Unintegrated gluon distribution from the Ciafaloni-Catani-Fiorani-Marchesini equation

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The gluon distribution $f(x,k_t^2,\mu^2)$, unintegrated over the transverse momentum k_t of the gluon, satisfies the angular-ordered CCFM equation which interlocks the dependence on the scale k_t with the scale μ of the probe. We show how, to leading logarithmic accuracy, the equation can be simplified to a single-scale problem. In particular we demonstrate how to determine the two-scale unintegrated distribution $f(x,k_t^2,\mu^2)$ from knowledge of the integrated gluon obtained from a unified scheme embodying both BFKL $[\log(1/x)]$ and DGLAP $(\log \mu^2)$ evolution.

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I. INTRODUCTION

Deep-inelastic electron-proton scattering is described in terms of the scale-dependent parton distributions $q(x,\mu^2)$ and $g(x,\mu^2)$. For less inclusive processes it is, however, necessary to consider distributions unintegrated over the transverse momentum k_t of the parton, which for the gluon, for example, we denote by $f(x,k_t^2,\mu^2)$. These distributions depend on two hard scales: k_t and the hard scale μ of the probe. The (conventional) integrated gluon distribution is given by

$$xg(x,\mu^2)=\int^{\mu^2}\frac{dk_t^2}{k_t^2}f(x,k_t^2,\mu^2). \tag{1}$$

Unintegrated distributions are required to describe measurements where transverse momenta are exposed explicitly, for example, to describe the p_T spectrum of prompt photons produced in high energy hadron collisions or for dijets or vector mesons produced at the DESY collider HERA.

At very low x, that is, to leading $\log(1/x)$ accuracy, the unintegrated distribution becomes independent of the hard scale μ, and so from Eq. (1) we have

$$f(x,k_t^2,\mu^2)\rightarrow\frac{\partial}{\partial\ln\lambda^2}[xg(x,\lambda^2)]|_{\lambda^2=k_t^2}. \tag{2}$$

Clearly Eq. (2) cannot remain true as x increases. Indeed we see that it would give negative values for f. Moreover, even at low x, there are significant subleading corrections which, to some level of approximation, modify Eq. (2) to the form [1,2]

$$f(x,k_t^2,\mu^2)\approx\frac{\partial}{\partial\ln\lambda^2}[xg(x,\lambda^2)T_g(\lambda,\mu)]|_{\lambda^2=k_t^2}, \tag{3}$$

where T_g is the Sudakov form factor. In fact Eq. (3) is oversimplified. As discussed below, the expression for $f(x,k_t^2,\mu^2)$ is more complicated than Eq. (3).

The natural framework for unifying the small and large x domains is the Ciafaloni-Catani-Fiorani-Marchesini (CCFM) formalism based on angular ordering [3–10], which follows from color coherence effects [11]. It reduces to the leading order Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) formalism at moderate x and it embodies the Balitskiĭ-Fadin-Kuraev-Lipatov (BFKL) formalism at small x. The unintegrated gluon distribution $f(x,k_t^2,\mu^2)$ satisfies the CCFM equation [3,4] which interlocks the two hard scales (k_t^2,μ^2) in a complicated way. The equation is based on the coherent radiation of gluons, which leads to an angular ordering of the gluon emissions along the chain. The ordering introduces a scale specifying the maximum angle of gluon emission, which turns out to be essentially the hard scale μ of the probe. At moderate x the angular ordering becomes an ordering in the gluon transverse momenta and the CCFM equation reduces to DGLAP evolution. At very small x the angular ordering does not provide any constraint on the transverse momenta along the chain and, in the leading $\log(1/x)$ approximation, $f(x,k_t^2,\mu^2)$ becomes the μ-independent distribution which satisfies the BFKL equation. On the other hand, although the dependence on the scale μ only enters at subleading $\log(1/x)$ level, f does depend on μ through leading $\log\mu^2$ evolution.

The outline of the paper is as follows. The angular-ordered CCFM equation is introduced in Sec. II. In Sec. III we simplify this evolution, yet staying within leading logarithmic accuracy, to show that the two-scale distribution $f(x,k_t^2,\mu^2)$ can be obtained in terms of the conventional one-scale $g(x,\mu^2)$ distribution. In this way, we are led to a procedure for determining $f(x,k_t^2,\mu^2)$ from a unified BFKL-DGLAP single-scale evolution equation. This is described in Sec. IV. Moreover, we are able to extend the formalism to incorporate important subleading $\log(1/x)$ effects, which are generated by the so-called consistency condition[1] [12,13] and which subsume the angular ordering constraint at low x. We also extend the formalism to include the contributions due to the quark distributions. For comparison, in Sec. V we present the pure DGLAP-type approach to determine $f(x,k_t^2,\mu^2)$, in which the gluon cascade evolves according to evolution strongly ordered in k_t. Section VI contains sample numerical results for $f(x,k_t^2,\mu^2)$ obtained from the

[1]Called the kinematic constraint in [12].

fully unified approach of Sec. IV. As expected, we have some diffusion of the gluon transverse momenta into the region $k_t > \mu$. This is in contrast to the pure DGLAP-type approximation in which the distribution $f(x,k_t^2,\mu^2)$ is limited to the domain $k_t < \mu$. Finally, Sec. VII contains a summary of the procedure that we have used to determine the unintegrated gluon distribution $f(x,k_t^2,\mu^2)$.

