - dr^2 \\ &-\Psi^2(r)d\theta^2. \end{aligned} \tag{26}$$

In order that ζ remain timelike and κ spacelike for all r, we define $\alpha(r)$ by setting $g_{\zeta\kappa}=0$, which gives

$$\tan[2\alpha(r)] = \frac{2\Phi(r)}{\Lambda(r)+\Omega(r)}. \tag{27}$$

This equation can be further simplified by using Eq. (5) to obtain

$$\tan[2\alpha(r)+\pi/4] = \frac{\Omega(r)}{\Lambda(r)}. \tag{28}$$

At the string core $\alpha(0)=0$, which is consistent with a Minkowski spacetime. As we move out radially from the string core, $\Omega(r)$ and $\Lambda(r)$ decreases (see Fig. 1), with $\Omega(r)$ vanishing at some distance. This behavior results in $\alpha(r)$ decreasing from zero to a negative value and describes the rotation of timelike vectors relative to the t-z coordinate system. Since $\Omega(r)/\Lambda(r)$ is always greater than -1, the angle $\alpha(r)$ can never decrease below $-\pi/4$. Timelike vectors are not forced to rotate into the z direction and not all photon trajectories are trapped by the string spacetime.

To explore the dependence of the cutoff function, $\beta(r)$, on the momentum per unit length, we plot $\beta(r)$ for a superconducting cosmic string with different values of k (see Fig. 3). In the limit $k\rightarrow 0$, the spacetime becomes conical and there is no impediment to particles moving outwards from the string. However, as k increases, geodesics which extend to a given distance, r, are more restricted. Therefore we can reduce the range of $\beta(r)$, describing geodesics which extend to a distance r, to arbitrarily small values by choosing an appropriate value of k.

The spacetime curvature prevents photons from leaving the vicinity of the string, but does not prevent photons from reaching the string core. For the string to be isolated from incoming photons, we require the solutions to $\beta(r)$ [see Eq. (24)] to be equal at some distance ρ, i.e.,

$$\Lambda(\rho) = -\Omega(\rho). \tag{29}$$

However, as discussed in Sec. III this corresponds to the determinant vanishing. Hence it is not possible for a realistic superconducting cosmic string to be cut off from photons in the Universe. This can only be achieved by an oscillatory spacetime for which the string must violate the DEC. Nevertheless, particles which reach the string are constrained to have a velocity in the z direction which is comparable to the speed of the current condensate.

We can extend our analysis to massive uncharged particles moving at speeds less than the local speed of light, for which

$$K = g_{\mu\nu}dx^\mu dx^\nu > 0, \tag{30}$$

where K is a constant. In this case the geodesic equation (21) becomes

$$\dot{r}^2 = \frac{\Lambda(r)[\Omega(r)-B\Phi(r)]^2 + 2\Phi(r)[\Omega(r)-B\Phi(r)][\Phi(r)+B\Lambda(r)] - \Omega(r)[\Phi(r)+B\Lambda(r)]^2}{[\Phi^2(r)+\Lambda(r)\Omega(r)]^2} - K. \tag{31}$$

The maximum radial velocity of a massive particle at a given distance r [i.e., $\dot{z}(r)=0$] is determined by the choice of K according to

$$\dot{r}_{max}=\left[\frac{1}{\Lambda(r)}-K\right]^{1/2}. \tag{32}$$

The speed of a massive particle (measured at $r=0$) is given by

$$v=[1-K]^{1/2}. \tag{33}$$

Furthermore, the maximum value of K for particles at a radius r is

$$K_{max}=\Lambda^{-1}(r). \tag{34}$$

The function $\beta(r)$, which defines where the geodesics become undefined, is modified for a nonzero value of K according to

$$\beta(r)=\frac{-\Phi(r)\pm(\Phi^2(r)+\Lambda(r)\{\Omega(r)-K[\Phi^2(r)+\Lambda(r)\Omega(r)]\})^{1/2}}{\Lambda(r)}. \tag{35}$$

In Figs. 4 and 5 we have plotted the function $\beta(r)$ for various values of K. Unlike photon geodesics in Fig. 2, uncharged massive particles which are moving sufficiently slowly cannot escape from the string core. Moreover, particles which originate in the ''outside'' Universe cannot reach the string core. This is because the maximum value of K is proportional to $\Lambda^{-1}(r)$. Since $\Lambda(r)$ decreases with increasing r, it is possible to choose geodesics which are well defined for $K>1$ (at sufficiently large r).

