U(1) symmetry and *R* parity violation

Anjan S. Joshipura, Rishikesh Vaidya, and Sudhir K. Vempati

Theoretical Physics Group, Physical Research Laboratory, Navarangpura, Ahmedabad, 380 009, India

The patterns of *R* violation resulting from the imposition of a gauged U(1) horizontal symmetry on the minimal supersymmetric standard model are systematically analyzed. We concentrate on a class of models with integer U(1) charges chosen to reproduce the quark masses and mixings as well as charged lepton masses exactly or approximately. The U(1) charges are further restricted by the requirement that very large bilinear lepton number violating terms should not be allowed in the superpotential. It is shown that this leads to severely constrained patterns of trilinear interactions. Specifically, the only choice compatible with phenomenological restrictions is the one in which all the trilinear λ'_{ijk} and all but at most two trilinear λ_{ijk} couplings vanish or are enormously suppressed. The U(1) symmetry can allow the effective generation of bilinear lepton number violating parameters through terms in the Kähler potential. The resulting models are identified and the structure of neutrino masses in some of these is briefly discussed.

I. INTRODUCTION

One of the attractive ways to understand the mysterious hierarchy among quark and lepton masses is to postulate the existence of a U(1) symmetry broken spontaneously at a scale much larger than that of weak interactions [1]. Most fermion masses and the entire Cabibbo-Kobayashi-Maskawa (CKM) matrix arise in this approach due to the breaking of the U(1) symmetry and are determined in terms of a parameter $\lambda \sim \langle \theta \rangle / M$ and the U(1) charges of the fermions. Here $\langle \theta \rangle$ determines the scale of U(1) breaking and M is some higher scale which could be the Planck scale M_P or the string scale if U(1) arises from an underlying string theory. The λ is usually identified with the Cabibbo angle ~ 0.22 and all the fermion mass matrices are represented as powers of λ. Although this mechanism is quite general, it becomes quite attractive to combine the virtues of this U(1) symmetry with that of the minimal supersymmetric standard model (MSSM) [2–8]. In this case, the U(1) can give information not only about the quark spectrum but also about the *R* parity violating couplings which can determine the neutrino masses through the pattern of *R* violation it dictates [6,7,9–11].

Lepton number violation in the MSSM is generated due to the presence of the supersymmetric partners of quarks and leptons. This can be characterized by the following *R* violating terms in the superpotential of the model:

$$W_{R_p} = \lambda'_{ijk} L_i Q_j D_k^c + \lambda_{ijk} L_i L_j E_k^c + \epsilon_i L_i H_2. \quad (1)$$

A priori, this involves 39 independent parameters. Each of these can contribute to the mass matrix for the three light neutrinos. It is desirable to restrict the number of allowed couplings from some symmetry principle and the U(1) symmetry can play a crucial role. By requiring that the U(1) charges of the MSSM field should be such that it leads to correct quark and charge lepton masses as well as the CKM matrix, one could considerably reduce the freedom in choosing the U(1) charges. A set of charges so determined would

lead to definite patterns of the *R* violating couplings appearing in Eq. (1). This in turn leads to specific structure for neutrino masses.

The purpose of this paper is to systematically search for all possible allowed patterns for the *R* violating couplings of Eq. (1) which result from U(1) charge assignments consistent with the successful predictions in the quark sector in case of integer U(1) charges for all fields. In a large class of such models [4,7,10,11], the U(1) symmetry tends to lead to very large and phenomenologically unacceptable values for the coefficient ϵ_i of the bilinear terms in Eq. (1). Requiring that this not happen restricts the allowed set of models in a stringent manner. We find a remarkable result that in all these restricted models, almost all the trilinear couplings in Eq. (1) are either zero, are highly suppressed, or their predicted magnitudes are inconsistent with phenomenology. Specifically, all the models we analyzed require zero λ'_{ijk} and at most one or two nonzero λ_{ijk} if they are to be phenomenologically consistent. The resulting theory still possesses lepton number violation since a significant amount of bilinear couplings can be generated through couplings in the Kähler potential using the mechanism proposed by Giudice and Masiero (GM) [12]. The neutrino mass patterns in this case get restricted in terms of only three or four independent lepton number violating parameters, making U(1) symmetry a very predictive scheme not only for descriptions of the quark spectrum but also for neutrino masses and mixing.

We start in the next section with a discussion of our framework and the basic assumptions and highlight the problem of the generation of large ϵ_i parameters within this framework. In the next section, we discuss the structure of trilinear interactions and their consistency with phenomenology in models which can explain the quark spectrum. Section IV contains a specific discussion of the consequences of models allowed on phenomenological grounds and we summarize the main results in the last section.

II. U(1) SYMMETRY AND THE ϵ PROBLEM

Let us consider the MSSM augmented with a gauged horizontal U(1) symmetry. The standard superfields

$(L_i, Q_i, D_i^c, U_i^c, E_i^c, H_1, H_2)$ are assumed to carry charges $(l_i, q_i, d_i, u_i, e_i, h_1, h_2)$, respectively, with i running from 1 to 3. The U(1) symmetry is assumed to be broken at a high scale by the vacuum expectation value (VEV) of one gauge singlet superfield θ with U(1) charge normalized to -1 or with two such fields $\theta, \bar{\theta}$ with charges -1 and 1, respectively. It is normally assumed that only the third generation of fermions has renormalizable couplings invariant under U(1). The rest of the couplings arise in the effective theory from higher dimensional terms [1]:

$$\Psi_i \Psi_j H \left(\frac{\theta}{M} \right)^{n_{ij}},$$

where Ψ_i is a chiral superfield, H is the Higgs doublet, M is some higher mass scale which could be the Planck scale M_p, and $n_{ij} = \psi_i + \psi_j$ are positive numbers representing the charges of Ψ_i, Ψ_j under U(1), respectively. A similar term is absent in case of a negative n_{ij} due to the holomorphic nature of W [2]. For positive n_{ij}, one gets an ijth entry of order $\lambda^{n_{ij}}$ in the mass matrix for the field Ψ. The identification $\lambda \sim 0.22$ and proper choice of U(1) charges leads to successful quark mass matrices [3–5].

