Quasirotational motions and stability problem in the dynamics of string hadron models

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For the relativistic string with massive ends (the meson model) and four various string baryon models (q-qq, q-q-q, Y and Δ) we consider the classical quasirotational motions, which are small disturbances of the planar uniform rotations of these systems. For the string meson model two types of these solutions are obtained. They describe oscillatory motions in the form of stationary waves in the rotational plane and in the orthogonal direction. This approach and the suggested method of determining an arbitrary motion of the system on the basis of initial data let us solve the stability problem for the rotational motions for all mentioned string configurations. It is shown that the classic rotational motions are *stable* for the string meson model (or its analog q-qq) and for the Δ baryon configuration, but they are *unstable* for the string baryon models Y and q-q-q. For the latter two systems any small asymmetric disturbances grow with increasing time. The motion of the q-q-q configuration becomes more complicated and quasiperiodic but the quarks do not merge. In the case of the Y model the evolution of disturbances results in a quark falling into the junction. These features of the classic behavior are important for describing hadron states on the Regge trajectories and for choosing and developing the most adequate QCD-based string hadron models.

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INTRODUCTION

The string hadron models [1–9] use the striking analogy between the QCD confinement mechanism at large interquark distances and the relativistic string with linearly growing energy connecting two material points. Such a string with massive ends may be regarded as the meson string model [2,3].

String models of the baryon were suggested in the following four variants (Fig. 1), differing from each other in the topology of the spatial junction of three massive points (quarks) by relativistic strings: (a) the quark-diquark model q-qq [4] (on the classic level it coincides with the meson model [2]), (b) the linear configuration q-q-q [5], (c) the "three-string" model or Y configuration [6,7], and (d) the "triangle" model or Δ configuration [8,9].

The problem of choosing the most adequate string baryon model among the four mentioned ones has not been solved yet. The investigation of this problem from the point of view of the QCD limit at large distances has not been completed. In particular, the QCD-motivated baryon Wilson loop operator approach gives some arguments in favor of the Y configuration [10] or the "triangle" model [11]. Leaving the Wilson loop analysis outside this paper, we concentrate on the classical dynamics of these configurations.

For all mentioned string hadron models the classical solutions describing the rotational motion (planar uniform rotations of the system) are known and widely used for modeling the orbitally excited hadron states on the main Regge trajectories [4,12–14]. The rotational motion of the meson model or the baryon configuration q-qq is a rotation of the rectilinear string segment [2,4]. For the model q-q-q the motion is the same but with the middle quark at the rotational center. The form of rotating three-string configuration is three rectilinear string segments joined in a plane at the angles 120° [6,7]. For the model "triangle" this form is the

rotating closed curve consisting of segments of a hypocycloid [8,9].

In this paper we consider the motions of all these systems which are close to rotational ones. They are interesting due to the following two reasons: (a) we are to search the motions describing the hadron states, which are usually interpreted as higher radially excited states in the potential models [14], in other words, we are to describe the daughter Regge trajectories; (b) the important problem of stability of rotational motions has not been solved yet for all mentioned string models. For the meson string model the quasirotational motions of slightly curved string with massive ends were searched in Refs. [15,16]. But some simplifying assumptions in these papers oblige us in Sec. I to verify the solutions [15] numerically and to suggest another approach for obtaining the quasirotational solutions in the form of stationary waves and solving the stability problem. In Secs. II, III, and IV the stability problem for the string baryon configurations Δ, q-q-q and Y (Fig. 1) is tested using the suggested method of solving the initial-boundary value problem for these systems.

The obtained classical solutions are applied in describing hadron states after the quantization. The standard quantization procedure for the open string is not applicable to the considered hadron models (in particular, to the string with massive ends) because of the essential nonlinearity of the boundary conditions [3]. To avoid these difficulties different authors restricted the dynamics of the system to some subset of the configurational space; i.e. they quantized separate classes of motions of the massless Y configuration [7] or of the string with massive ends [12,16] by using a different technique. In particular, the method developed in Ref. [17] for the relativistic flux tube model was used in Ref. [16] for quantizing the motions of the curved relativistic string with massive ends. For this purpose the authors consider only one dynamical coordinate — the distance R between the quarks (with the conjugated momentum p_R). However, the adequate quantization procedure for the string's quasirotational mo-
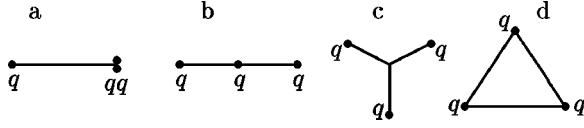
FIG. 1. String baryon models.

tions has to include the infinite number of dynamical variables describing all possible string oscillations and is connected, for example, with the mentioned stationary waves. So the profound problem of quantization for the quasirotational motions of various string hadron models requires special consideration beyond the scope of this paper.

I. STRING WITH MASSIVE ENDS

The relativistic string with the tension γ and the masses m_1, m_2 at the ends (the meson or q-qq baryon model) is described by the action [2,3,13]

$$S = -\gamma \int_\Omega \sqrt{-g}\, d\tau d\sigma - \sum_{i=1}^{N} m_i \int \sqrt{\dot{x}_i^2(\tau)}\, d\tau, \qquad (1)$$

written here in the general form with N material points for all mentioned hadron models. For the string with massive ends $N=2$, $X^\mu(\tau,\sigma)$ are coordinates of a string point in D-dimensional Minkowski space $R^{1,D-1}$ with the signature $+,-,-,\ldots$, $g = \dot{X}^2 X'^2 - (\dot{X},X')^2$, where $(a,b) = a^\mu b_\mu$ is the (pseudo)scalar product, $\dot{X}^\mu = \partial_\tau X^\mu$, $X'^\mu = \partial_\sigma X^\mu$, $\Omega = \{(\tau,\sigma): \tau_1 < \tau < \tau_2, \quad \sigma_1(\tau) < \sigma < \sigma_2(\tau)\}$, $\sigma_i(\tau)$ are inner coordinates of the quark[1] world lines, their coordinates in $R^{1,D-1}$ are $x_i^\mu(\tau) = X^\mu(\tau,\sigma_i(\tau))$, $\dot{x}_i^\mu = (d/d\tau)x_i^\mu(\tau)$, and the speed of light $c=1$. The first summand in Eq. (1) is proportional to the world surface area and may be rewritten in the equivalent form [3,15,16].

The equations of motion

$$\frac{\partial}{\partial \tau} \frac{\partial \sqrt{-g}}{\partial \dot{X}^\mu} + \frac{\partial}{\partial \sigma} \frac{\partial \sqrt{-g}}{\partial X'^\mu} = 0, \qquad (2)$$

and the boundary conditions for the quark trajectories

$$\frac{d}{d\tau} \frac{\dot{x}_{i\mu}}{\sqrt{\dot{x}_i^2}} - \frac{(-1)^i \gamma}{m_i} \left[\frac{\partial \sqrt{-g}}{\partial X'^\mu} - \sigma_i'(\tau) \frac{\partial \sqrt{-g}}{\partial \dot{X}^\mu} \right] \Bigg|_{\sigma=\sigma_i} = 0 \qquad (3)$$

are deduced from action (1) [3].

The exact solution of Eq. (2) satisfying conditions (3) and describing the rotational motion of the rectilinear string is well known [2–4] and may be represented as

$$X^0 \equiv t = b\tau, \quad X^1 + iX^2 = \omega^{-1}\sin(\omega b\sigma) \cdot e^{i\omega t}. \qquad (4)$$

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[1] We use the term ''quark'' for brevity, here and below quarks, antiquarks, and diquarks are material points on the classic level.

Here ω is the angular velocity, $\sigma \in [\sigma_1, \sigma_2]$, $\sigma_i = $ const, $\sigma_1 \leq 0 < \sigma_2$; the substitution $\tilde{\sigma} = \sin(\omega b\sigma)$ can be made.

The authors of Refs. [15,16] search motions of this system close to the rotational one (4) in the form

$$X^0 = t, \quad X^1 + iX^2 = \sigma R(t)e^{i[\omega t + \phi(t) + f(\sigma)]}, \qquad (5)$$

where $\phi(t)$, $f(\sigma)$, $\dot{R}(t)$ are assumed to be small. After substituting this formula into Eq. (2) and dropping the second-order terms they obtain the expression for $f(\sigma)$

$$f(\sigma) = \ddot{\phi}(t)\omega^{-3}R^{-1}(t)[f_1(\sigma,v_\perp) - f_1(1,v_\perp)], \qquad (6)$$

where $f_1(\sigma,v_\perp) = (\frac{1}{2}v_\perp A + \sigma^{-1}\sqrt{1-\sigma^2 v_\perp^2})A$, $A = \arcsin\sigma v_\perp$, $v_\perp = \omega R(t)$, $\sigma_1 = 0 \leq \sigma \leq 1 = \sigma_2$, the first heavy quark with $m_1 \to \infty$ is at rest.

Deducing Eq. (6) the authors of Refs. [15,16] ignore that (a) the obtained function $f(\sigma)$ (6) depends on t essentially [one may assume the dependences $R(t)$, $v_\perp(t)$ weak but this cannot be valid for the multiplier $\ddot{\phi}(t)$], so all previous calculations appear to be wrong; (b) expressions (5), (6) do not satisfy the boundary condition (3) for the moving quark. The dependencies $\phi(t)$, f, $R(t) - \langle R \rangle$ on time should also be analyzed. In particular, the following important question remains without answer: do these disturbances grow with increasing time, in other words, is the rotational motion (4) stable?

In our opinion, a better way of searching quasirotational motions and solving the stability problem includes the choice of the coordinates τ, σ on the world surface (that can always be made [3]), in which the orthonormality conditions

$$\dot{X}^2 + X'^2 = 0, \quad (\dot{X},X') = 0, \qquad (7)$$

are satisfied. Under restrictions (7) the equations of motion (2) become linear

$$\ddot{X}^\mu - X''^\mu = 0, \qquad (8)$$

and the boundary conditions (3) take the simplest form

$$m_i \frac{d}{d\tau} U_i^\mu(\tau) + (-1)^i \gamma X'^\mu(\tau,\sigma_i) = 0, \quad i=1,2, \qquad (9)$$

where $U_i^\mu(\tau) = \dot{x}_i^\mu(\tau)/\sqrt{\dot{x}_i^2}$ is the unit $R^{1,D-1}$-velocity vector of ith quark.

