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Cosmological electromagnetic fields due to gravitational wave perturbations

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We consider the dynamics of electromagnetic fields in an almost-Friedmann-Robertson-Walker universe using the covariant and gauge-invariant approach of Ellis and Bruni. Focusing on the situation where deviations from the background model are generated by tensor perturbations only, we demonstrate that the coupling between gravitational waves and a weak magnetic test field can generate electromagnetic waves. We show that this coupling leads to an initial pulse of electromagnetic waves whose width and amplitude are determined by the wavelengths of the magnetic field and gravitational waves. A number of implications for cosmology are discussed; in particular we calculate an upper bound of the magnitude of this effect using limits on the quadrupole anisotropy of the cosmic microwave background.

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I. INTRODUCTION

There have been numerous investigations on the scattering of electromagnetic waves off gravitational fields (see Refs. [1] for a representative sample). Most of this research has been focused on the effect gravitational waves have on vacuum electromagnetic fields. Other papers, see e.g. Refs. [2], also consider situations in astrophysics where plasma effects are taken into account.

Magnetic fields play an important role in our Universe, appearing on all scales from the solar system, through interstellar and extragalactic scales, to intracluster scales of several Mpc. Although magnetic field inhomogeneities have not yet been observed on scales as large as those exhibited by cosmic microwave background (CMB) anisotropies, it is natural to expect that magnetic fields exist on such scales [3], and that they could play a role in the formation of large-scale structure. Indeed many mechanisms have been proposed to explain how these fields may be generated in the early universe—a process called *primordial magnetogenesis*. For example on small scales (less than the Hubble radius), QCD and electroweak transitions can give rise to local charge separation leading to local currents which can generate magnetic fields [4]. Large-scale magnetic fields can be generated during inflation or in pre-big-bang models based on string theory [5], in which vacuum fluctuations are amplified via the inflaton or dilaton.

The effect magnetic fields have on density perturbations has been studied extensively by a number of authors, both in the context of Newtonian and relativistic cosmology [6,7], but as yet there have been no studies of their effect on gravitational wave perturbations.

In what follows, we use the well-known covariant and gauge-invariant approach of Ellis and Bruni [8] to investigate this interaction in the context of cosmology by considering the dynamics of electromagnetic fields in an almost-Friedmann-Robertson-Walker (FRW) universe, focusing on the situation where deviations from the FRW background are generated by tensor or gravitational wave perturbations [9,10].

We show that in the presence of a weak (near-homogeneous) magnetic test field propagating on the background FRW model [11], the gravitational waves couple nonlinearly to this field to produce a pulse of gravitationally induced electromagnetic waves. In particular, because of the different ways in which tensor perturbations enter the wave equations for the electric and magnetic fields, respectively, there will be, in the case of long wavelength gravitational waves and large-scale magnetic fields, a growth in the expansion normalized electric field, as the expansion normalized shear grows in time.

This paper is organized as follows. After a short discussion of notation and conventions, in Sec. III we outline in detail the linearization procedure used to approximate the Einstein-Maxwell equations in cosmology. In Sec. IV we derive a set of nonlinear wave equations which show how electromagnetic fields can be generated when gravitational waves couple to a near-homogeneous magnetic test field propagating on the FRW background. Finally in Sec. V we solve the equations perturbatively and use our results to put

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an upper bound on the size of these gravitationally induced fields.

II. NOTATION AND CONVENTIONS

Notation and conventions are taken to be the same as in [12]. In particular $8\pi G=c=1$; the projected spatial covariant derivative a tensor $T^{cd\ldots}{}_{ef\ldots}$ is given by $\tilde{\nabla}_a T^{cd\ldots}{}_{ef\ldots} \equiv h^b{}_a h^c{}_p \ldots h^d{}_q h^r{}_e \ldots h^s{}_f \nabla_b T^{p\ldots q}{}_{r\ldots s}$, where u^a is the four-velocity of the matter, g_{ab} is the metric tensor, and $h_{ab}=g_{ab}+u_a u_b$ is the spatial projection tensor ($h_{ab}u^b=0$). A dot denotes the covariant derivative along u^a, so for any tensor $\dot{T}^{cd\ldots}{}_{ef\ldots}\equiv u^a\nabla_a T^{cd\ldots}{}_{ef\ldots}$. We assume that the matter is described by *irrotational dust* [13] so that the *pressure p*, *acceleration vector* $\dot{u}^a$, and *vorticity tensor* ω_{ab} all vanish exactly. In this case the first covariant derivative of the four-velocity can be written as $\nabla_a u_b=\sigma_{ab}+\frac{1}{3}\Theta h_{ab}$, where σ_{ab} and Θ are respectively the usual shear and volume expansion of the matter congruence. We also define the Hubble parameter H terms of the expansion Θ and scale factor a in the usual way: $H=\Theta/3=\dot{a}/a$.

