Renormalization of the Cabibbo-Kobayashi-Maskawa matrix

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Using the on-shell scheme and the general linear R_ξ gauge we calculate the one-loop amplitude $W^+ \to u_I \overline{d}_j$. In agreement with previous work, we show that the Cabibbo-Kobayashi-Maskawa (CKM) matrix ought to be renormalized. We show how to renormalize the CKM matrix and, at the same time, obtain a gauge-independent W decay amplitude.

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I. INTRODUCTION

The electroweak sector of the standard model (SM) has been the subject of extensive studies during the last 25 years. Since the renormalizability of the SM was proved [1], an immense effort has been made to implement this renormalization program at the one-loop level (cf. Refs. [2] and [3] for a review). The agreement between these calculations and the experimental results is impressive.

Despite these facts, the renormalization of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [4] was done only by one group, Denner and Sack [5] (DS) in 1990. They have shown that, as soon as one takes into account the nondegeneracy of the quark masses, the CKM matrix ought to be renormalized. However, recently Gambino, Grassi, and Madricardo [6] (GGM) have raised some doubts about the DS renormalization prescription. In particular, they have claimed that the on-shell conditions used by DS lead to a gauge-dependent width for the decay $W \to q\overline{q}$. Then, they propose an alternative renormalization prescription.

In view of this situation, we decided that it is appropriate to carry out another independent calculation of the renormalization of the CKM matrix. This is our aim. We repeat the work of DS, but with a fundamental difference. Rather than using the common 't Hooft–Feynman gauge ($\xi = 1$) we do our calculation in the general linear R_ξ gauge. Hence, we will be able to show, explicitly, the problem raised by GGM and make a proposal to solve it.

To address the question of the CKM renormalization one has to consider a process where this matrix appears at the tree level. To be precise, let us consider the decay $W^+ \to q_I \overline{q}_j$, where I and j are generation indices. We use uppercase letters for the *up*-type quarks and lowercase letters for the *down*-type quarks. Then, at the tree level the decay amplitude T_0 is

$$T_0 = V_{Ij} A_L , \tag{1}$$

with

$$A_L = \frac{g\, N_c}{\sqrt{2}} \overline{u}_I(p_1) \not{\varepsilon}\, \gamma_L v_j(q - p_1). \tag{2}$$

V_{Ij} are the elements of the CKM matrix, N_c is the number of colors, and g is the SU(2) coupling constant.

At one loop, Eq. (1) is modified in several different ways. First, one has to sum all one-loop irreducible vertex diagrams. This gives a contribution proportional to V_{Ij} but not entirely proportional to A_L. Second, we have the counterterms stemming from the usual variation of the Lagrangian parameters. The counterterms δg and δZ_W (W-wave-function renormalization) also give rise to contributions proportional to the tree-level amplitude. However, since the quarks get mixed by the renormalization procedure, this is not true for the quark wave function renormalization constants $\delta Z^L_{II'}$ and $\delta Z^L_{jj'}$. Finally, an additional counterterm δV_{Ij} has to be included.

For a real W that decays into on-shell quarks, it is easy to show that the vertex diagrams can be written in terms of four independent form factors. Each one is associated with a given Lorentz structure for the spinors. Denoting by q^μ the four-momentum of the incoming W^+ and by p_1^μ the four-momentum of the outgoing *up* quark I, let us define

$$B_L = \frac{g\, N_c}{\sqrt{2}} \overline{u}_I(p_1) \frac{\varepsilon \cdot p_1}{m_W} \gamma_L v_j(q - p_1), \tag{3}$$

where ε^μ is the W polarization vector. Similarly, replacing in Eqs. (2) and (3) γ_L by γ_R we define A_R and B_R, respectively. Now, the one-loop amplitude T_1 is

