Effects of neutrino mixing on high-energy cosmic neutrino flux

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Several cosmologically distant astrophysical sources may produce high-energy cosmic neutrinos ($E \geq 10^6$ GeV) of all flavors above the atmospheric neutrino background. We study the effects of vacuum neutrino mixing in the three flavor framework on this cosmic neutrino flux. We also consider the effects of possible mixing between the three active neutrinos and the (fourth) sterile neutrino with or without big-bang nucleosynthesis constraints and estimate the resulting final high-energy cosmic neutrino flux ratios on Earth compatible with currently existing different neutrino oscillation hints in a model independent way. Further, we discuss the case where the intrinsic cosmic neutrino flux does not have the standard ratio.

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I. INTRODUCTION

High-energy neutrino ($E \geq 10^6$ GeV) astroparticle physics is now a rapidly developing field impelled by the need for improved flux estimates as well as a good understanding of detector capabilities for all neutrino flavors, particularly in light of recently growing experimental support for flavor oscillations [1].

Currently envisaged astrophysical sources of high-energy cosmic neutrinos include, for instance, active galactic nuclei (AGN) and gamma ray burst (GRB) fireballs [2]. The production of high-energy cosmic neutrinos other than the AGNs and GRBs may also be possible [3].

High-energy neutrino production in cosmologically distant astrophysical systems such as AGNs and GRBs follow mainly from the production and decay of relevant unstable hadrons. These unstable hadrons may be produced mainly when the accelerated protons in these environments interact with the ambient photon field ($p\gamma$) and/or protons (pp) present there. The electron and muon neutrinos (and corresponding anti neutrinos) are mainly produced in the decay chain of charged pions whereas the tau neutrinos (and anti neutrinos) are mainly produced in the decay chain of charmed mesons in the same collisions at a suppressed level [4].

Previously the effects of vacuum neutrino mixing on the intrinsic ratios of high-energy cosmic neutrinos, in the context of three flavors, were briefly considered in [5]. Here we update the description [6] and consider three and four neutrino schemes with the most up-to-date constraints from the terrestrial, solar and atmospheric neutrino experiments. We also discuss the case where the intrinsic high-energy cosmic

neutrino flux has nonstandard ratio which may not be obtained in charged pion decays.

The present study is particularly useful as the new underice or water Čerenkov light neutrino telescopes, namely, AMANDA and Baikal (also the ANTARES and NESTOR) as well as the large (horizontal) shower array detectors will have energy, angle and possibly flavor resolutions for high-energy neutrinos originating at cosmological distances [7]. Several discussions are now available pointing towards the possibility of flavor identification for high-energy cosmic neutrinos [8].

The plan of this paper is as follows. In Sec. II, we discuss the effects of vacuum neutrino mixing in three as well as four flavor schemes and numerically estimate the final (downward going) ratios of flux for high-energy cosmic neutrinos using the ranges of neutrino mixing parameters implied by recent searches for neutrino oscillations. In Sec. III, we discuss the consequences of a hypothesis in which the intrinsic cosmic neutrino flux does not have the standard ratio [$F^0(\nu_e):F^0(\nu_\mu):F^0(\nu_\tau) = 1:2:0$]. In Sec. IV, we summarize our results.

II. EFFECTS OF NEUTRINO MIXING ON HIGH-ENERGY COSMIC NEUTRINO FLUX

In $p\gamma$ and in pp collisions, typically one obtains the following ratio of intrinsic high-energy cosmic neutrinos flux: $F^0(\nu_e):F^0(\nu_\mu):F^0(\nu_\tau) = 1:2:<10^{-5}$. For simplicity, here we take these ratios as $F^0(\nu_e):F^0(\nu_\mu):F^0(\nu_\tau) = 1:2:0$. We also discuss the effects of vacuum neutrino mixing by varying first two of these ratios from their above quoted values as this might be the case under some circumstances [9]. We count both neutrinos and anti neutrinos in the symbol for neutrinos.

We consider an order of magnitude energy interval essentially around 10^6 GeV since the currently available models for high-energy cosmic neutrinos give neutrino flux above

the relevant atmospheric neutrino background specifically from AGN cores within this energy interval. Also in this energy range the flavor identification for high-energy cosmic neutrinos may be conceivable in new km^2 surface area neutrino telescopes [8]. We have checked that for high-energy neutrinos originating from cosmologically distant sources, the change in the flavor composition of the high-energy cosmic neutrinos due to vacuum mixing is essentially energy independent for the entire energy range relevant for observations as the energy effects are averaged out in the relevant neutrino flavor oscillation probability expressions.