III. APPROXIMATIONS

In order to simplify the nonlinear dynamics of the coupled Einstein-Maxwell equations and to isolate the effects we are looking for, we will adopt the following *approximation scheme* based on two parameters: $\varepsilon_{\rm g}$ will refer to quantities occurring in the gravitational equations, while $\varepsilon_{\rm em}$ characterizes the electromagnetic field. We assume that the gravitational equations follow the *almost FRW* conditions [8], so that the energy density μ and expansion Θ have a nonzero contribution in the background model and can therefore be considered $\mathcal{O}(0_{\rm g})$ while σ_{ab}, E_{ab}, and H_{ab} vanish in the background and are $\mathcal{O}(\varepsilon_{\rm g})$. In the case of the Maxwell field, we assume that there is a *weak magnetic test field* B_0^a at $\mathcal{O}(0_{\rm em})$ which propagates on the background FRW model, whose gravitational influence is given by the Alfvén parameter $\varepsilon\equiv(B_0^a B_a^0/\mu)^{1/2}$. On the other hand the electric field E^a vanishes in the background and is considered to be $\mathcal{O}(\varepsilon_{\rm em})$. The perturbation scheme we adopt is to drop terms of $\mathcal{O}(\varepsilon^2)$ (so that the magnetic field does not contribute to the gravitational dynamics [7]), $\mathcal{O}(\varepsilon_{\rm g}^2)$, $\mathcal{O}(\varepsilon_{\rm em}^2)$, and $\mathcal{O}(\varepsilon_{\rm g}\varepsilon_{\rm em})$.

In the covariant approach to linear perturbations of FRW models [8], *pure tensor* or *gravitational wave* perturbations are characterized by the following covariant conditions [9,10]:

$$\tilde{\nabla}^b E_{ab}=0\Rightarrow\tilde{\nabla}_a\mu=0, \tag{1a}$$

$$\tilde{\nabla}^b H_{ab}=0\Rightarrow\omega_a=0; \tag{1b}$$

the first one excludes scalar (density) perturbations and the second, vector (rotational) perturbations. The conditions that the terms on the right-hand side vanish, are analogous to the transverse condition on tensor perturbations in the metric approach. In addition, we notice that since the Weyl tensor is the trace-free part of the Riemann tensor, both E_{ab} and H_{ab} are trace-free, again like the tensor perturbations of the Bardeen approach [14].

Given the assumed equation of state, these conditions also imply that the spatial gradient of the expansion $\tilde{\nabla}_a\Theta$ vanishes (see [8]). Together with Eqs. (1) these conditions provide a unique characterization of tensor perturbations.

IV. EINSTEIN-MAXWELL EQUATIONS

We assume overall charge neutrality and use the Bianchi identities and Maxwell's equations as presented in [12]. Then, with the above prerequisites, we obtain a set of nonlinear wave equations for the gravitational (σ_{ab}: shear) and electromagnetic (E^a, B^a: electric and magnetic fields) degrees of freedom:

$$\Delta\sigma_{ab}+5H\dot{\sigma}_{ab}+\tfrac{3}{2}H^2\sigma_{ab}=0, \tag{2a}$$

$$\Delta E^a+5H\dot{E}^a+3H^2E^a+\dot{j}^a+\Theta j^a=j_E^a, \tag{2b}$$

$$\Delta B^a+5H\dot{B}^a+3H^2B^a-\epsilon^{abc}\tilde{\nabla}_b j_c=j_B^a, \tag{2c}$$

where

$$j_E^a=\epsilon^{abc}\tilde{\nabla}_b(\sigma_c^d B_d)+\epsilon^{abc}\sigma^d{}_b\tilde{\nabla}_d B_c+H^a{}_b B^b,$$

$$j_B^a=2H\sigma^a{}_b B^b-2E^a{}_b B^b+\sigma^a{}_b\dot{B}^b \tag{3}$$

are gravitational induced magnetic and electric currents, and $E_{ab}=-\dot{\sigma}_{ab}-\frac{2}{3}\Theta\sigma_{ab}$ and $H^{ab}=\epsilon^{cd(a}\tilde{\nabla}_c\sigma^{b)}{}_d$ are the electric and magnetic parts of the Weyl tensor. Also $\Delta f\equiv\ddot{f}-\tilde{\nabla}^2 f$ where f is any tensor orthogonal to u^a.

