Axial baryon number nonconserving antisymmetric tensor four-quark effective interaction

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We discuss the phenomenological consequences of the $U_A(1)$ symmetry-breaking two-flavor four-fermion antisymmetric (AS) Lorentz tensor interaction Lagrangians. We use the recently developed methods that respect the "duality" symmetry of this interaction. Starting from the Fierz transform of the two-flavor 't Hooft interaction (a four-fermion Lagrangian with AS tensor interaction terms augmented by a Nambu–Jona-Lasinio-type Lorentz scalar interaction responsible for dynamical symmetry breaking and quark mass generation), we find the following. (1) Four antisymmetric tensor and four antisymmetric pseudotensor bosons exist which satisfy a mass relation previously derived for scalar and pseudoscalar mesons from the 't Hooft interaction. (2) Antisymmetric tensor bosons mix with vector bosons via one-fermion-loop effective couplings so that both kinds of bosons have their masses shifted and the fermions (quarks) acquire anomalous magnetic moment form factors that explicitly violate chiral symmetry. (3) The mixing of massive antisymmetric tensor fields with vector fields leads to two sets of spin-one states. The second set of spin-one mesons is heavy and has not been observed. Moreover at least one member of this second set is tachyonic, under standard assumptions about the source and strength of the antisymmetric tensor interaction. The tachyonic state also shows up as a pole in the spacelike region of the EM form factors. (4) The axial-vector fields' mixing with antisymmetric tensor bosons is proportional to the (small) isospin-breaking up-down quark mass difference, so the mixing-induced mass shift of axial vector mesons is negligible. (5) The antisymmetric tensor version of the Veneziano-Witten $U_A(1)$ symmetry-breaking interaction does not lead to tachyons, or any antisymmetric tensor field propagation to leading order in N_C.

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I. INTRODUCTION

Axial baryon number nonconserving, also known as $U_A(1)$ symmetry-breaking, quark self-interactions were introduced into modern physics as a solution to the "$U_A(1)$ problem," the fact that QCD naively predicts a light ($\simeq 135$ MeV) pseudoscalar (P) flavor singlet state (η'). The first connection with the underlying gauge theory was established by 't Hooft who derived one particular kind of such an interaction ("the 't Hooft interaction") from color SU(2) instantons [1]. This analysis was extended four years later to color SU(3), i.e., to QCD, and three flavors [2] where new kinds of effective interactions depending on Pauli's antisymmetric tensor $\sigma_{\mu\nu}=(i/2)\,[\gamma_\mu,\gamma_\nu]$ appear (such interactions are also a product of the Fierz rearrangement of the two-flavor 't Hooft interaction). Whereas the 't Hooft interaction, as well as another scalar-P $U_A(1)$ symmetry-breaking quark self-interaction due to Veneziano and Witten [3] have been extensively examined [4,5] using methods introduced by Nambu and Jona-Lasinio (NJL) [6], the "antisymmetric tensor" [AS T] and "mixed" interactions have not, except in two special cases [7,8]. One reason for this is the technically challenging nature of this interaction, which has an additional "duality" symmetry which has only recently received a proper treatment [9]. There is also a widespread lack of familiarity with AS T field theories to contend with.

Another reason may have been some apparently paradoxical results that emerge from such an analysis. Klimt *et al.* [8] have examined antisymmetric (AS) tensor self-interaction in the context of the mixing of the AS T modes with the vector ones. They found a small ω-ρ mass splitting and a constituent quark anomalous magnetic moment that was discovered somewhat earlier by Blin, Hiller, and Schaden [7]. Unfortunately neither of these papers reports details of the respective calculations. The present calculation shows that the AS T interaction creates a second set (octet or nonet) of spin-one states that mix with the "usual" vector states and thus shift their masses. This does not mean that this "second" set of states disappears, quite the contrary, these states survive and lead to dramatic consequences especially in the EM form factors and more generally in the spin-one mass spectra. Their masses are also shifted by the mixing. One peculiarity of their spectra is the apparent inevitability of spacelike (tachyonic) excitations. This feature can be directly related to the duality symmetry mentioned earlier that precludes the existence of $U_A(1)$ symmetry-conserving AS T quark self-interactions. The problem can be dodged by pushing these states to ever higher energies, though in that case one needs fine-tuning of the free parameters and one loses some of the apparent benefits, such as the explanation of the ω-ρ mass splitting. We shall not explore the group-theoretical ramification of these results here, but confine ourselves to the two-flavor case.

The AS T and PT states satisfy the same 't Hooft interaction mass sum rule derived for scalar and pseudoscalar mesons in Ref. [4]. But the AS T states of opposite parities do not couple to the rest of the world in a symmetrical way, specifically the AS PT states decouple and do not mix with ordinary axial-vector states. So the observability of AS PT states seems questionable.

The purpose of this paper is to explore the phenomenological consequences of antisymmetric tensor quark self-

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interaction terms in NJL-type models with dynamical symmetry breaking. We use a formalism developed in Ref. [9] that preserves the duality symmetry and show that the Lagrangian obtained from the two-flavor 't Hooft interaction by a Fierz rearrangement, leads to composite antisymmetric tensor and pseudotensor NG bosons. The AS tensor modes mix with vector bosons coupled to this system, which leads to a mass shift of both and to observable effects in the EM form factors. We shall confine ourselves to two flavors so as to keep the treatment as simple as possible. The same issues arise with three flavors, but they are obscured by the more complicated algebra.

