Measuring $|V_{ub}|$ with $B\to D_s^+X_u$ transitions

R. Aleksan and M. Zito
Commissariat à l'Energie Atomique, Saclay, DSM/DAPNIA/SPP, 91191 Gif-sur-Yvette Cedex, France

A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal
Laboratoire de Physique Théorique, Université de Paris XI, Bât. 211, 91405 Orsay Cedex, France
(Received 22 July 1999; revised manuscript received 13 April 2000; published 11 October 2000)

We propose the determination of the CKM matrix element $|V_{ub}|$ by the measurement of the spectrum of $B\to D_s^+X_u$, dominated by the spectator quark model mechanism $\overline{b}\to D_s^{(*)+}\overline{u}$. The interest in considering $B\to D_s^+X_u$ versus the semileptonic decay is that more than 50% of the spectrum for $B\to D_s^+X_u$ occurs above the kinematical limit for $B\to D_s^+X_c$, while most of the spectrum $B\to l\nu X_u$ occurs below the $B\to l\nu X_c$ one. Furthermore, the measure of the hadronic mass M_X is easier in the presence of an identified D_s than when a ν has been produced. As a consistency check, we point out that the rate $\overline{b}\to D_s^{(*)+}\overline{c}$ (including QCD corrections) is consistent with the measured BR$(B\to D_s^\pm X)$ by CLEO. Although the hadronic complications may be more severe in the decay mode that we propose than in the semileptonic inclusive decay, the end of the spectrum in $B\to l\nu X_u$ is not well understood on theoretical grounds. We argue that, in our case, the excited mesons D_s^{**}, decaying into DK, do not contribute and, if there is tagging of the B meson, the other mechanisms to produce a D_s of the right sign are presumably small, of $O(10^{-2})$ relative to the spectator amplitude, or can be controlled by kinematical cuts. We discuss in detail the hadronic uncertainties of the method and present an error calculation on $|V_{qb}|$ $(q=u,c)$ and on the ratio $|V_{ub}|/|V_{cb}|$, for which most of the systematic errors cancel. In the absence of tagging, other hadronic backgrounds deserve careful study. We present a feasibility study with the BaBar detector.

PACS number(s): 12.15.Hh, 13.25.Hw

I. INTRODUCTION

The determination of the strength of the transition between b and u quarks is a very important goal for understanding the sector of the theory involving flavor mixing. Indeed, the value of the element $|V_{ub}|$ in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [1] is a key ingredient which is used to determine the unitarity triangle and thus test the consistency of the standard model in the sector responsible for CP violation. It is also one of the most difficult measurements in B physics, in particular due to the large and model dependent theoretical uncertainties. The methods which have been used so far to extract $|V_{ub}|$ involve semileptonic B decays. The first method uses the inclusive lepton spectrum above the kinematical limit for $b\to c$ transitions while the second technique requires the exclusive reconstruction of $B\to\pi l\nu$ or $\rho l\nu$. The errors in the first case are due to the fact that only a tiny fraction of the lepton energy spectrum from $b\to ul\nu$ is observed, that parton model evaluation is questionable in this kinematical region and that a large model dependent extrapolation is necessary to extract the total rate. An improvement based on studying the hadronic mass spectrum increases the signal but is not free of problems related to the $b\to c$ background [2]. In the second case, the uncertainties are mainly due to the limited statistics and the theoretical uncertainty in the form factors for the $B\to\pi$ and $B\to\rho$ transitions.