In the above equations, the electric and magnetic fields consist of two parts, a contribution due to the magnetic test field B_0^a which gives rise to the current j^a [15] and contributions generated by the nonlinear coupling of this test field to gravitational waves via the gravitationally induced currents j_E^a and j_B^a.

$$E^a=E^a_{\rm grav},\quad B^a=B_0^a+B^a_{\rm grav}. \tag{4}$$

V. ANALYTIC SOLUTIONS AND NUMERICAL INTEGRATION

We solve the above equations perturbatively by first calculating the gravitationally induced currents j_E^a and j_B^a and then solving Eqs. (2b) and (2c) together with Eq. (2a) for the gravitationally induced electric and magnetic fields.

To $\mathcal{O}(0_{\rm em})$, Maxwell's equations [12] give $E=0$ and

$$\dot{B}_0^a+2HB_0^a=0, \tag{5}$$

which we can integrate to obtain

$$B_0^a=a^{-2}A_{(n)}^a,\quad \tilde{\nabla}^a B_0^b=a^{-3}A_{(n)}^{ab},\quad A_{(n)}^{ab}\equiv a\tilde{\nabla}^a A_{(n)}^b, \tag{6}$$

where $A^{a}_{(n)}$ and $A^{ab}_{(n)}$ determine the spatial variation of the magnetic test field and are constant along the fluid flow lines: $\dot{A}^{a}_{(n)}=\dot{A}^{ab}_{(n)}=0$ [10]. Furthermore we assume that the spatial functions $A^{a}_{(n)}$ and $A^{ab}_{(n)}$ satisfy the Helmholtz equation

$$\tilde{\nabla}^2 A^{a}_{(n)}=-\frac{n^2}{a^2}A^{a}_{(n)}, \quad \tilde{\nabla}^2 A^{ab}_{(n)}=-\frac{n^2}{a^2}A^{ab}_{(n)}, \tag{7}$$

in this way defining a specific length scale $\lambda_{B_0}=2\pi a/n$ associated with the magnetic field determining its scale of inhomogeneity, where n is a *fixed* wave number associated with that scale.

In order to solve Eqs. (2) it is standard to decompose physical (perturbed) fields into a spatial and temporal part using eigenfunctions which are solutions of the Helmholtz equation [16]. In the case of the shear tensor we write

$$\sigma_{ab}=\sum_k \sigma_{(k)}Q^{(k)}_{ab}, \quad \dot{Q}^{(k)}_{ab}=0, \tag{8}$$

where $Q^{(k)}_{ab}$ is a tensor harmonic satisfying

$$\tilde{\nabla}^2 Q^{(k)}_{ab}=-\frac{k^2}{a^2}Q^{(k)}_{ab}. \tag{9}$$

We can also define higher order harmonics by taking comoving spatial derivatives of the lower order harmonics; for example, $Q^{(k)}_{abc}\equiv a\tilde{\nabla}_a Q^{(k)}_{bc}$ can easily be shown to satisfy

$$\tilde{\nabla}^2 Q^{(k)}_{abc}=-\frac{k^2}{a^2}Q^{(k)}_{abc}. \tag{10}$$

Using the above solution (6) (dropping the index n which indicates the scale length of B^a_0), the decomposition (8), and writing

$$E^a_{\rm grav}=\sum_k \mathcal{E}_{(k)}\mathcal{E}^a_{(k)}, \quad B^a_{\rm grav}=\sum_k \mathcal{H}_{(k)}\mathcal{H}^a_{(k)}, \tag{11}$$

the wave equations (2) become

$$\ddot{\sigma}_{(k)}+5H\dot{\sigma}_{(k)}+\left(\tfrac{3}{2}H^2+\frac{k^2}{a^2}\right)\sigma_{(k)}=0, \tag{12a}$$

$$\ddot{\mathcal{E}}_{(k)}+5H\dot{\mathcal{E}}_{(k)}+\left(3H^2+\frac{k^2}{a^2}+\frac{n^2}{a^2}\right)\mathcal{E}_{(k)}=a^{-3}\sigma_{(k)}, \tag{12b}$$