In Eqs. (9) the functions $\sigma_i(\tau)$ are chosen in the form $\sigma_i(\tau) = $ const. One can always fix them, in particular, as

$$\sigma_1 = 0, \quad \sigma_2 = \pi \qquad (10)$$

with the help of the reparametrizations [3]

$$\tilde{\tau} \pm \tilde{\sigma} = f_\pm(\tau \pm \sigma) \qquad (11)$$

($f_\pm$ are arbitrary smooth monotone functions), which keep the invariance of Eqs. (7) and (8).

Using the general solution of Eq. (8)

$$X^\mu(\tau,\sigma) = \frac{1}{2}[\Psi_+^\mu(\tau+\sigma) + \Psi_-^\mu(\tau-\sigma)], \qquad (12)$$

we can reduce the problem in a natural way to solve ordinary differential equations. In particular, this approach is fruitful for considering the initial-boundary value problem (IBVP) for the string with massive ends [18]. This problem implies obtaining the motion of the string on the basis of two given initial conditions: an initial position of the system in Minkowski space and initial velocities of string points. In other words, we are to determine the solution of Eq. (8) $X^\mu(\tau,\sigma)$ satisfying the orthonormality (7), boundary (9), and initial conditions.

An initial position of the string can be given as the parametric curve in Minkowski space

$$x^\mu=\rho^\mu(\lambda), \quad \lambda\in[\lambda_1,\lambda_2], \quad \rho'^2<0. \qquad (13)$$

The initial velocity of a string point is a timelike vector on this curve $v^\mu(\lambda)$, $\lambda\in[\lambda_1,\lambda_2]$, $v^\mu(\lambda)$ may be multiplied by an arbitrary scalar function $\chi(\lambda)>0$.

To solve the problem we set parametrically the initial curve on the world sheet

$$\tau=\check{\tau}(\lambda), \quad \sigma=\check{\sigma}(\lambda), \quad \lambda\in[\lambda_1,\lambda_2], \qquad (14)$$

and use the following general form for the initial position of the string [18]:

$$X^\mu(\check{\tau}(\lambda),\check{\sigma}(\lambda))=\rho^\mu(\lambda), \quad \lambda\in[\lambda_1,\lambda_2]. \qquad (15)$$

Here $|\check{\tau}'|<\check{\sigma}'$, $\check{\tau}(\lambda_1)=\check{\sigma}(\lambda_1)=0$, $\check{\sigma}(\lambda_2)=\pi$. There is the freedom in choosing the functions $\check{\tau}(\lambda)$, $\check{\sigma}(\lambda)$ [18,19] connected with the invariance of Eqs. (7), (8), (9), and (10) with respect to the substitutions (11) where

$$f_+(\xi)=f_-(\xi)=f(\xi), \quad f(\xi+2\pi)=f(\xi)+2\pi \qquad (16)$$

and $f'(\xi)>0$.

Using the formulas [18]

$$\frac{d}{d\lambda}\Psi_\pm^\mu(\check{\tau}(\lambda)\pm\check{\sigma}(\lambda))=\left[1\pm\frac{(v,\rho')}{\Delta}\right]\rho'^\mu\mp\frac{\rho'^2}{\Delta}v^\mu, \qquad (17)$$

where $\Delta(\lambda)=\sqrt{(v,\rho')^2-v^2\rho'^2}$, we can determine from the initial data the function Ψ_+^μ in the initial segment $[0,\check{\tau}(\lambda_2)+\pi]$ and the function Ψ_-^μ in the segment $[\check{\tau}(\lambda_2)-\pi,0]$. The constants of integration are fixed from the initial condition (15).

The functions $\Psi_\pm^\mu$ are to be continued beyond the initial segments with the help of boundary conditions (9) which may be reduced to the equations [18,19]

$$U_1'^\mu(\tau)=\gamma m_1^{-1}[\delta_\nu^\mu-U_1^\mu(\tau)U_{1\nu}(\tau)]\Psi_+'^\nu(\tau), \qquad (18)$$

$$U_2'^\mu(\tau)=\gamma m_2^{-1}[\delta_\nu^\mu-U_2^\mu(\tau)U_{2\nu}(\tau)]\Psi_-'^\nu(\tau-\pi), \qquad (19)$$

$$\Psi_-'^\mu(\tau)=\Psi_+'^\mu(\tau)-2m_1\gamma^{-1}U_1'^\mu(\tau), \qquad (20)$$

$$\Psi_+'^\mu(\tau+\pi)=\Psi_-'^\mu(\tau-\pi)-2m_2\gamma^{-1}U_2'^\mu(\tau). \qquad (21)$$

where

$$\delta_\nu^\mu=\begin{cases}1, & \mu=\nu\\ 0, & \mu\neq\nu.\end{cases}$$

Solving the systems of Eqs. (18), (19) with the initial conditions

$$U_i^\mu(\check{\tau}(\lambda_i))=v^\mu(\lambda_i)/\sqrt{v^2(\lambda_i)}, \quad i=1,2, \qquad (22)$$

we obtain the velocities $U_i^\mu(\tau)$ on the base of the functions $\Psi_\pm'^\mu(\tau)$ known from Eqs. (17) in the initial segments. Simultaneously we continue (with no limit) the functions $\Psi_\pm^\mu$ outside these segments with the help of Eqs. (20), (21) and determine the world surface (12).

The above described procedure of solving the initial boundary value problem (IBVP) for the string with the fixed end (the infinitely heavy quark with $m_1\to\infty$ is at rest) and $m_2=1$, $\gamma=1$ is illustrated in Fig. 2. This is a typical example of a slightly disturbed motion close to the rotational one (4). In the case $m_1\to\infty$ Eqs. (18), (20) are to be substituted for the equations [18] $U_1^\mu(\tau)=U_1^\mu=$ const (the infinite mass moves at a constant velocity) and

$$\Psi_-'^\mu(\tau)=[2U_1^\mu U_{1\nu}-\delta_\nu^\mu]\Psi_+'^\nu(\tau), \quad m_1\to\infty. \qquad (23)$$

Equation (19) with initial condition (22), $i=2$ was solved numerically.

For the motion in Fig. 2 the initial distance between the quarks is $R(0)=1$, the initial string shape [the curve 1 in Figs. 2(a) and 2(c)] is determined by Eq. (6) with the amplitude multiplier $\beta_0=\ddot{\phi}(0)\omega^{-3}R^{-1}(0)=1$. The initial velocities of string points correspond to rotating in the xy plane at the angular velocity $\vec{\omega}=\{0;0;\omega\}$, $\omega=1/\sqrt{2}$:

$$v^\mu(\lambda)=\{1;\vec{v}(\lambda)\}, \quad \vec{v}(\lambda)=[\vec{\omega}\times\vec{\rho}(\lambda)]. \qquad (24)$$

The initial speed of the moving end $|\vec{v}(\lambda_2)|=v_\perp=\omega R(0)=1/\sqrt{2}$ satisfies the relation

$$v_\perp^2+\omega v_\perp m_i/\gamma=1 \qquad (25)$$

($i=2$), that is valid for the rotational motion (4) [4,13].

In Figs. 2(a) and 2(b) the positions of the string in xy-plane (sections $t=$ const of the world surface) are shown. They are numbered in order of increasing t with the step in time $\Delta t=0.2$ and these numbers are near the position of the moving quark marked by the small circle. The first turn of the string is shown in Fig. 2(a) and the fourth one (after the missed time interval) is represented in Fig. 2(b). We can see that main features of the motion are kept: the string rotates and its shape (slightly deviating from straightness) changes quasiperiodically.

The evolution of this shape is shown in detail in Fig. 2(c), where the first 9 curves from Fig. 2(a) (with the same step $\Delta t=0.2$) are turned in the xy plane so that the second end of each curve lies on the axis of abscissas. In other words the curves in Fig. 2(c) are the string positions in the frame of reference rotating with the string. One may conclude that the shape (6) of the string is not conserved with increasing time. Changing the shape looks like spreading waves along the
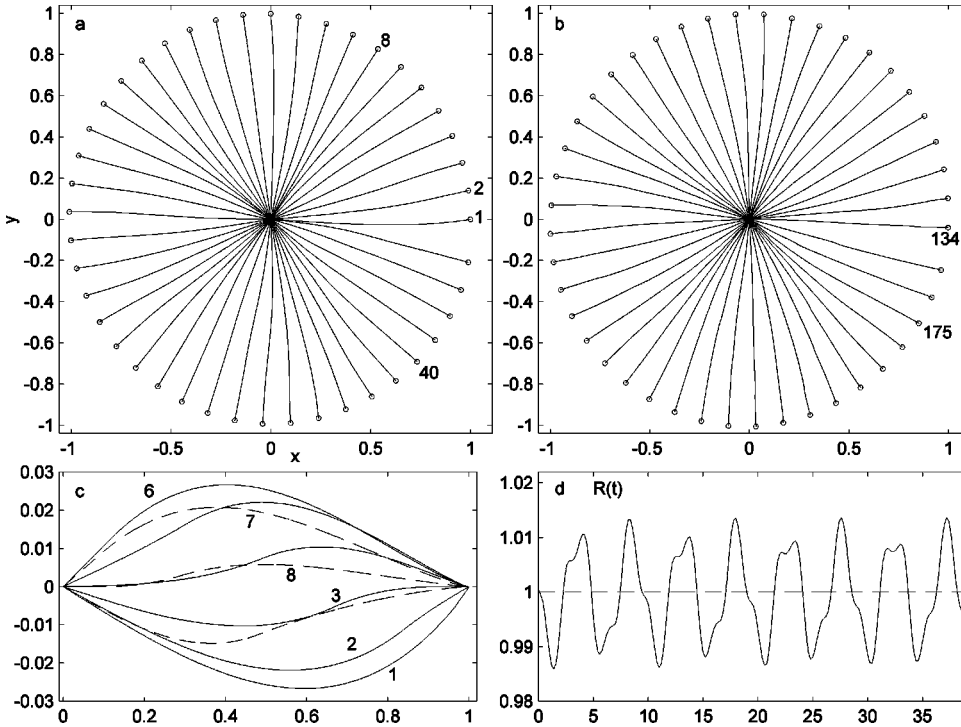
FIG. 2. The curved string motion of type (5), (6).

string (that uniformly rotates). Drawing other curves in this manner we see that the disturbances of the shape do not grow with growing time t. This also concerns the distance $R(t)$ between the quarks shown in Fig. 2(d). The distance $R(t)$ varies quasiperiodically near the initial value $R(0)=1$. It is interesting that the period $T_R \simeq 4.75$ when varying R does not coincide with the rotational period $T=2\pi/\omega \simeq 8.886$ and the period $T_s \simeq 2$ when changing the string shape. This hierarchy of the periods or frequencies and will be clarified below.