II. PRELIMINARIES

It has long been known that four-fermion contact interactions of the Nambu and Jona-Lasinio (NJL-) type can lead to dynamical symmetry breaking along with associated composite spinless Nambu-Goldstone (NG) bosons [6]. Such interactions have been extended to include all but six of the 16 independent Dirac matrix bilinears. The six still unexplored terms correspond to the antisymmetric (AS) tensor self-interaction, which leads after bosonization to antisymmetric tensor bosonic excitations [10]. We shall use the results and notation of Ref. [9], with minimal modifications.

We shall work with a chirally symmetric field theory described by

$$\mathcal{L}_{\rm ENJL}=\bar{\psi}[i\partial\!\!\!/-m_0]\psi+G_S[(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2] \\ -G_T[(\bar{\psi}\sigma_{\mu\nu}\psi)^2+(\bar{\psi}i\gamma_5\sigma_{\mu\nu}\boldsymbol{\tau}\psi)^2], \tag{1}$$

where ψ is an isospin-doublet and a color-triplet Dirac field.[1] There are no color-dependent forces, m_0 is the current quark mass matrix, and $\boldsymbol{\tau}$ are the isospin Pauli matrices. The antisymmetric tensor self-interaction in the second line of Eq. (1) preserves the $SU_L(2)\times SU_R(2)$ chiral symmetry, but violates the axial baryon number $U_A(1)$ symmetry [5]. This self-interaction is related to the two-flavor 't Hooft interaction [1] by a Fierz transformation, which in this case must be performed in the color space as well.

The whole tensor term in the Lagrangian (1) vanishes identically in the Abelian, i.e., single-flavor ($N_f=1$) case, due to another, hidden symmetry which we shall call the ''duality symmetry.'' It follows from the identity, see Refs. [10,11]

$$\gamma_5\sigma_{\mu\nu}=\frac{i}{2}\varepsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}=i\sigma^{\star}_{\mu\nu}=i\tilde{\sigma}_{\mu\nu} \tag{2}$$

which allows the second line in the Lagrangian (1) to be written as

$$G_{\rm T}[(1+\lambda)(\bar{\psi}\sigma_{\mu\nu}\psi)^2 \\ +\lambda(\bar{\psi}i\gamma_5\sigma_{\mu\nu}\boldsymbol{\tau}\psi)^2+\lambda(\bar{\psi}i\gamma_5\sigma_{\mu\nu}\psi)^2 \\ +(1+\lambda)(\bar{\psi}\sigma_{\mu\nu}\boldsymbol{\tau}\psi)^2], \tag{3}$$

where λ is an arbitrary (real) ''duality-symmetry gauge fixing parameter.'' Of course, observable predictions of this model must be independent of λ. The duality-symmetry has several curious consequences, such as the structure of the effective propagators for the composite AS T fields, see Ref. [9].

For any given Lagrangian one can write down the exact SD equations which form an infinite set of coupled integral equations for (infinitely many) n-point Green functions. They describe exactly the nonperturbative dynamics of the quantum fields, but are also intractable in their exact form. For this reason they must be truncated in any practical application. In this model the truncation can be accomplished using N_C (number of colors) counting as the guiding principle. To leading order in $1/N_C$ the truncation leads to two Schwinger-Dyson [SD] equations:[2] (i) the gap equation and (ii) the Bethe-Salpeter (BS) equation. These two equations have been solved in Ref. [9].

There are four AS T,PT states altogether: two isospin channels, isoscalar and isovector, which differ only in the overall sign of the tensor coupling constant G_T and two parities. In Ref. [9] we found the poles in the BS propagators, which determine the masses of the T,PT states, while the residues determine their coupling constants to the quarks as follows:

$$g_p^2=\left(\frac{m}{f_p}\right)^2=\frac{2}{3}g_{\rm T}^2\left[1+\left(\frac{g_p}{2\pi}\right)^2\right]=\frac{2}{3}g_{PT}^2\left[1+\left(\frac{g_p}{2\pi}\right)^2\right], \tag{4}$$

where f_p is the ''bare'' pion decay constant. A remarkable pattern exists in the mass spectrum: the four poles are symmetrically placed about the origin with locations at

$$M_T^{(1)\ 2}\simeq 6m^2+m_T^2, \tag{5a}$$

$$M_T^{(0)\ 2}\simeq 6m^2-m_T^2, \tag{5b}$$

$$M_{PT}^{(1)\ 2}\simeq -(6m^2-m_T^2), \tag{5c}$$

$$M_{PT}^{(0)\ 2}\simeq -(6m^2+m_T^2), \tag{5d}$$

where the bracketed superscript indicates the isospin channel. Here the (gauge invariant) ''tensor mass'' m_T is

$$m_T^2=\frac{3g_p^2}{4G_T}. \tag{6}$$

Hence follows the two-flavor AS tensor version of the 't Hooft interaction sum rule [4]

[1]The sign of the tensor coupling constant G_T has been changed from that in Ref. [9], so as to conform with the sign of the vector coupling constant G_T.