It has been pointed out that there are no matter effects on vacuum neutrino oscillations for relevant mass squared difference values $[\mathcal{O}(10^{-10}) \lesssim \Delta m^2/\mathrm{eV}^2 \lesssim \mathcal{O}(1)]$ for high-energy cosmic neutrinos scattering over the matter particles in the vicinity of sources [10]. Nevertheless, for high-energy cosmic neutrinos scattering over the relic neutrinos, in halos or other wise, during their propagation, there may be relatively small (at most, of the order of few percent) matter enhanced flavor oscillation effects under the assumption of rather strong CP asymmetric neutrino background [11]. However, given the current status of high-energy cosmic neutrino flux measurements, vacuum flavor oscillations remain the dominant effect and therefore from now on we consider here the effects of vacuum flavor oscillations only. For some other possible mixing effects, see [12]. The typical distance to these astrophysical sources is taken as $L \simeq 10^2$ Mpc (where 1 pc $\simeq 3 \times 10^{18}$ cm). It is relevant here to mention that our following analysis is not necessarily restricted to high-energy cosmic neutrinos originating only from AGNs or GRBs as the above mentioned ratios of intrinsic neutrino flux can in principle be conceivable from some other cosmologically distant astrophysical sources as well [3]. For simplicity, we assume the absence of relatively dense objects between the cosmologically distant high-energy neutrino sources and the prospective neutrino telescopes so as not to change significantly the vacuum oscillation pattern.

A. Three neutrino scheme

It has been known in the two flavor framework that the solar and the atmospheric neutrino problems are accounted for by neutrino oscillations with $\Delta m^2_{\mathrm{smaller}} \sim 10^{-5}$ eV^2 or 10^{-10} eV^2 and $\Delta m^2_{\mathrm{larger}} \sim 10^{-2.5}$ eV^2, respectively. Without loss of generality we assume that $|\Delta m^2_{21}| < |\Delta m^2_{32}| < |\Delta m^2_{31}|$ where $\Delta m^2_{ij} \equiv m^2_i - m^2_j$ ($i \neq j$; $i,j = 1,2,3$). If both the solar and the atmospheric neutrino problems are to be solved by energy dependent solutions, we have to have $\Delta m^2_{21} \simeq \Delta m^2_\odot$ and $\Delta m^2_{32} \simeq \Delta m^2_{\mathrm{atm}}$, i.e., we have mass hierarchy in this case. Here, the subscripts $\odot$ and atm stand for the mixing angles for the solar and atmospheric neutrino oscillations, respectively.

In the three flavor framework, the flavor and mass eigenstates are related by the MNS matrix U [13]:

$$
\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad U \equiv \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}. \quad (1)
$$

In the present convention for the mass pattern, the disappearance probability for the CHOOZ experiment [14] is given by

$$
P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - 4|U_{e3}|^2(1 - |U_{e3}|^2)\sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right), \quad (2)
$$

and combining the negative result by the CHOOZ experiment [14] with the atmospheric neutrino data, we have (for quantitative discussions, see [15])

$$
|U_{e3}|^2 \ll 1. \quad (3)
$$

As was mentioned in the Introduction, the matter effect is irrelevant in our discussions, so let us consider the vacuum oscillation probability in the case where the oscillation length is very small as compared to the distance between the source and the detector [hereafter referred to as far distance (approximation)]. In vacuum, it is well known that the flavor oscillation probability is given by

$$
P(\nu_\alpha \to \nu_\beta; L) = \delta_{\alpha\beta} - \sum_{j \neq k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k}(1 - e^{-i\Delta E_{jk} L}). \quad (4)
$$

In the limit $L \to \infty$, we have

$$
P(\nu_\alpha \to \nu_\beta; L = \infty) = \delta_{\alpha\beta} - \sum_{j \neq k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k}
$$

$$
= \sum_j |U_{\alpha j}|^2 |U_{\beta j}|^2, \quad (5)
$$

where we have averaged over rapid oscillations. Thus, we can represent the oscillation probability as a symmetric matrix P and P can be written as a product of a matrix A:

$$
P \equiv \begin{pmatrix} P_{ee} & P_{e\mu} & P_{e\tau} \\ P_{e\mu} & P_{\mu\mu} & P_{\mu\tau} \\ P_{e\tau} & P_{\mu\tau} & P_{\tau\tau} \end{pmatrix} \equiv AA^T, \quad (6)
$$

with

$$
A \equiv \begin{pmatrix} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu1}|^2 & |U_{\mu2}|^2 & |U_{\mu3}|^2 \\ |U_{\tau1}|^2 & |U_{\tau2}|^2 & |U_{\tau3}|^2 \end{pmatrix}. \quad (7)
$$

Now, the cosmic neutrino flux in the far distance can be expressed as a product of P and the intrinsic flux $F^0(\nu_\alpha)$ ($\alpha = e, \mu, \tau$):

$$
\begin{pmatrix} F(\nu_e) \\ F(\nu_\mu) \\ F(\nu_\tau) \end{pmatrix} = P \begin{pmatrix} F^0(\nu_e) \\ F^0(\nu_\mu) \\ F^0(\nu_\tau) \end{pmatrix} = AA^T \begin{pmatrix} F^0(\nu_e) \\ F^0(\nu_\mu) \\ F^0(\nu_\tau) \end{pmatrix}. \quad (8)
$$