$$\ddot{\mathcal{H}}_{(k)}+5H\dot{\mathcal{H}}_{(k)}+\left(3H^2+\frac{k^2}{a^2}+\frac{n^2}{a^2}\right)\mathcal{H}_{(k)}=2a^{-2}(\dot{\sigma}_{(k)}+2H\sigma_{(k)}), \tag{12c}$$

where

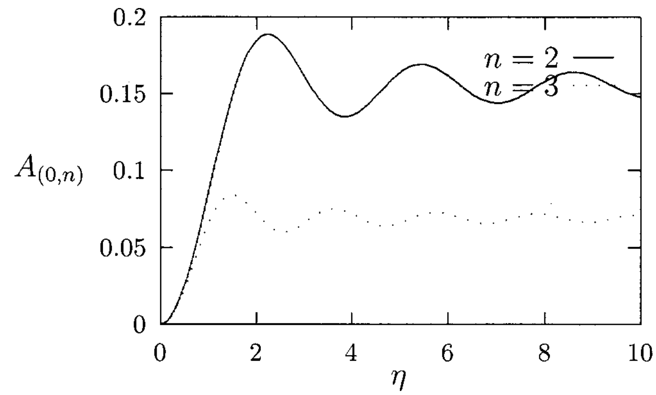

FIG. 1. Expansion normalized gravitationally induced electric field for different values of the magnetic wave number n.

$$\mathcal{E}^{(k)}_a=\tfrac{3}{2}\epsilon_{abc}Q^{bcd}_{(k)}A_d+\tfrac{1}{2}\epsilon^{bcd}Q^{(k)}_{cda}A_b+\epsilon_{abc}Q^{cd}_{(k)}A_{bd}+\epsilon_{abc}Q^{bd}_{(k)}A^c{}_d, \tag{13a}$$

and

$$\mathcal{H}^{(k)}_a=Q^{(k)}_{ab}A^b. \tag{13b}$$

It is straightforward to verify that the spatial functions $\mathcal{E}^a_{(k)}$ and $\mathcal{H}^a_{(k)}$ also satisfy the Helmholtz equation.

In order to estimate the dynamical importance of our fields we introduce *expansion normalized variables*

$$\Sigma_{(k)}\equiv\frac{\sigma_{(k)}}{H}, \quad \tilde{\mathcal{E}}_{(k)}\equiv\frac{\mathcal{E}_{(k)}}{H}, \quad \tilde{\mathcal{H}}_{(k)}\equiv\frac{\mathcal{H}_{(k)}}{H}, \tag{14}$$

giving us a set of scale invariant functions (see, e.g., [17]). Introducing the conformal time parameter η (whose defining equation is $\dot{\eta}=a^{-1}$), the scale factor and Hubble parameter for a dust FRW background are given by $a(\eta)=\eta^2$, $H=2\eta^{-3}$ (see, e.g., [18]).

Substituting these into Eqs. (12) we obtain

$$\Sigma''_{(k)}+2\eta^{-1}\Sigma'_{(k)}+(-6\eta^{-2}+k^2)\Sigma_{(k)}=0, \tag{15a}$$

$$\tilde{\mathcal{E}}''_{(k)}+2\eta^{-1}\tilde{\mathcal{E}}'_{(k)}+(k^2+n^2)\tilde{\mathcal{E}}_{(k)}=\eta^{-2}\Sigma_{(k)}, \tag{15b}$$

and

$$\tilde{\mathcal{H}}''_{(k)}+2\eta^{-1}\tilde{\mathcal{H}}'_{(k)}+(k^2+n^2)\tilde{\mathcal{H}}_{(k)}=2\eta^{-2}(\Sigma'_{(k)}+\eta^{-1}\Sigma_{(k)}). \tag{15c}$$

Equations (15) can be solved exactly in the long wavelength gravitational wave limit (i.e., the term k^2/a^2 is small compared to the other terms in the wave equations), but a numerical investigation gives more transparent results. It turns out to be convenient to introduce the variables $A_{(k,n)}\equiv\tilde{\mathcal{E}}_{(k)}\eta_0^2/\Sigma(\eta_0)$, $B_{(k,n)}\equiv\tilde{\mathcal{H}}_{(k)}\eta_0^2/\Sigma(\eta_0)$, where η_0 and $\Sigma(\eta_0)$ are respectively the initial values of the conformal time and the normalized shear, and we have reinstated the index n indicating the scale length of B^a_0. These variables are