[2]For an extension beyond the leading order in $1/N_C$, see Ref. [12].

$$M_T^{(1)\ 2}-M_T^{(0)\ 2}=M_{PT}^{(0)\ 2}-M_{PT}^{(1)\ 2}. \tag{7}$$

This sum rule says that the isoscalar-isovector mass splittings in the pseudotensor and tensor channels are equal in size and opposite in sign.

It is clear that in the chiral limit one has an isovector massless Nambu-Goldstone the antisymmetric (pseudo) tensor state, at $k^2=0$, provided that

$$m_T^2=6m^2, \tag{8}$$

holds, which is equivalent to $G_T^{-1}=8f_p^2=-2G_S^{-1}$. The relations (6),(8) bear remarkable similarity to analogous relations for the vector mass and coupling constant in the extended NJL (ENJL) model [16]. In other words, Eq. (8) defines a critical point in the space of AS T coupling constants of this theory. Change of G_T or G_S can lead to (phase) transitions to other phases of the theory, and thence to tachyons.

III. BOSONIZATION OF THE MODEL

The bosonization procedure allows one to find a bosonic ''effective Lagrangian'' that describes the low-energy dynamics of the bound state mesons in the underlying fermionic ENJL model. It amounts to finding the effective equations of motion (EOM) for the bosonic bound states in the fermionic model and the appropriate bosonic Lagrangian [10]. The corresponding EOM then leads to the ENJL propagator, Eq. (28) in Ref. [9], which is rather unusual, as AS T propagators go. One can find two standard forms of AS T propagators in the literature, corresponding to the two standard forms of the kinetic energy: (1) the Kemmer-Proca form [13] corresponding to the kinetic energy

$$\mathcal{L}_{\text{boson}}=\frac{1}{2}(\partial^\mu\Phi_{\mu\nu})^2-\frac{1}{4}M_T^2\Phi_{\mu\nu}^2, \tag{9}$$

and (2) the Kalb-Ramond form [14]

$$\mathcal{L}_{\text{boson}}=-\frac{1}{12}(\Phi_{\mu\nu\rho})^2+\frac{1}{4}M_T^2\Phi_{\mu\nu}^2, \tag{10}$$

which is related to Eq. (9) by the duality

$$\tilde{\Phi}_{\mu\nu}=\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\Phi_{\alpha\beta},$$

$$\Phi_{\mu\nu\rho}=\partial_\mu\Phi_{\nu\rho}+\partial_\nu\Phi_{\rho\mu}+\partial_\rho\Phi_{\mu\nu}=\varepsilon_{\mu\nu\rho\alpha}\partial_\beta\tilde{\Phi}^{\alpha\beta},$$

$$\partial^\mu\tilde{\Phi}_{\mu\nu}=\frac{1}{6}\varepsilon_{\alpha\beta\gamma\nu}\Phi^{\alpha\beta\gamma}. \tag{11}$$

Consequently, the two kinds of AS T propagator are (double) duals of each other. This simple relationship seems to have gone unnoticed thus far. The AS T propagators of either Kemmer-Proca or the dual kind are ill-defined as the mass goes to zero: they have a momentum-independent singularity (a pole) there. This is commonly thought of as a symptom of the Ogievetskii-Polubarinov spin-mass discontinuity [15]. Compare this with the smooth zero-mass limit of the ENJL AS T propagator Eq. (28) in Ref. [9], which follows, in the long wavelength limit, from the effective kinetic Lagrangian

$$\mathcal{L}_{\text{boson}}=\frac{1}{2}(\partial^\mu\Phi_{\mu\nu})^2-\frac{1}{12}(\Phi_{\mu\nu\rho})^2+\frac{1}{4}M_T^2\Phi_{\mu\nu}^2. \tag{12}$$

This Lagrangian is an unusual beast, since normally in the theory of AS T fields one finds one (Kemmer-Proca) or another (Kalb-Ramond) form of the kinetic term, but never both. The absence of a term can be construed as its being identically zero. This assumption is rather important, however, since the vanishing of one of these two forms of kinetic energy amounts to the assumption that the AS T field is closed, or co-closed. Closure is a mathematical term for the vanishing of $\Phi_{\mu\nu\rho}=\partial_\mu\Phi_{\nu\rho}+\partial_\nu\Phi_{\rho\mu}+\partial_\rho\Phi_{\mu\nu}=0$, which is equivalent to $\partial_\alpha\tilde{\Phi}^{\alpha\beta}=0$. Co-closure implies the same, but for the dual field, i.e., $\partial^\mu\Phi_{\mu\rho}=0$. *Either of these two constraints reduces the number of independent components of* $\Phi_{\mu\nu}$ *from six to three*. Closed, or co-closed AS tensors can be written in terms of a four-vector with one subsidiary condition, i.e., in terms of three independent field components, as is well known from the example of the EM field strength tensor. This corresponds to a massive spin-one field. An unconstrained AS T field describes *two* such objects of opposite parities. Kemmer was explicit about the closure assumption, though he did not discuss it further in his paper [13] (he was specifically interested in AS theories describing vector particles), whereas other authors do not seem to have been aware of the constrained nature of their Lagrangians in the first place.