A priori, the model has 15 independent U(1) charges for matter and 2 charges for Higgs fields. Of these, all but four can be fixed from different requirements discussed in the literature which we list below [5].

(1) The fermions in the third generation are assumed to have the following couplings invariant under U(1):

$$W_Y = \beta_t Q_3 U_3^c H_2 + \beta_b Q_3 D_3^c H_1 \left(\frac{\theta}{M} \right)^x + \beta_\tau L_3 E_3^c H_1 \left(\frac{\theta}{M} \right)^x.$$
(2)

This is possible if

$$q_3 + u_3 + h_2 = 0, \quad q_3 + d_3 + h_1 = l_3 + e_3 + h_1 = x. \quad (3)$$

This determines $h_2 = -q_3 - u_3$ and $h_1 = -q_3 - d_3 + x$ with $\tan\beta \sim \lambda^x (m_t/m_b)$. The phenomenological requirement of $\tan\beta \gtrsim O(1)$ implies $0 \leq x \leq 2$. b-τ unification has been implicitly assumed in writing Eq. (3).

(2) The charge differences $q_{i3} \equiv q_i - q_3$, $u_{i3} \equiv u_i - u_3$, and $d_{i3} \equiv d_i - d_3$ ($i = 1,2$) are determined by requiring that the quark masses and the CKM matrix come out to be exactly or approximately correct. Various possible values for these differences have been classified in [5] and we shall use these results.

(3) The U(1) symmetry being gauged is required to be anomaly free. It has been shown [4] that all relevant U(1) anomalies cannot be zero in models with a single θ if one is to require the correct structure for the quark and lepton masses. These anomalies then needs to be canceled by the Green-Schwarz (GS) mechanism [13]. This requirement imposes three nontrivial relations among the U(1) charges.

(4) The prediction of an approximately correct hierarchy among the charged lepton masses requires

$$l_{13} + e_{13} = 4 \quad \text{or} \quad 5, \qquad l_{23} + e_{23} = 2. \quad (4)$$

After imposing the above-listed requirements, the successful model is fixed in terms of the four independent charges. Each choice of these charges would imply different patterns for R violation. Since the U(1) is capable of predicting orders of magnitudes of various couplings, it is not guaranteed that all the patterns of R violation predicted in this way would be phenomenologically consistent. In fact very few can meet the constraints from phenomenology. The most stringent constraint on possible choice of R charges is provided by the parameters ϵ_i. The U(1) symmetry can lead to the following term in W:

$$M L_i H_2 \left(\frac{\theta}{M} \right)^{l_i + h_2}. \quad (5)$$

This leads to

$$\epsilon_i \sim M \left(\frac{\langle \theta \rangle}{M} \right)^{l_i + h_2} \sim M \lambda^{l_i + h_2}. \quad (6)$$

Unless the charges $l_i + h_2$ are appropriately chosen, the predicted value for ϵ_i can grossly conflict with (a) the scale of SU(2)×U(1) breaking which would require a sneutrino VEV $\leq O(M_W)$ and (b) neutrino masses. A bilinear parameter ϵ would imply a neutrino mass [14] of order [15]:

$$m_\nu \sim \left(\frac{\epsilon}{\mu} \right)^2 \frac{M_Z^2}{M_{SUSY}} \sin^2\phi. \quad (7)$$

Here, $\sin^2\phi$ is O(1) if supersymmetry (SUSY) breaking is not characterized by the universal boundary conditions at a high scale. In the converse case, this factor gets enormously suppressed due to the fact that ϵ_i can be rotated away from the full Lagrangian in the limit of vanishing down quark and charged lepton couplings. This issue is discussed in number of papers [16]. A typical order of magnitude estimate of $\sin^2\phi$ is [17]

$$\sin^2\phi \sim \left(\frac{3 h_b^2 \ln \frac{m_X^2}{m_Z^2}}{16\pi^2} \right)^2 \sim 10^{-7}. \quad (8)$$

These equations are very rough estimates. The exact values depend upon the MSSM parameters. But these rough estimates are sufficient to show that the phenomenologically required ϵ_i are grossly in disagreement with typical predictions, for, e.g., even with $\sin\phi^2 \sim 10^{-7}$, $m_\nu < 1$ eV would need $\epsilon \sim 1$ GeV for $\mu \sim M_{SUSY} \sim 100$ GeV.

In order to prevent very large ϵ_i from being generated, one must ensure one of the following.

(a) $l_i + h_2$ is bounded by

$$l_i + h_2 \gtrsim 24. \quad (9)$$

This can lead to ϵ_i in the GeV range and neutrinos with mass in the eV range in the case of models with universal boundary conditions and $M \sim 10^{16}$ GeV. In models without universal boundary conditions, the required magnitude for $l_i + h_2$ would be even larger.

TABLE I. We present here all the possible models which generate correct quark and lepton mass hierarchies as well as the CKM matrix.

Models	$l_{13}+e_{13}$	$l_{23}+e_{23}$	q_{13}	q_{23}	u_{13}	u_{23}	d_{13}	d_{23}
IA	4	2	3	2	5	2	1	0
IIA	4	2	4	3	4	1	1	-1
IIIA	4	2	4	3	4	1	-1	-1
IVA	4	2	-2	-3	10	7	6	5
IB	5	2	3	2	5	2	1	0
IIB	5	2	4	3	4	1	1	-1
IIIB	5	2	4	3	4	1	-1	-1
IVB	5	2	-2	-3	10	7	6	5

(b) U(1) is broken by only one superfield θ and l_i+h_2 is negative. The terms in Eq. (6) are then not allowed in W by the U(1) symmetry and by the analyticity of W.