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$$T_1 = A_L \Bigg[V_{Ij} \left(F_L + \frac{\delta g}{g} + \frac{1}{2}\delta Z_W + \frac{1}{2}\delta Z_{II}^{L*} + \frac{1}{2}\delta Z_{jj}^{L} \right) + \sum_{I'\neq I} \frac{1}{2}\delta Z_{I'I}^{L*} V_{I'j} + \sum_{j'\neq j} V_{Ij'} \frac{1}{2}\delta Z_{j'j}^{L} + \delta V_{Ij} \Bigg] + V_{Ij}[A_R F_R + B_L G_L + B_R G_R], \tag{4}$$

where $F_{L,R}$ and $G_{L,R}$ are the form factors. We calculate the different terms in Eq. (4) using the general R_ξ gauge for the W propagators. However, to simplify the calculation, we use the 't Hooft–Feynman gauge for the Z and photon propagators. This is not inconsistent, since the ξ parameters of the gauge-fixing Lagrangian,

$$\mathcal{L}_{GF} = -\frac{1}{2\xi_\gamma}(\partial\cdot A)^2 - \frac{1}{2\xi_Z}(\partial\cdot Z - \xi_Z m_Z G^0)^2 - \frac{1}{\xi_W}|\partial\cdot W^+ + i\xi_W m_W G^+|^2,$$

are independent. For our purpose it is sufficient to set $\xi_\gamma = \xi_Z = 1$ but to keep ξ_W as a free parameter. From this point onwards it will be denoted simply by ξ. For the numerical calculations we used the values from Particle Data Group [7].

II. IRREDUCIBLE VERTEX DIAGRAMS

In Fig. 1 we show the irreducible diagrams that give the one-loop $W^+ \to u_I \bar{d}_j$ amplitude. The calculation of these diagrams using dimensional regularization is standard. It was done using the XLOOPS program [8]. To keep track of the divergences it is convenient to introduce the notation

$$\zeta = \frac{2}{D-4} - \gamma_E + \ln 4\pi^2 - \ln\left(\frac{m_W}{\mu}\right)^2,$$

where D is the dimension of momentum space ($D \to 4$), γ_E is the Euler constant, and μ is the arbitrary renormalization mass.

It is not particularly instructive to show in detail the form factors. So we have decided to show explicitly the divergent contributions and plot the finite parts as a function of ξ. In Fig. 2 we display the ξ dependence of the real part of F_R, G_L, and G_R for the decay $W^+ \to u\bar{d}$. As one can see, these form factors are ξ independent and finite as they should be. In fact, any divergence or gauge dependence here would be impossible, given the gauge structure of the theory. On the contrary, F_L is both divergent and ξ dependent, i.e.,

$$F_L = \frac{e^2}{64\pi^2}\zeta\left[\frac{3\xi+8}{\sin^2\theta_W} + \frac{1}{9\cos^2\theta_W} + \frac{m_I^2+m_j^2}{m_W^2}\frac{1}{\sin^2\theta_W}\right] + \hat{F}_L, \tag{5}$$

where $\hat{F}_L$ is finite but ξ dependent. This is clearly seen in Fig. 2. Notice, that the form factors F_R, G_L, and G_R are smaller than $\hat{F}_L$ because they are proportional to the quark masses divided by the W mass.

III. COUNTERTERMS

A. W-wave-function renormalization δZ_W

Calculating the W-boson self-energy at one-loop and imposing the on-shell renormalization conditions one obtains [2]

$$\delta Z_W = \frac{e^2}{96\pi^2\sin^2\theta_W}\zeta[22 - 3\xi - 2N_g(1+N_c)] + \delta\hat{Z}_W. \tag{6}$$

As before $\delta\hat{Z}_W$ denotes the finite contribution. We will follow this notation for all counterterms. $N_g = N_c = 3$ are the number of generations and the number of colors, respectively. We found that it is convenient to show these parameters explicitly in order to keep track of the contributions of lepton and quark loops.