Throughout this section we assume the standard ratio of the intrinsic cosmic neutrino flux, i.e., $F^0(\nu_e):F^0(\nu_\mu):F^0(\nu_\tau) = 1:2:0$, so that, we get
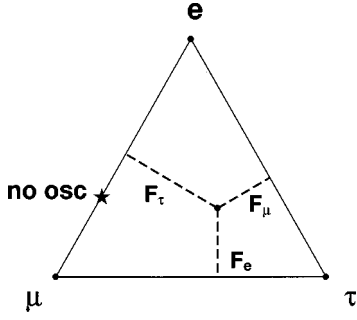
FIG. 1. The representation of the ratio of the final flux of (downward going) high-energy cosmic neutrinos on earth with a unit regular triangle. The x and y coordinates of the points are given by $x=(2F_\tau+F_e)/\sqrt{3}$, $y=F_e$ and the inside region of the triangle is given by $0\leqslant x\leqslant 2/\sqrt{3}$, $0\leqslant y\leqslant 1$. The point with an asterisk stands for the ratio without oscillations $F(\nu_e):F(\nu_\mu):F(\nu_\tau)=1:2:0$ and is given by the coordinate $(1/3\sqrt{3},1/3)$.

$$A^T\begin{pmatrix} F^0(\nu_e) \\ F^0(\nu_\mu) \\ F^0(\nu_\tau) \end{pmatrix} = \begin{pmatrix} |U_{e1}|^2 & |U_{\mu 1}|^2 & |U_{\tau 1}|^2 \\ |U_{e2}|^2 & |U_{\mu 2}|^2 & |U_{\tau 2}|^2 \\ |U_{e3}|^2 & |U_{\mu 3}|^2 & |U_{\tau 3}|^2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} F^0(\nu_e)$$

$$= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} F^0(\nu_e) + \begin{pmatrix} |U_{\mu 1}|^2-|U_{\tau 1}|^2 \\ |U_{\mu 2}|^2-|U_{\tau 2}|^2 \\ |U_{\mu 3}|^2-|U_{\tau 3}|^2 \end{pmatrix} F^0(\nu_e),$$

(9)

where we have used the unitarity condition, i.e., $\Sigma_j|U_{\alpha j}|^2 =1$. When $|U_{e3}|^2\ll 1$ and $|U_{\mu 3}|\simeq|U_{\tau 3}|$, we have $||U_{\mu j}|^2 -|U_{\tau j}|^2|\ll 1\,(j=1,2,3)$, so the second term in Eq. (9) is small. Hence with the constraints of the solar and atmospheric neutrino and the reactor data, we obtain

$$\begin{pmatrix} F(\nu_e) \\ F(\nu_\mu) \\ F(\nu_\tau) \end{pmatrix} \simeq A\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} F^0(\nu_e)\simeq \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} F^0(\nu_e), \quad (10)$$

where we have used the unitarity condition again. Therefore, we conclude that the ratio of the cosmic high-energy neutrino fluxes in the far distance is $1:1:1$, irrespective of which solar solution is chosen. This is because the MNS matrix elements satisfy $|U_{e3}|^2\ll 1$, $|U_{\mu 3}|\simeq|U_{\tau 3}|$ and $F^0(\nu_e):F^0(\nu_\mu):F^0(\nu_\tau)=1:2:0$. This is the feature that is expected to be observed at the new km^2 surface area neutrino telescopes in the case of the three neutrino oscillation scheme.

Using the allowed region for the MNS matrix elements given in [15], we have evaluated the final ratio of the cosmic neutrino flux numerically. To plot the ratio of the three neutrino flavors, we introduce the triangle representation, which is introduced by Fogli, Lisi, and Scioscia [16] in a different context. In Fig. 1, a unit regular triangle is drawn and the position of the point gives the ratio of the final high-energy neutrino flux with $F_\alpha\equiv F(\nu_\alpha)$. One reason that we adopt this representation is because currently we do not know the precise total cosmic neutrino flux, while we may observe the
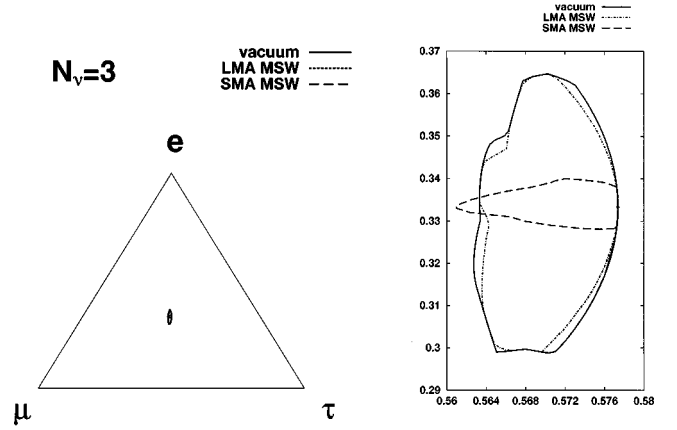