(c) l_i+h_2 is fractional, forbidding coupling of the bilinear term to θ.

(d) Impose some additional symmetry, e.g., modular invariance which may prevent the occurrence of dangerous terms [18].

Note that models containing two θ-like fields with opposite U(1) charges would lead to large ϵ_i independent of the sign of l_i+h_2. Thus these models can be made phenomenologically consistent only by choosing fractional or unnaturally high values for $|l_i+h_2|$. We shall therefore not consider these models and concentrate only on models with a single θ and also assume only integer U(1) charges. Then ϵ_i can be suppressed either through (a) or through (b) if no other symmetry is imposed.

Although the structure of R violating interactions following from a U(1) symmetry alone has been discussed in a number of papers [4,7,9–11], the requirement that the U(1) symmetry should not generate large ϵ_i has not always been imposed [4,7,10]. It is argued customarily that ϵ_i are unphysical as they can be rotated away by redefining the new H_1 as a linear combination of the original H_1 and L_i appearing in Eq. (1). This, however, changes the original μ parameter to $(\mu^2+\epsilon_i^2)^{1/2}$. Thus, if the models do allow large ϵ_i, then rotating them away generates equally large μ which is also phenomenologically inconsistent. One must therefore allow only U(1) charge assignments corresponding to zero or suppressed ϵ_i in W.

III. STRUCTURES OF TRILINEAR COUPLINGS

In this section, we shall enumerate possible U(1) models leading to the correct quark mass spectrum and investigate structures for the trilinear couplings in these models, keeping the phenomenological constraints in mind.

After imposing Eqs. (3), the quark mass ratios and the CKM mixing angles are determined in terms of the quark charge differences. A systematic search for the possible charge differences led to the eight models [5,7] reproduced in Table I.

Model I exactly reproduces the quark mass ratios and all three CKM mixing angles. Since the predictions of U(1) symmetry are exact only up to coefficients of $O(1)$, one has to allow for models which may deviate from the exact predictions by a small amount. The charge differences in models II, III, and IV represent models which deviate from the exact predictions by $O(\lambda)$ [5]. The leptonic mixing analogous to the CKM matrix is still arbitrary in these models but the charged lepton masses are required to satisfy $m_e/m_\tau \sim \lambda^4$, $m_\mu/m_\tau \sim \lambda^2$ in models A and $m_e/m_\tau \sim \lambda^5$, $m_\mu/m_\tau \sim \lambda^2$ in models B.

The U(1) charges are still subject to the anomaly constraint. The anomalies generated due to the presence of the extra U(1) are as follows:

$$[\mathrm{SU}(3)]^2 \mathrm{U}(1)_X: \quad A_3=\sum_{i=1}^3 (2q_i+u_i+d_i),$$

$$[\mathrm{SU}(2)]^2 \mathrm{U}(1)_X: \quad A_2=\sum_{i=1}^3 (3q_i+l_i)+h_1+h_2,$$

$$[\mathrm{U}(1)_Y]^2 \mathrm{U}(1)_X: \quad A_1=\sum_{i=1}^3 \left(\frac{1}{3}q_i+\frac{8}{3}u_i+\frac{2}{3}d_i+l_i+2e_i\right)$$
$$+h_1,+h_2$$

$$\mathrm{U}(1)_Y[\mathrm{U}(1)]_X^2: \quad A_1'=\sum_{i=1}^3 (q_i^2-2u_i^2+d_i^2-l_i^2+e_i^2)$$
$$-h_1^2+h_2^2. \tag{10}$$

These can be canceled in string theory through the Green-Schwartz mechanism [13] by requiring

$$A_2=A_3=\tfrac{3}{5}A_1, \quad A_1'=0. \tag{11}$$

The above constraints on A_1,A_2,A_3 can be solved to give

$$h\equiv h_1+h_2=\sum_{i=1}^3 (q_{i3}+d_{i3})-\sum_{i=1}^3 (l_{i3}+e_{i3}),$$

$$l_2=m-(l_1+l_3+9q_3+4h-3x), \tag{12}$$

where

$$m=\sum_{i=1}^3 (u_{i3}+d_{i3}-q_{i3}). \tag{13}$$

Also, from Eqs. (3),

$$u_3=x-2q_3-d_3-h. \tag{14}$$

Note that the parameter h determines whether the μ term is allowed in W. Positive h will result in too large μ unless h is also correspondingly large.[1] Negative h does not allow the μ term in W but a phenomenologically consistent value can be

---

[1]See, however, Ref. [18] which imposes additional modular invariance.

generated through GM mechanism in this case. $h=0$ allows arbitrary μ in W. The anomaly constraint determines h completely in terms of the charge differences fixed by the models in Table I and is insensitive to the overall redefinition of the $U(1)$ charges. It is seen that all except model IIA lead to zero or negative h and thus are phenomenologically consistent.

The magnitudes and structure of the trilinear couplings are determined by the following equations:

$$\lambda'_{ijk} = \theta(c_i + n^d_{jk})\lambda^{c_i + n^d_{jk}},$$

$$\lambda_{ijk} = \theta(c_i + n^l_{jk})\lambda^{c_i + n^l_{jk}}, \qquad (15)$$

where $c_i = l_i + x + h_2 - h$, $n^d_{jk} = q_{j3} + d_{k3}$, $n^l_{jk} = l_{j3} + e_{k3}$ with n^d_{jk}, n^l_{jk} being completely fixed for a given model displayed in Table I. Note that some of the trilinear couplings may be zero if the corresponding exponent is negative. They may still be generated due to a nonminimal contribution to the kinetic energy term of different fields [5–7]. Such contributions do not, however, affect the order of magnitudes of those couplings which are nonzero to start with [6].