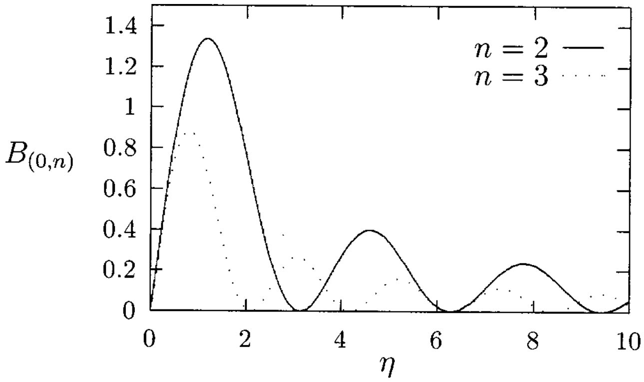

FIG. 2. Expansion normalized gravitationally induced magnetic field for different values of the magnetic wave number *n*.

invariant with respect to changes in η_0 and $\Sigma(\eta_0)$, thus giving us a scale-invariant measure of the generated electromagnetic field. Performing the integration for various values of the wave number *n* for the magnetic test field, we find that the normalized electric field $A_{(k,n)}$ tends to a constant value which depends linearly on the initial value of the shear perturbation, while the normalized magnetic field $B_{(k,n)}$ tends asymptotically to zero (see Figs. 1 and 2 below).

In the long wavelength gravitational wave case, the expansion normalized shear is given by $\Sigma_{(k_{\rm long})}=\frac{3}{4}C_1\eta^2-3C_2\eta^{-3}$, where C_1 and C_2 are integration constants. For late times, during the matter dominated era (when the equation of state $p=0$ applies), the second term in $\Sigma_{(k_{\rm long})}$ can be neglected. In this way we can easily obtain the late time behavior of the expansion normalized gravitationally induced electric field:

$$\tilde{\mathcal{E}}_{(k_{\rm long})}=\begin{cases}\dfrac{1}{12\pi^2}\left(\dfrac{\lambda_{B_0}}{\lambda_H}\right)\Sigma_{(k_{\rm long})}, & n\neq 0,\\ \frac{1}{6}\Sigma_{(k_{\rm long})}, & n=0,\end{cases}\tag{16}$$

where $\lambda_H=1/H$ is the Hubble radius during that epoch. It follows the generated electric field is proportional to the expansion normalized shear. Since the normalized magnetic field asymptotically tends to zero, the above results demonstrate that electric fields produced by this effect could play an important dynamical role in the early universe, possibly causing charge separation. Furthermore, because the asymptotic value of the electric field is proportional to the magnitude of the shear, we can use the CMB anisotropy limits on Σ to give an upper bound on the size of this effect [19]:

$$\tilde{\mathcal{E}}_{(k_{\rm long})}\lesssim\begin{cases}\left(\dfrac{\lambda_{B_0}}{\lambda_H}\right)\times 10^{-6}, & n\neq 0,\\ 2\times 10^{-5}, & n=0.\end{cases}\tag{17}$$

VI. DISCUSSION

In this paper we derived a set of nonlinear wave equations which demonstrate how electromagnetic fields can be generated when gravitational waves couple to a near-homogeneous magnetic test field propagating on a FRW background. In particular we found that for long wavelength gravitational waves, the gravitationally induced fields are proportional to the magnitude of the expansion normalized shear which characterizes tensor perturbations. This allows a simple determination of an upper bound on the magnitude of these fields based on the quadrupole anisotropy of the cosmic microwave background.

We note that this paper has not considered the back-reaction of this effect on the gravitational dynamics, which although small may also give rise to interesting results. This issue will be explored in a forthcoming paper [20].

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[1] H. Stephani, *General Relativity* (Cambridge University Press, Cambridge, England, 1982); B. S. DeWitt and R. W. Brehme, Ann. Phys. (N.Y.) **9**, 220 (1960); F. I. Cooperstock, *ibid.* **47**, 173 (1968); Ya. B. Zeldovich, Zh. Eksp. Teor. Fiz. **65**, 1311 (1973) [Sov. Phys. JETP **38**, 652 (1974)]; U. N. Gerlach, Phys. Rev. Lett. **32**, 1023 (1974); V. I. Denisov, Zh. Eksp. Teor. Fiz. **74**, 401 (1978) [Sov. Phys. JETP **47**, 209 (1978)]; L. P. Grishchuk and A. G. Polnarev, in *General Relativity and Gravitation*, edited by A. Held (Plenum, New York, 1980), Vol. 2; M. Demiański, *Relativistic Astrophysics* (Pergamon, New York, 1985), pp. 256–257; G. A. Lupanov, Zh. Eksp. Teor. Fiz. **52**, 118 (1967) [Sov. Phys. JETP **25**, 76 (1967)]; V. B. Braginskiĭ *et al.*, *ibid.* **65**, 1729 (1973) [**38**, 865 (1974)]; L. P. Grishchuk and M. V. Sazhin, *ibid.* **68**, 1569 (1975) [**41**, 787 (1976)].