The result of bosonization of the AS T fields part of the extended two-flavor NJL model Eq. (1) is the following bosonic effective Lagrangian:

$$\mathcal{L}_{\text{boson}}=\frac{1}{2}[(\partial^\mu\Sigma_{\mu\nu})^2+(\partial^\mu\mathbf{\Pi}_{\mu\nu})^2+(\partial^\mu\Pi_{\mu\nu})^2+(\partial^\mu\mathbf{\Sigma}_{\mu\nu})^2]$$
$$+\frac{1}{8}(6m^2+m_T^2)[\Sigma_{\mu\nu}^2-\Pi_{\mu\nu}^2]+\frac{1}{8}(6m^2-m_T^2)$$
$$\times[\mathbf{\Sigma}_{\mu\nu}^2-\mathbf{\Pi}_{\mu\nu}^2]+\cdots, \tag{13}$$

where the ellipsis stands for cubic and higher-order AS T or PT field interaction, as well as the scalar and p field terms. All AS T or PT fields here ought to be thought of as closed/co-closed. Several comments are in order now.

(i) Note that due to the duality relations

$$\Sigma_{\mu\nu}=\tilde{\Pi}_{\mu\nu}=\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\Pi_{\alpha\beta},$$

$$\tilde{\Sigma}_{\mu\nu}=-\Pi_{\mu\nu}=\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\Sigma_{\alpha\beta},$$

$$\Sigma_{\mu\nu\rho}=\partial_\mu\Sigma_{\nu\rho}+\partial_\nu\Sigma_{\rho\mu}+\partial_\rho\Sigma_{\mu\nu},$$

$$\partial^\mu\tilde{\Sigma}_{\mu\nu}=\frac{1}{6}\varepsilon_{\alpha\beta\gamma\nu}\Sigma^{\alpha\beta\gamma}, \tag{14}$$

and similarly for the isovector fields $\boldsymbol{\Sigma}_{\mu\nu},\boldsymbol{\Pi}_{\mu\nu}$, this model Lagrangian can be written using only a single (unconstrained) AS T field of either parity

$$\mathcal{L}_{\rm boson}=\frac{1}{2}[(\partial^{\mu}\Sigma_{\mu\nu})^2+(\partial^{\mu}\boldsymbol{\Sigma}_{\mu\nu})^2]-\frac{1}{12}[(\Sigma_{\mu\nu\rho})^2+(\boldsymbol{\Sigma}_{\mu\nu\rho})^2]+\frac{1}{4}(6m^2+m_T^2)[\Sigma_{\mu\nu}^2]+\frac{1}{4}(6m^2-m_T^2)[\boldsymbol{\Sigma}_{\mu\nu}^2]+\cdots$$
$$=\frac{1}{2}[(\partial^{\mu}\Pi_{\mu\nu})^2+(\partial^{\mu}\boldsymbol{\Pi}_{\mu\nu})^2]-\frac{1}{12}[(\Pi_{\mu\nu\rho})^2+(\boldsymbol{\Pi}_{\mu\nu\rho})^2]-\frac{1}{4}(6m^2+m_T^2)[\Pi_{\mu\nu}^2]-\frac{1}{4}(6m^2-m_T^2)[\boldsymbol{\Pi}_{\mu\nu}^2]+\cdots. \quad (15)$$

(ii) These alternate forms of the effective Lagrangian may give rise to questions about the number of independent degrees of freedom ($N_{\rm DF}$). In view of our previous remarks about the closed and unconstrained AS T fields, it ought to be clear that the number of the $N_{\rm DF}$ is always the same.

(iii) In the limit $m_T^2\to 6m^2$ we find massless states, Goldstone bosons, in the isovector channel and massive states in the isoscalar one, in accord with the broken $U_A(1)$ symmetry. (We do not expect $m_T^2\to 6m^2$ to hold in general.) The fact that massless spinless fields (Goldstone bosons) can be described by massless closed AS T fields is well known [18], only the doubling of the Goldstone bosons in this case is a surprise. This, however, can be explained by the fact that our massless AS T field is not closed, but rather can be thought of as containing two such components with opposite parities.

(iv) Note that there is always at least one tachyon in the spectrum: States of opposite parity have opposite signs of mass squared; one of these masses can be arranged to vanish, but the other cannot. Tachyons are usually signs of an unstable, i.e., badly chosen ''ground state,'' although there are also tachyonic states, such as the Landau ghosts, which are not associated with a bad choice of ground state. In our case it is difficult to see how one could have chosen a ''better'' ground state, as the gap Eq. (5) in Ref. [9], which determines the vacuum, does not depend on the AS tensor interaction to leading order in $1/N_C$. Nor is this tachyon related to the Landau ghost. The reason for the tachyon(s) lies entirely with the duality symmetry which forbids the existence of an $U_A(1)$ symmetric AS T interaction at this level: the same problem appears with the original (scalar and pseudoscalar) 't Hooft interaction when left without an $U_A(1)$ symmetric interaction.[3] The source of the problem can be most easily seen from the isospin form of the 't Hooft interaction

$$[\Sigma_{\mu\nu}^2-\Pi_{\mu\nu}^2-\boldsymbol{\Sigma}_{\mu\nu}^2+\boldsymbol{\Pi}_{\mu\nu}^2]. \quad (16)$$

This term is proportional to the mass squared: now note that with either overall sign there are always two terms with nonpositive curvature. As explained above, one of these two curvatures can be arranged to go to zero, but the second one cannot. In the case of the scalar 't Hooft interaction, Ref. [4], this problem was solved by having a (stronger) $U_A(1)$-invariant interaction that stabilizes the total Hamiltonian [4]. In the AS tensor case such an $U_A(1)$-invariant interaction Hamiltonian vanishes identically due to duality symmetry, Eq. (3), and such a stabilization measure is impossible.