The picture of motion (including the behavior of R and the string shape) is similar for various values of ω, $R(0)$, m_2 connected by Eq. (25), and for various amplitudes β_0 of the disturbance (6). If β_0 is small enough the string motion is visually identical with the pure rotational one. But the disturbance of the string shape does not keep its form. So we may conclude that the expressions (5), (6) from Refs. [15,16] do not describe a motion of the curved string during an appreciable time interval.

The problem of searching the quasirotational motions of the curved string, for which the string shape behaves like stationary waves is solved in this paper with the more convenient (in comparison with Refs. [15,16]) analytical approach. This method also confirms analytically the numerical investigation of the stability problem for the rotational motions.

For this purpose we consider the string with the infinitely heavy end $m_2 \to \infty$ (or with the fixed end in the frame of reference moving at the velocity $U_2^\mu = \text{const}$) for the sake of simplicity. It is convenient to use the unit velocity vector of the moving end

$$U_1^\mu(\tau) \equiv U^\mu(\tau), \quad U^2(\tau)=1 \qquad (26)$$

for describing the string motion because the world surface is

totally determined from the given function $U^\mu(\tau)$ and the value γ/m_1 using the formulas [19,20]

$$\Psi_\pm'^\mu(\tau)=m_1\gamma^{-1}[\sqrt{-U'^2(\tau)}\,U^\mu(\tau)\pm U'^\mu(\tau)]. \quad (27)$$

If we substitute the analog of Eq. (23) for the second end

$$\Psi_+'^\mu(\tau)=P_\nu^\mu\Psi_{(-)}'^\nu, \quad P_\nu^\mu=2U_2^\mu U_{2\nu}-\delta_\nu^\mu, \quad m_2 \to \infty$$

and substitute Eq. (27) into Eq. (18), we get the relation

$$U'^\mu(\tau)=(\delta_\nu^\mu-U^\mu U_\nu)P_\kappa^\nu[\sqrt{-U_{(-)}'^2}\,U_{(-)}^\kappa-U_{(-)}'^\kappa]. \tag{28}$$

Here and below $_{(-)} \equiv (\tau-2\pi)$, the argument (τ) may be omitted.

The vector-function $U^\mu(\tau)$ given in a segment with the length 2π (if γ/m_1, U_2^μ are also given) contains all the information about this motion of the system [20]. Indeed, the system of ordinary differential equations with shifted argument (28) lets us continue $U^\mu(\tau)$ beyond the given initial segment and obtain the world surface with using Eqs. (27) and (12).

For the rotational motion (4) the velocity of the moving quark satisfying Eq. (28) may be written in the form

$$\bar{U}^\mu(\tau)=\Gamma[e_0^\mu+v_0\acute{e}^\mu(\tau)], \quad \Gamma=(1-v_0^2)^{-1/2}. \quad (29)$$

Here $v_0=v_\perp=\sin \pi\theta$ is the constant speed of this quark; $\acute{e}^\mu=\theta^{-1}(d/d\tau)e^\mu(\tau)$, $e^\mu=e_1^\mu \cos \theta\tau+e_2^\mu \sin \theta\tau$ are unit spacelike rotating vectors, $\theta=\omega b=\pi^{-1}\arcsin v_0=\text{const}$ is the "frequency" with respect to τ, the ends are renumerated in comparison with Eq. (4). The second quark in this frame of reference is at rest: $U_2^\mu=e_0^\mu$.

To study the stability of the rotational motion (4) we consider arbitrary small disturbances of this motion or of the vector (29) in the form

$$U^\mu(\tau) = \bar{U}^\mu(\tau) + u^\mu(\tau). \tag{30}$$

The disturbance $u^\mu(\tau)$ is given in the initial segment, for example, $I = [-2\pi, 0]$; it is to be small $|u^\mu| \ll 1$ (we neglect the second order terms) and satisfies the condition

$$[\bar{U}(\tau), u(\tau)] = 0, \tag{31}$$

resulting from Eq. (26) for both U^μ and $\bar{U}^\mu$.

Substituting expressions (30), (29) into Eq. (28) we obtain the equation describing the evolution of u^μ

$$u'^\mu + Qu^\mu + \bar{U}^\mu(\bar{U}', u)$$
$$= (\delta^\mu_\nu - \bar{U}^\mu\bar{U}_\nu)P^\nu_\kappa[Qu^\kappa_{(-)} - u'^\kappa_{(-)}]$$
$$+ Q^{-1}[(1 + 2v_0^2)\bar{U}^\mu - P^\mu_\nu\bar{U}^\nu_{(-)}](\bar{U}'_{(-)}, u'_{(-)}), \tag{32}$$

where

$$Q = \sqrt{-\bar{U}'^2} = \Gamma\theta v_0 = \frac{v_0 \arcsin v_0}{\pi\sqrt{1-v_0^2}} = \text{const.}$$

Projecting the disturbance u^μ onto the basic vectors e_0^μ, $e^\mu(\tau)$, $\acute{e}^\mu(\tau)$, e_3^μ we denote its three independent components in the following manner:

$$u_0(\tau) = (e_0, u), \quad u_e(\tau) = (e, u), \quad u_z(\tau) = (e_3, u).$$

The fourth component is expressed due to Eq. (31): $(\acute{e}, u) = -v_0^{-1}u_0$. Considering corresponding projections one can transform the system (32) into the following one:

$$u'_0 + Qu_0 - \Gamma Qu_e = u'_{0(-)} - Qu_{0(-)} - \Gamma Qu_{e(-)}, \tag{33}$$

$$u'_e + Qu_e + \theta v_0^{-1}u_0 = u'_{e(-)} + Qu_{e(-)} + F(u_{0(-)}), \tag{34}$$

$$u'_z + Qu_z = u'_{z(-)} - Qu_{z(-)}, \tag{35}$$

where $F(u_{0(-)}) = \theta(v_0^{-1} + 2v_0)u_{0(-)} - 2\sqrt{1-v_0^2}u'_{0(-)}$.

Equation (35) for z-axial disturbances is independent of others and may be easily solved: if $u_z(\tau)$ is given in the initial segment $I = [-2\pi, 0]$, the continuation of this function for $\tau > 0$ is [here $\Delta u_z = u_z(0) - u_z(-2\pi)$]:

$$u_z(\tau) = u_{z(-)} + e^{-Q\tau}\left[\Delta u_z - 2Q\int_0^\tau e^{Q\tilde\tau}u_{z(-)}d\tilde\tau\right]. \tag{36}$$

The pure harmonic solutions (36) $u_z(\tau) = e^{i\vartheta\tau}$ exist if the "frequency" ϑ satisfies the relation

$$\vartheta/Q = \cot\pi\vartheta, \tag{37}$$

resulting from substitution $u_z = e^{i\vartheta\tau}$ into Eq. (35). The transcendental Eq. (37) has the countable set of roots ϑ_n, $n - 1 < \vartheta_n < n$, the minimal positive root $\vartheta_1 = \theta = \pi^{-1}\arcsin v_0$. These pure harmonic z disturbances corresponding to various ϑ_n result in the following correction to the motion (4) [due to Eqs. (27), (12) there is only a z or e_3^μ component of the correction]:

$$X^3(\tau, \sigma) = B\sin\vartheta_n\sigma \cdot \cos(\vartheta_n\tau + \varphi_0). \tag{38}$$

Here the string ends have been again renumerated so $\sigma = 0$ corresponds to the fixed end as in Eq. (4). The amplitude B is small in comparison with ω^{-1}.

Expression (38) describes the oscillating string in the form of orthogonal (with respect to the rotational plane) stationary waves with $n-1$ nodes in the interval $0 < \sigma < \pi$. Note that the moving quark is not in a node, it oscillates along the z axis at the frequency $\omega_n = \omega\vartheta_n/\theta$. The shape $F = B\sin\vartheta_n\sigma$ of the z oscillation (38) is not pure sinusoidal with respect to the distance $s = \omega^{-1}\tilde\sigma = \omega^{-1}\sin\theta\sigma$ from the center to a point "σ":

$$F(s) = B\sin(\vartheta_n\theta^{-1}\arcsin\omega s), \quad 0 \leq s \leq v_0/\omega. \tag{39}$$

If $n = 1$ this dependence is linear. In this trivial case the motion is pure rotational (4) with a small tilt of the rotational plane. But the motions (38) with excited higher harmonics $n = 2, 3, \ldots$ are nontrivial.
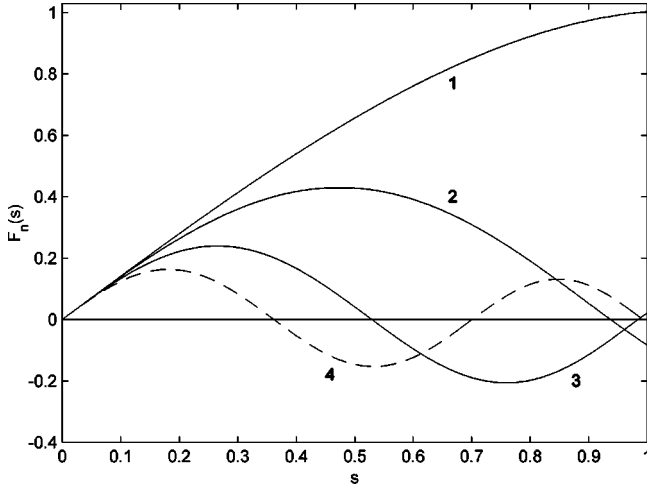
Equation (35), its harmonic solutions, the "frequencies" ϑ_n as roots of Eq. (37) were investigated in Refs. [19,21] where the motions of the meson string with linearizable boundary conditions (9) were studied and classified. Those solutions [in $(3+1)$-dimensional Minkowski space] with ϑ_n, $n \geq 2$ described the exotic motions of the $n-1$ times folded rectilinear string with $n-1$ points moving at the speed of light. But in Eq. (38) these higher harmonics became apparent as (much more physical) excitations of the rotating string.