From the W self-energy one also obtains the mass counterterm, namely,

$$\delta m_W^2 = \frac{-e^2}{96\pi^2\sin^2\theta_W}\zeta\Bigg[(34-3\xi)m_W^2 - 6m_Z^2 - 2N_g(1+N_c)m_W^2 + 3\sum_l m_l^2 + 3N_c\sum_{I'j'}|V_{I'j'}|^2(m_{I'}^2 + m_{j'}^2)\Bigg] + \delta\hat{m}_W^2. \tag{7}$$

B. Coupling counterter δg

It is discussed in great detail in Ref. [2] how to obtain δg. So, again, we simply summarize our results, which agree with those in Ref. [2] for $\xi = 1$. It is easy to show that

$$\frac{\delta g}{g} = \frac{\delta e}{e} - \frac{\delta\sin\theta_W}{\sin\theta_W}, \tag{8a}$$

where

$$\frac{\delta\sin\theta_W}{\sin\theta_W} = \frac{m_W^2\delta m_Z^2 - m_Z^2\delta m_W^2}{2\,m_Z^2(m_Z^2 - m_W^2)}. \tag{8b}$$

From the Z self-energy one obtains δm_Z^2. Like the analogue result shown in Eq. (7), δm_Z^2 depends on ξ. However, the combination given by Eq. (8b) is ξ independent. Furthermore, δe is also ξ independent. This makes the final result

$$\frac{\delta g}{g} = \frac{e^2}{96\pi^2\sin^2\theta_W}\zeta\left[N_g(1+N_c) - \frac{43}{2}\right] + \frac{\delta\hat{g}}{g} \tag{9}$$

fully ξ independent.