After imposing the constraints of Eqs. (11), one is still left with four independent parameters including x. One would thus expect considerable freedom in the choice of $\lambda'_{ijk}, \lambda_{ijk}$. Typically, more than one such couplings are allowed to be nonzero simultaneously in various models. Thus they lead to flavor violating transitions which are known to be enormously suppressed. It is these constraints on the product of trilinear couplings which lead to stringent restrictions on the allowed $U(1)$ charges. It turns out that the constraint following from the $K^0 - \bar{K}^0$ mass difference alone is sufficient to rule out the presence of nonzero trilinear couplings in most models. The $K^0 - \bar{K}^0$ mass difference constrains the product $\lambda'_{i12}\lambda'_{i21}$ to be $\leq 10^{-9}$ [19] for slepton masses of $O(100$ GeV). Allowing for some variation in these masses, we shall use the following conservative limit:

$$\lambda'_{i12}\lambda'_{i21} \leq \lambda^{12} \sim 1.3 \times 10^{-8}. \qquad (16)$$

We now analyze the magnitudes of the product in Eq. (16) predicted by models of Table I, when one imposes the additional requirement that the $l_i + h_2$ be negative or have the large value given in Eq. (9). These requirements result in zero or suppressed ϵ_i, respectively. But they would also lead to zero or suppressed trilinear interactions as we now discuss. Let us consider these two cases separately.

A. $l_i + h_2 \gtrsim 24$

In this case, ϵ_i are artificially forced to be small by choosing a very large value of $l_i + h_2$ as in Eq. (9). But the large value of these charges also results in an enormous suppression in the allowed magnitudes of the trilinear couplings. This is easily seen from Eqs. (15). Since h is zero or negative for all allowed models and $x \leq 2$, it follows that

$$c_i = l_i + h_2 + x - h \geq l_i + h_2 \geq 22.$$

TABLE II. Here we display the allowed models where the following constraints have been imposed: (a) the requirement of correct quark and lepton mass hierarchies as per model IA in Table I, (b) GS anomaly cancellations, (c) $f_i = l_i + h_2 \leq 0$, (d) phenomenological constraints from K^0-$\bar{K}^0$ mixing on λ'_{ijk} couplings, and (e) $|q_3, u_3, d_3, l_i| \leq 10$.

No.	x	q_3	u_3	d_3	l_1	l_2	l_3	f_1	f_2	f_3	If λ_{ijk} allowed
1	0	2	1	-5	-6	-3	-6	-9	-6	-9	No
2	0	2	1	-5	-5	-5	-5	-8	-8	-8	No
3	0	2	1	-5	-4	-7	-4	-7	-10	-7	No
4	0	2	1	-5	-3	-9	-3	-6	-12	-6	$\lambda_{132} \sim 4.8 \times 10^{-2}$
5	0	2	2	-6	-10	-4	-1	-14	-8	-5	$\lambda_{231} \sim 5.1 \times 10^{-4}$
6	0	3	2	-8	-10	-4	-10	-15	-9	-15	No
7	0	3	2	-8	-9	-6	-9	-14	-11	-14	No
8	0	3	2	-8	-8	-8	-8	-13	-13	-13	No
9	0	3	2	-8	-7	-10	-7	-12	-15	-12	No
10	2	3	2	-6	-7	-3	-8	-12	-8	-13	No
11	2	3	2	-6	-6	-5	-7	-11	-10	-12	No
12	2	3	2	-6	-5	-7	-6	-10	-12	-11	No
13	2	3	2	-6	-4	-9	-5	-9	-14	-10	No
14	2	4	3	-9	-9	-8	-10	-16	-15	-17	No
15	2	4	3	-9	-8	-10	-9	-15	-17	-16	No

It follows from Table I that the $n^{d,l}_{jk}$ are positive or small negative numbers in all models. As a consequence, all trilinear couplings are $\leq \lambda^{19} \sim 10^{-12}$ in this case. This value is too small to have any phenomenological consequence.

B. $l_i + h_2 < 0$

We shall first show that the most preferred model IA can be phenomenologically consistent in this case only when all λ'_{ijk} are zero and then generalize this result to other cases. The λ'_{ijk} are explicitly given as follows in this model:

$$\lambda'_{ijk} = \lambda^{l_i + h_2 + x} \begin{bmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{bmatrix}, \qquad (17)$$

where it is implicit that some element is zero if the corresponding exponent is negative [2]. The matrix in the above equation (17) coincides with $\epsilon^{-x}(M_d)_{jk}$. Hence, for negative $l_i + h_2$, it follows that λ'_{ijk} is either larger than the matrix element $(M_d)_{jk}$ or is zero for every i. In the former case, one cannot easily meet the phenomenological requirement in Eq. (16). Specifically, the equation for c_i gets translated to

$$c_i \equiv l_i + h_2 + x < -3 \quad \text{or} \quad \geq 3. \qquad (18)$$

This condition ensures that $\lambda'_{i12}\lambda'_{i21}$ either satisfies Eq. (16) (when $c_i > 3$) or is identically zero when $c_i < -3$. But $c_i \geq 3$ is untenable since $l_i + h_2 \leq 0$ and $\tan\beta \sim \lambda^x(m_t/m_b) \geq O(1)$ needs $x \leq 2$ leading to $c_i \leq 2$. As a result one must restrict c_i to less than -3 for *all* i. It can be easily seen that $c_i = -4$ is also ruled out. As follows from Eq. (17), all λ'_{ijk}

TABLE III. Same as Table II, but for values given by model IB.