It was shown in Ref. [21] that any smooth function $u(\tau)$ in a segment I with the length 2π may be expanded in the series

$$u(\tau) = \sum_{n=-\infty}^{+\infty} u_n\exp(i\vartheta_n\tau), \quad \tau \in I = [\tau_0, \tau_0 + 2\pi]. \tag{40}$$

So if we expand any given disturbance $u_z(\tau)$ in the initial segment I, we obtain solution (36) of Eq. (35) in the form (40) for all $\tau \in R$. If the initial function $u_z(\tau)$ satisfies Eq. (35) at the ends of I, $u_z(\tau) \in C^2(I)$, this series converges absolutely in I, hence its sum (40) is a limited function for all $\tau \in R$. This is the proof of stability of the rotational motions (4) for the string with fixed end with respect to the z oscillations.

In Fig. 2 we observe another type of oscillation. These planar disturbances are described by Eqs. (33) and (34). Their solutions in the form $u_0 = \beta_0 e^{i\Theta\tau}$, $u_e = \beta_1 e^{i\Theta\tau}$ exist only if the "frequency" Θ satisfies the equation

FIG. 3. Shapes of planar stationary waves for $n=1,2,3,4$.

$$\Theta^2 - Q^2(1+v_0^{-2}) = 2Q\Theta \cot \pi\Theta. \tag{41}$$

The roots of Eq. (41) $\Theta_n \in (n-1,n)$, $n \geq 1$ behave like the roots ϑ_n of Eq. (37) but $\Theta_n > \vartheta_n$. The disturbances (30) $u^\mu = \beta^\mu \exp(i\Theta_n \tau)$ correspond to the following small ($B \ll \omega^{-1}$) harmonic planar oscillations or planar stationary waves:

$$X^\mu = e_0^\mu t + e^\mu(\tau)[\omega^{-1}\sin\theta\sigma + Bf_r(\sigma)\cos\Theta_n\tau]$$

$$+ B[e_0^\mu f_0(\sigma) + \acute{e}^\mu(\tau)f_s(\sigma)]\sin\Theta_n\tau, \tag{42}$$

$$f_0 = 2(\theta^2 - \Theta_n^2)\cos\Theta_n\sigma,$$

$$B = \frac{m_i\beta_0}{2\gamma(\Theta_n^2 - \theta^2)\sin\pi\Theta_n},$$

$$f_s = (\Theta_n + \theta)^2\sin(\Theta_n - \theta)\sigma - (\Theta_n - \theta)^2\sin(\Theta_n + \theta)\sigma,$$

$$f_r = (\Theta_n + \theta)^2\sin(\theta - \Theta_n)\sigma - (\Theta_n - \theta)^2\sin(\Theta_n + \theta)\sigma.$$

The shape of these stationary waves in the corotating frame of reference (where the axes x and y are directed along e^μ and $\acute{e}^\mu$) is approximately described by the function $B[f_s(\sigma) - f_0(\sigma)\sin\theta\sigma]$ if the deflection is maximal. These shapes $F = F_n(s)$ for $n=1,2,3,4$ are shown in Fig. 3 with indicated numbers n. Here the parameters m_i/γ are the same as for the example in Fig. 2. For each n this curved string oscillates at the frequency $\omega_n = \omega\Theta_n/\theta$, it has $n-1$ nodes in $(0,\pi)$ (which are not strictly fixed because f_0 and f_r are nonzero), and the moving quark is not in a node, especially for the main mode with $n=1$. The latter feature [similar to the motion (38)] radically differs from that in expression (6) [15,16] where the disturbance in this endpoint is forcedly nullified. Note that Eq. (42) describes both the deflection of this endpoint $B\acute{e}^\mu f_s(\pi)\sin\Theta_n\tau$ and its radial motion $Be^\mu f_r(\pi)\cos\Theta_n\tau$.

Any smooth disturbance $u(\tau)$ may be expanded in the series similar to Eq. (40) with the roots Θ_n of Eq. (41) [21]. So any quasirotational motion of this sting, in particular, the

motion in Fig. 2 is the superposition of the stationary waves (42) and (for nonplanar motions) (38). This fact let us conclude that the rotational motions (4) for the considered model are *stable*, and we explain the above-mentioned problem with the "hierarchy of the periods" for the example in Fig. 2. Now, when we expand this solution in the combination of expressions (42), it becomes obvious that the radial period T_R [Fig. 2(d)] corresponds to the main "frequency" $\Theta_1 \simeq 0.463$ with the background of higher harmonics, whereas the "shape" period T_s is connected mainly with the following "frequency" $\Theta_2 \simeq 1.149$, because the string shape for the mode Θ_1 is closer to the rectilinear one (Fig. 3) and it is "not observable" in Fig. 2(c). We note that $T_R\Theta_1 = T_s\Theta_2 = T\theta$ where T is the rotational period and $\theta = 0.25$.

The obtained results are generalized for the string with both finite masses $0 \leq m_1, m_2 < \infty$ at the ends. For this purpose we are to substitute the disturbed expression (30) into Eqs. (18)–(21), (27) and deduce the equations generalizing Eqs. (32). The search of their oscillatory solutions $u^3 = B_z\exp(i\vartheta\tau)$, $u^\mu = B^\mu\exp(i\Theta\tau)$ results in the following generalizations of Eqs. (37) and (41):

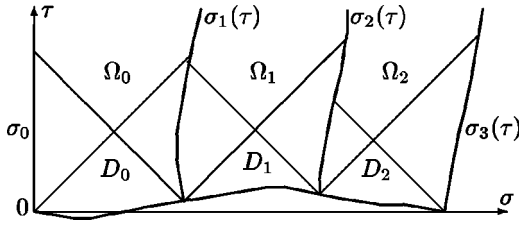$$(\vartheta - Q_1Q_2/\vartheta)/(Q_1 + Q_2) = \cot \pi\vartheta, \tag{43}$$

$$\frac{\Phi_1(\Theta)\Phi_2(\Theta) - 4Q_1Q_2\Theta^2}{2\Theta[Q_1\Phi_2(\Theta) + Q_2\Phi_1(\Theta)]} = \cot \pi\Theta, \tag{44}$$

where $Q_i = \theta v_i/\sqrt{1-v_i^2}$, v_i are the constant speeds of the ends for Eq. (4), $\Phi_i(\Theta) = \Theta^2 - Q_i^2(1+v_i^{-2})$. The corresponding disturbances of the rotational motion (4) are the stationary waves behaving similar to Eqs. (38) and (42). In particular, in Eq. (38) one should substitute $\sin\vartheta_n\sigma$ for $\cos(\vartheta_n\sigma + \phi_n)$. In this case the endpoints also oscillate at satisfying Eqs. (43) and (44) "frequencies" ϑ_n or Θ_n.

All roots of Eqs. (43), (44) are real numbers. Hence, the amplitudes of these harmonics are not increasing (and not decreasing) and the quasirotational motions constructed as combinations of these harmonics are *stable* (but they are not asymptotically stable). Note that the minimal positive root of both Eqs. (43) and (44) equals $\theta = \omega b$. In both cases the corresponding solutions are trivial: they describe a small tilt of the rotational plane or a small shift of the rotational center in this plane.

The obtained single-mode oscillatory motions for the finite masses m_i are simulated numerically and are represented with typical graphs for Eqs. (43), (44) and shapes of the harmonics, corresponding to Θ_i. Arbitrary slightly disturbed rotational motions may also be obtained as solutions of the IBVP with certain initial data. Numerical experiments show their stability and that they are always superpositions of the above-described stationary waves.

The stability problem for the rotational motions for the string baryon models q-q-q, Y and Δ (Fig. 1) is more complicated than that for the meson string model. We consider all string baryon configurations in turn starting from the model "triangle" [8,9].

FIG. 4. The domain Ω for the model Δ.

II. ''TRIANGLE'' BARYON MODEL

This configuration may be regarded as a closed relativistic string with the tension γ carrying three pointlike masses m_1, m_2, m_3 [8,9]. In action (1) for the ''triangle'' model $N=3$ and the domain Ω is divided by the quark trajectories into three domains: $\Omega=\Omega_0\cup\bar\Omega_1\cup\Omega_2$, $\Omega_i=\{(\tau,\sigma):\sigma_i(\tau)<\sigma<\sigma_{i+1}(\tau)\}$ (Fig. 4). The equations $\sigma=\sigma_0(\tau)$ and $\sigma=\sigma_3(\tau)$ determine the trajectory of the same (third) quark. It is connected with the fact that the string is closed (so the world surface is tube-like) and may be written in the following general form [8]:

$$X^\mu(\tau,\sigma_0(\tau))=X^\mu(\tau^*,\sigma_3(\tau^*)). \quad (45)$$

The parameters τ and τ^* in these two parametrizations of just the same line are not equal in general.