FIG. 2. The ratio of the final flux of high-energy cosmic neutrinos in the far distance in the three neutrino scheme. The allowed region is the inside of each contour; (b) is an enlarged figure of (a). The allowed region lies near the midpoint $(1/\sqrt{3},1/3)$.

ratio of flux of different cosmic neutrino flavors experimentally to a certain extent. Using this triangle representation, the allowed region in the three flavor framework with the constraints from the terrestrial, solar and atmospheric neutrino data, is given in Figs. 2(a) and 2(b). The allowed region is a small area around the midpoint $F(\nu_e)=F(\nu_\mu)=F(\nu_\tau) =1/3$ and the small deviation from the midpoint indicates the smallness of $|U_{e3}|$ and $||U_{\mu j}|^2-|U_{\tau j}|^2|$.

B. Four neutrino scheme

Let us now consider the case with three active and one sterile neutrino. Schemes with sterile neutrinos have been studied by a number of authors [17–20]. Here, we are interested in the four neutrino scheme in which the solar, atmospheric and Liquid Scintillation Neutrino Detector (LSND) experiment [21] are all explained by neutrino oscillations. By generalizing the discussion on big-bang nucleosynthesis (BBN) from the two neutrino scheme [22] to four neutrino case, it has been shown [19,20] that the neutrino mixing angles are strongly constrained not only by the reactor data [18] but also by BBN if one demands that the number N_ν of effective neutrinos be less than four. In this case, the 4×4 Maki-Nakagawa-Sakata (MNS) matrix splits approximately into two 2×2 block diagonal matrices.

On the other hand, some authors have given conservative estimate for N_ν [23] and if their estimate is correct then we no longer have strong constraints on the neutrino mixing angles. In this section, we discuss the final ratio of the cosmic neutrino flux components with and without BBN constraints separately. Throughout this section we assume the following mass pattern:

$$m_1^2\lesssim m_2^2\ll m_3^2\lesssim m_4^2, \quad (11)$$

with corresponding Δm_{21}^2, Δm_{43}^2 and Δm_{32}^2 standing for the mass squared differences suggested by the solar, atmospheric neutrino deficits and the LSND experiment, respectively.

1. Four neutrino scheme with BBN constraints

Here, we adopt the notation in [19] for the 4×4 MNS matrix:

$$U \equiv \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

$$\equiv R_{34}\left(\frac{\pi}{2} - \theta_{34}\right) R_{24}(\theta_{24}) R_{23}\left(\frac{\pi}{2}\right) e^{2i\delta_1 \lambda_3} R_{23}(\theta_{23})$$

$$\times e^{-2i\delta_1 \lambda_3} e^{\sqrt{6} i \delta_3 \lambda_{15}/2} R_{14}(\theta_{14})$$

$$\times e^{-\sqrt{6} i \delta_3 \lambda_{15}/2} e^{2i\delta_2 \lambda_8 / \sqrt{3}} R_{13}(\theta_{13}) e^{-2i\delta_2 \lambda_8 / \sqrt{3}} R_{12}(\theta_{12}),$$

$$(12)$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$ and

$$R_{jk}(\theta) \equiv \exp(iT_{jk}\theta), \tag{13}$$

is a 4×4 orthogonal matrix with

$$(T_{jk})_{lm} = i(\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}), \tag{14}$$

and $2\lambda_3 \equiv \mathrm{diag}(1,-1,0,0)$, $2\sqrt{3}\lambda_8 \equiv \mathrm{diag}(1,1,-2,0)$, $2\sqrt{6}\lambda_{15} \equiv \mathrm{diag}(1,1,1,-3)$ are diagonal elements of the $SU(4)$ generators. From the constraint of the reactor data of Bugey [24], which strongly constrain the disappearance probability $P(\bar{\nu}_e \to \bar{\nu}_e)$ for the entire region of mass squared difference implied by the data of LSND [21] and E776 [25], we have [18–20]

$$|U_{e3}|^2 + |U_{e4}|^2 \ll 1. \tag{15}$$

On the other hand, from the BBN constraint that sterile neutrino be not in thermal equilibrium, we get

$$|U_{s3}|^2 + |U_{s4}|^2 \ll 1. \tag{16}$$

In this case, the oscillation probabilities in the far distance are given essentially by the following two neutrino flavor formulas:

$$P(\nu_e \to \nu_e; L=\infty) = 1 - \frac{1}{2}|U_{e1}|^2|U_{e2}|^2 = 1 - \frac{1}{2}\sin^2 2\theta_{12},$$

$$P(\nu_e \to \nu_s; L=\infty) = \frac{1}{2}|U_{e1}|^2|U_{e2}|^2 = \frac{1}{2}\sin^2 2\theta_{12},$$