No.	x	q_3	u_3	d_3	l_1	l_2	l_3	f_1	f_2	f_3	If λ_{ijk} allowed
1	0	2	2	-5	-6	-3	-2	-10	-7	-6	$\lambda_{131} \sim 1.0$, $\lambda_{231} \sim 10^{-2}$
2	0	3	2	-7	-4	-6	-10	-9	-11	-15	No
3	1	3	2	-6	-3	-5	-9	-8	-10	-14	No
4	0	3	3	-8	-10	-1	-9	-16	-7	-15	$\lambda_{231} \sim 1.0$
5	0	3	3	-8	-8	-6	-6	-14	-12	-12	No
6	1	3	3	-7	-8	-4	-5	-14	-10	-11	$\lambda_{231} \sim 1.0$
7	1	3	3	-7	-6	-9	-2	-12	-15	-8	No
8	2	3	3	-6	-8	-2	-4	-14	-8	-10	$\lambda_{121} \sim 1.0$, $\lambda_{231} \sim 2.3 \times 10^{-3}$
9	1	4	4	-10	-10	-7	-9	-18	-15	-17	No
10	2	4	4	-9	-10	-5	-8	-18	-13	-16	No
11	2	4	4	-9	-8	-10	-5	-16	-18	-13	No

except λ'_{i11} are zero in this case to start with. But the mixing of superfields in kinetic terms can regenerate other λ'_{ijk}. Specifically, one gets

$$\lambda'_{i12} = V^D_{12} \lambda'_{i11} \sim \lambda,$$

$$\lambda'_{i21} = V^Q_{12} \lambda'_{i11} \sim \lambda,$$

$$\lambda'_{i12} \lambda'_{i21} \sim \lambda^2, \qquad (19)$$

where V^ψ rotates the matter field Ψ_i to bring kinetic terms to canonical form [6]:

$$\Psi_i \rightarrow V^\psi_{ij} \Psi_j,$$

$$V^\psi_{ij} \sim \left(\frac{\langle \theta \rangle}{M} \right)^{|\psi_i - \psi_j|}. \qquad (20)$$

It follows from the above that one must require $c_i < -4$ for all i. One concludes from Eq. (17) that the only phenomenologically viable possibility in model IA is to require vanishing λ'_{ijk} for all values of i,j,k. We emphasize that a nontrivial role is played in the above argument by the requirement of zero or negative $l_i + h_2$ and by the value of h determined from the anomaly constraints.

The above argument also serves to restrict the trilinear couplings λ_{ijk}. Defining the antisymmetric matrices $(\Lambda_k)_{ij} \equiv \lambda_{ijk}$, one could rewrite Λ_k as follows:

$$(\Lambda_1)_{ij} = \lambda^4 \begin{pmatrix} 0 & \lambda^{c_2} & \lambda^{c_3} \\ -\lambda^{c_2} & 0 & \lambda^{c_3+l_2-l_1} \\ -\lambda^{c_3} & -\lambda^{c_3+l_2-l_1} & 0 \end{pmatrix},$$

$$(\Lambda_2)_{ij} = \lambda^2 \begin{pmatrix} 0 & \lambda^{c_1} & \lambda^{c_3+l_1-l_2} \\ -\lambda^{c_1} & 0 & \lambda^{c_3} \\ -\lambda^{c_3+l_1-l_2} & -\lambda^{c_3} & 0 \end{pmatrix},$$

$$(\Lambda_3)_{ij} = \begin{pmatrix} 0 & \lambda^{c_2+l_1-l_3} & \lambda^{c_1} \\ -\lambda^{c_2+l_1-l_3} & 0 & \lambda^{c_2} \\ -\lambda^{c_1} & -\lambda^{c_2} & 0 \end{pmatrix}, \qquad (21)$$

where c_i are the same coefficients defined in the context of λ' and are required to be < -4 as argued above. It then immediately follows from Table I that all λ_{ijk} except $\lambda_{123}, \lambda_{231}$, and λ_{312} are forced to be zero. Moreover, λ_{312} and λ_{231} cannot simultaneously be zero. Thus one reaches an important conclusion that model IA can be consistent with phenomenology only if all λ'_{ijk} and all λ_{ijk} except at most two are zero. We have not made use of one of the anomaly equations, namely, $A'_1 = 0$. Use of this does not allow even one λ_{ijk} to be nonzero in a large number of models.

Essentially the same argument can be repeated also in the case of other models. The structure of λ'_{ijk} is determined in these models by

$$\lambda'_{ijk} \sim \lambda^{c_i + q_{j3} + d_{k3}}, \qquad (22)$$

where $c_i \equiv l_i + h_2 + x - h$; The main difference compared to an earlier model is that the h appearing in c_i is not forced to be zero but is given by Eq. (12) and can take values -1 (model IB, model IIIA, model IVB) or -2 (model IIIB). The $h = 0$ for model IIB and the above argument made in the case of model IA also remain valid in this case. Because $h \leq 0$ in these models, they allow somewhat larger values for c_i compared to $c_i \leq 2$ in the case of model IA. These larger values of c_i result in an extreme case corresponding to $l_i + h_2 = 0$ and $x = 2$. It is possible to satisfy the constraint coming from Δm_K in these extreme cases; e.g., for model IB, $l_i + h_2 = 0$, $x = 2$ lead[2] to

$$(\lambda'_i)_{jk} \approx \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^4 & \lambda^3 & \lambda^3 \end{pmatrix}. \qquad (23)$$

This structure is consistent with Eq. (16) as well as all other constraints on λ'_{ijk}. This possibility cannot be therefore ruled out purely on phenomenological grounds. But as we will show, $A'_1 = 0$ plays an important role and does not allow these marginal cases.