The equations of string motion and the boundary conditions at three quark trajectories result from action (1) [8,9]. Derivatives of X^μ can have discontinuities on the lines $\sigma=\sigma_i(\tau)$ (except for tangential). However, by choosing coordinates τ, σ the induced metric on all the world surface may be made continuous and conformally flat [8], i.e., satisfying the orthonormality conditions (7). Under these conditions the equations of motion for all Ω_i take the form (8) and the boundary conditions are [8,9]

$$m_i\frac{d}{d\tau}U_i^\mu(\tau)-\gamma[X'^\mu+\sigma_i'(\tau)\dot X^\mu]|_{\sigma=\sigma_i+0}+\gamma$$

$$\times[X'^\mu+\sigma_i'(\tau)\dot X^\mu]|_{\sigma=\sigma_i-0}=0, \quad (46)$$

where the same notation U_i^μ for the unit velocity vector of the ith quark is used. In accordance with Eq. (45) for the third quark one should set $\sigma=\sigma_0(\tau)$ in the second term of Eq. (46) and replace τ by τ^* in the last term.

At this stage we have five undetermined functions in this model: $\tau^*(\tau)$ in the closure condition (45) and four trajectories $\sigma_i(\tau)$, $i=0$, 1, 2, 3. Using the invariance of Eqs. (7), (8), (46) with respect to the reparametrizations (11) we fix two of these functions as follows (Fig. 4):

$$\sigma_0(\tau)=0, \quad \tau^*(\tau)=\tau. \quad (47)$$

The remaining three functions will be calculated with solving the IBVP.

The first Eq. (47) may be obtained by the substitution (11) with the required f_+ and $f_-(\eta)=\eta$. At the second step one can get the equality $\tau^*=\tau$ by repeating the procedure (11) with the function $f_+=f_-\equiv f$ that satisfies the condition

$$2f(\tau)=f(\tau^*(\tau)+\sigma_3(\tau))+f(\tau^*(\tau)-\sigma_3(\tau)).$$

The constraint $\tau^*(\tau)=\tau$ (that is the closure of any coordinate line $\tau=$const on the world surface) was not fulfilled in Refs. [8,9].

Because of discontinuities of X'^μ at $\sigma=\sigma_i(\tau)$ the general solutions of Eq. (8) in three domains Ω_i are described by three different functions ($i=0,1,2$)

$$X^\mu(\tau,\sigma)=\tfrac12[\Psi_{i+}^\mu(\tau+\sigma)+\Psi_{i-}^\mu(\tau-\sigma)], \quad (\tau,\sigma)\in\Omega_i. \quad (48)$$

Nevertheless, the function X^μ (with the tangential derivatives) is continuous in Ω.

The initial-boundary value problem (IBVP) for the ''triangle'' string configuration is stated similarly to that for the meson model in Sec. I. The procedure of its solving is briefly described below. One can find the details in e-print [22].

An initial position of the string can be given as the curve (13) in Minkowski space but for the Δ model $\lambda\in[\lambda_0,\lambda_3]$ and this curve is closed: $\rho^\mu(\lambda_0)=\rho^\mu(\lambda_3)$. The function $\rho^\mu(\lambda)$ is piecewise smooth, ρ'^μ may have discontinuities at the quark positions $\lambda=\lambda_1$, λ_2.

Initial velocities on the initial curve can be given as a timelike vector $v^\mu(\lambda)$, $\lambda\in[\lambda_0,\lambda_3]$, $v^\mu(\lambda)$ may be multiplied by an arbitrary scalar function $\phi(\lambda)>0$. The condition $v^\mu(\lambda_0)=v^\mu(\lambda_3)\cdot$const is fulfilled.

To solve the problem we use the parametrization (14), (15), $\lambda\in[\lambda_0,\lambda_3]$ of the initial curve on the world surface (Fig. 4), and the formulas (17) for determining the functions $\Psi_{i\pm}'^\mu$ from the initial data in the finite segments

$$\Psi_{i+}^\mu(\xi), \quad \xi\in[\check\tau(\lambda_i)+\check\sigma(\lambda_i),\check\tau(\lambda_{i+1})+\check\sigma(\lambda_{i+1})],$$

$$\Psi_{i-}^\mu(\xi), \quad \xi\in[\check\tau(\lambda_{i+1})-\check\sigma(\lambda_{i+1}),\check\tau(\lambda_i)-\check\sigma(\lambda_i)], \quad (49)$$

which lets us find the solution of the problem in the form (48) in the zones D_i shown in Fig. 4. In these zones bounded by the initial curve and the characteristic lines $\tau-\sigma\le\check\tau(\lambda_i)-\check\sigma(\lambda_i)$, $\tau+\sigma\le\check\tau(\lambda_{i+1})+\check\sigma(\lambda_{i+1})$ the solution depends only on initial data without the influence of the boundaries. In others parts of the domains Ω_i the solution is obtained with the help of the boundary conditions (46) which may be reduced to the form [22]

$$U_i'^\mu=\gamma m_i^{-1}[\delta_\nu^\mu-U_i^\mu(\tau)U_{i\nu}(\tau)]$$

$$\times\frac{d}{d\tau}[\Psi_{i+}^\nu(\tau+\sigma_i)+\Psi_{(i-1)-}^\nu(\tau-\sigma_i)]. \quad (50)$$

For $i=3$ in Eq. (50) one should replace $\Psi_{3+}^\mu(\tau+\sigma_3)$ by $\Psi_{0+}^\mu(\tau)$ in accordance with Eqs. (45) and (47).

Integrating systems (50) with the initial conditions (22), $i=1,2,3$ we can determine unknown vector functions $U_i^\mu(\tau)$ for $\tau\in[\check\tau(\lambda_i),\tau_i^c]$ with the help of the functions $\Psi_{i\pm}'^\mu$ known in the segments (49) from the initial data. Here τ_i^c are (minimal) ordinates of the points in which the trajectories $\sigma=\sigma_i(\tau)$ cross the characteristic lines $\tau\pm\sigma=$const (Fig. 4).
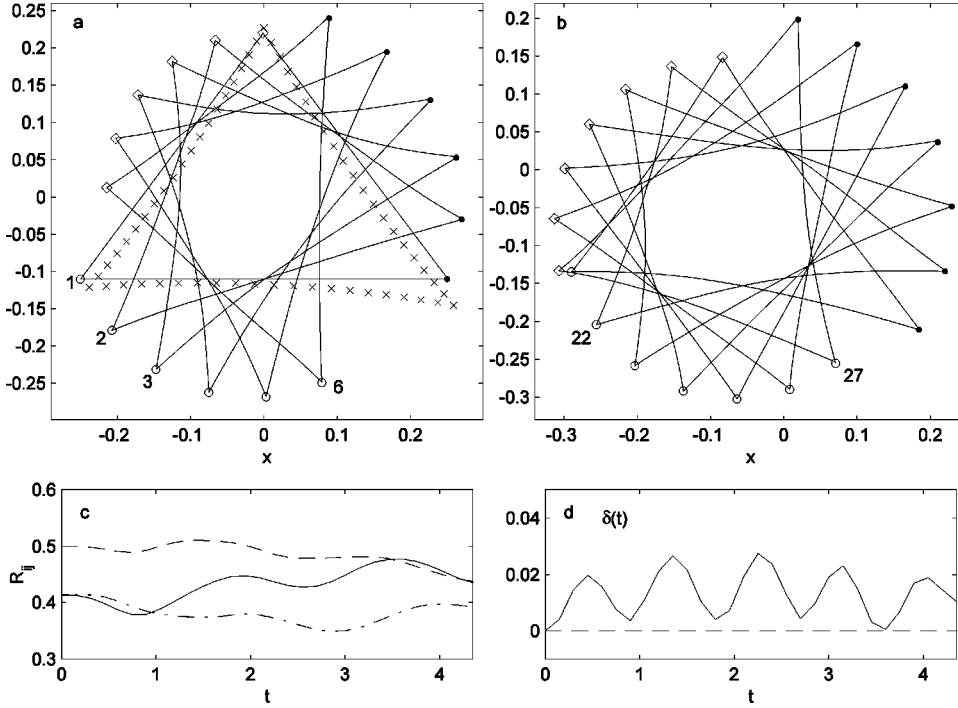
FIG. 5. The quasirotational motion of the model Δ.

However we can continue this procedure for $\tau > \tau_i^c$ if for every value of τ [after calculating $U_i'^\mu$ from Eq. (50)] we determine $\Psi_{i\pm}'^\mu$ outside segments (49) from the equations ($i = 1,2$)

$$\Psi_{(i-1)+}^\mu(\tau+\sigma_i) = \Psi_{i+}^\mu(\tau+\sigma_i) - \frac{m_i}{\gamma} U_i^\mu(\tau) + C_i^+ ,$$

$$\Psi_{i-}^\mu(\tau-\sigma_i) = \Psi_{(i-1)-}^\mu(\tau-\sigma_i) - \frac{m_i}{\gamma} U_i^\mu(\tau) + C_i^- . \tag{51}$$

They are obtained from Eqs. (46), (48) and from continuity of X^μ [22]. The similar relations for $i=3$ are

$$\Psi_{2+}^\mu(\tau+\sigma_3) = \Psi_{0+}^\mu(\tau) - m_3\gamma^{-1}U_3^\mu(\tau) + C_3^+ ,$$

$$\Psi_{0-}^\mu(\tau) = \Psi_{2-}^\mu(\tau-\sigma_3) - m_3\gamma^{-1}U_3^\mu(\tau) + C_3^- .$$

The constants of integration $C_i^\pm$ are fixed from Eq. (15).