We would like in the following to propose a new approach to measure $|V_{ub}|$ which involves inclusive $B\to D_s^+$ transitions where we make use as much as possible of experimentally measured parameters in order to reduce the uncertainties. In these decays the D_s meson is essentially produced via the virtual W emitted by the b quark (see Fig. 1). We shall discuss later the other possibilities to produce a D_s meson and make a preliminary survey of the backgrounds and hadronic uncertainties of our method to measure $|V_{ub}|$. The $b\to u$ transitions are identified by requiring the momentum of the D_s meson to be in the range above the kinematical limit for the decay $B\to D_s^+\overline{D}$ (i.e., $\sim$1.82 GeV in the B meson center of mass) and up to 2.27 GeV corresponding to the transition $B\to D_s^+\pi$. It is very important to note here that in contrast to the inclusive semileptonic case this range includes the majority of the $\overline{b}\to D_s^+\overline{u}$ transitions and therefore a smaller extrapolation is needed to obtain the total rate. Of course, a drawback of this new method is that, since it concerns purely hadronic transitions, it is subject to other hadronic uncertainties than the semileptonic end spectrum $B\to l\nu X_u$. After calculating the inclusive rate for $B\to D_s^+X_q$ we discuss how $|V_{ub}|$ is extracted and then enumerate and try to estimate the uncertainties in Sec. III. Various sources of background are studied and rejection methods are proposed in Sec. IV for tagged events and in Sec. V for untagged events. Finally, in Sec. VI we present a feasibility study for the BaBar detector, and in Sec. VII we conclude.

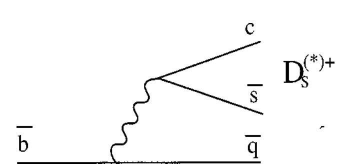

FIG. 1. Spectator diagram for the decay $\overline{b}\to D_s^+\overline{q}$.

When this paper was finished, we noticed that other methods to measure V_{ub} have been proposed using channels that involve also the $(\overline{s}c)(\overline{u}b)$ weak coupling. Namely, the totally *inclusive B* decays through $b\to \overline{c}su$ have been proposed [3] or rare exclusive decays of the type $B^+\to D_s^+\gamma$ [4]. However, although the weak coupling is the same, these methods do not overlap with the proposition of our paper to measure $|V_{ub}|$.

II. THE $B\to D_s^+X_q$ RATE

The inclusive decay rate of a B meson decaying into a D_s^+ meson is obtained using the spectator quark model by writing

$$\Gamma(B\to D_s^+X_q)\simeq\Gamma(\overline{b}\to D_s^+\overline{q})+\Gamma(\overline{b}\to D_s^{*+}\overline{q}), \tag{1}$$

where $\overline{q}$ is the outgoing quark as shown in Fig. 1 (other diagrams exists and will be discussed later). One should note that decays to the lowest P wave D_s^{**} states do not lead to D_s mesons since their main decays are $D_s^{**}\to D^{(*)}K$.

Extending the standard vacuum insertion approximation, successful in exclusive decays, the effective matrix element used for the weak decay $\overline{b}\to D_s^{(*)+}\overline{q}$ reads

$$\langle D_s^{(*)+}\overline{q}|\mathcal{H}_{\text{eff}}|\overline{b}\rangle=\frac{G_F}{\sqrt{2}}a_1V_{qb}^*V_{cs}\langle D_s^{(*)+}|A^\mu(V^\mu)|0\rangle \times\langle\overline{q}|J_{\mu q}|\overline{b}\rangle, \tag{2}$$

where G_F is the Fermi constant, V_{ij} are the Cabibbo-Kobayashi-Maskawa matrix elements, and

$$a_1=c_1+\frac{c_2}{N_c} \tag{3}$$

is a combination of short distance QCD factors, and the current $J_{\mu q}$ reads

$$J_{\mu q}=\overline{q}\gamma_\mu(1-\gamma_5)b. \tag{4}$$

We have, for the emission of a pseudoscalar

$$\langle D_s^+|A^\mu|0\rangle=-if_Pp_P^\mu \tag{5}$$

and for the emission of a vector meson

$$\langle D_s^{*+}|V^\mu|0\rangle=m_Vf_V\epsilon_V^{*\mu}. \tag{6}$$

Here ϵ^* is the polarization quadrivector of the meson. In Eq. (3), the Wilson coefficients are [5]

$$c_1=\frac{c_++c_-}{2}\quad\text{and}\quad c_2=\frac{c_+-c_-}{2},\qquad c_\pm=\left[\frac{\alpha_s(\mu)}{\alpha_s(m_W)}\right]^{d_\pm}, \tag{7}$$