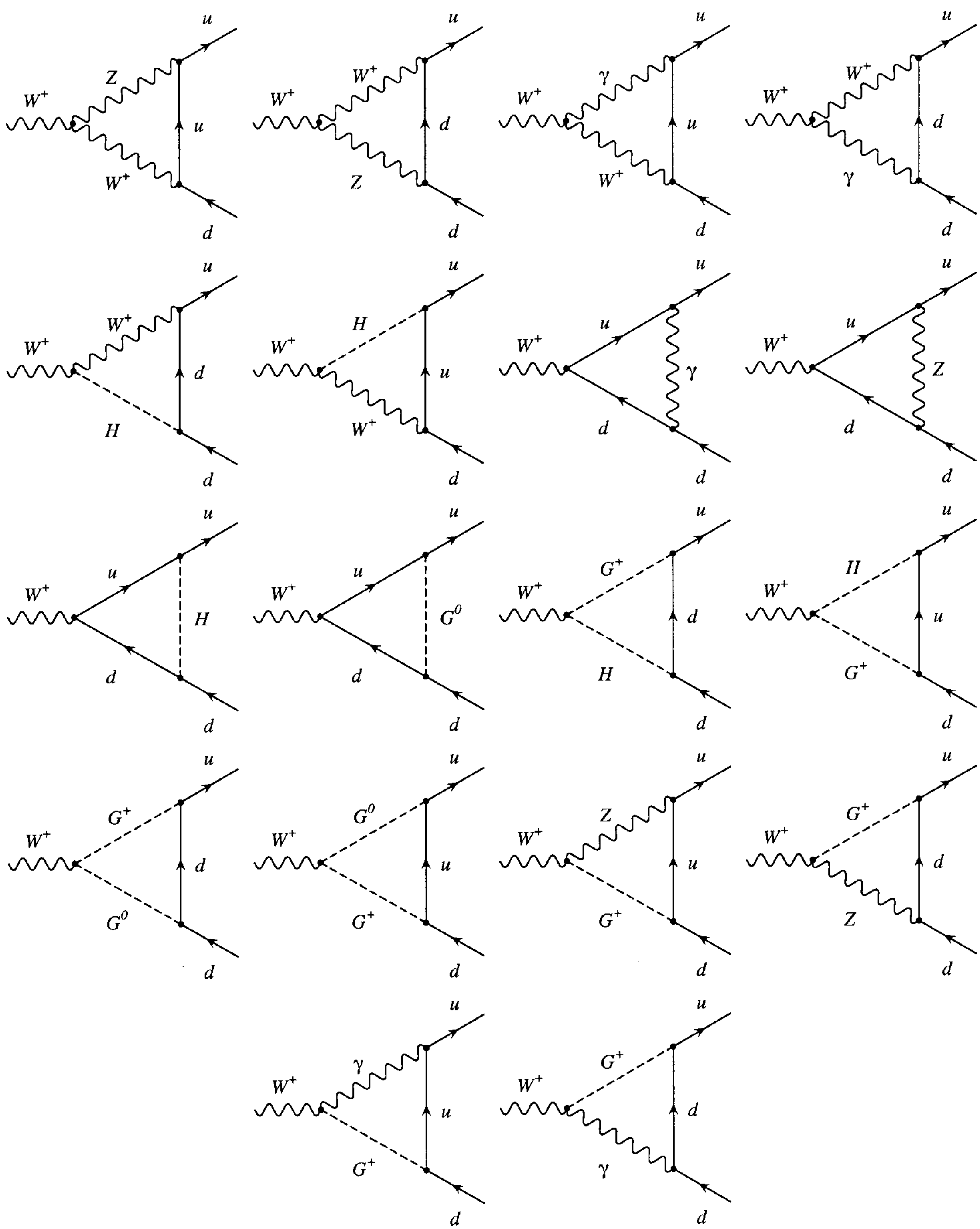

FIG. 1. Irreducible electroweak one-loop diagrams for $W^+ \to u\overline{d}$.

C. Quark-field renormalization

As is well known, under renormalization the quark fields are mixed. Let us write the self-energy of an *up*-type quark in the general form

$$\Sigma_{II'} = \Sigma^L_{II'}(p^2)\not{p}\gamma_L + \Sigma^R_{II'}(p^2)\not{p}\gamma_R + \Sigma^M_{II'}(p^2)[m_I\gamma_L + m_{I'}\gamma_R]. \tag{10}$$

Then, using the on-shell renormalization condition one obtains the matrix elements of the wave function renormalization constants δZ^L [9], namely,

$$\delta Z^L_{II} = -\Sigma^L(m_I^2) - m_I^2[\Sigma^{L'}(m_I^2) + \Sigma^{R'}(m_I^2) + 2\Sigma^{M'}(m_I^2)], \tag{11a}$$

where $\Sigma^{L'}$ denotes the derivative $(\partial/\partial q^2)\Sigma^L$ and, for $I' \neq I$,

$$\delta Z^L_{II'} = 2\frac{(m_I^2 + m_{I'}^2)\Sigma^M(m_{I'}^2) + m_I m_{I'}\Sigma^R(m_{I'}^2) + m_{I'}^2\Sigma^L(m_{I'}^2)}{m_I^2 - m_{I'}^2}. \tag{11b}$$

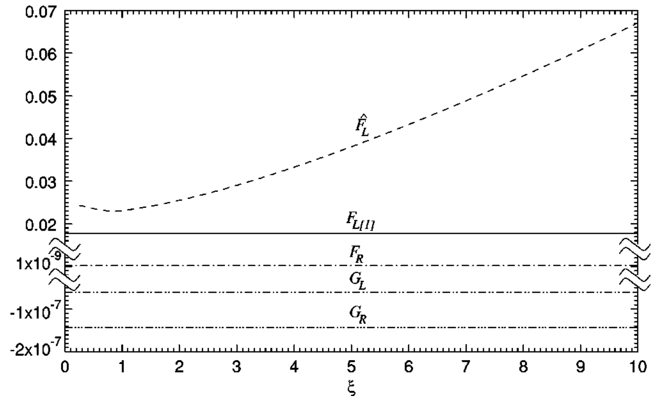

FIG. 2. The real part of F_R, G_L, G_R, $\hat{F}_L$, and $F_{L[1]}$ for the $W^+ \to u\overline{d}$ decay as a function of ξ.