For solving the system (50) we are to determine the functions $\sigma_i(\tau)$ for $\tau > \check{\tau}(\lambda_i)$. Multiplying $U_i^\mu(\tau)$ by $(d/d\tau)\Psi_{j\pm}^\mu(\tau\pm\sigma_i)$ we obtain the equalities

$$(1-\sigma_i')[U_i,\Psi_{(i-1)-}'(\tau-\sigma_i)] = (1+\sigma_i')[U_i,\Psi_{i+}'(\tau+\sigma_i)],$$

which let us express σ_i' in the following way (separately for $i=1,2$ and $i=3$):

$$\frac{d\sigma_i}{d\tau} = \frac{\{U_i,[\Psi_{(i-1)-}'(\tau-\sigma_i) - \Psi_{i+}'(\tau+\sigma_i)]\}}{\{U_i,[\Psi_{(i-1)-}'(\tau-\sigma_i) + \Psi_{i+}'(\tau+\sigma_i)]\}} , \tag{52}$$

$$\frac{d\sigma_3}{d\tau} = 1 - \frac{[U_3,\Psi_{0+}'(\tau)]}{[U_3,\Psi_{2-}'(\tau-\sigma_3)]} .$$

Here $U_i^\mu \equiv U_i^\mu(\tau)$.

Equations (50)–(52) allow us to continue the functions $\Psi_{i\pm}^\mu$ unambiguously beyond the segments (49). This algorithm solves (numerically in general) the considered IBVP with arbitrary initial conditions $\rho^\mu(\lambda)$, $v^\mu(\lambda)$.

For the model ''triangle'' the exact solutions of Eqs. (8), (7), (45), (46) describing rotational motions of the system (with hypocycloidal segments of the strings between the quarks) are obtained in Refs. [8,9]. World surfaces with the parametrization $\sigma_i(\tau) = \sigma_i = $ const and the form of closure condition (45) $\tau^* - \tau = T = $ const may be represented as follows ($X^1 \equiv x$, $X^2 \equiv y$):

$$X^0 = \tau - \alpha\sigma, \quad X^1 + iX^2 = w(\sigma)e^{i\omega\tau}. \tag{53}$$

Here $w(\sigma) = A_i \cos\omega\sigma + B_i \sin\omega\sigma$, $\sigma \in [\sigma_i,\sigma_{i+1}]$; $\alpha = T/(\sigma_3 - \sigma_0) = $ const, ω is the angular frequency of this rotation. Real (σ_i, T, ω, m_i/γ) and complex (A_i, B_i) constants are connected by certain relations [8,9,14]. There are many topologically different types[2] of solutions (53) but we consider motions close to the simple states [9] in which three quarks are connected by smooth segments of hypocycloids.

The simple motions (53) may be obtained by solving the IBVP with corresponding initial data. To research the stability of these motions we consider here the disturbed initial conditions $\rho^\mu(\lambda)$ and $v^\mu(\lambda)$. With an illustrative view in Fig. 5 the example of such a motion [close to the simple state (53)] of the ''triangle'' configuration is represented. Here the initial string position is the rectilinear equilateral triangle with the base $a = 0.5$ and the altitude $h = 0.33$ in the xy plane

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[2]The so called exotic states [9] contain string points moving at the speed of light.

[position 1 in Fig. 5(a)]. The initial velocities $v^\mu(\lambda)$ in the form (24) correspond to the uniform rotation of the system at the frequency $\vec{\omega}=\{0;0;\omega\}$, $\omega=2$ about the origin of coordinates. The quarks with masses $m_1=1$, $m_2=1.5$, $m_3=1.2$, $\gamma=1$ are placed at the corners of the triangle.

The results of computing are represented as projections of the world surface level lines $t=X^0(\tau,\sigma)=$ const onto the xy plain as in Fig. 2 and numbered with the time step $\Delta t=0.15$. The values $\rho^\mu(\lambda)$ and $v^\mu(\lambda)$ are close to those which give the exact hypocycloidal solution (53) for this system (with the same m_i, γ and $\omega=2$) describing the uniform rotation of the system with the string shape that is x marked in Fig. 5(a). The positions of the third quark with $m_3=1.2$ are marked by circles.

The further evolution of the system after one turn of the triangle is represented in Fig. 5(b) (one can find the omitted phases of this motion in Ref. [22]). In Fig. 5(c) the dependence of three mutual distances R_{ij} between the three quarks on time t is shown, in particular, $R_{12}(t)$ is denoted by the full line. In Fig. 5(d) the deflection $\delta(t)$ from rectilinearity of the string segment with $R_{13}(t)$ is shown. We see that when the system rotates the distances between the quarks and the configuration of the string segments fluctuate near the values corresponding to the motion (53) [x marks in Fig. 5(a)].

This situation is typical for slightly disturbed rotational motions of the string baryon model ''triangle.'' Numerous tests (with various values m_i, the energy, $\rho^\mu(\lambda)$, $v^\mu(\lambda)$ and various types of disturbances [22]) show that the simple rotational motions (53) are *stable*. That is small disturbances of the motions (initial conditions) do not grow with increasing time.

We emphasize that the simple motions (53) are stable with respect to transforming into the ''quark-diquark'' states of the Δ string configuration [9] with merging two quarks into the diquark. It is shown in Ref. [22] that such a transformation may be obtained only through very strong disturbances of the initial conditions, for example, by essentially reducing one of the sides of the initial triangle. However, in this case the two nearest quarks do not merge but revolve with respect to each other.

When one of the quark masses m_i is larger than the sum of two others the shape of the triangle configuration for the simple rotational motion (53) tends to a rectilinear segment with the position of the heaviest mass at the rotational center when the energy of the system decreases [9]. In other words the Δ configuration tends to the q-q-q one. But the motion of the ''triangle'' model remains stable unlike that of the q-q-q model described in the next section.

III. LINEAR STRING CONFIGURATION

The dynamics of the linear (q-q-q) string baryon model is described by action (1) where $N=3$ and the domain Ω is divided by the middle quark trajectory into two parts Ω_1 and Ω_2: $\Omega_i=\{(\tau,\sigma):\sigma_i(\tau)<\sigma<\sigma_{i+1}(\tau)\}$, $\sigma_1(\tau)<\sigma_2(\tau)<\sigma_3(\tau)$ [5]. The equations of string motion under conditions (7) may be reduced to the same form (8). The boundary conditions at the ends $\sigma=\sigma_1$ and $\sigma=\sigma_3$ look like Eqs. (9) but for the middle quark they take the form (46), $i=2$. At

this line the derivatives of X^μ are not continuous in general.

Using the reparametrizations (11) we fix two (from three) functions $\sigma_i(\tau)$ in a form similar to Eq. (10): $\sigma_1=0$, $\sigma_3=\pi$. The third function $\sigma_2(\tau)$ is obtained from Eq. (52) with solving IBVP that is described in detail in Ref. [5]. The main stages of this procedure are the same as in Secs. I, II: determining the functions $\Psi_{i\pm}^\mu$, $i=1,2$ in Eq. (48) with the help of given $\rho^\mu(\lambda)$, $v^\mu(\lambda)$ and Eq. (17) in the initial segments (49). The further continuation of $\Psi_{i\pm}^\mu$ beyond Eq. (49) includes equations similar to Eqs. (18)–(21) for the endpoints (here $i=3$ for the second end) and Eqs. (50) and (51) for the middle quark.

The rotational motion of the q-q-q system is described by Eq. (4) but with the middle quark at the center of rotation. The authors of Ref. [4] assumed that motion is unstable with respect to centrifugal moving away of the middle quark and transforming this configuration into the quark-diquark one. The numerical experiments were made in Ref. [5] in accordance with the above scheme of solving the IBVP. They showed that the rotational motion of the q-q-q system is *unstable* indeed. Any arbitrarily small disturbances of the initial data result in the complicated motion of the middle quark including its centrifugal moving away but the material points never merge and the configuration never transforms into a q-qq one on the classic level.

In Fig. 6 the example of such a motion of the q-q-q system is represented [Figs. 6(a)–6(d)] in comparison with the similar motion of the model ''triangle'' [Figs. 6(e)–6(g)]. For both models the quark masses are $m_1=m_3=1$, $m_2=3$, $\Delta t=0.15$ the tension is $\gamma_\Delta=1$ in the ''triangle'' and $\gamma=2$ in the q-q-q configuration. The initial shape of the string is the rectilinear segment, for the Δ configuration it is a particular case of the hypocycloidal motion (53). The initial velocities satisfies Eqs. (24) and (25) where $v_\perp=v_0=0.5$. The position of the middle quark (marked by the square) is slightly displaced with respect to the center of rotation so it uniformly moves at the initial stage [Figs. 6(a) and 6(e)] where the behavior of both systems practically coincides.

Further [starting with position 17 in Fig. 6(b)] one can see that in the q-q-q model the middle heavy quark moves to the string end, while in the ''triangle'' model [Figs. 6(f), 6(g)] it remains in the vicinity of the rotational center. The latter configuration is stable unlike the q-q-q one. The axes are omitted here for saving space.

But the minimal distance between the nearest two quarks for the q-q-q system never equals zero. The middle quark begins to play a role of rotational center for this string segment [Fig. 6(c)] and then it returns to the center of the system [Fig. 6(d)] and the process recurs. Such a quasiperiodic motion of the q-q-q system is the qualitatively universal result of the evolution for motions (4) with various types of disturbances [5].