Before drawing any conclusions let us see if these tachyons are observable. In case they decouple, one might be able to ignore them. In case they are observable, it will serve as an indictment against the 't Hooft effective interaction as the source of $U_A(1)$ symmetry breaking: the Veneziano-Witten effective interaction does not lead to tachyons because it is *quartic* in the AS T fields and hence does not contribute to the AS T propagator to leading order in N_C. Next we shall see that AS T interactions/fields give anomalous magnetic moments to external vector currents and that is where we shall find observable effects of the aforementioned tachyons.

IV. INDUCED ANOMALOUS MAGNETIC MOMENT

The place to look for the tachyon is in the EM form factors due to the mixing of T states with the vector mesons. Blin, Hiller, and Schaden [7] pointed out that the AS T interaction leads to anomalous magnetic moments of the constituent quarks via mixing with vector currents

$$\mathcal{L}_{\rm V}=\frac{1}{2m}[\kappa^{(1)}(\bar{\psi}\sigma_{\mu\nu}\boldsymbol{\tau}\psi)\cdot\mathbf{F}^{\mu\nu}+\kappa^{(0)}(\bar{\psi}\sigma_{\mu\nu}\psi)F^{\mu\nu}]. \quad (17)$$

It then stands to reason that the (Pauli) magnetic form factor will have a pole at the AS T field mass.

To evaluate the anomalous magnetic moment all we need is one-way conversion V to T, not the other way round. We need the vector-pseudotensor (V-PT) transition matrix element $\Pi_{\mu\nu\alpha}^{V\text{-}PT}$, which is again given by a simple one-loop graph. One finds

$$\Pi_{\mu\nu\alpha}^{V\text{-}PT}(s)=2N_C\int\frac{d^4p}{(2\pi)^4}{\rm tr}[\gamma_5\sigma_{\mu\nu}S(p+q)\gamma_{\alpha}S(p)]=2iemg_p^{-2}F(s)\varepsilon_{\mu\nu\alpha\beta}q^{\beta}. \quad (18)$$

(The analogous A-T transition tensor is proportional to the up-down quark difference, i.e., in the good isospin limit AS tensors do *not* couple to axial-vector currents [8]. This is a sign of asymmetry between the two sectors with opposite parities.) Attach an AS PT coupling at the end of this and find the following addition to the vector current ($s=q^2$):

[3] This is precisely what was found in Ref [4]: '' . . . note that as $G_1\to 0$, i.e., with 't Hooft interaction only, . . . at least one bound state (η^*) necessarily becomes a tachyon, . . . *the 't Hooft interaction is not consistent by itself, and must be accompanied by an $U_L(2)\times U_R(2)$ symmetric interaction.*''

$$\delta V_\alpha = -2iG_T\gamma_5\sigma^{\mu\nu}\Pi^{V\text{-}PT}_{\mu\nu\alpha}(q) = 4G_T emg_p^{-2}F(s)\varepsilon_{\mu\nu\alpha\beta}q^\beta\gamma_5\sigma^{\mu\nu} = 16G_T f_p^2 F(s)\frac{i}{2m}\sigma_{\alpha\beta}q^\beta \tag{19}$$

which yields $\kappa = 16G_T f_p^2$. Manifestly one ought to iterate the PT interaction, which leads to the PT propagator. The Levi-Civita symbol in Eq. (18) leads to the dual of the PT propagator which equals the T one

$$\delta V_\delta = -iD^{PT}_{\mu\nu;\alpha\beta}\Pi^{V\text{-}PT}_{\alpha\beta\delta}(q)\gamma_5\sigma^{\mu\nu} = F_2(s)\frac{i}{2m}\sigma_{\delta\beta}q^\beta,$$

$$F_2(s) = \frac{-12m^2F(s)}{\{(s-6m^2)F(s)+2m^2[F(s)-1]+m_T^2\}}. \tag{20}$$

Therefore the anomalous magnetic Pauli form factor $F_2(s)$ must have a pole at the T mass in the respective isospin channel.

Now remember Eq. (5b), from which it follows that the T pole is tachyonic in the isoscalar channel. Since we know that the Pauli form factor acquires a pole at the T meson mass we see that a tachyon pole will show up in the (deeply) spacelike Euclidean region of the EM form factor (the isoscalar part). Thus, the tachyon will be observable as a pole in elastic and inelastic electron scattering cross section when the momentum transfer q^2 is spacelike. But this kinematic region has been well examined in deeply inelastic scattering (DIS) experiments and no deviations from the usual photon propagator, i.e., from the Coulomb law have been observed. DIS has been measured upward of $Q^2 = -q^2 = 100\ \text{GeV}^2$ with no indication of an enhancement in the Mott cross section, not to mention a pole. We may safely conclude that $m_T \geqslant 60m$ from here. This value is substantially larger than the Fierz value/prediction of $m_T \simeq 9m$, based on

$$G_T = \frac{1}{4N_C}G_{\text{tH}}, \tag{21}$$

We know from Ref. [4] that the requirement that there be no tachyons in the scalar-P meson sector leads to the inequality $2|G_{\text{tH}}| \leqslant |G_S|$, which together with Eq. (19) in Ref. [9] and Eq. (21) above leads to

$$|G_T| \leqslant \frac{1}{32N_C f_p^2}, \tag{22}$$

or equivalently to

$$m_T^2 \geqslant 72m^2. \tag{23}$$

In other words, the DIS result leads to an unusually small T coupling constant, i.e., one must fine-tune the constant to reach consistency with observation.