In our case we obtain

$$\delta Z_{II}^L = \frac{-e^2}{64\pi^2}\zeta\left[\frac{1+2\xi}{\sin^2\theta_W} + \frac{m_I^2 + \sum_{i'} |V_{Ii'}|^2 m_{i'}^2}{m_W^2 \sin^2\theta_W} + \frac{1}{9\cos^2\theta_W}\right] + \delta\hat{Z}_{II}^L \tag{12a}$$

for the diagonal terms and

$$\delta Z_{IJ}^L = \frac{-e^2}{32\pi^2\sin^2\theta_W}\zeta\frac{m_I^2+2m_J^2}{m_J^2-m_I^2}\sum_{i'} V_{Ii'}V_{Ji'}^*\frac{m_{i'}^2}{m_W^2} + \delta\hat{Z}_{IJ}^L \tag{12b}$$

for the off-diagonal terms. In the latter equation a ξ-dependent term in the divergent part was canceled due to the unitarity of the CKM matrix. The corresponding result for the *down*-type quarks is

$$\delta Z_{ij}^L = \frac{-e^2}{32\pi^2\sin^2\theta_W}\zeta\frac{m_i^2+2m_j^2}{m_j^2-m_i^2}\sum_{I'} V_{I'i}V_{I'j}^*\frac{m_{I'}^2}{m_W^2} + \delta\hat{Z}_{ij}^L \tag{13}$$

and the diagonal part is identical to Eq. (12a) replacing I by i and I' by i'. It is interesting to point out that the matrices δZ^L are neither Hermitian nor anti-Hermitian. Of course, they can be decomposed in a sum of such matrices, $\delta Z^L = \delta Z^{LH} + \delta Z^{LA}$. However, one should realize that the divergence is present both in δZ^{LH} and in δZ^{LA}. In fact, from Eq. (12b) it is straightforward to obtain

$$\delta Z_{IJ}^{LH} = \frac{-e^2}{64\pi^2\sin^2\theta_W}\zeta\sum_{i'} V_{Ii'}V_{Ji'}^*\frac{m_{i'}^2}{m_W^2} + \text{finite} \tag{14a}$$

and

$$\delta Z_{IJ}^{LA} = \frac{-3e^2}{64\pi^2\sin^2\theta_W}\zeta\frac{m_I^2+m_J^2}{m_J^2-m_I^2}\sum_{i'} V_{Ii'}V_{Ji'}^*\frac{m_{i'}^2}{m_W^2} + \text{finite.} \tag{14b}$$

Clearly, Eq. (12a) shows that the diagonal terms of δZ^L are real. These remarks will be important in Sec. V, when we consider the renormalization of the CKM matrix.

IV. W^+ DECAY INTO LEPTONS

Using Eqs. (5), (6), (9), (12a), (12b), and (13) it is easy to obtain

$$F_L + \frac{\delta g}{g} + \tfrac{1}{2}\delta Z_W + \tfrac{1}{2}\delta Z_{II}^{L*} + \tfrac{1}{2}\delta Z_{jj}^L = \frac{e^2}{128\pi^2\sin^2\theta_W}\zeta \times \frac{m_I^2 - \sum_{i'}|V_{Ii'}|^2 m_{i'}^2 + m_j^2 - \sum_{I'}|V_{I'j}|^2 m_{I'}^2}{m_W^2} + \text{finite.} \tag{15}$$

Notice that there are no divergences proportional to the gauge parameter ξ. If $V_{Ii'} = \delta_{Ii'}$ and $V_{I'j} = \delta_{I'j}$, i.e., if the CKM matrix is the unit matrix, the divergent term is identically zero. In this case, we call the above combination of F_L and counterterms $F_{L[1]}$.[1]

From Eq. (4) it is now clear that the one-loop leptonic decay amplitude $W^+ \to l^+\nu_l$ can be written as

$$T_1^l = A_L F_{L[1]} + B_R G_R, \tag{16}$$

where in $F_{L[1]}$ and G_R the leptonic masses are used and in Eqs. (2) and (3) we set $N_c = 1$. The form factors F_R and G_L are proportional to m_I. Hence, they vanish for massless neutrinos. As we have shown T_1^l is finite, as it should be. Furthermore, Fig. 2, where we show $F_{L[1]}$ as a function of ξ, clearly proves that the one-loop leptonic amplitude T_1^l is also gauge independent. Having established the finiteness and the gauge independence of $F_{L[1]}$ we are now in a position to return to Eq. (4) and consider the δV_{Ij} counterterm.