II. CCFM EQUATION

The unintegrated gluon distribution satisfies the CCFM evolution equation [3,4] based on angular ordering. In unfolded[2] form the equation is

$$f(x,k_t^2,\mu^2)=f_0(x,k_t^2)+\frac{\alpha_S}{2\pi}\int_0^1 dz\int\frac{d^2q}{\pi q^2}\Theta(1-z-q_0/q)\times\left[\Theta(z-x)\Theta(\mu-qz)\frac{k_t^2}{k_t'^2}P(z)\times f\left(\frac{x}{z},k_t'^2,q^2\right)-zP(z)\Theta(\mu-q)\times f(x,k_t^2,q^2)\right]-\frac{\alpha_S}{2\pi}2N_C\int_x^1\frac{dz}{z}\int\frac{d^2q}{\pi q^2}\times\Theta(q-q_0)\Theta(k_t^2-q^2)f\left(\frac{x}{z},k_t^2,q^2\right), \tag{4}$$

where $k_t' \equiv |\boldsymbol{k}_t+(1-z)\boldsymbol{q}|$. For simplicity, α_S is taken outside the integrals, but the scale will be specified carefully in the final equations [in particular see Eq. (23)]. The driving term f_0 is of nonperturbative origin and is assumed to contribute only for $k_t^2 < q_0^2$. The remaining terms contribute in the $k_t^2 > q_0^2$ domain. The driving term thus gives the nonperturbative starting gluon distribution

$$xg(x,q_0^2)=\int^{q_0^2}\frac{dk_t^2}{k_t^2}f_0(x,k_t^2). \tag{5}$$

This angular-ordered equation for f, which embodies both DGLAP and BFKL evolution, is shown schematically in Fig. 1. The first term in the square brackets in Eq. (4) describes real gluon emission with angular ordering imposed. The term containing $f(x,k_t^2,q^2)$ is related to the virtual corrections corresponding to the unfolded Sudakov form factor, while the last term in Eq. (4) represents the virtual corrections which, when resummed, give rise to gluon Reggeization. The latter correspond to the BFKL part of the (unfolded) non-Sudakov form factor. Angular ordering along the chain, a portion of which is shown in Fig. 2, requires

[2]The folded form (which actually is the CCFM equation [3,4]) contains Sudakov and non-Sudakov form factors, which arise from the resummation of virtual corrections and screen the singularities as $z\to 1$ and $z\to 0$, respectively.

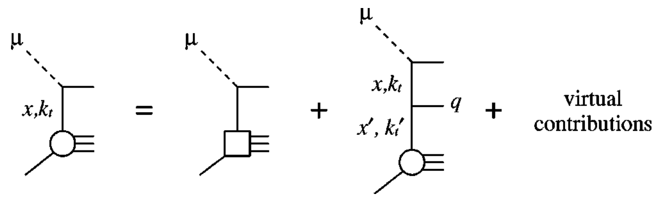

FIG. 1. Schematic representation of the CCFM evolution equation (4) for the unintegrated gluon distribution $f(x,k_t^2,\mu^2)$. The variable q is defined by $q \equiv q_t/(1-z)$ where $z = x/x'$ and $\boldsymbol{k}_t' = \boldsymbol{k}_t + (1-z)\boldsymbol{q}$.

$$z_{n-1}q_{n-1} < q_n \quad \text{where} \quad q_n \equiv q_{tn}/(1-z_n) \tag{6}$$

and $z_n = x_n/x_{n-1}$. The angular ordering continues up to a maximum angle with the limit expressed in terms of the hard scale via $\Theta(\mu - qz)$. When z is away from the $z\sim 0$ and $z\sim 1$ domains, angular ordering is equivalent to the strong k_t ordering of pure DGLAP leading order (LO) evolution. At first sight there appear to be three types of large logarithms in Eq. (4). First the usual DGLAP logarithms coming from the

$$|\boldsymbol{k}_t'| \equiv |\boldsymbol{k}_t + (1-z)\boldsymbol{q}| \ll q \tag{7}$$

domain. Second there are the BFKL-type log(1/x) contributions originating from the 1/z part of the real emission term in Eq. (4) and the gluon Reggeization contribution. These two terms can be combined together in the function

$$F(x,k_t^2)=\frac{\alpha_S}{2\pi}2N_C\int\frac{d^2q}{\pi q^2}\int_x^1\frac{dz}{z}\left[\frac{k_t^2}{|\boldsymbol{k}_t+\boldsymbol{q}|^2}f\left(\frac{x}{z},|\boldsymbol{k}_t+\boldsymbol{q}|^2,q^2\right)-\Theta(k_t^2-q^2)f\left(\frac{x}{z},k_t^2,q^2\right)\right], \tag{8}$$

where we have assumed $z \ll 1$, in k_t' of Eq. (7). Finally there is a danger that in Eq. (4) we have a large logarithm from the region $q^2 \ll k_t^2$. However, we see from Eq. (4) that the function $f(x,k_t^2,q^2)$ only extends into the region $k_t^2 > q^2$ as a result of the BFKL small x effects, which are subleading at finite x. That is, to leading logarithmic accuracy, it can be shown, in the so-called ''single-loop'' approximation, that if the last term in Eq. (4) is neglected and $\Theta(\mu - qz)$ is replaced by $\Theta(\mu - q)$, then the function $f(x,k_t^2,q^2)$ vanishes for $k_t^2 > q^2$. Thus we limit the integration regions in Eq. (4) to the strongly ordered domain $k_t'^2 \ll q^2$ for those contribu-

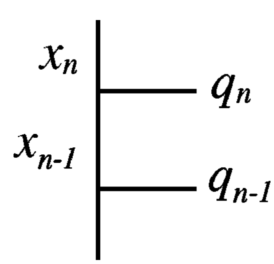

FIG. 2. A portion of the evolution chain. Angular ordering requires $z_{n-1}q_{n-1} < q_n$, where $q_n \equiv q_{tn}/(1-z_n)$ and $z_n = x_n/x_{n-1}$.

tions in which the unintegrated gluon is multiplied by the part $\bar{P}(z)$ of the splitting function $P(z)$ which is nonsingular at low z. That is

$$\bar{P}(z)=P(z)-\frac{2N_C}{z}. \tag{9}$$

It should be noted that the BFKL part, Eq. (8), of Eq. (4) (for which this approximation is not justified) is free from singularities as $q\to 0$, since the potential singularity of the real emission term is canceled by the virtual contribution.