IV. THREE-STRING MODEL

In the three-string baryon model or Y configuration [6,7,23,24] three world sheets (swept up by three segments of the relativistic string) are parametrized with three different functions $X_i^\mu(\tau_i,\sigma)$. It is convenient to use the different no-
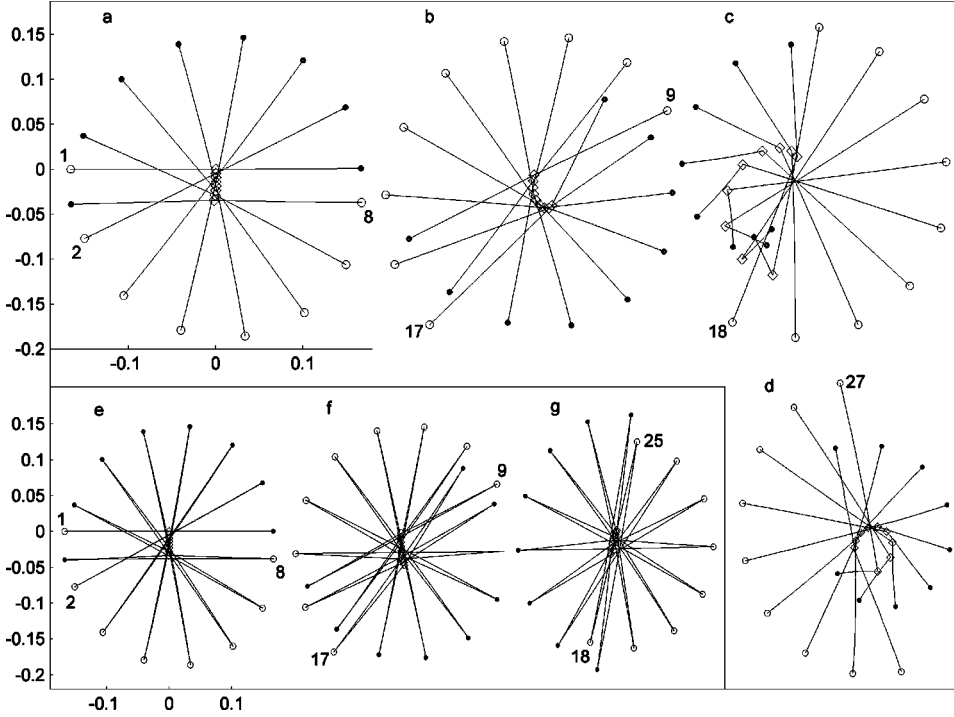
FIG. 6. Behavior of the q-q-q system (a)–(d) in comparison with the model Δ (e)–(g).

tations τ_1, τ_2, τ_3 for the ''timelike'' parameters [23,25]. But the ''spacelike'' parameters are denoted here by the same symbol σ. These three world sheets are joined along the world line of the junction that may be set as $\sigma=0$ for all sheets without loss of generality (see below and Ref. [25]).

Under these notations the action of the three-string is close to Eq. (1):

$$S=-\sum_{i=1}^{3}\int d\tau_i\left[\gamma\int_0^{\sigma_i(\tau_i)}\sqrt{-g_i}d\sigma+m_i\sqrt{\dot{x}_i^2(\tau_i)}\right]. \quad (54)$$

This action with different τ_i generalizes the similar expressions in Refs. [6,7,23] (where $m_i=0$) and in Ref. [24], where $m_i\neq0$ but the class of motions is limited.

There is the additional boundary condition at the junction taking the form

$$X_1^{\mu}(\tau,0)=X_2^{\mu}(\tau_2(\tau),0)=X_3^{\mu}(\tau_3(\tau),0), \quad (55)$$

if the parameters τ_i on the three world sheets are connected at this line in the following manner:

$$\tau_2=\tau_2(\tau), \quad \tau_3=\tau_3(\tau), \quad \tau_1\equiv\tau.$$

The equations of motion (2) and the boundary conditions for the junction and the quark trajectories are deduced from action (54). Using its invariance one may choose the coordinates in which the orthonormality conditions (7) are satisfied and string equations of motion for all X_i^{μ} take the form (8). The junction condition (55) unlike more rigid conditions with $\tau_1=\tau_2=\tau_3$ on the junction line [6,7], let us choose these coordinates independently on each world sheet. After this substitution (new coordinates are also denoted τ_i,σ) the inner equations of the junction line will be more general $\sigma=\sigma_{0i}(\tau_i)$ (in comparison with the previous ones $\sigma=0$) and

the boundary conditions in the junction and on the quark trajectories $\sigma=\sigma_i(\tau_i)$ will take the following form:

$$\sum_{i=1}^{3}[X_i'^{\mu}(\tau_i,\sigma_{0i})+\sigma_{0i}'(\tau_i)\dot{X}_i^{\mu}(\tau_i,\sigma_{0i})]\tau_i'(\tau)=0, \quad (56)$$

$$m_iU_i'^{\mu}(\tau_i)+\gamma[X_i'^{\mu}+\sigma_i'(\tau_i)\dot{X}_i^{\mu}]|_{\sigma=\sigma_i(\tau_i)}=0. \quad (57)$$

Here in Eq. (56) $\tau_i=\tau_i(\tau)$, $\sigma_{0i}=\sigma_{0i}(\tau_i(\tau))$.

The reparametrizations similar to Eq. (11)

$$\tilde{\tau}_i\pm\tilde{\sigma}=f_{i\pm}(\tau_i\pm\sigma), \quad i=1,2,3 \quad (58)$$

(with six arbitrary smooth monotone functions $f_{i\pm}$) keep invariance of Eqs. (7), (8), (56), and (57). Choosing the functions $f_{i\pm}$ we can fix the equations of the junction and quark trajectories on each world sheet independently in the form (10)

$$\sigma_{0i}(\tau_i)=0, \quad \sigma_i(\tau)=\pi, \quad i=1,2,3, \quad (59)$$

One can obtain the first Eq. (59) like that in Eq. (47) and the equalities $\sigma_i=\pi$ through the substitution (58) with $f_{i+}=f_{i-}$ keeping the invariance of the equation $\sigma=0$.

In this paper the parametrization satisfies the conditions (59) and (7). But the ''time parameters'' τ_1, τ_2 and τ_3 in Eq. (55) are not equal in general. The possible alternative approach implies introducing the condition $\tau_2(\tau)=\tau_3(\tau)=\tau$ on the junction line (55) in conjunction with the condition $\sigma_{0i}=0$ (or $\sigma_{0i}=$ const). But under these restrictions two of the functions $\sigma_i(\tau)$ on the quark trajectories are not equal to constants in general.

If under orthonormal gauge (7) we demand satisfying the conditions (59), as the equalities $\tau_1=\tau_2=\tau_3$ on the junction

line (55) (as, for example, in Ref. [7]), then we actually restrict the class of motions of the system, which the model describes. In other words, not all physically possible motions satisfy the above mentioned conditions.

The proof of these statements in Ref. [25] uses the fact that under restrictions (7) only reparametrizations (58) with the functions $f_{i+}=f_{i-}=f_i$ satisfying the condition (16) keep Eqs. (59) [21,25]. These functions have the properties that: if $f(\xi)$ and $g(\xi)$ satisfy conditions (16), then the inverse function $f^{-1}(\xi)$ and the superposition $f(g(\xi))$ also satisfy Eq. (16). To obtain the equalities $\tilde{\tau}_2=\tilde{\tau}_3=\tilde{\tau}_1$ on the junction line $\sigma=0$, we have to use transformations (58), (16) satisfying the relations $\tau_i(\tau)=f_i^{-1}(f_1(\tau))$, $i=2,3$. This is possible only if the functions $\tau_2(\tau)$, $\tau_3(\tau)$ satisfy conditions (16), which are not fulfilled for an arbitrary motion in general [25].

For describing an arbitrary motion of the three-string in the suggested approach the unknown functions $\tau_i(\tau)$ are determined from dynamic equations with solving the IBVP for this system. This approach is similar to the above-formulated one for other string models. In particular, for the Y configuration we have the general solution (48) where the index i in $\Psi_{i\pm}^{\mu}$ numerates the world sheets. Using the given initial position of the three-string in the form of three joined curves in Minkowski space

$$x^{\mu}=\rho_i^{\mu}(\lambda), \quad \lambda \in [0,\lambda_i], \quad \rho_1^{\mu}(0)=\rho_2^{\mu}(0)=\rho_3^{\mu}(0)$$

and initial velocities $v_i^{\mu}(\lambda)$, we obtain the functions Ψ_{i+}^{μ} and Ψ_{i-}^{μ} in the initial segments $[0,\check{\tau}_i(\lambda_i)+\pi]$ and $[\check{\tau}_i(\lambda_i)-\pi,0]$ correspondingly from Eq. (17).

The functions $\Psi_{i\pm}^{\mu}$ are to be continued with using the boundary conditions (55), (56), (57). In particular, the conditions on the quark trajectories (57) are reduced to the form (19), (21), (22):

$$U_i'^{\mu}=\gamma m_i^{-1}[\delta_{\nu}^{\mu}-U_i^{\mu}(\tau_i)U_{i\nu}(\tau_i)]\Psi_{i-}'^{\nu}(\tau_i-\pi),$$
(60)

$$\Psi_{i+}'^{\mu}(\tau_i+\pi)=\Psi_{i-}'^{\mu}(\tau_i-\pi)-2m_i\gamma^{-1}U_i'^{\mu}(\tau_i).$$
(61)

Substituting Eq. (48) for X_i^{μ} into the boundary conditions in the junction (55), (56), (59) we express the function $\Psi_{i-}'^{\mu}(\tau_i)$ through $\Psi_{i+}'^{\mu}(\tau_i)$:

$$\frac{d}{d\tau}\Psi_{i-}^{\mu}(\tau_i(\tau))=\sum_{j=1}^{3} T_{ij}\frac{d}{d\tau}\Psi_{j+}^{\mu}(\tau_j(\tau)),$$
(62)

where

$$T_{ij}=\begin{cases} -1/3, & i=j \\ 2/3, & i\neq j \end{cases}$$

Equations (62), (61), and (60) let us infinitely continue the functions $\Psi_{i\pm}^{\mu}$ outside the initial segments if the functions $\tau_2(\tau)$ and $\tau_3(\tau)$ are known. They can be found by using the isotropy condition $\Psi_{i\pm}'^2(\tau_i)=0$ [resulting from Eqs. (7)] and the following consequences of Eqs. (55):

$$[\tau_i'(\tau)]^2[\Psi_{i+}'(\tau_i),\Psi_{i-}'(\tau_i)]=[\Psi_{1+}'(\tau),\Psi_{1-}'(\tau)].$$

Substituting $\Psi_{i-}'^{\mu}$ from Eq. (62) into these relations we obtain the formulas for calculating the functions $\tau_i(\tau)$

$$\tau_2'(\tau)=\frac{(\Psi_{1+}',\Psi_{3+}')}{(\Psi_{2+}',\Psi_{3+}')}, \quad \tau_3'(\tau)=\frac{(\Psi_{1+}',\Psi_{2+}')}{(\Psi_{2+}',\Psi_{3+}')}.$$
(63)

Here the functions $\Psi_{i+}'^{\mu}\equiv\Psi_{i+}'^{\mu}(\tau_i)$ are taken from Eqs. (17) and (61) during solving the IBVP.