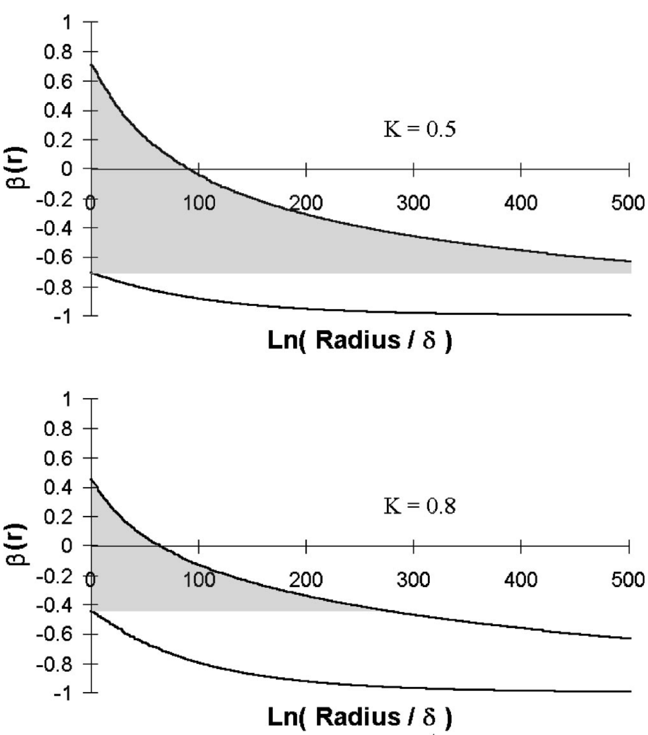

FIG. 4. The function $\beta(r)$ is plotted for various choices of K. The shaded region represents values of B corresponding to geodesics which extend from $r=0$. Note that for $K=0.5$ (corresponding to $0.87c$ at $r=0$), massive particles can still escape from the string core. However, for $K=0.8$ (corresponding to $0.6c$ at $r=0$), uncharged massive particles which originate at $r=0$ cannot escape the string, and particles corresponding to $K=0.8$ in the ''outside'' Universe cannot reach the string core.

V. STRINGS WITH SPIN-0 SUPERCURRENTS

Although the previous analysis applies to strings supporting fermionic supercurrents, exotic spacetime effects are also apparent in models with spin-0 supercurrents. We can illustrate this for a neutral spin-0 current, based on the Witten model [22]. Our numerical scheme for the calculation of the spacetime of a string with a spin-0 supercurrent has been discussed in [8]. To calculate the string vortex, we employ the *Ansätze*

$$\phi(r,\theta)=f(r)e^{i\theta} \tag{36a}$$

$$A_\theta(r)=\frac{a_\theta(r)}{q\Psi(r)} \tag{36b}$$

$$\sigma(t,r,z)=s(r)e^{-i\omega t}, \tag{36c}$$

where $\sigma(t,r,z)$ is the spin-0 current condensate (see [8]) and ω is a constant. Note that we have chosen a frame in which the current condensate is stationary.

Figure 6 shows the spacetime behavior of a string with a spin-0 current condensate. The spacetime in the boosted

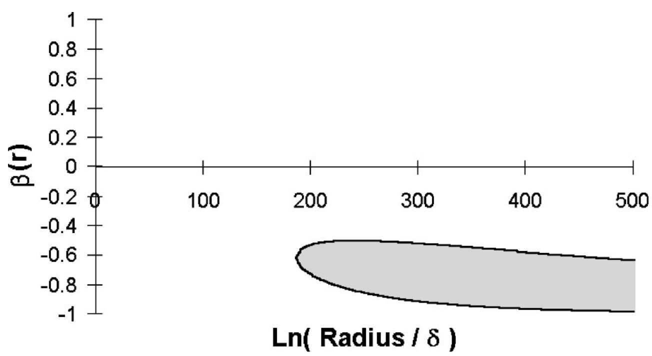

FIG. 5. The function $\beta(r)$ is plotted for $K=2$. The shaded region corresponds to values of B for which geodesics are well defined. When $K=2$ geodesics are only defined for uncharged massive particles at large radial distances from the string core.

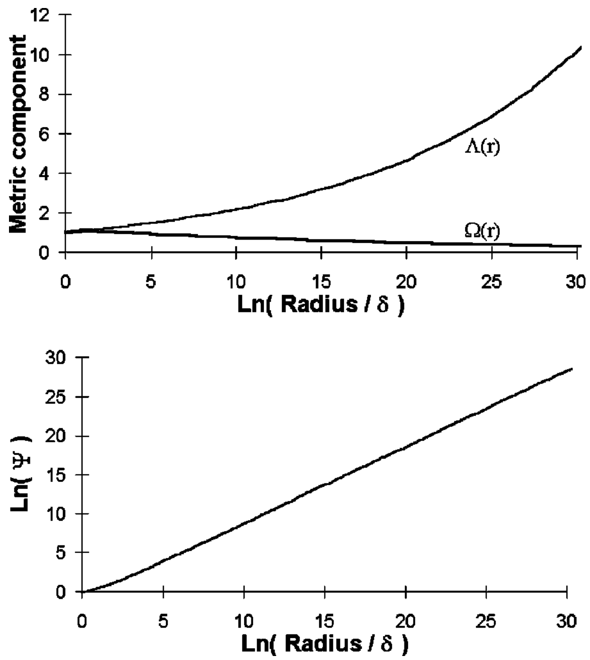

FIG. 6. The metric components for a cosmic string with a spin-0 supercurrent [$\omega = 0.2\delta$, $k=0$, $\beta = 2.381(\delta\eta_\phi)^{-2}$, $\lambda_\sigma = 20.928(\delta\eta_\phi)^{-2}$, $\eta_\sigma = 0.466\eta_\phi$, $q\delta\eta_\phi = 1$, and $e=0$]. The spacetime has been calculated for a frame in which the momentum is zero.