$$P(\nu_\mu \to \nu_\mu; L=\infty) = 1 - \frac{1}{2}|U_{\mu 3}|^2|U_{\mu 4}|^2 = 1 - \frac{1}{2}\sin^2 2\theta_{24},$$

$$P(\nu_\mu \to \nu_\tau; L=\infty) = \frac{1}{2}|U_{\mu 3}|^2|U_{\mu 4}|^2 = \frac{1}{2}\sin^2 2\theta_{24}, \tag{17}$$

where, we have substituted $\theta_{13} = \theta_{14} = \theta_{23} = \theta_{34} = 0$ in Eq. (12) and θ_{12}, θ_{24} correspond to the mixing angles of the solar and atmospheric neutrino oscillations, respectively. It

has been known [26] that only the small mixing angle (SMA) Mikheyev-Smirnov-Wolfenstein (MSW) solution is allowed in the $\nu_e \leftrightarrow \nu_s$ solar oscillation scheme, so in the present case, we take $|\theta_{12}| \ll 1$. Thus, we have the following ratio of the final cosmic high-energy neutrino flux:

$$\begin{pmatrix} F(\nu_e) \\ F(\nu_\mu) \\ F(\nu_\tau) \\ F(\nu_s) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & c_{\mathrm{atm}}^2 & s_{\mathrm{atm}}^2 \\ 0 & 0 & s_{\mathrm{atm}}^2 & c_{\mathrm{atm}}^2 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & c_{\mathrm{atm}}^2 & s_{\mathrm{atm}}^2 & 0 \\ 0 & s_{\mathrm{atm}}^2 & c_{\mathrm{atm}}^2 & 0 \end{pmatrix}$$

$$\times \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix} F^0(\nu_e) = \begin{pmatrix} 1 \\ 2 - \sin^2 2\theta_{\mathrm{atm}} \\ \sin^2 2\theta_{\mathrm{atm}} \\ 0 \end{pmatrix} F^0(\nu_e),$$

$$(18)$$

where, we have taken $\theta_{24} \equiv \theta_{\mathrm{atm}}$ which satisfies [27]

$$0.88 \lesssim \sin^2 2\theta_{\mathrm{atm}} \leqslant 1. \tag{19}$$

In this case, therefore, we again have $F(\nu_e):F(\nu_\mu):F(\nu_\tau) \simeq 1:1:1$.

We have also numerically obtained the allowed region by letting θ_{12}, θ_{13}, θ_{14}, θ_{23}, θ_{34} and θ_{24} to vary within the constraints obtained from the reactor, solar and atmospheric neutrino data. In the four neutrino scheme, the total flux of active neutrino is in general not the same as that at the production point, and in principle we have to use a unit tetrahedron to express the ratio of the four neutrino flux. In practice, however, we may not observe the cosmic sterile neutrino events nor do we know the precise total flux of the cosmic high-energy neutrinos, so it is useful to normalize the flux of each active neutrino by the total flux of active neutrinos:

$$\begin{pmatrix} \widetilde{F}(\nu_e) \\ \widetilde{F}(\nu_\mu) \\ \widetilde{F}(\nu_\tau) \end{pmatrix} \equiv \frac{1}{F(\nu_e) + F(\nu_\mu) + F(\nu_\tau)} \begin{pmatrix} F(\nu_e) \\ F(\nu_\mu) \\ F(\nu_\tau) \end{pmatrix}. \tag{20}$$

After redefining the flux, we can plot the ratio of each active neutrino with the same triangle graph as in the three neutrino case, and the result is depicted in Fig. 3. The allowed region is again small and the reason that the region lies horizontally is because of possible deviation of $\theta_{24} \equiv \theta_{\mathrm{atm}}$ from $\pi/4$ [see Eq. (19)].

2. Four neutrino scheme without BBN constraints

In this subsection, we discuss what happens to the ratio of the final cosmic high-energy neutrino flux if we lift BBN constraints. Without BBN constraints, the only conditions we have to take into account are the solar and atmospheric neutrino deficit data and the results of experiment of LSND (the appearance experiment of $\bar{\nu}_\mu \to \bar{\nu}_e$) [21], Bugey (the disappearance experiment of $\bar{\nu}_e \to \bar{\nu}_e$) [24] and CDHSW (the disappearance experiment of $\bar{\nu}_\mu \to \bar{\nu}_\mu$) [28]. For the range of the Δm^2 suggested by the LSND data, which is given by 0.2
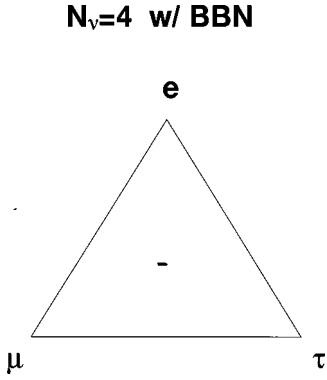
$N_\nu = 4$ w/ BBN