---

[2] Similar marginal cases are also found for models IIIB,IVB.

TABLE IV. Same as Table II, but for values given by model IIB.

No.	x	q_3	u_3	d_3	l_1	l_2	l_3	f_1	f_2	f_3	If λ_{ijk} allowed
1	0	2	2	-6	-3	-8	-9	-7	-12	-13	No
2	0	2	3	-7	-8	-5	-7	-13	-10	-12	No
3	0	2	3	-7	-6	-10	-4	-11	-15	-9	No
4	1	2	3	-6	-8	-2	-7	-13	-7	-12	$\lambda_{231} \sim 1.0$
5	1	2	3	-6	-6	-7	-4	-11	-12	-9	No
6	2	2	3	-5	-6	-4	-4	-11	-9	-9	$\lambda_{231} \sim 1.0$
7	1	3	4	-9	-9	-10	-7	-16	-17	-14	No
8	2	3	4	-8	-9	-7	-7	-16	-14	-14	No

TABLE V. Same as Table II, but for values given by model IIIA.

No.	x	q_3	u_3	d_3	l_1	l_2	l_3	f_1	f_2	f_3	If λ_{ijk} allowed
1	0	2	3	-6	-7	-2	-9	-12	-7	-14	No
2	0	2	3	-6	-6	-4	-8	-11	-9	-13	No
3	0	2	3	-6	-5	-6	-7	-10	-11	-12	No
4	0	2	3	-6	-4	-8	-6	-9	-13	-11	No
5	0	2	3	-6	-3	-10	-5	-8	-15	-10	$\lambda_{132} \sim 1.0$
6	1	2	3	-5	-6	-2	-7	-11	-7	-12	No
7	1	2	3	-5	-5	-4	-6	-10	-9	-11	No
8	1	2	3	-5	-4	-6	-5	-9	-11	-10	No
9	1	2	3	-5	-3	-8	-4	-18	-13	-9	$\lambda_{132} \sim 1.0$
10	1	2	3	-5	-2	-10	-3	-7	-15	-8	$\lambda_{132} \sim 2.3 \times 10^{-3}$
11	2	2	3	-4	-4	-4	-4	-9	-9	-9	No
12	2	2	3	-4	-3	-6	-3	-8	-11	-8	$\lambda_{132} \sim 1.0$

IV. MODELS

Let us now discuss specific models which successfully meet all the phenomenological constraints. An important role is played in categorizing these models by the anomaly constraint $A_1' = 0$ which has been not yet imposed. Imposition of this further constrains the model.

It is possible to give a general solution of all anomaly constraints for all models listed in Table I. We outline the solution for the $A_1' = 0$ condition in the Appendix. We have numerically looked for integer solutions of the anomaly constraints satisfying the criteria (1) $l_i + h_2 \lesssim 0$, (2) c_i are chosen to satisfy the constraint Eq. (16), e.g., $c_i < -4$ in the case of model IA, and (3) the absolute values of $q_3, u_3, d_3, l_1, l_2, l_3$ are restricted to be less than or equal to 10. The last requirement is imposed for simplicity. Moreover, in practice, higher values of these charges will generically result in suppressed R violating couplings which may not be of phenomenological interest. Although all U(1) couplings can be specified using only four parameters, we have displayed values of x, q_3, u_3, d_3, l_i, and $l_i + h_2$ in Tables II–VIII. We draw the following conclusions from the tables.

(1) None of the models displayed allow the value $l_i + h_2 = 0$, ruling out the marginal models displayed in Eqs. (23) at least for the ranges of parameters considered here.

(2) While all λ_{ijk}' are forced to be zero, some of the models allow one or two nonzero λ_{ijk}. We have shown this in the last column which also gives the order of magnitude for the allowed λ_{ijk}. This need not always be compatible with phenomenology particularly after taking care of the mixing of kinetic energy terms. Thus some of the models displayed in the tables would not be allowed.

(3) Although the term $L_i H_2$ is not directly allowed, it can be generated from the Kahler potential through the mechanism proposed by GM [12] in order to explain the μ parameter. The order of magnitudes of ϵ_i is given in this case by

$$\epsilon_i \sim m_{3/2} \lambda^{|l_i + h_2|}, \qquad (24)$$

where $m_{3/2}$ is the gravitino mass. This can be read off from the tables in all cases. The uniformly large magnitudes of $l_i + h_2$ found in the tables imply that R violation through the effective bilinear term is also quite suppressed but it can still be of phenomenological relevance.

(4) We did not impose baryon parity in the above analysis. A look at the solutions presented in the tables, however, shows that the operator $U_i^c D_j^c D_k^c$ carries large negative charge in all models. Thus baryon number violating terms are automatically forbidden from the superpotential. These terms will be generated from the effective U(1) violating D term

$$\frac{1}{M_P} \left(\frac{\theta^*}{M} \right)^{|q_{ijk}|} (U_i^c D_j^c D_k^c),$$

where q_{ijk} is the negative U(1) charge of the combination $U_i^c D_j^c D_k^c$. This leads to baryon number violating couplings

$$\lambda_{ijk}'' \sim \frac{m_{3/2}}{M_P} \lambda^{|q_{ijk}|},$$

which are extremely suppressed, $\leq O(10^{-15})$ for $m_{3/2} \sim 1$ TeV. Thus proton stability gets automatically explained in all the models.

(5) Trilinear lepton number violating terms are not allowed in the superpotential from analyticity. But they will be effectively generated in the same way as λ'' discussed above. Their magnitudes will also be enormously suppressed, $\leq 10^{-15}$, depending upon the model.