where $d_+=-6/23$ and $d_-=12/23$. In writing Eq. (2), factorization has been assumed. This assumption is justified since the diagram involved here (Fig. 1) is the spectator diagram with external emission of the W. Indeed no internal emission diagram nor penguin diagrams exist. Factorization is so far consistent with the experimental data in exclusive decays where only Fig. 1 type diagrams are involved and the parameter $|a_1|$ has been extracted using a combined fit of several measured modes, and is consistent within errors with $|a_1|\cong 1$. We discuss this important parameter in Sec. IV. Our approach relies on the hypothesis of quark-hadron duality in nonleptonic decays [7,8].

TABLE I. Fraction of tranverse to longitudinal polarized D_s^* mesons in inclusive decays.

	$\overline{b}\to D_s^{*+}\overline{c}$	$\overline{b}\to D_s^{*+}\overline{u}$
Γ_T/Γ_L	$\sim 1/2$	$\sim 1/3$

The width of the inclusive $\overline{b}\to D_s^+\overline{q}$ is calculated easily by evaluating the diagram in Fig. 1 using Eq. (2). One finds (an earlier calculation was performed by Palmer and Stech [6]):

$$\Gamma^{(0)}(\overline{b}\to D_s^+\overline{q})=\frac{G_F^2}{8\pi}|V_{qb}^*V_{cs}|^2f_{D_s}^2\frac{(m_b^2-m_q^2)^2}{m_b^2}\times\left(1-\frac{m_{D_s}^2(m_b^2+m_q^2)}{(m_b^2-m_q^2)^2}\right)p_{D_s}a_1^2, \tag{8}$$

where $p_{D_s}=\sqrt{[m_b^2-(m_{D_s}+m_q)^2][m_b^2-(m_{D_s}-m_q)^2]}/2m_b$ is the momentum of the outgoing D_s meson in the b rest frame, G_F is the Fermi constant, and f_{D_s} is the D_s decay constant. The notation $\Gamma^{(0)}$ is used for the width without including the radiative corrections. A similar formula is obtained for $\Gamma^{(0)}[\overline{b}\to D_s^{*+}(\lambda=0)\overline{q}]$ where the D_s^{*+} is longitudinally polarized by replacing, in Eq. (8), f_{D_s} by $f_{D_s^*}$, m_{D_s} by $m_{D_s^*}$. For the transverse polarization $(\lambda=\pm 1)$ we find

$$\Gamma^{(0)}[\overline{b}\to D_s^{*+}(\lambda=\pm 1)\overline{q}]=\frac{G_F^2}{4\pi}|V_{qb}^*V_{cs}|^2f_{D_s^*}^2m_{D_s^*}^2\times\frac{m_b^2+m_q^2}{m_b^2}\left(1-\frac{m_{D_s^*}^2}{m_b^2+m_q^2}\right)p_{D_s^*}a_1^2. \tag{9}$$

It is interesting to note that neglecting $m_{D_s^*}^2$ compared to m_b^2 and for $m_q^2\ll m_b^2$, one obtains

$$\frac{\Gamma_T}{\Gamma_L}=\frac{\Gamma^{(0)}(\overline{b}\to D_s^{*+}(\lambda=\pm 1)\overline{q})}{\Gamma^{(0)}(\overline{b}\to D_s^{*+}(\lambda=0)\overline{q})}\simeq\frac{2m_{D_s^*}^2}{m_b^2-4m_q^2} \tag{10}$$

and therefore transverse polarizations are suppressed.