So finally we have just the large logarithms coming from either $k_t'\ll k_t$ or from $z\ll 1$. Our aim is to develop an approximate treatment of the CCFM equation which incorporates both types of large logarithms.

III. SIMPLIFICATION OF THE CCFM EQUATION

To simplify the angular-ordered equation (4), we rearrange the equation and retain only terms which generate large logarithms. To achieve this it is convenient to add and subtract the term

$$\frac{\alpha_S}{2\pi}\int_0^1 dz\int\frac{d^2q}{\pi q^2}\Theta\left(1-z-\frac{q_0}{q}\right)\Theta(\mu-q)\Theta(k_t^2-k_t'^2)\times zP(z)\frac{k_t^2}{k_t'^2}f(x,k_t'^2,q^2) \tag{10}$$

from the right-hand side of Eq. (4), and to group together contributions containing the singular $2N_C/z$ part of the splitting function

$$P(z)\equiv\bar{P}(z)+\frac{2N_C}{z}. \tag{11}$$

In this way we obtain the approximate form

$$\begin{aligned}f(x,k_t^2,\mu^2)=&f_0(x,k_t^2)+\frac{\alpha_S}{2\pi}\int_0^1 dz\,\Theta(k_t-q_0)\Theta\left(\mu-\frac{k_t}{1-z}\right)\\&\times\left[\bar{P}(z)\Theta(z-x)\frac{x}{z}g\left(\frac{x}{z},\left(\frac{k_t}{1-z}\right)^2\right)-zP(z)xg\left(x,\left(\frac{k_t}{1-z}\right)^2\right)\right]+\frac{\alpha_S}{2\pi}\int_0^1 dz\\&\times\int\frac{dq^2}{q^2}\Theta\left(1-z-\frac{q_0}{q}\right)\Theta(\mu-q)zP(z)\\&\times\left[\int\frac{dk_t'^2}{k_t'^2}\Theta(k_t^2-k_t'^2)f(x,k_t'^2,q^2)\frac{q}{2}\times\delta\left(q-\frac{k_t}{1-z}\right)-f(x,k_t^2,q^2)\Theta\left(q-\frac{k_t}{1-z}\right)\right]\\&+F(x,k_t^2),\end{aligned} \tag{12}$$

where the BFKL-type $\log(1/x)$ contribution $F(x,k_t^2)$ is given by Eq. (8). The second term on the right-hand side is the pure DGLAP contribution in the large x limit. It comes from the strongly ordered configuration

$$k_t'^2\equiv|\boldsymbol{k}_t+(1-z)\boldsymbol{q}|^2\ll q^2 \tag{13}$$

in the second term on the right-hand side of Eq. (4). In this configuration the variable q becomes $k_t/(1-z)$. We have also made the large z approximation such that

$$\bar{P}(z)\Theta(\mu-qz)\approx\bar{P}(z)\Theta(\mu-q)\approx\bar{P}(z)\Theta\left(\mu-\frac{k_t}{1-z}\right). \tag{14}$$

With these approximations we may rewrite the second term using

$$\begin{aligned}\int^{k_t^2/(1-z)^2}\frac{dk_t'^2}{k_t'^2}&\left[\Theta(z-x)\bar{P}(z)f\left(\frac{x}{z},k_t'^2,\left(\frac{k_t}{1-z}\right)^2\right)-zP(z)f\left(x,k_t'^2,\left(\frac{k_t}{1-z}\right)^2\right)\right]\\&=\Theta(z-x)\bar{P}(z)\frac{x}{z}g\left(\frac{x}{z},\left(\frac{k_t}{1-z}\right)^2\right)-zP(z)xg\left(x,\left(\frac{k_t}{1-z}\right)^2\right),\end{aligned} \tag{15}$$

where the upper limit k_t^2 of the $dk_t'^2$ integration has, to leading $\log\mu^2$ accuracy, been replaced by $[k_t/(1-z)]^2$.

Finally, the third term on the right-hand side of Eq. (12) corresponds to the difference between Eq. (10) and the virtual Sudakov (DGLAP-type) contribution given by the second term in the square brackets in Eq. (4), that is, to the contribution

$$\frac{\alpha_S}{2\pi}\int_0^1 dz\int\frac{d^2q}{\pi q^2}\Theta\left(1-z-\frac{q_0}{q}\right)\Theta(\mu-q)zP(z)\times\left[\frac{k_t^2}{k_t'^2}f(x,k_t'^2,q^2)-f(x,k_t^2,q^2)\right]. \tag{16}$$

The first integral is evaluated using the strongly ordered configuration $[k_t'^2\ll k_t^2,q\approx k_t/(1-z)]$, while the second integration is restricted to the region $q(1-z)>k_t$. It should be noted that for $q(1-z)\ll k_t$ the two terms in the integrand of Eq. (16) cancel, since then $k_t'\approx k_t$.

The contribution (16) represents the virtual corrections which have to be resummed. The resummation is performed in the Appendix. We obtain

$$f(x,k_t^2,\mu^2)=\frac{\partial}{\partial\ln k_t^2}\left[T_g(k_t,\mu)xg(x,k_t^2)+\int_{k_t^2}^{\mu^2}\frac{dq^2}{q^2}T_g(q,\mu)\times\int_{q_0^2}^{k_t^2}\frac{dk_t'^2}{k_t'^2}\frac{\partial L(x,k_t'^2,q^2)}{\partial\ln q^2}\right], \tag{17}$$

where

$$L(x,k_t^2,\mu^2)=\frac{\alpha_S}{2\pi}\Theta(k_t-q_0)\int_0^1 dz\Theta\left(\mu-\frac{k_t}{1-z}\right)\times\left[\bar{P}(z)\Theta(z-x)\frac{x}{z}g\left(\frac{x}{z},\left(\frac{k_t}{1-z}\right)^2\right)-zP(z)xg\left(x,\left(\frac{k_t}{1-z}\right)^2\right)\right], \tag{18}$$

and where the Sudakov form factor

$$T_g(q,\mu)=\exp\left(-\int_{q^2}^{\mu^2}\frac{dp^2}{p^2}\frac{\alpha_S(p^2)}{2\pi}\int_0^{1-k_t/p}dz'z'P(z')\right). \tag{19}$$