It follows from the forgoing discussions that consistently implemented U(1) symmetry allows very simple R violating interactions, namely, three bilinear terms and at most two trilinear couplings λ_{ijk}. The constraints coming from the $K^0 - \bar{K}^0$ mass difference were instrumental in arriving at this conclusion. It is worth emphasizing that the effective bilinear interactions generated from the GM mechanism in this case are not subject to such stringent constraint from the flavor violating process. A priori, the bilinear terms can be rotated away in favor of trilinear λ' and λ interactions. It turns out that one does not generate dangerous flavor violating terms in the process. Specifically, one finds, for the flavor structure [17],

TABLE VI. Same as Table II, but for values given by model IIIB.

No.	x	q_3	u_3	d_3	l_1	l_2	l_3	f_1	f_2	f_3	If λ_{ijk} allowed
1	0	2	3	-5	-2	-5	-7	-7	-10	-12	No
2	0	2	4	-6	-7	-3	-4	-13	-9	-10	$\lambda_{231} \sim 0.22$
3	0	2	4	-6	-5	-8	-1	-11	-14	-7	$\lambda_{131} \sim 1.0$, $\lambda_{132} \sim 1.0$
4	2	3	4	-6	-3	-4	-10	-10	-11	-17	$\lambda_{123} \sim 1.0$
5	0	3	5	-9	-8	-6	-9	-16	-14	-17	No
6	1	3	5	-8	-9	-3	-8	-17	-11	-16	No
7	1	3	5	-8	-7	-8	-5	-15	-16	-13	No
8	2	3	5	-7	-8	-5	-4	-16	-13	-12	$\lambda_{231} \sim 1.0$
9	2	3	5	-7	-6	-10	-1	-14	-18	-9	$\lambda_{131} \sim 1.0$, $\lambda_{132} \sim 0.22$
10	2	4	6	-10	-9	-8	-9	-19	-18	-19	No

$$W = -\frac{\tan\theta_3}{\langle H_1 \rangle}[(O_L^T)_{3\alpha}L_\alpha](m_\beta^l L_\beta e_\beta^c + m_i^D Q_i d_i^c), \quad (25)$$

where all the fields are in the physical, i.e., the mass, basis. (O_L^T) represents a mixing matrix determined solely by the ratios of ϵ_i and $\tan\theta_3 = \sqrt{(\Sigma_i \epsilon_i^2)}/\mu$ and α, β run over e, μ, τ. It is seen that the resulting trilinear interactions are flavor diagonal and thus the parameters ϵ_i are not severely constrained.[3] The major effect of the bilinear terms is to generate the neutrino masses and leptonic Kobayashi-Maskawa matrix.

The neutrino masses in the presence of bilinear terms alone have been discussed in many papers [16]. A large number of these concentrated on universal boundary conditions since they provide a natural means to understand the smallness of neutrino masses even when the bilinear parameters are not suppressed [16,17]. The soft SUSY breaking terms are also subject to U(1) symmetry and need not follow a universal structure [18]. But the smallness of the neutrino masses follows here from the U(1) symmetry itself without invoking universal boundary conditions since the allowed values of $|l_i + h_2|$ in various tables are large, leading to suppressed ϵ/μ and hence neutrino masses, Eq. (7). The detailed structure of neutrino masses and mixing will be more model dependent here than in the case of universal boundary conditions. It seems possible to obtain a reasonable mixing and masses in some of the models. As an example, consider model 2 in Table III. This is characterized by three bilinear terms of equal magnitudes. Thus in the absence of any fine-tuning one can expect to get large mixing angles naturally. The heaviest neutrino would have mass of the order of

$$m_\nu \sim \lambda^{18}\frac{M_Z^2}{M_{SUSY}} \sim 10^{-1} \text{ eV},$$

which is in the right range for solving the atmospheric neutrino anomaly. The other mass gets generated radiatively through Eq. (25) and would be suppressed compared to the above mass. The detailed predictions of the neutrino spectrum would depend upon the structures of soft symmetry breaking terms which themselves would be determined by the U(1) symmetry. We shall not discuss it here.

V. SUMMARY

The supersymmetric standard model allows 39 lepton number violating parameters which are not constrained theoretically. We have shown in this paper that the U(1) symmetry invoked to understand fermion masses can play an important role in constraining these parameters. We restricted ourselves to integer U(1) charges and considered different U(1) charge assignments compatible with the fermion spectrum. We have shown that the only phenomenologically consistent possibility in this context is that all the trilinear λ'_{ijk} and all but two λ_{ijk} couplings are zero or extremely small of $O(10^{-15})$. While the patterns of R violation have been earlier discussed in the presence of U(1) symmetry the systematic confrontation of these patterns with phenomenology leading to this important conclusion was not made to the best of our knowledge. In fact, some works [11] which neglected the important constraint of $l_i + h_2 \lesssim 0$ concluded to the contrary that it is possible to obtain phenomenologically consistent and nonzero trilinear couplings.

Our work is restricted to only U(1) symmetry which is by far most popular and to integer U(1) charges. Use of other horizontal symmetries can allow nonzero trilinear interactions and still be consistent with phenomenology. An example of this can be found in [20]. Our work is closely related to and complements the analysis presented in [9]. It

TABLE VII. Same as Table II, but for values given by model IVA.

No.	x	q_3	u_3	d_3	l_1	l_2	l_3	f_1	f_2	f_3	If λ_{ijk} allowed
1	0	6	-3	-9	-10	-4	-7	-13	-7	-10	$\lambda_{231} \sim 1.0$
2	0	6	-3	-9	-9	-6	-6	-12	-9	-9	No
3	0	6	-3	-9	-8	-8	-5	-11	-11	-8	No
4	0	6	-3	-9	-7	-10	-4	-10	-13	-7	No
5	2	7	-2	-10	-8	-6	-10	-13	-11	-15	No
6	2	7	-2	-10	-7	-8	-9	-12	-13	-14	No
7	2	7	-2	-10	-6	-10	-8	-11	-15	-13	No

_____

[3]The same conclusion was also drawn in Ref. [5] by using a different leptonic basis.