As an illustration, Table I shows the expected order of magnitude of the ratio Γ_T/Γ_L for $\overline{b}\to D_s^{*+}\overline{c}$ and $\overline{b}\to D_s^{*+}\overline{u}$ transitions. Experimental verifications of Table I would be useful and would give further confidence in the method proposed here. Adding both longitudinal and transverse polarizations, one has

$$\Gamma^{(0)}(\overline{b}\to D_s^{*+}\overline{q})=\frac{G_F^2}{8\pi}|V_{qb}^*V_{cs}|^2 f_{D_s^*}^2\frac{(m_b^2-m_q^2)^2}{m_b^2}\times\left(1+\frac{m_{D_s^*}^2(m_b^2+m_q^2-2m_{D_s^*}^2)}{(m_b^2-m_q^2)^2}\right)\times p_{D_s^*}a_1^2. \tag{11}$$

From Eqs. (3) and (7) it can be seen that the *short distance* QCD factor $a_1=1+O(\alpha_s^2)$; i.e., the correction to the tree rate is of second order in α_s. We have computed elsewhere [9] the radiative corrections to $\overline{b}\to D_s^{(*)+}\overline{u}$ at order α_s that involve vertex, self-energy and bremsstrahlung diagrams. These radiative corrections are evaluated at the order α_s in the same way as for the semileptonic decays [10], i.e., on the lower quark legs in Fig. 1. This is because the $D_s^{(*)}$ is a color singlet. We have obtained, within the on-shell renormalization scheme [9],

$$\Gamma(\overline{b}\to D_s^{(*)+}\overline{q})=\Gamma^{(0)}(\overline{b}\to D_s^{(*)+}\overline{q})\times\left[1+\frac{4}{3}\frac{\alpha_s}{\pi}\eta^{(*)}(\xi_{D_s^{(*)}},r_q)\right], \tag{12}$$

where $\xi=q^2/m_b^2$ and $r_q=m_q/m_b$, with $q^2=m_{D_s}^2$ or $m_{D_s^*}^2$. As shown in Ref. [9], in the limit $\xi\to 0$, $r_q\to 0$ one finds

$$\eta(0,0)=\eta^*(0,0)=\frac{5}{4}-\frac{\pi^2}{3}. \tag{13}$$

The functions $\eta^{(*)}(\xi,r)$ are slowly varying with r and ξ. In the Appendix we give the expressions of the radiative corrections for arbitrary ξ and $r=0$. For any ξ and r, these can be found in Ref. [9].

For the quark masses, we take pole masses from a fit to the semileptonic decay rate $b\to cl^-\overline{\nu_l}$ with QCD corrections at one loop [10], to be consistent with the same order that we compute here. The semileptonic decay rate reads, in this approximation

$$\Gamma(b\to cl^-\overline{\nu_l})=\frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 f_{PS}(r)\left[1+\frac{4}{3}\frac{\alpha_s}{\pi}f_{RC}(r)\right], \tag{14}$$

where the phase space $f_{PS}(r)$ and radiative correction $f_{RC}(r)$ functions depend on $r=m_c/m_b$ and are given in Ref. [10]. Setting $r=0.3$, we obtain, from the semileptonic branching ratio 11%,

$$m_b=4.85\ \text{GeV},\quad m_c=1.45\ \text{GeV}. \tag{15}$$

The mass difference $m_b-m_c=3.40$ GeV compares well with the value $m_b-m_c=(3.43\pm 0.04)$ GeV obtained in the $1/m_Q$ expansion of the heavy quark effective theory [11]. With this order of magnitude values $m_u\cong 0$ and $\alpha_s(m_b)=0.2$, the radiative corrections take the following values (the mass dependence is discussed in Ref. [9]) for $q=c$

$$\frac{4}{3}\frac{\alpha_s}{\pi}\eta(\xi_{D_s},r_c)=-0.095,$$

$$\frac{4}{3}\frac{\alpha_s}{\pi}\eta^*(\xi_{D_s^*},r_c)=-0.108 \tag{16}$$

and for $q=u$

$$\frac{4}{3}\frac{\alpha_s}{\pi}\eta(\xi_{D_s},0)=-0.168,$$

$$\frac{4}{3}\frac{\alpha_s}{\pi}\eta^*(\xi_{D_s^*},0)=-0.159. \tag{17}$$

Using $\tau_B=1.6$ ps and $|V_{cb}|=0.04$, one calculates, including the QCD corrections, and the decay constants (uncertainties on these numbers will be discussed below)