There is an apparent solution, however, viz. $m_T^2 \leqslant 6m^2$. This solution has its problems, too: the tachyon moves to the AS PT mesons, which mix weakly with axial currents, via a II class current. Such currents are badly known, only an upper bound on the static value of the form factor exist at the moment, but it seems highly unlikely that a spacelike pole in the cross section would have been missed. Even if this were the case, this choice merely shifts the problem to another sector, for in that case Eq. (21) implies a violation of the $2|G_{\text{tH}}| \leqslant |G_S|$ inequality and thus leads to tachyons in the scalar-P meson sector [4].

V. MIXING WITH VECTOR MESONS

We have shown the role AS T excitations play in external, e.g., EM, vector currents. This suggests a possibility of mixing with preexisting vector mesons, such as the ρ and the ω. Indeed this mixing has already been addressed in the literature [8], but several key aspects of the results have not been mentioned, specifically that the number of vector mesons, some of which are tachyonic, is doubled. Moreover, in the meantime, the treatment of vector mesons in the ENJL model has undergone a significant change [16], which leads us to believe that a new treatment has its merits.

A. Preliminaries

The extension to include the vector (V) and axial-vector (A) N_{DF} is straightforward: we add the following terms to the Lagrangian (1):

$$\mathcal{L}_{\text{V}} = -G_{\text{V}}^{(1)}[(\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi)^2] - [G_V^{(0)}(\bar{\psi}\gamma_\mu\psi)^2 + G_A^{(0)}(\bar{\psi}\gamma_\mu\gamma_5\psi)^2]. \tag{24}$$

In the isotriplet channel one must have $G_V^{(1)} = G_A^{(1)}$ in order to preserve the chiral symmetry, whereas in the isosinglet channel the two coupling constants need not coincide $G_V^{(0)} \neq G_A^{(0)}$.

The mixing of the P and longitudinal axial-vector modes implies a finite renormalization of the ''bare'' ($G_A = 0$) P meson decay constant f_p to f_π and of the constituent quark axial coupling g_A [16] according to

$$g_A^{(I)} = (1+8G_A^{(I)}f_p^2)^{-1} = \left(\frac{f_\pi^{(I)}}{f_p}\right)^2 = \left(\frac{g_p}{g_\pi^{(I)}}\right)^2. \tag{25}$$

This leads to the relation

$$g_A^{(I)} = 1 - 8G_A^{(I)}f_\pi^{(I)\ 2} \tag{26}$$

between $g_A^{(I)}$ and $G_A^{(I)}$ and $f_\pi^{(I)}$, the last of which is kept constant. An $f_\pi^{(I)}$-fixing procedure analogous to the one described in Sec. II now yields a separate m vs Λ curve for any given value of $g_A^{(I)}$, see Fig. 1 in Ref. [16]. An important consequence of the relation (26) and of the second line of Eq. (25) is the inequality $0 \leqslant g_A^{(I)} \leqslant 1$. This imposes a new upper bound on $G_A^{(I)}$:

$$G_A^{(I)} \leqslant 1/(8f_\pi^{(I)\ 2}), \tag{27}$$

apart from the trivial lower bound $G_A^{(I)} \geqslant 0$. $G_A^{(I)}$ values exceeding the bound imply imaginary values of g_p and f_p, which in turn imply complex cutoff Λ and/or mass m. A physical interpretation of such complex objects is lacking.

We see from Eq. (26) that $G_A^{(I)}$ can be determined from the value of the constituent quark axial coupling constant $g_A^{(I)}$, at constant $f_\pi^{(I)}$. One common prescription for estimating $g_A^{(1)}$ is based on the SU(6) symmetric nucleon wave function and the impulse approximation result for the nucleon axial coupling in the quark model

$$g_{A,N}^{(1)} = \frac{5}{3} g_{A,Q}^{(1)} = 1.25|_{\rm expt}, \tag{28}$$

which yields $g_{A,Q}^{(1)} = 0.75$ if we neglect the two-quark axial current contributions to the nucleon isovector axial current matrix element $g_{A,2Q}^{(1)}$. Completely analogous results hold in the flavor-singlet channel where the SU(6) factor is 1 (instead of 5/3 in the isotriplet channel):

$$g_{A,Q}^{(0)} = g_{A,Q}^{(0)} = 0.12 \pm 0.23|_{\rm expt}. \tag{29}$$

This is also subject to the axial MEC corrections, which are not known in detail at the moment, except that they are of $\mathcal{O}(1/N_C) \simeq 30\%$, and *must* be included if chiral symmetry is to be conserved.