TABLE VIII. Same as Table II, but for values given by model IVB.

No.	x	q_3	u_3	d_3	l_1	l_2	l_3	f_1	f_2	f_3	If λ_{ijk} allowed
1	0	6	-2	-9	-8	-5	-4	-12	-9	-8	$\lambda_{231} \sim 0.22$
2	2	7	-1	-10	-8	-5	-7	-14	-11	-13	No
3	2	7	-1	-10	-6	-10	-4	-12	-16	-10	No

was assumed in this paper that bilinear R violating interactions come from the GM mechanism and are absent in the superpotential. Assuming that there are no trilinear interactions in the superpotential it was shown that flavor violating transitions in the model are adequately suppressed. We have systematically shown that this is the only allowed possibility except for the occurrence of one or two trilinear λ_{ijk} couplings. This way, U(1) symmetry is shown to require that only 4 or 5 of the total 39 lepton number violating couplings could have magnitudes in the phenomenologically interesting range.

APPENDIX

Here we give the most general solutions for the Green-Schwarz anomaly conditions in terms of the four independent charges. The constraints $A_3 = A_2$ and $A_3 = \frac{3}{5}A_1$ give us Eq. (12). The condition $A'_1 = 0$ can be solved to give

$$l_3 = Ad_3 + Bq_3 + Cl_1 + Dx + E, \tag{A1}$$

where

$$A = \frac{-1}{k_2}\left(\sum_i (d_{i3} + 2u_{i3}) - h + k_1 + k_2 - m + 3x\right),$$

$$B = \frac{-1}{k_2}\left(\sum_i (q_{i3} + 4u_{i3}) - 7h + k_1 + 10k_2 - m + 9x\right),$$

$$C = \frac{-1}{k_2}(k_2 - k_1),$$

$$D = \frac{-1}{k_2}\left(5h - 4\sum_i (u_{i3}) - 3(k_2 + x)\right),$$

$$E = \left(\sum_i (d_{i3}^2 + q_{i3}^2 - 2u_{i3}^2 + k_i^2) - 5h^2 + 2k_2(4h - m)\right) \tag{A2}$$

and

$$k_1 = l_{13} + e_{13},$$

$$k_2 = l_{23} + e_{23}. \tag{A3}$$

In the above we have taken q_3, d_3, l_1, and x as four independent parameters and l_3 has been expressed in terms of them. m and u_3 are, respectively, given by Eqs. (13),(14) of the text and remaining charges by Table I defining the models. This way all U(1) charges get fixed in terms of q_3, d_3, l_1 and x once a model displayed in the table is chosen.

[1] C. D. Frogatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979).

[2] M. Leurer, Y. Nir, and N. Seiberg, Nucl. Phys. **B398**, 319 (1993); **B420**, 468 (1994).

[3] L. E. Ibanez and G. G. Ross, Phys. Lett. B **332**, 100 (1994).

[4] P. Binetruy and P. Ramond, Phys. Lett. B **350**, 49 (1995).

[5] E. Dudas, S. Pokorski, and C. A. Savoy, Phys. Lett. B **356**, 45 (1995).

[6] P. Binetruy, S. Lavignac, and P. Ramond, Nucl. Phys. **B477**, 353 (1996).

[7] E. J. Chun and A. Lukas, Phys. Lett. B **387**, 99 (1996); K. Choi, E. J. Chun, and K. Hwang, Phys. Rev. D **60**, 031301 (1999).

[8] Y. Nir, Phys. Lett. B **354**, 107 (1995); V. Jain and R. Shrock, hep-ph/9507238; E. J. Chun, Phys. Lett. B **367**, 226 (1996); K. Choi, E. J. Chun, and H. Kim, *ibid.* **394**, 89 (1997). Also see J. M. Mira, E. Nardi, and D. A. Restrepo, Phys. Rev. D **62**, 016002 (2000).

[9] P. Binetruy, E. Dudas, S. Lavignac, and C. A. Savoy, Phys. Lett. B **422**, 171 (1998).

[10] J. Ellis, S. Lola, and G. G. Ross, Nucl. Phys. **B526**, 115 (1998).

[11] R. Barbier *et al.*, ''Report of the group on R-parity violation,'' hep-ph/9810232.

[12] G. F. Giudice and A. Masiero, Phys. Lett. B **206**, 480 (1988).

[13] M. B. Green and J. H. Schwarz, Phys. Lett. **149B**, 117 (1984); L. E. Ibanez, Phys. Lett. B **303**, 55 (1993).

[14] L. J. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984).

[15] A. S. Joshipura and M. Nowakowski, Phys. Rev. D **51**, 2421 (1995).

[16] See, for example, M. Hirsch *et al.*, Phys. Rev. D (to be published), hep-ph/0004115 and references therein.

[17] A. S. Joshipura and K. S. Babu (unpublished); A. S. Joshipura and S. K. Vempati, Phys. Rev. D **60**, 095009 (1999).

[18] E. Dudas, C. Grojean, S. Pokorski, and C. A. Savoy, Nucl. Phys. **B481**, 85 (1996).

[19] For a review see G. Bhattacharyya, hep-ph/9709395; B. Allanach, A. Dedes, and H. Dreiner, Phys. Rev. D **60**, 075014 (1999).

[20] T. Banks, Y. Grossman, E. Nardi, and Y. Nir, Phys. Rev. D **52**, 5319 (1995).