Equations (60)–(63) form the closed system for infinite continuation of $\Psi_{i\pm}^{\mu}$ or for solving the IBVP in the three-string model. The described method is used here (and with more details in Ref. [25]) for investigating the rotational stability of this configuration. For this purpose we consider the IBVP with disturbed initial conditions $\rho_i^{\mu}(\lambda)$ and $v_i^{\mu}(\lambda)$.

As was mentioned above the rotational motion of the three-string is the uniform rotating of three rectilinear string segments joined in a plane at the angles 120° [6,7]. Their lengths R_i or the speeds $v_i=\omega R_i$ are connected with the angular velocity ω by the relation (25) or

$$R_i\omega^2(R_i+m_i/\gamma)=1.$$
(64)

This motion and slightly disturbed motions may be obtained (numerically, in general) by solving the IBVP with the appropriate initial position $\rho_i^{\mu}(\lambda)$ in the form of three rectilinear segments with lengths R_i and velocities (24) with some disturbances $\delta\rho_i^{\mu}(\lambda)$ or $\delta v_i^{\mu}(\lambda)$.

The typical example of a quasirotational motion of the three-string with masses $m_1=1$, $m_2=2$, $m_3=3$, $\gamma=1$ is represented in Fig. 7. Here the positions of the system in the xy plane are numbered in order of increasing t with the step in time $\Delta t=0.125$ and these numbers are near the position of the first quark marked by the small square. This motion is close to the rotational one: the initial velocities satisfy the relation (24), $\delta v_i^{\mu}=0$, the angular velocity $\omega\simeq1.6$ and the different lengths $R_1=0.3$, $R_3\simeq0.125$ are connected by Eqs. (64). But the assigned value $R_2=0.22$ does not satisfy Eq. (64) (that gives $R_2\simeq0.179$) so this difference plays a role of the disturbance for the motion in Fig. 7.

The evolution of this disturbance includes the motion of the junction [Figs. 7(a) and 7(b)] with varying the lengths of the string segments unless one of these lengths becomes equal to zero, i.e., the third quark falls into the junction and merges with the junction after the shown position 31 in Fig. 7(c). They move together [Fig. 7(d)] during the finite time. The waves from the point of merging spread along the strings and complicate the picture of motion. The falling ith material point into the junction is simultaneous with becoming infinite in the corresponding ''time'' ($\tau_i\to\infty$). This is not a ''bad parametrization'' but the geometry of the system changes: the three-string transforms into the q-q-q configuration after merging a quark with the junction. The lifetime of this ''q-q-q stage'' is finite but nonzero because the material point with the mass m_i moving at a speed $v<1$ cannot slip through the junction instantaneously. Otherwise under three noncompensated tension forces the massless junction
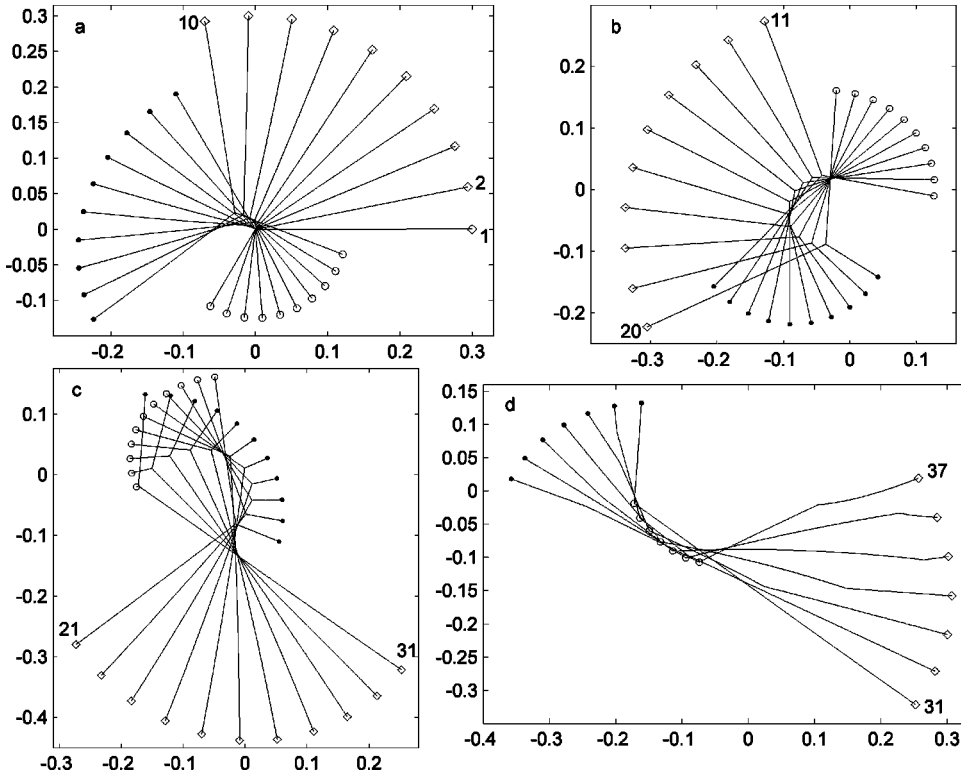
FIG. 7. The disturbed motion of the three-string model.

will move at the speed of light. But we must note that this description is a purely classical one: it will unlikely be the same after developing a more general QCD-based theory.

Nevertheless on the classic level the numerical experiments in Ref. [25] show that the picture of motion in Fig. 7 is qualitatively identical for any small asymmetric disturbance $\delta\rho_i^\mu(\lambda)$ or $\delta v_i^\mu(\lambda)$. Starting from some point in time the junction begins to move. During this complicated motion the distance between the junction and the rotational center increases and the lengths of the string segments vary quasi-periodically unless one of the material points inevitably merges with the junction. So one may conclude that rotational motions of the three-string are *unstable*. The evolution of the instability is slow at the first stage if the disturbance is small, but the middle and final stages are rather similar to the motion in Figs. 7(b) and 7(c). The dependences $\tau_i(\tau)$ for these motion do not satisfy the periodicity conditions (16) in general [25]. This fact does not allow describing these motions in the frameworks of the parametrization [6,7] with $\tau_1 = \tau_2 = \tau_3$. The above-described behavior of slightly disturbed rotational motions takes place also for the massless ($m_i = 0$) three-string model [25].

CONCLUSION

In this paper the classic motions of the various meson and baryon (Fig. 1) string models close to the rotational motions (4) or (53) are investigated. For the meson string model (or the q-qq baryon configuration) it is made both analytically and numerically but in each of these methods we use the orthonormal conditions (7) which let us reduce the problem to solving the systems of ordinary differential equations (18), (19), (28). Using this approach we obtained a set of solutions

(38), (42) describing small oscillatory excitations of the rotating string in the form of stationary waves. They are divided into two classes: the orthogonal or z oscillations (38) and the planar oscillations (42). Each class contains a countable set of solutions with different ''frequencies'' ϑ_n, Θ_n, which are the roots of Eqs. (37), (41), (43), and (44). For these stationary waves the moving quarks are not in a node of oscillation, they also oscillate. This was one of the reasons resulting in the wrong expression (6) in Refs. [15,16].

The energy M and the angular momentum J of the oscillatory excited motions (38), (42) are close to the values M and J [13] for the pure rotational motions (4) because the disturbances are small. In Ref. [16] the latter expressions M and J were used for the quantization of the curved string motions in the framework of the approach suggested in Ref. [17] for the relativistic flux tube model. But this procedure was based upon the questionable assumption that only one specified motion (5), (6) of the curved string exists (or can be the subject of interest). So the quantization in Ref. [16] includes only one dynamical variable R for describing the considerable set of motions.

However, in Sec. I of this paper we show that the spectrum of string quasirotational motions is very rich and an arbitrary motion from this class is the superposition of the stationary waves (38) and (42). So for the adequate quantization one should describe all infinite spectrum of these modes taking, for example, their amplitudes $B = B_n$ as dynamical variables. This procedure of quantization requires the special study for each string model which appears in separate papers. At the final stage of this study some generalizations of the oscillatory states (38), (42) may be used for describing the radial, hybrid or other hadron excitations. Note that the planar oscillations (42) include also the radial

motions of the string endpoints (a quark trajectory in the corotating frame of reference is an ellipse). This especially concerns the mode with the ''frequency'' Θ_1 of Eq. (41) or (44).

A search of quasirotational motions in a form similar to Eq. (30) for the string baryon model ''triangle,'' q-q-q and Y encounters essential difficulties connected with nonfixed quark trajectories $\sigma = \sigma_i(\tau)$ in Eq. (46) or the expressions $\tau_i(\tau)$ in Eq. (55) for the three-string configuration. But for each string baryon model the method of solving the initial-boundary value problem with arbitrary initial position and velocities is suggested. Using this approach we numerically simulated various quasirotational motions for all the models and obtained that the simple [9] rotational states of the string model ''triangle'' are stable (i.e., small disturbances behave like in the meson model) and the rotational motions of the systems q-q-q and Y are unstable. In the latter two cases any small asymmetric disturbances grow with increasing time. For the model q-q-q the middle quark moves away from the center under the centrifugal force but then it quasiperiodi-

cally returns without merging with an endpoint. The evolution of the three-string instability includes the complicated motion of the junction and inevitably results in one of the quarks falling into the junction.

These features of the classical behavior of the string baryon models give some advantage for the q-qq and ''triangle'' systems over the q-q-q and Y configurations. But this does not mean ''closing'' the latter two models, in particular, because of the fact that the majority of orbitally excited baryon states are resonances so their classical stability is a problem of minor importance. So the final choice of the string baryon model is to depend on all aspects of this problem, including QCD-based grounds [10,11] and describing the baryon Regge trajectories [13,14].

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