B. Mixing formalism

All the objects in the BS equation are 2×2 matrices. The solutions to the BS matrix Eq. (6) in Ref. [9] for the propagator matrix **D** are

$$\mathbf{D} = 2\mathbf{G}(\mathbf{1} - 2\mathbf{\Pi}\,\mathbf{G})^{-1} = (\mathbf{1} - 2\mathbf{G}\,\mathbf{\Pi})^{-1}(2\mathbf{G}). \tag{30}$$

The vector-pseudotensor (V-PT) transition matrix element $\Pi_{\mu\nu\alpha}^{V-PT}$, Eq. (18) couples the vector states to the PT ones, but the Levi-Civita symbol contained therein leads to the dual of the PT propagator which is just the T one. Thus we are led to the following coupled channel problem.

The matrix $(\mathbf{1} - 2\mathbf{G\Pi})$ has to be inverted. Its explicit structure is

$$\mathbf{1} - 2\mathbf{G\Pi} = \begin{pmatrix} [1 + 2G_V\Pi_V(s)] & 2G_V\Pi_{V\text{-}T}(s) \\ 2G_T\Pi_{V\text{-}T}(s) & [1 + 2G_T\Pi_T(s)] \end{pmatrix}, \tag{31}$$

where

$$\Pi_{V\text{-}T}(s) = 4m\sqrt{s}g_p^{-2}F(s) = 4\frac{m}{g_A}\sqrt{s}g_\pi^{-2}F(s). \tag{32}$$

This leads to the solution for the matrix propagator **D** in the form

$$-\mathbf{D} = (\mathbf{1} - 2\mathbf{G}\,\mathbf{\Pi})^{-1}(2\mathbf{G}) = \mathcal{D}^{-1}\begin{pmatrix} -2G_V[1 + 2G_T\Pi_T(s)] & 4G_TG_V\Pi_{V\text{-}T}(s) \\ 4G_TG_V\Pi_{V\text{-}T}(s) & -2G_T[1 + 2G_V\Pi_V(s)] \end{pmatrix}, \tag{33}$$

where

$$\mathcal{D} = Det(\mathbf{1} - 2\mathbf{G\Pi}) = [1 + 2G_V\Pi_V(s)][1 + 2G_T\Pi_T(s)] - 4G_TG_V\Pi_{V\text{-}T}^2(s). \tag{34}$$

The meson masses are determined by the poles, i.e., by the zeros of the determinant $\mathcal{D}$. In order to get some feeling for the behavior of the masses as functions of the coupling constants we may take as a first approximation

$$\frac{1}{2G_T} + \Pi_T(s) \simeq -\frac{2}{3}g_p^{-2}(s - M_T^2), \tag{35a}$$

$$\frac{1}{2G_V} + \Pi_V(s) \simeq -\frac{2}{3}g_p^{-2}(s - m_V^2), \tag{35b}$$

$$\Pi_{V\text{-}T}(s) \simeq 4m\sqrt{s}g_p^{-2}, \tag{35c}$$

where

$$m_{\rm V}^2 = \frac{3g_p^2}{4G_{\rm V}}, \tag{36}$$

and M_T^2 is given in Eqs. (5a),(5b). Hence, the two poles are at

$$s_\pm = \frac{1}{2}[m_V^2 + M_T^2 + 36m^2 \pm \sqrt{(m_V^2 + M_T^2 + 36m^2)^2 - 4m_V^2M_T^2}]. \tag{37}$$

In the weak T coupling limit ($G_T \to 0$), i.e., as $M_T^2 \gg 36m^2, m_V^2$, which roughly corresponds to the Fierz prediction $G_S \gg G_T$, we may write

$$s_+ = m_V^2 + M_T^2 + 36m^2 \simeq m_T^2,$$

$$s_- = \frac{m_V^2M_T^2}{(m_V^2 + M_T^2 + 36m^2)} \simeq m_V^2\left[1 - \left(\frac{m_V^2 + 36m^2}{M_T^2}\right)\right] \tag{38}$$

which shows that the vector mass is shifted by the tensor interaction, even in the weak T coupling limit. As the sign in $M_T^2 = 6m^2 \pm m_T^2$ alternates depending on the isospin channel, one finds two different values for the lighter meson solution (s_-). This leads to the ρ-ω mass splitting, though it is *not* its only possible source: chiral symmetry does not demand that the isovector and isoscalar vector (or axial vector) mesons masses be identical in the first place $G_V^{(1)} \neq G_V^{(0)} \neq G_A^{(0)}$. If we assume that, for some reason $G_V^{(1)} = G_V^{(0)}$ and therefore the unperturbed isovector and isoscalar vector masses are degenerate $m_V^{(1)} = m_V^{(0)} \simeq 2m$ before turning on of the AS T interaction, then we may use the experimental mass splitting to determine the T coupling G_T, or, what is the same, see Eq. (6), the T mass as $m_T \simeq 45m$. This is in qualitative agreement with our estimate from electron scattering and substantially larger than the ''Fierz value'' of $m_T \simeq 9m$. Hence, in this scenario one needs either a dynamical justification of the crossed-channel suppression, or a fine-tuning of coupling constants in the Lagrangian. Both solutions seem *ad hoc* at the present time.