FIG. 3. The ratio of the final flux of high-energy cosmic neutrinos in the far distance in the four neutrino scheme with BBN constraints. The allowed region lies near the midpoint $(1/\sqrt{3}, 1/3)$.
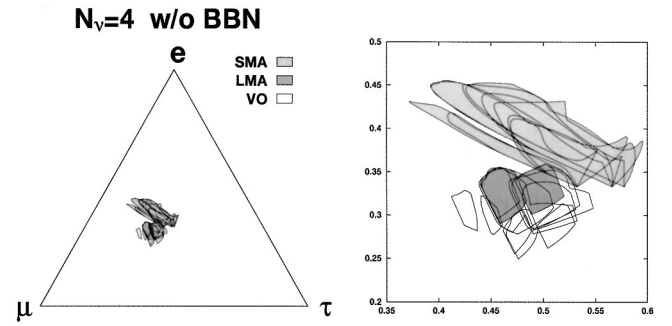
$N_\nu = 4$ w/o BBN

FIG. 4. The ratio of the final flux of high-energy cosmic neutrinos in the far distance in the four neutrino scheme without BBN constraints; (b) is an enlarged figure of (a). The allowed region of the LOW solution is a subset of that of the LMA solution.

$\lesssim \Delta m_{\text{LSND}}^2 / \text{eV}^2 \lesssim 2$ when combined with the data of Bugey [24] and E776 [25], the constraint by the Bugey data is very stringent and

$$|U_{e3}|^2 + |U_{e4}|^2 \lesssim 10^{-2}, \tag{21}$$

has to be satisfied [18–20]. Therefore, for simplicity, we take $U_{e3} = U_{e4} = 0$, in the following discussion.

The analysis of the solar neutrino data in the four neutrino scheme with ansatz $U_{e3} = U_{e4} = 0$ has been done recently by Giunti, Gonzalez-Garcia and Peña-Garay [29]. They have shown that the scheme is reduced to the two neutrino framework in which only one free parameter $c_s \equiv |U_{s1}|^2 + |U_{s2}|^2$ appears in the analysis.[1] Their conclusion is that the SMA (MSW) solution exists for the entire region of $0 \leq c_s \leq 1$, while the large mixing angle (LMA) and vacuum oscillation (VO) solutions survive only for $0 \leq c_s \lesssim 0.2$ and $0 \leq c_s \lesssim 0.4$, respectively.

The analysis of the atmospheric neutrino data in the four neutrino framework has been done by one of the authors [30] more recently again with ansatz $U_{e3} = U_{e4} = 0$. The conclusion of [30] is that the region $-\pi/2 \lesssim \theta_{34} \lesssim \pi/2$ and $0 \leq \theta_{23} \lesssim \pi/6$ as well as $0 \leq \delta_1 \leq \pi$ is consistent with the Superkamiokande atmospheric neutrino data of the contained and upward going μ events, where θ_{34} and θ_{23} stand for, roughly speaking, the ratio of $\nu_\mu \leftrightarrow \nu_\tau$ versus $\nu_\mu \leftrightarrow \nu_s$ and the ratio of the contributions of $\sin^2(\Delta m_{\text{atm}}^2 L/4E)$ versus $\sin^2(\Delta m_{\text{LSND}}^2 L/4E)$ in the flavor oscillation probability, respectively, and δ_1 is the only *CP* phase left in this scheme. Notice that the recent claim by the Superkamiokande group [27] that $\nu_\mu \leftrightarrow \nu_s$ is almost completely excluded is based on the hypothesis of two neutrino $\nu_\mu \leftrightarrow \nu_s$ oscillations with only one mass squared difference Δm_{atm}^2, and their claim is completely consistent with the results in [30], where the region $\theta_{34} \simeq \pm \pi/2$ and $\theta_{23} \simeq 0$, which would lead to pure $\nu_\mu \leftrightarrow \nu_s$ oscillations, is excluded. For generic mixing angles of θ_{34}

and θ_{23}, the ansatz in [30] implies hybrid of $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_s$ oscillations in general, and one has to take into account the constraint of the CDHSW experiment [28] also. Following [30], we take $\Delta m_{32}^2 \equiv \Delta m_{\text{LSND}}^2 = 0.3$ eV^2, which is consistent with the negative result of the disappearance experiment of CDHSW [28] for the entire region of the mixing angles obtained in [30].