V. CKM COUNTERTERM

Let us consider the W-quark coupling in the standard model Lagrangian. Introducing an obvious matrix notation we write

$$\mathcal{L} = -\frac{g}{\sqrt{2}}\overline{U}_L V D_L W_\mu + \text{H.c.}, \tag{17}$$

[1]Obviously in $F_{L[1]}$ δg and δZ_W are not calculated with a unit CKM matrix.

where U_L and D_L are the left-handed up and down quark fields, respectively. Leaving aside the renormalization of g and of the W field, let us focus our attention on the renormalization of the quark fields and V. In the former work of DS the matrix V is multiplicatively renormalized, i.e.,

$$V \to U_1 V U_2 = V + \delta U_1 V + V \delta U_2 , \tag{18}$$

where U_1 and U_2 are unitary matrices. Then, introducing the usual quark wave-function renormalization

$$\overline{U}_L \to \overline{U}_L Z_{UL}^{1/2\dagger},$$

$$D_L \to Z_{DL}^{1/2} D_L ,$$

Eq. (17) becomes

$$\begin{aligned}\overline{U}_L V D_L \to & \overline{U}_L Z_{UL}^{1/2\dagger} U_1 V U_2 Z_{DL}^{1/2} D_L \\ &= \overline{U}_L \Big[V + \tfrac{1}{4}[\delta Z_{UL}^\dagger + \delta Z_{UL}]V + \tfrac{1}{4} V[\delta Z_{DL} + \delta Z_{DL}^\dagger] \\ &\quad + \frac{1}{4}[\delta Z_{UL}^\dagger - \delta Z_{UL}]V + \tfrac{1}{4} V[\delta Z_{DL} - \delta Z_{DL}^\dagger] \\ &\quad + \delta U_1 V + V \delta U_2 \Big] D_L,\end{aligned} \tag{19}$$

where, for convenience, we have split the δZ matrices into its Hermitian and anti-Hermitian parts. Because the unitarity of the matrixes U_i implies that the δU_i are anti-Hermitian, DS concluded that $\delta V = \delta U_1 V + V \delta U_2$ is required to absorb the divergence in the anti-Hermitian parts of δZ_L. Hence, they have introduced the following renormalization condition:

$$\delta V = -\tfrac{1}{4}[\delta Z_{UL}^\dagger - \delta Z_{UL}]V - \tfrac{1}{4} V[\delta Z_{DL} - \delta Z_{DL}^\dagger]. \tag{20}$$

Of course, there are still divergences in the Hermitian part of δZ, but, as we will see, they are the ones needed to cancel the divergences in the vertex contribution to F_L. In fact, using Eqs. (14a),(14b) and (12a),(12b) it is straightforward to obtain

$$\begin{aligned}\frac{1}{2}[\delta Z_{UL}^H V + V \delta Z_{DL}^H]_{Ij} &= \frac{-e^2}{128\pi^2 \sin^2\theta_W}\zeta \Bigg[\sum_{i'J} V_{Ii'} V_{Ji'}^* V_{Jj} \frac{m_{i'}^2}{m_W^2} + \frac{m_I^2}{m_W^2} V_{Ij} \\ &\quad + \sum_{I'i'} V_{Ii'} V_{I'i'}^* V_{I'j} \frac{m_{I'}^2}{m_W^2} + \frac{m_j^2}{m_W^2} V_{Ij} \Bigg] + \text{finite}.\end{aligned} \tag{21}$$

In the equation above, when using the diagonal elements of the matrix δZ^H, only the contribution of the second term of Eq. (12a) is explicitly shown. The other two terms are irrelevant for the discussion since they cancel with similar divergences coming from F_L, δZ_W, and δg.

Now, the unitarity of V reduces Eq. (21) to the form:

$$\frac{1}{2}[\delta Z_{UL}^H V + V \delta Z_{DL}^H]_{Ij} = \frac{-e^2}{64\pi^2 \sin^2\theta_W}\zeta \frac{m_I^2 + m_j^2}{m_W^2} V_{Ij} , \tag{22}$$

which is exactly what we need to cancel a similar divergence in $V_{Ij} F_L$, namely the third term in Eq. (5).