The cutoff $z'<1-k_t/p$ enters on account of the kinematic structure of the real emission term, where the upper limit of the p integration is given by $k_t/(1-z')$. Note that $T_g(q,\mu)$ therefore implicitly depends on k_t. The integration limits defining the Sudakov form factor should be understood as arising from the Θ function constraints

$$\Theta(1-k_t/p)\Theta(\mu-p)\Theta(p-q). \tag{20}$$

This implies $T_g(q,\mu)=1$ if these constraints are not satisfied. In particular $T_g=1$ for $k_t>\mu$ or $q>\mu$.

A nice feature of the result (17) is that the unintegrated gluon $f(x,k_t^2,\mu^2)$ is entirely specified in terms of the integrated gluon xg. The next step is to introduce a single-scale unified equation, which embodies both BFKL- and DGLAP-type effects, to determine xg.

IV. STRATEGY FOR DETERMINING THE UNINTEGRATED GLUON

We have emphasized that the angular-ordered equation (12) is a ''two-scale'' evolution equation for $f(x,k_t^2,\mu^2)$. That is, the scales k_t^2 and μ^2 are intertwined by angular ordering. In the previous section we have shown how the two-scale unintegrated gluon $f(x,k_t^2,\mu^2)$ can be determined once we know the integrated gluon xg. Here we describe the procedure to obtain xg from a unified evolution equation for a single-scale auxiliary distribution:

$$h(x,\mu^2)\equiv\mu^2\frac{\partial(xg)}{\partial\mu^2}=\mu^2\frac{\partial}{\partial\mu^2}\left(\int^{\mu^2}\frac{dk_t^2}{k_t^2}f(x,k_t^2,\mu^2)\right). \tag{21}$$

If we integrate both sides of Eq. (12) over k_t^2 up to μ^2 and differentiate with respect to $\log\mu^2$, then we find that $h(x,\mu^2)$ satisfies the evolution equation

$$h(x,\mu^2)=\frac{\alpha_S}{2\pi}\int_0^{1-q_0/\mu}dz\left\{\Theta(z-x)\bar{P}(z)\frac{x}{z}g\left(\frac{x}{z},\mu^2\right)-zP(z)xg(x,\mu^2)\right\}+F(x,k_t^2=\mu^2), \tag{22}$$

where $F(x,k_t^2)$ is defined by Eq. (8). Note that the integral in Eq. (22) has no singularity close to $z=1$. We can now derive a relation expressing the two-scale unintegrated gluon distribution $f(x,k_t^2,\mu^2)$ in terms of the one-scale distribution xg (or h). From Eq. (17) we get the following expression for $f(x,k_t^2,\mu)$:

$$f(x,k_t^2,\mu^2)=T_g(k_t,\mu)h(x,k_t^2)+T_g(k_t,\mu)\times\int_0^{1-k_t/\mu}dzzP(z)\frac{\alpha_S(k_t^2/(1-z)^2)}{2\pi}xg(x,k_t^2)-\frac{\alpha_S}{2\pi}T_g(k_t,\mu)\int_0^{1-q_0/k_t}dz\left[\Theta(z-x)\bar{P}(z)\times\frac{x}{z}g\left(\frac{x}{z},k_t^2\right)-P(z)zxg(x,k_t^2)\right]+\Theta(\mu-k_t)\times\int_0^{1-k_t/\mu}dz\frac{\alpha_S}{2\pi}T_g\left(\frac{k_t}{1-z},\mu\right)\times\left[\Theta(z-x)\bar{P}(z)\frac{x}{z}g\left(\frac{x}{z},\left(\frac{k_t}{1-z}\right)^2\right)-P(z)zxg\left(x,\left(\frac{k_t}{1-z}\right)^2\right)\right], \tag{23}$$

where the scale of α_S is taken to be the scale of the appropriate gluon, except for the second term on the right-hand side. We may safely set the $1-z$ cutoff q_0/k_t to zero, but not the cutoffs k_t/μ.

Note that in the leading ln(1/x) approximation we may set $T_g=1$ and neglect all the integral terms in Eq. (23), since they do not generate ln(1/x) contributions. In this approximation the unintegrated gluon is simply

$$f(x,k_t^2,\mu^2)=h(x,k_t^2),$$

with no dependence on the scale μ.

When obtaining Eq. (23) from Eq. (17), we have chosen to neglect a contribution coming from the derivative of the Sudakov form factor $T_g(q,\mu)$ with respect to k_t, which arises from the k_t dependence of the regulator; see Eq. (19). For this reason the unintegrated gluon of Eq. (23) does not precisely integrate to $xg(x,\mu^2)$, although the corrections are subleading in $\log\mu^2$. The discrepancy is indeed negligible at low x, but can become of the order of 20% or so for large values of $x\gtrsim0.1$. Rather than complicating Eq. (23) by including the derivative, we eliminate the discrepancy by changing the regulator in the form factor (19) from k_t/p to q/p; that is, we take the Sudakov form factor

$$T_g(q,\mu)=\exp\left(-\int_{q^2}^{\mu^2}\frac{dp^2}{p^2}\frac{\alpha_S(p^2)}{2\pi}\int_0^{1-q/p}dz'z'P(z')\right). \tag{24}$$

This approximation is justified since in our case either $q=k_t$ or $q\sim k_t$. Within this approximation it is evident that the unintegrated gluon (23) integrates exactly to $xg(x,\mu^2)$;

the sum of the first two terms on the right-hand side of Eq. (23) forms the total derivative

$$k_t^2 \frac{\partial}{\partial k_t^2}[T_g(k_t,\mu) x g(x,k_t^2)], \tag{25}$$

and the integrals of the third and fourth terms cancel each other.