Combining the results of [29] on solar neutrinos and [30] on atmospheric neutrinos, we have evaluated the final ratio of the cosmic high-energy neutrino flux and the result is given in Figs. 4(a) and 4(b), where the allowed region with the SMA (MSW), LMA (MSW) and VO solutions are shown separately. Almost the entire allowed region with the SMA (MSW) solution lies above the line $\tilde{F}(\nu_e) = 1/3$, and this is because the total normalization $F(\nu_e) + F(\nu_\mu) + F(\nu_\tau) = 1 - F(\nu_s)$ becomes less than 1 while $F(\nu_e) = (1 - \sin^2 2\theta_{12})F^0(\nu_e)$ hardly changes due to smallness of $|\theta_{12}| = |\theta_\odot|$. On the other hand, for most of the region with LMA (MSW) and VO solutions, $F(\nu_e)/F^0(\nu_e)$ becomes smaller than $1 - F(\nu_s)$ and the region lies below the line $\tilde{F}(\nu_e) = 1/3$. This four neutrino scheme without BBN constraint gives us clearly a distinctive pattern for the final ratio of the cosmic high-energy neutrino flux, and observationally, if we have good precision then it may be even possible to distinguish the SMA (MSW) solution from the LMA (MSW) or VO solutions. The allowed regions in Figs. 4(a) and 4(b) are plotted for $\delta_1 = 0$ and $\delta_1 = \pi/2$. We observe that most of the allowed regions for $\delta_1 = 0$ and $\delta_1 = \pi/2$ overlap with each other and it implies that distinction between $\delta_1 = 0$ and $\delta_1 = \pi/2$ is difficult in this scheme of four neutrinos.

We note in passing that the allowed region of the low probability low mass (LOW) solution to the solar netrino problem is contained in that of the LMA solution as far as $\sin^2 2\theta_\odot$ is concerned, and therefore the LOW solution gives us only a subset of the allowed region of the LMA solution in Figs. 4(a) and 4(b).

III. NONSTANDARD RATIO OF THE HIGH-ENERGY COSMIC NEUTRINO FLUX

In Sec. II, we have seen that the schemes of three neutrinos and of four neutrinos with BBN constraints give us the

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[1] In the notation of [29] this parameter is given by $c_s = c_{23}^2 c_{24}^2$. We adopt different notation from [29] for the parametrization of the mixing angles, however, will use c_s for $|U_{s1}|^2 + |U_{s2}|^2$.
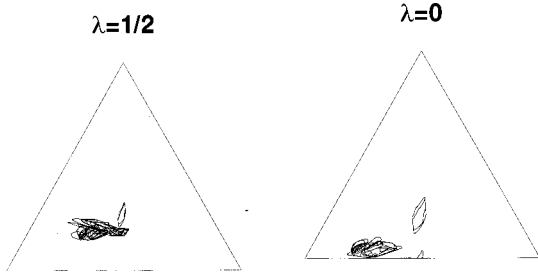
λ=**1/2**   λ=**0**

FIG. 5. The ratio of the final flux of high-energy cosmic neutrinos in the far distance in a nonstandard scenario characterized by $F^0(\nu_e):F^0(\nu_\mu):F^0(\nu_\tau)=\lambda/3:1-\lambda/3:0$. In (a), $\lambda=1/2$, the cases of $N_\nu=3$ with SMA and $N_\nu=4$ with BBN constraints have small region near the point $F(\nu_e):F(\nu_\mu):F(\nu_\tau)=2:5:5$, the region for the cases of $N_\nu=3$ with LMA and VO lie above this point, whereas most of the region for the case of $N_\nu=4$ without BBN constraints lie to the left of this point. As in Fig. 4, the allowed region of the LOW solution is a subset of that of the LMA solution. In (b), $\lambda=0$, the cases of $N_\nu=3$ with SMA and $N_\nu=4$ with BBN constraints have small region near the point $F(\nu_e):F(\nu_\mu):F(\nu_\tau)=0:1:1$, the region for the cases of $N_\nu=3$ with LMA and VO lie above this point, whereas most of the region for the case of $N_\nu=4$ without BBN constraints lie to the left of this point.

ratio $F(\nu_e):F(\nu_\mu):F(\nu_\tau)\simeq1:1:1$, irrespective of which solar solution is chosen. This is due to the fact that the intrinsic high-energy cosmic neutrino flux has the ratio $F^0(\nu_e):F^0(\nu_\mu):F^0(\nu_\tau)=1:2:0$ and the oscillation of atmospheric neutrinos is with maximal mixing while $|U_{e3}|^2\ll1$. It has been pointed out [9] that the intrinsic flux of the cosmic high-energy neutrinos may not have the standard ratio $F^0(\nu_e):F^0(\nu_\mu):F^0(\nu_\tau)=1:2:0$, mainly because some of muons may lose their energy in a magnetic field. Here, we discuss in a model independent way the consequences of a generic scenario which is characterized by $F^0(\nu_e):F^0(\nu_\mu):F^0(\nu_\tau)=\lambda/3:1-\lambda/3:0$, where λ is a free parameter $0\leq\lambda\leq1$ (the standard ratio is obtained for $\lambda=1$).