Hence, from the point of view of canceling the divergences in T_1, the renormalization proposal by DS works. In other words, it is sufficient to choose δV as the divergent part of the right-hand side of Eq. (20) to obtain a finite one-loop amplitude. DS have also included in δV the finite contributions stemming from δZ^A. We have checked that this gives rise to a gauge-dependent result.

To solve this problem let us define the quantity

$$\begin{aligned}\delta X_{ud} = & \frac{1}{2} V_{ud}[\delta Z_{uu}^{L*} - \delta Z_{uu[1]}^{L*} + \delta Z_{dd}^L - \delta Z_{dd[1]}^L] \\ & + \frac{1}{2}\sum_{I'\neq u} \delta Z_{I'u}^{L*} V_{I'd} + \frac{1}{2}\sum_{j'\neq d} V_{uj'} \delta Z_{j'd}^L,\end{aligned} \tag{23}$$

which obviously represents the difference between the ''leptonic''[2] and the quark transition amplitude. Notice that $\delta Z_{uu[1]}^{L*}$ is given by Eq. (12a) but replacing the CKM by the unit matrix. After introducing the quantity δX_{Ij} it is clear that Eq. (4) can be rewritten as

$$\begin{aligned}T_1 = & V_{Ij}[A_L F_{L[1]} + A_R F_R + B_L G_L + B_R G_R] \\ & + A_L[\delta X_{Ij} + \delta V_{Ij}].\end{aligned} \tag{24}$$

Having proved that the first term of Eq. (24), proportional to V_{Ij}, is both finite and gauge independent, it is obvious that the CKM counterterm should be

$$\delta V_{Ij} = -\delta X_{Ij} . \tag{25}$$

This is our main result. In physical terms what we are saying is that all contributions to the T_1 amplitude arising from the renormalization of the quark mixing are canceled by the CKM counterterm. This δV is an alternative to the one proposed by GGM which requires the use of quark wave-function renormalization constants at zero momentum. Both schemes lead to gauge-invariant results. In fact, the unitarity of the CKM matrix implies that δX is gauge independent.

Obviously, if we impose Eq. (25) for the nine decay processes $W^+ \to q_I \overline{q}_j$,[3] we are overconstraining the system. Despite this, if one is simply interested in one-loop results, this

[2]Here leptonic means that no mixing takes place among the different generations. Of course, for calculating the $\delta Z_{[1]}$ renormalization constants, massive quarks were used.

[3]Simply kinematics prevents $W^+ \to t\overline{q}_j$, but one can take the equivalent process $t \to W^+ \overline{q}_j$.

prescription could be of some practical use. But at two-loop order the unitarity of the CKM is lost, giving rise to serious renormalization problems.

The CKM matrix has four independent parameters. So the renormalization condition (25) should be enforced only in four independent processes. Let us stress that the divergences of the counterterm matrix δV are entirely due to the anti-Hermitian parts of the δZ_{UL} and δZ_{DL} matrixes. Then, our renormalization prescription, which introduces a gauge-independent finite part in δV, can be forced to obey the unitarity of the CKM matrix to all orders.

VI. CONCLUSIONS

Beyond the tree level, quarks with the same electric charge get mixed under renormalization. Then, the amplitude for the $W^+ \rightarrow u\overline{d}$ explicitly depends on these flavor-changing renormalization constants. Therefore, to obtain a finite amplitude it is essential to renormalize the corresponding element of the CKM matrix, V. Using the on-shell renormalization scheme and the R_ξ gauge we have shown how to construct the CKM counterterm matrix δV. Our final result is given in Eq. (25). With this prescription the tree-level relation

$$T(W^+ \rightarrow u\overline{d}) = V_{ud} N_c T(W^+ \rightarrow e^+ \nu_e),$$

residing aside α_s corrections and obvious kinematic differences, is maintained at the next order. We have proved that at one loop one obtains a finite and gauge independent amplitude. It is interesting to point out that a finite amplitude T_1 can only be obtained if the CKM matrix is unitary. This is particularly important in view of some recent discussions about the possible nonunitarity of this matrix [10].

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