From Eq. (21) we see that the integrated gluon distribution g can be expressed in terms of h, namely,

$$xg(x,k_t^2) = xg(x,q_0^2) + \int_{q_0^2}^{k_t^2} \frac{d\mu^2}{\mu^2} h(x,\mu^2). \tag{26}$$

Equations (22) and (23), together with Eqs. (8) and (26), form a system of coupled equations. If we substitute f of Eq. (23) into the F term on the right-hand side of Eq. (22), and take account of Eq. (26), then we obtain an integral equation for h. We may solve this equation for the single-scale auxiliary distribution $h(x,\mu^2)$, and then compute the two-scale unintegrated gluon $f(x,k_t^2,\mu^2)$ from Eq. (23).

It is convenient to simplify the integral equation (22) for $h(x,\mu^2)$ using approximations which are valid to leading logarithmic accuracy. To be precise we simplify the computation of $F(x,k_t^2)$ of Eq. (8). First instead of allowing the scale q^2 of f to vary, we note that in Eq. (8) the dominant values of q^2 are such that $q^2 \approx k_t^2$. Moreover, we notice that in the strongly ordered domain $k_t'^2$ $(\equiv |\boldsymbol{k}_t + \boldsymbol{q}|^2) \ll k_t^2$, the first term on the right-hand side of Eq. (8) can be simplified using

$$\begin{aligned}\int \frac{d^2q}{\pi q^2} \frac{k_t^2}{k_t'^2} f\left(\frac{x}{z}, |\boldsymbol{k}_t + \boldsymbol{q}|^2, q^2\right) &\approx \int^{k_t^2} \frac{dk_t'^2}{k_t'^2} f\left(\frac{x}{z}, k_t'^2, k_t^2\right) \\ &\equiv \frac{x}{z} g\left(\frac{x}{z}, k_t^2\right);\end{aligned} \tag{27}$$

see Eq. (1). In the remaining contributions to $F(x,k_t^2)$ of Eq. (8) we can use Eq. (23) to approximate f by the first term,

$$f(x,k_t^2,q^2) \approx h(x,k_t^2), \tag{28}$$

noting that $q^2 \approx k_t^2$ and $T_g(k_t,k_t) = 1$. The other terms in Eq. (23) give only subleading $\log(1/x)$ contributions to the BFKL kernel. Since this contribution to $F(x,k_t^2)$ goes beyond the strongly ordered part of the kernel, it is also subleading in $\log \mu^2$. After these approximations, Eq. (22) for $h(x,\mu^2)$ for $\mu^2 > q_0^2$ may be written as[3]

[3]In order to be consistent with Eq. (22), the upper limit of the z integration in the second term on the right-hand side of Eq. (29) should be $1 - q_0/q$ rather than 1. The integrals are, of course, regular at $z = 1$ and so the discrepancy is subleading in $\ln \mu^2$.

$$\begin{aligned} h(x,\mu^2) = {} & h_0(x,\mu^2) + \frac{\alpha_S(\mu^2)}{2\pi} \int_0^1 dz \int_{q_0^2}^{\mu^2} \frac{dq^2}{q^2} \\ & \times \left\{ \Theta(z-x) \bar{P}(z) h\left(\frac{x}{z}, q^2\right) - zP(z) h(x,q^2) \right\} \\ & + \frac{\alpha_S(\mu^2)}{2\pi} 2N_C \int_x^1 \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(k_t'^2 - q_0^2) \\ & \times \left\{ \frac{\mu^2}{k_t'^2} h\left(\frac{x}{z}, k_t'^2\right) - \Theta(\mu^2 - q^2) h\left(\frac{x}{z}, \mu^2\right) \right\}, \end{aligned} \tag{29}$$

where $k_t'^2 = |\boldsymbol{k}_t + \boldsymbol{q}|^2$ with $k_t^2 = \mu^2$. The driving term, which arises from the substitution of Eq. (26) for xg in Eqs. (22) and (27), is given by

$$\begin{aligned} h_0(x,\mu^2) = \frac{\alpha_S(\mu^2)}{2\pi} \int_0^1 dz \bigg\{ & \Theta(z-x) P(z) \frac{x}{z} g\left(\frac{x}{z}, q_0^2\right) \\ & - zP(z) xg(x,q_0^2) \bigg\}. \end{aligned} \tag{30}$$

Notice that the strongly ordered contribution of $F(x,k_t^2 = \mu^2)$ has combined with the residual DGLAP contribution in Eq. (22) with the effect that $\bar{P}(z) \to P(z)$.

Equation (29) is the single-scale unified BFKL-DGLAP equation for the gluon that was proposed in Ref. [14]. There it was shown that it is straightforward to incorporate a major part of the subleading order $\log(1/x)$ (or BFKL) effects by imposing a consistency condition to ensure that the virtuality of the exchanged gluon is dominated by its transverse momentum squared. This is achieved by including the theta function $\Theta(\mu^2 - zq^2)$ in the real emission contribution in the last term of Eq. (29). Other important subleading terms arising from using the complete DGLAP splitting function and from the running of α_S are automatically included in our framework [14]. This formalism was used to fit to deep-inelastic scattering data and the auxiliary function $h(x,\mu^2)$ was determined [14]. It was checked that the corresponding integrated gluon $xg(x,k_t^2)$ computed from Eq. (22) was compatible with the gluons obtained in the Martin-Roberts-Stirling (MRS), CTEQ global parton analyses [15].