frame exhibits exotic behavior, as can be seen in Fig. 7, where we have plotted $\beta(r)$ for the numerical solution corresponding to the supercurrent shown in Fig. 6. Although the current condensate is stationary, photon geodesics are undefined at sufficiently large r. Furthermore, objects at large radial distances from the string are constrained to move in the z direction at speeds comparable to the speed of the current condensate (as measured by an observer located at the string core). There is no requirement for the current condensate to be moving at the speed of light in order to exhibit exotic spacetime behavior.

However, there are some differences between the spacetime generated by a cosmic string with a fermionic supercur-

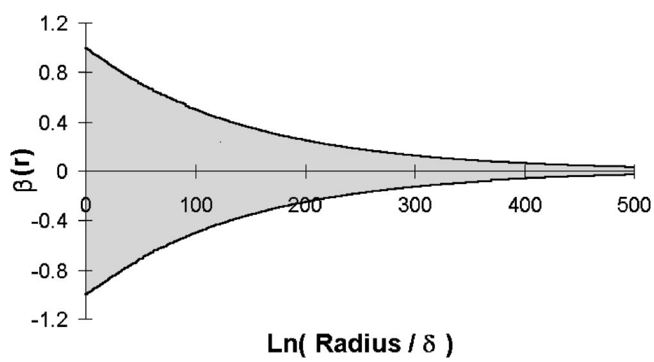

FIG. 7. The function $\beta(r)$ (corresponding to photons for which $K=0$) is plotted for the numerical spacetime shown in Fig. 6. Although the geodesics are calculated in a frame where the momentum is zero, photons are still deflected and the string exhibits exotic behavior.

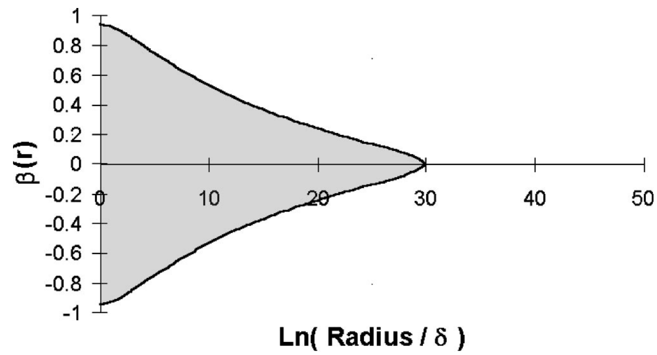

FIG. 8. The function $\beta(r)$ is plotted for $K=0.1$, corresponding to massive particle geodesics. The shaded region indicates geodesics which are well defined at some distance r. Although the current condensate is stationary, nonrelativistic massive particles are prevented from escaping from the vicinity of the string core.

rent (Fig. 1) and that generated by a spin-0 supercurrent (Fig. 6). For example, in Fig. 6 the magnitude of the metric component, $\Lambda(r)$, increases with r, compared to the fermionic current condensate solution in Fig. 1, where the magnitude of the metric component, $\Lambda(r)$, decreases with r. As a consequence massive uncharged particles moving slower than the local speed of light ($K>0$) have K_{max} decreasing with increasing radius. This is the opposite situation to the fermionic current condensate, where K_{max} increases with radius. This means that the spacetime of a string supporting a spin-0 supercurrent has K restricted to

$$0 \leqslant K \leqslant 1, \tag{37}$$

and all geodesics can reach $r=0$. Hence it is not possible for particles to be isolated from the string core (i.e., no geodesics correspond to $K>1$). Nevertheless, massive particles are still trapped in the vicinity of the cosmic string as illustrated in Fig. 8.

VI. CONCLUSION

In this paper we have used two numerical models to show that exotic effects are associated with string defects which do not violate the DEC. The principal difference between the strings considered in this paper and the spacetime discussed in [8,15] is that strings which violate the DEC are completely isolated from particles in the ''outside'' Universe, whereas for strings which satisfy the DEC, particles can escape or reach the string core if they possess a velocity component parallel to the current condensate (which is comparable to the speed of the condensate). Since the trapping of particles by supermassive superconducting cosmic strings is a consequence of frame dragging, the results of this work can be generalized to other types of superconducting cosmic strings.

We have also considered the behavior of nonrelativistic particles in the spacetime of supermassive superconducting cosmic strings. For both superconducting string models nonrelativistic particles are trapped by the string spacetime and are prevented from escaping to the ''outside'' Universe. Significantly, in the case of a cosmic string with a fermionic supercurrent it is possible for nonrelativistic massive particles to be isolated from the string core, despite the string satisfying the DEC. This differs dramatically from the conical spacetime of a nonsuperconducting cosmic string for which there is no impediment to particles reaching the string core.

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