Finally we need to take a closer look at the second scenario $6m^2 \geqslant m_T^2$, and strong tensor 't Hooft interaction coupling constant G_T. The tachyon poles still show up in the badly known isovector class II axial current, but can be pushed up in mass out of reach of extant experiments. The corresponding light tachyon in the isoscalar axial current is unobservable, as the transition matrix element vanishes (isoscalar current turns a quark into itself, so with no up-down mass difference there is no transition matrix element). One can make this spectrum conform with the observed one using the freedom in $G_V^{(0)}$. This comes at a high price, however. The issue of scalar-P tachyons reappears at such strong coupling G_T, or again one faces the question of arbitrary Fierz suppression. Whereas this does not constitute a ''watertight'' case against the 't Hooft interaction, it does take us closer to the conclusion that the Veneziano-Witten interaction [3] ought to be preferred to the 't Hooft one [5].

C. Chiral symmetry breaking by the anomalous magnetic moment

To our surprise, it turns out that the induced one-loop interaction leads to the anomalous magnetic moment Lagrangian Eq. (17) which explicitly breaks chiral symmetry. Perhaps the easiest way to see this is to evaluate the divergence of the axial current

$$\partial^\mu(\bar{\psi}\gamma_\mu\gamma_5\psi) = -\frac{i}{m}[\kappa^{(1)}(\bar{\psi}\sigma_{\mu\nu}\gamma_5\boldsymbol{\tau}\psi)\cdot\mathbf{F}^{\mu\nu} + \kappa^{(0)}(\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi)F^{\mu\nu}],$$

$$\partial^\mu(\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi) = -\frac{i}{m}[\kappa^{(1)}(\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi)\mathbf{F}^{\mu\nu} + \kappa^{(0)}(\bar{\psi}\sigma_{\mu\nu}\gamma_5\boldsymbol{\tau}\psi)F^{\mu\nu}]. \quad (39)$$

These divergences can be rewritten using identity (2) in terms of the dual field tensors

$$\partial^\mu(\bar{\psi}\gamma_\mu\gamma_5\psi) = \frac{1}{m}[\kappa^{(1)}(\bar{\psi}\sigma_{\mu\nu}\boldsymbol{\tau}\psi)\cdot\tilde{\mathbf{F}}^{\mu\nu} + \kappa^{(0)}(\bar{\psi}\sigma_{\mu\nu}\psi)\tilde{F}^{\mu\nu}],$$

$$\partial^\mu(\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi) = \frac{1}{m}[\kappa^{(1)}(\bar{\psi}\sigma_{\mu\nu}\psi)\tilde{\mathbf{F}}^{\mu\nu} + \kappa^{(0)}(\bar{\psi}\sigma_{\mu\nu}\boldsymbol{\tau}\psi)\tilde{F}^{\mu\nu}], \quad (40)$$

very similar to the Adler-Bell-Jackiw (ABJ) axial anomaly. Their physical consequences might also be similar to those of the ABJ anomaly: In some applications this explicit chiral symmetry breaking appears to be *soft*, e.g., it does *not* lead to violations of low-energy elastic πN scattering theorems, such as the Weinberg-Tomozawa one. In other applications, such as pion photoproduction and electroproduction, it is *hard* as it leads to observable differences between P and PV LETs at the Born approximation level.

VI. DISCUSSION AND CONCLUSIONS

In conclusion, we have shown the following. (1) A model with non-Abelian symmetry, dynamical symmetry breaking of the NJL type and an antisymmetric tensor fermion self-interaction leads to a few massive and at least one tachyonic composite antisymmetric tensor boson. (2) Vector gauge bosons coupled to this system mix with the antisymmetric tensor bosons so that both have their masses shifted. The AS tensor bosons couple to vector bosons via one-fermion-loop diagrams and the fermions (quarks) acquire anomalous magnetic moments that explicitly violate chiral symmetry, just like the Adler-Bell-Jackiw axial anomaly. (3) Bosonization procedure has been applied and a bosonic Lagrangian involving AS T fields has been constructed. Massive AS tensor fields can be used to describe massive vector bosons, hence the mixing of massive AS tensor fields with vector fields leads to two sets of spin-1 excitations. The second spin-1 set is usually much heavier than the first one and is not readily identifiable with anything in the particle tables. The isovector ''second'' vector state is almost always tachyonic, however, and shows up as a pole in the spacelike region of the EM form factors. Alternative scenarios exist, but they expel tachyons from the vector sector only at the cost of introducing them into the scalar mesons and/or poorly known class II axial-vector current. Thus preponderance of evidence points towards the Veneziano-Witten effective interaction as the more viable alternative to the 't Hooft one. (4) Axial-vector fields mixing with antisymmetric tensor bosons is proportional to the isospin-violating up-down quark mass difference, therefore contributes only to class II axial currents, which are not well known experimentally. (5) The number of independent degrees of freedom and their spin-parity content are another curiosity: One may say that there is one unconstrained field of either parity, and the duality symmetry relates it to another completely dependent field of opposite parity. Or one can effectively view this situation as containing two independent but constrained (closed or coclosed) fields of each parity. In the massless limit the AS T fields turn into an effective description of spinless, rather than spin-one states of both parities in accord with expectations [17,18].

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