[2] D. Macdonald and K. S. Thorne, Mon. Not. R. Astron. Soc. **198**, 345 (1982); G. Brodin and M. Marklund, Phys. Rev. Lett. **82**, 3012 (1999); M. Marklund, G. Brodin, and P. K. S. Dunsby, Astrophys. J. **536**, 875 (2000); Yu. G. Ignat'ev, Phys. Lett. A **230**, 171 (1997); R. Bingham *et al.*, Phys. Scr. **T75**, 61 (1998); J. Daniel and T. Tajima, Phys. Rev. D **55**, 5193 (1997).

[3] E. Battaner and H. Lesch, ''On the Physics of Primordial Magnetic Fields,'' astro-ph/0003370 (2000).

[4] R. Brustein and D. H. Oaknin, Phys. Rev. D **60**, 023508 (1999); M. Giovannini and M. Shaposhnikov, *ibid.* (to be published), hep-ph/0004269.

[5] M. Gaperini, M. Giovannini, and G. Veneziano, Phys. Rev. Lett. **75**, 3796 (1995).

[6] T. V. Ruzmaikina and A. A. Ruzmaikin, Astron. Zh. **47**, 1206 (1971) [Sov. Astron. **14**, 963 (1971)]; I. Wasserman, Astrophys. J. **224**, 337 (1978); E. Kim, A. Olinto, and R. Rosner, *ibid.* **468**, 28 (1996); E. Battaner, E. Florido, and J. Jiménez-Vicente, Astron. Astrophys. **326**, 13 (1997); K. Jedamzik, V. Katalinic, and A. Olinto, Phys. Rev. D **57**, 3264 (1998); C. G. Tsagas and R. Maartens, *ibid.* **61**, 083519 (2000); S. Hobbs

and P. K. S. Dunsby, ''Dynamical systems analysis of magnetised density perturbations,'' gr-qc/0007010 (2000).

[7] C. G. Tsagas and J. D. Barrow, Class. Quantum Grav. **15**, 3523 (1998).

[8] G. F. R. Ellis and M. Bruni, Phys. Rev. D **40**, 1804 (1989).

[9] S. W. Hawking, Astrophys. J. **145**, 544 (1966).

[10] P. K. S. Dunsby, B. A. C. C. Bassett, and G. F. R. Ellis, Class. Quantum Grav. **14**, 1215 (1997).

[11] Although this in principle breaks the homogeneity and isotropy of the FRW spacetime, a weak magnetic test field can be shown to be consistent in such a background provided the spatial variations it generates are negligible.

[12] G. F. R. Ellis and H. van Elst, Cosmological models (Cargèse lectures 1998), in *Theoretical and Observational Cosmology*, edited by M. Lachièze-Rey (Kluwer, Dordrecht, 1999), p. 1.

[13] R. Maartens, Phys. Rev. D **55**, 463 (1997).

[14] J. Bardeen, Phys. Rev. D **22**, 1882 (1980).

[15] Note that the inhomogeneity variable $\tilde{\nabla}_a B_b$ is determined by the background current j^a.

[16] E. R. Harrison, Rev. Mod. Phys. **39**, 862 (1967).

[17] J. Wainwright and G. F. R. Ellis, *Dynamical Systems in Cosmology* (Cambridge University Press, Cambridge, England, 1997).

[18] G. F. R. Ellis, Relativistic Cosmology, in *Cargèse Lectures in Physics, Volume 6*, edited by E. Schatzmann (Gordon and Breach, New York, 1973).

[19] W. Stoeger, M. Araujo, and T. Gebbie, Astrophys. J. **476**, 435 (1997); **522**, 559 (1999).

[20] P. K. S. Dunsby, M. Marklund, and G. Brodin, Electromagnetic Fields in the Early Universe (in preparation).