We have plotted the allowed region in Figs. 5(a) and 5(b) for $\lambda=1/2$ and $\lambda=0$, respectively. We observe that the cosmic high-energy neutrino flux with the nonstandard ratio gives relatively lower value of $\tilde{F}(\nu_e)$. If relatively lower value of cosmic ν_e flux is observed in the future experiments, then it may imply that the possible oscillation scenario is the four neutrino scheme without BBN constraints and the intrinsic cosmic neutrino flux with the nonstandard ratio. Independent information on neutrino mixing parameters as well as on relevant astrophysical inputs may be needed here to arrive at a more definite conclusion.

IV. RESULTS AND DISCUSSION

In general, the final flux of high-energy cosmic neutrinos is expected to be almost equally distributed among the three (active) cosmic neutrino flavors because of vacuum flavor mixing or oscillations, provided the astrophysical sources for these high-energy cosmic neutrinos are cosmologically distant, essentially irrespective of the neutrino flavors (three or four). Nevertheless, this may not be the case if the intrinsic

high-energy cosmic neutrino flux ratios differ from the standard one, namely from $F^0(\nu_e):F^0(\nu_\mu):F^0(\nu_\tau)=1:2:0$. In the examples considered in this work for the nonstandard intrinsic high-energy cosmic neutrino flux, a relatively lower final flux for cosmic ν_e is obtained. The situation of nonstandard intrinsic cosmic neutrino flux may arise, for instance, if some of the muons lose their energy in the relatively intense magnetic field in the vicinity of the source.

A simultaneous measurement of the three cosmic neutrino flavors may be useful to obtain information about a particular neutrino (mass and) mixing scheme depending on the relevant achievable resolutions for typical km^2 surface area neutrino telescopes.

Irrespective of the numbers of neutrino flavors, in each of the neutrino (mass and) mixing schemes discussed in this work, the final flux of high-energy cosmic tau neutrinos is essentially comparable to that of non tau (active) neutrinos, even if it is intrinsically negligible. This may, at least in principle, be useful to constrain the relevant nonstandard particle physics or astrophysics scenarios.

In this work, we have considered the effects of vacuum neutrino flavor oscillations on high-energy cosmic neutrino flux in the context of three as well as four flavors. These oscillations result in an energy independent ratio, $R_{\alpha\beta} \equiv N_\alpha/N_\beta$ ($\alpha\neq\beta;\alpha=\beta=\nu_e,\nu_\mu,\nu_\tau$) of the number of events detected for the neutrino flavors α and β. It is so because the various neutrino flavor precession probabilities given in Sec. II [see Eq. (5) and Eq. (17)] are essentially independent of neutrino energy. In the following paragraph, we briefly describe the prospects offered by the typical km^2 surface area underice or water neutrino telescopes which are currently under construction or planning to possibly identify the cosmic neutrino flavor and hence the ratio, $R_{\alpha\beta}$ [8].

We ignore the possible observational difference between cosmic neutrinos and anti neutrinos for simplicity in the following discussion and assume that the flavor content in the cosmic neutrino flux is equally distributed because of vacuum flavor oscillations. Several of the recent discussions suggest that the absorption of high-energy cosmic neutrino flux by earth is neutrino flavor dependent [8]. The upward going electron and muon neutrino fluxes are significantly attenuated typically for $E_0\gtrsim5\times10^4$ GeV, whereas the upward going tau neutrinos with $E>E_0$ may reach the detector with $E\lesssim E_0$ because of the short lifetime of the associated tau lepton and may appear as a small pile up with fairly flat zenith angle dependence. For $E\gtrsim E_0$, the upward going cosmic neutrino event rates range typically as $N_{\nu_\mu}\sim\mathcal{O}(10^1)$ whereas $N_{\nu_\tau}\sim\mathcal{O}(10^0)$ in units of per year per steradian for typical km^2 surface area neutrino telescopes, if one uses the current upper high-energy cosmic neutrino flux limits [2]. Let us note that for $E\gtrsim E_0$, presently the high-energy cosmic neutrino fluxes from AGNs dominate above the atmospheric neutrino background. For downward going high-energy cosmic neutrino flux, the event rate ranges typically as $N_{\nu_e}\sim\mathcal{O}(10^{1.5}),N_{\nu_\mu}\sim\mathcal{O}(10^2)$, whereas $N_{\nu_\tau}\sim\mathcal{O}(10^1)$ in units of per year per steradian for the same high-energy cosmic neutrino fluxes. For $E>E_0$, the downward going cosmic tau

neutrinos typically produce a two bang event topology such that the two bangs are connected by a μ-like track. The size of the second bang is on the average a factor of two larger than the first bang. The downward going electron neutrinos produce a single bang at these energies whereas the muon neutrinos typically produce a single shower along with a zipping μ-like track in km^2 surface area neutrino telescopes. Based on these rather distinct event topologies, cosmic neutrino flavor identification may be conceivable. The above order of magnitude estimates indicate that the typical km^2 surface area neutrino telescopes do offer some prospects for observations of high-energy cosmic neutrino flavor ratio, $R_{\alpha\beta}$, or at least may constrain it meaningfully.

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