In [14] the contribution of the quark distributions was included in Eq. (29) for $h(x,\mu^2)$. To incorporate the quarks in the present analysis we must also include the contribution of the singlet quark distribution Σ in Eq. (23) for $f(x,k_t^2,\mu^2)$. That is, we make the replacement

$$\bar{P}(z) \frac{x}{z} g \to \bar{P}(z) \frac{x}{z} g\left(\frac{x}{z}, \mu'^2\right) + P_{gq}(z) \frac{x}{z} \Sigma\left(\frac{x}{z}, \mu'^2\right) \tag{31}$$

in the real emission part of the third and fourth terms on the right-hand side of Eq. (23), where μ' is the appropriate scale. Recall that $P(z) \equiv P_{gg}(z)$. In the second term and in the virtual part of the third and fourth terms, we make the replacement

$$zP(z)xg \to [zP(z)+2n_f zP_{qg}(z)]xg(x,\mu'^2), \qquad (32)$$

where n_f is the number of active flavors, and the scale $\mu'^2 = k_t^2$ or $k_t^2/(1-z)^2$ as appropriate. Finally we have to modify the Sudakov form factor so that Eq. (24) becomes

$$T_g(q,\mu) = \exp\left(-\int_{q^2}^{\mu^2} \frac{\alpha_S(p^2)}{2\pi} \frac{dp^2}{p^2} \times \int_0^{1-q/p} z' \left[P(z') + \sum_q P_{qg}(z') \right] dz' \right). \qquad (33)$$

The above procedure allows the determination of the approximate solution $f(x,k_t^2,\mu^2)$ of the CCFM equation, which incorporates both a full (or so-called ''all-loop'' [5,6]) resummation of leading $\ln(1/x)$ contributions, as well as the resummation of the leading $\ln \mu^2$ contribution and the inclusion of dominant subleading $\ln(1/x)$ terms.

V. PURE DGLAP LIMIT

It is informative to compare the predictions above for the unintegrated gluon $f(x,k_t^2,\mu^2)$ with those obtained in the DGLAP (or so-called ''single-loop'' [5,6]) approximation, in which $\Theta(\mu - qz)$ in Eq. (4) is replaced by $\Theta(\mu - q)$, and the last term in Eq. (4) is neglected. After making these modifications we repeat the procedures of Secs. III and IV and obtain the DGLAP form

$$f(x,k_t,\mu^2) = \int_x^{1-k_t/\mu} dz \frac{\alpha_S[k_t^2/(1-z)^2]}{2\pi} T_g\left(\frac{k_t}{1-z},\mu\right) \times P(z) \frac{x}{z} g\left(\frac{x}{z}, \left(\frac{k_t}{1-z}\right)^2\right) + \int_0^{1-k_t/\mu} dz zP(z) \times \frac{\alpha_S[k_t^2/(1-z)^2]}{2\pi} \left[T_g(k_t,\mu) xg(x,k_t^2) - T_g\left(\frac{k_t}{1-z},\mu\right) xg\left(x,\left(\frac{k_t}{1-z}\right)^2\right) \right], \qquad (34)$$

with $f=0$ if $k_t > \mu$. Apart from the last term, this is the equation for the unintegrated gluon introduced in Ref. [2]. The last term, which is only nonzero on account of different scales, introduces subleading corrections. Its inclusion improves the accuracy of the integration of $f(x,k_t^2,\mu^2)$ to reproduce $xg(x,\mu^2)$; see Eq. (1). Note that the DGLAP or ''single-loop'' unintegrated gluon vanishes for $k_t \geqslant \mu$, as indeed it must [5,6]. In the results presented below we include the quark contributions as described in Eqs. (31)–(33).

It is informative to see how the full equation (23) for $f(x,k_t^2,\mu)$ reduces to the DGLAP limit (34). A crucial observation is that in the DGLAP domain ($k_t < \mu$) it is possible, within leading $\ln(1/x)$ and $\ln(\mu^2)$ accuracy, to replace $\overline{P}(z)$ by $P(z)$ in Eq. (23). Thus, *provided* $k_t < \mu$, we obtain the following more symmetric formula:

$$f(x,k_t^2,\mu^2) = T_g(k_t,\mu) h(x,k_t^2) + T_g(k_t,\mu) \times \int_0^{1-k_t/\mu} dz zP(z) \frac{\alpha_S(k_t^2/(1-z)^2)}{2\pi} xg(x,k_t^2) - \frac{\alpha_S(k_t^2)}{2\pi} T_g(k_t,\mu) \int_0^{1-q_0/k_t} dz P(z) \times \left[\Theta(z-x) \frac{x}{z} g\left(\frac{x}{z},k_t^2\right) - zxg(x,k_t^2) \right] + \Theta(\mu - k_t) \int_0^{1-k_t/\mu} dz \frac{\alpha_S}{2\pi} T_g\left(\frac{k_t}{1-z},\mu\right) P(z) \times \left[\Theta(z-x) \frac{x}{z} g\left(\frac{x}{z}, \left(\frac{k_t}{1-z}\right)^2\right) - zxg\left(x,\left(\frac{k_t}{1-z}\right)^2\right) \right]. \qquad (35)$$

In the DGLAP limit the first and third terms on the right-hand side of Eq. (35) exactly cancel[4]; they simply represent the DGLAP equation for $h(x,k_t^2)$ of Eq. (21). Thus Eq. (35) reduces to Eq. (34).

VI. NUMERICAL EVALUATION OF THE UNINTEGRATED GLUON

In Fig. 3 we show the k_t distributions of three different unintegrated gluons at each of four different values of x at a hard scale $\mu^2 = 100\ \mathrm{GeV}^2$.

(i) The solid curves are the gluons $f(x,k_t^2,\mu^2)$ obtained from Eq. (23), with the quark terms included, using the auxiliary function $h(x,k_t^2)$ of Ref. [14], which was itself obtained from a fit to deep inelastic scattering data using a unified BFKL-DGLAP equation. (The curves have been smoothed in the transition region $k_t \sim \mu$.)

(ii) The dot-dashed curves show $h(x,k_t^2)$ itself [14], which is independent of μ^2,

(iii) The dashed curves show $f(x,k_t^2,\mu^2)$ calculated from the pure DGLAP equation (35) using in this case the auxiliary function $h(x,k_t^2)$ obtained in [14] from pure DGLAP evolution from exactly the same starting distributions [$xg(x,q_0^2)$, etc.] as those found in the unified fit.

Note that the third set of gluons is shown solely to illustrate the difference between the two types of evolution. The gluons of the third set have not been constrained by a fit to the data, so should not be regarded as realistic.

In the pure DGLAP case (iii), we see that the distributions are confined to the domain $k_t < \mu$, as anticipated from strong ordering. On the other hand, the distributions $f(x,k_t^2,\mu^2)$ obtained in the unified BFKL-DGLAP framework develop a more and more extensive $k_t > \mu$ tail as x decreases. At small k_t and low x the magnitude of the unintegrated gluon calcu-

[4]Strictly speaking the cancellation is only exact when $q_0 \to 0$ in Eq. (35).

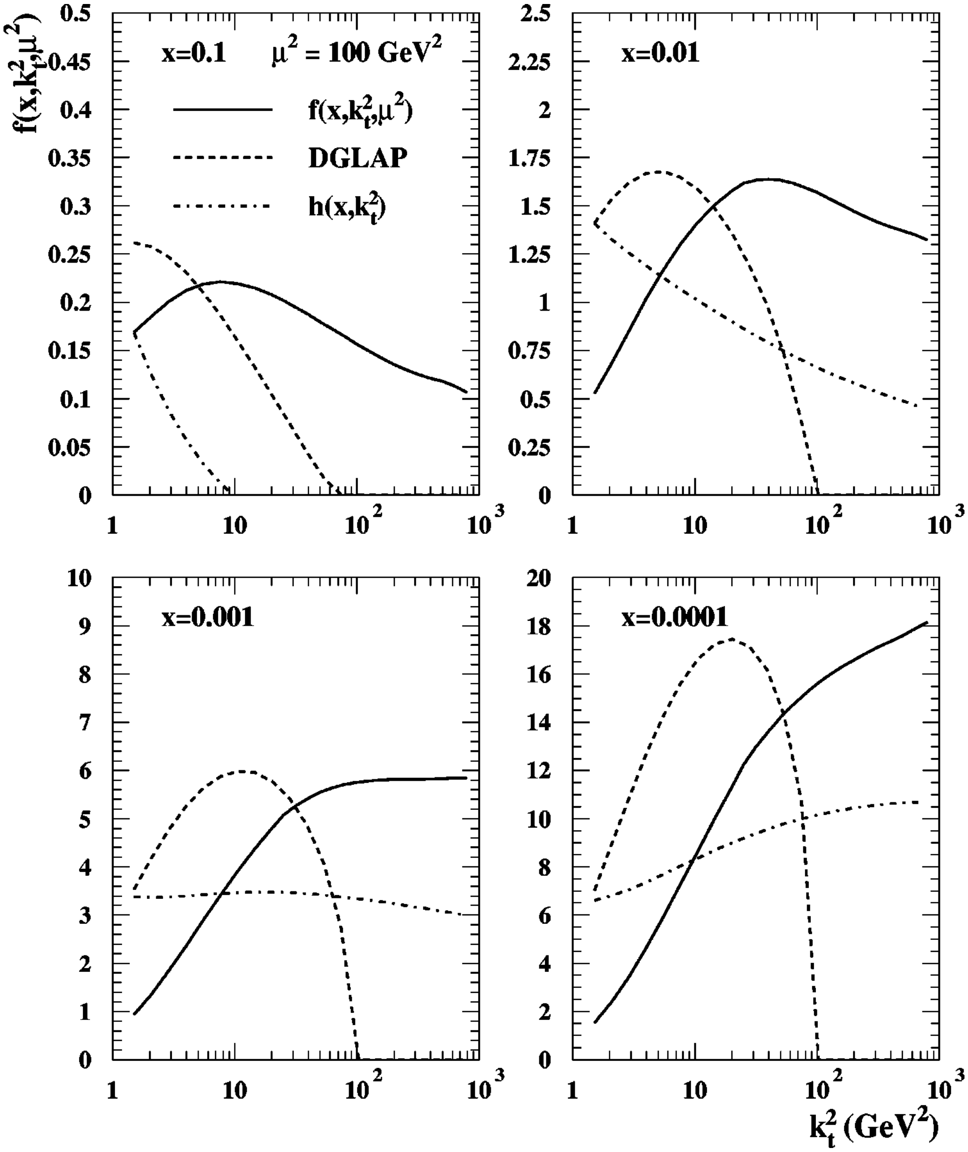

FIG. 3. The solid curves show the k_t dependence of the unintegrated gluon distribution $f(x,k_t^2,\mu^2)$ for $\mu^2=100$ GeV^2. For comparison we also show the input auxiliary function $h(x,k_t^2)$ (dot-dashed curves) [14] and the k_t dependence coming from pure DGLAP evolution (dashed curves).

lated from the unified scheme is about a factor of 2 less than that of the gluon coming from the pure DGLAP approach of case (iii). This is due to the imposition of the consistency constraint in case (i) which suppresses the magnitude of the gluon. If this constraint were absent, the distributions of cases (i) and (iii) would not be that different. We note that the auxiliary function h of case (ii) remains different from the unintegrated gluon f of case (i) down to very small values of x.

For $k_t>\mu$ we see that f is greater than h, whereas the DGLAP-driven unintegrated gluon vanishes, as it must. In this domain inspection of Eq. (22) and (23) shows that f comes purely from the BFKL contribution:

$$f(x,k_t^2,\mu^2)=F(x,k_t^2). \tag{36}$$

On the other hand, h is smaller than f due to the negative contribution of the integral term in Eq. (22). The latter is a DGLAP contribution which is ruled out when angular ordering is imposed. It is negative because the $2N_C/z$ contribution has been subtracted from the real emission contribution, but not from the virtual term. We see that for $x<0.01$ that there is about a factor of 2 discrepancy between h and the true unintegrated gluon f.

VII. SUMMARY

Here we have addressed the issue of obtaining a reliable determination of the (scale-dependent) gluon distribution $f(x,k_t^2,\mu^2)$, unintegrated over the gluon transverse momentum k_t, where μ denotes the hard scale of the probe. In the leading $\log(1/x)$ approximation the distribution is given simply by the derivative of the unintegrated gluon with respect to its scale $\lambda=k_t$ [see Eq. (2)] and satisfies the BFKL equation. We correct this simple relation by going beyond the leading $\log(1/x)$ approximation to include both subleading contributions and DGLAP effects. The final result for $f(x,k_t^2,\mu^2)$ is given in Eq. (23). To obtain this result we use the appropriate gluon cascade formalism based on angular ordering, which leads to the CCFM equation embodying both BFKL and DGLAP evolution. It is important to note that the CCFM equation gives a well-defined framework to calculate the very quantity that we seek: the unintegrated gluon distribution $f(x,k_t^2,\mu^2)$. Using this formalism we devise a procedure to determine $f(x,k_t^2,\mu^2)$ from the integrated gluon distribution $xg(x,q^2)$, its derivative $h(x,q^2)$, and the Sudakov form factor $T_g(q,\mu)$; cf. Eq. (23). An important ingredient is the solution of a (single-scale) evolution equation for $h(x,q^2)$ which embodies both BFKL and DGLAP effects. From the low x viewpoint it includes subleading effects from (i) the consistency constraint which limits the available phase space to the region in which the virtuality of the exchanged gluon is dominated by its transverse momentum squared, (ii) DGLAP effects generated by that part of the splitting function $P_{gg}(z)$ which is not singular in the limit $z\to 0$, (iii) the inclusion of the quark contribution, and (iv) allowing the coupling α_S to run and depend on the local scale(s) characteristic of the vertices of the cascade.

We presented sample results to show that the structure of the k_t distribution of the gluon $f(x,k_t^2,\mu^2)$ can be significantly different from that of $h(x,k_t^2)$, down to very small values of x. There are important consequences for the description of hadron-initiated hard processes in which the k_t of the gluon is probed locally.

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APPENDIX

Equation (17) for the unintegrated gluon $f(x,k_t^2,\mu^2)$ is obtained from Eq. (12) by resumming the virtual corrections given in the third term on the right-hand side of Eq. (12). Here we show how the resummation is performed. First we note that this virtual correction term can be written as a derivative, that is,

$$\frac{\alpha_S}{2\pi}\int_0^1 dz\,zP(z)\left[\int\frac{dk_t'^2}{k_t'^2}\Theta(k_t^2-k_t'^2)f(x,k_t'^2,q^2)\right.$$
$$\left.\times\frac{q}{2}\,\delta\left(q-\frac{k_t}{1-z}\right)-f(x,k_t^2,q^2)\Theta\left(q-\frac{k_t}{1-z}\right)\right]$$
$$=-k_t^2\frac{\partial}{\partial k_t^2}[A(k_t,q)R(x,k_t^2,q^2)], \tag{A1}$$

where

$$R(x,k_t^2,q^2)=\int^{k_t^2}\frac{dk_t'^2}{k_t'^2}f(x,k_t'^2,q^2), \tag{A2}$$

$$A(k_t,q)=\frac{\alpha_S}{2\pi}\int_0^1 dz\,zP(z)\Theta(1-z-k_t/q). \tag{A3}$$

The function $R(x,k_t^2,q^2)$ has a simple physical meaning. It is the gluon distribution for fixed impact parameter $b\sim 1/k_t$ at scale q. Note that if $b=1/q$ then the distribution R reduces to the integrated gluon $xg(x,q^2)$. Using Eq. (A1) we see that Eq. (12) can be expressed as an integro-differential equation for R:

$$f(x,k_t^2,\mu^2)\equiv k_t^2\frac{\partial R(x,k_t^2,\mu^2)}{\partial k_t^2}$$
$$=L(x,k_t^2,\mu^2)+F(x,k_t^2)-k_t^2\frac{\partial}{\partial k_t^2}$$
$$\times\int_{k_t^2}^{\mu^2}\frac{dq^2}{q^2}A(k_t,q)R(x,k_t^2,q^2), \tag{A4}$$

where L and F are defined by Eqs. (18) and (8), respectively.

In order to solve Eq. (A4) for R we integrate both sides over dk_t^2/k_t^2 up to k_t^2 and obtain the following integral equation:

$$R(x,k_t^2,\mu^2)=xg_0(x)+\int_{q_0^2}^{k_t^2}\frac{dk_t'^2}{k_t'^2}[L(x,k_t'^2,\mu^2)+F(x,k_t'^2)]$$
$$-\int_{k_t^2}^{\mu^2}\frac{dq^2}{q^2}A(k_t,q)R(x,k_t^2,q^2). \tag{A5}$$

From Eqs. (21) and (22) we see, if $\mu=k_t$, that the first two terms on the right-hand side of Eq. (A5) are just $xg(x,k_t^2)$. The solution of Eq. (A5) may be therefore written as

$$R(x,k_t^2,\mu^2)=T_g(k_t,\mu)xg(x,k_t^2)+\int_{k_t^2}^{\mu^2}\frac{dq^2}{q^2}T_g(q,\mu)$$
$$\times q^2\frac{\partial}{\partial q^2}\left(\int_{q_0^2}^{k_t^2}\frac{dk_t'^2}{k_t'^2}L(x,k_t'^2,q^2)\right), \tag{A6}$$

where the Sudakov form factor

$$T_g(q,\mu)=\exp\left(-\int_{q^2}^{\mu^2}\frac{dp^2}{p^2}A(k_t,p)\right) \tag{A7}$$

is in agreement with Eq. (19). Equation (23) then follows from Eq. (A6) after differentiation by $\partial/\partial\ln k_t^2$.

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