$$f_{D_s}\cong 230\ \text{MeV},\quad f_{D_s^*}\cong 280\ \text{MeV}, \tag{18}$$

$$\text{Br}(\overline{b}\to D_s^{(*)+}\overline{c})\simeq 8.0\% \tag{19}$$

where $\text{Br}(\overline{b}\to D_s^+\overline{c})\simeq 2.6\%$ and $\text{Br}(\overline{b}\to D_s^{*+}\overline{c})\simeq 5.4\%$ and with $|V_{ub}|/|V_{cb}|=0.08$

$$\text{Br}(\overline{b}\to D_s^{(*)+}\overline{u})\simeq 6.8\times 10^{-4}, \tag{20}$$

where $\text{Br}(\overline{b}\to D_s^+\overline{u})\simeq 2.3\times 10^{-4}$ and $\text{Br}(\overline{b}\to D_s^{*+}\overline{u})\simeq 4.5\times 10^{-4}$.

At this stage, several points should be underlined.

The sensitivity of the rate to the b quark mass goes as m_b^3 instead of m_b^5 in the case of the semileptonic decay.

The sensitivity of the decay rate with respect to the mass m_q is negligible for the light quarks. It is not dramatic for the c quarks, in particular if m_b-m_c is known to a good accuracy [Eq. (8)].

The calculated overall branching fraction for $\text{Br}(\overline{b}\to D_s^{(*)+}\overline{c})\simeq 8.0\%$ is in agreement within 1σ with the value measured by CLEO [12]:

$$\text{BR}(B\to D_s^{\pm}X)=(10.0\pm 2.5)\%. \tag{21}$$

The observed agreement is encouraging as it shows that the very simple approach at the quark level accounts rather well for the data. Equivalently, one could extract $|V_{cb}|$. Using $m_b=(5.0\pm 0.20)$ GeV/c^2 and the relative error $\sigma(f_{D_s^{(*)}})/f_{D_s^{(*)}}=0.1$, we find $|V_{cb}|=0.044\pm 0.008$.

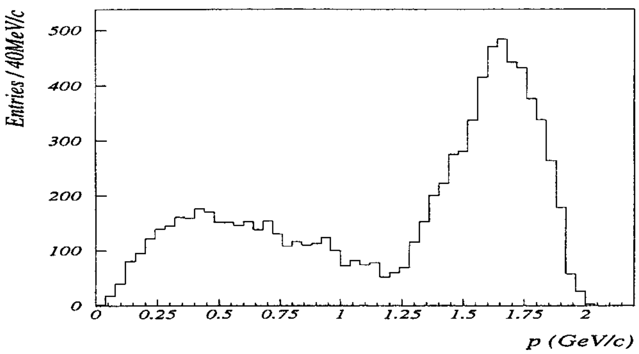

FIG. 2. Momentum spectrum for D_s^+ mesons produced from the reaction $\overline{b}\to D_s^+\overline{c}$ (i.e., upper vertex). Decays with a D^{**} meson from the lower vertex have not been included in this plot. These decays tend to fill the slight deep at 1.25 GeV/c but do not affect the end of the spectrum.

III. MEASUREMENT OF $|V_{ub}|$ USING $B\overline{B}$ PAIRS FROM $\Upsilon(4S)$ DECAYS

In a similar way than for the measurement of $|V_{cb}|$, it should be possible to determine $|V_{ub}|$ by selecting D_s mesons with momentum above the kinematical limit for $B\to D_s^+\overline{D}$. The D_s momentum in the latter case is 1.82 GeV/c in the B rest frame. However, for B pair production at the $\Upsilon(4S)$, B mesons are generated with a momentum of about 300 MeV/c and therefore the latter limit is of the order of 2.0 GeV/c as can be seen in Fig. 2. To extract $|V_{ub}|$, it is thus necessary to estimate the fraction of $B\to D_s^+X_u$ decays with $p_{D_s}>2.0$ GeV/c. We have computed the expected momentum spectrum of D_s^+ produced via the spectator diagram in Fig. 1, taking into account the b-quark Fermi motion inside the B meson using the ACCMM model [13] at tree level, neglecting for the moment the radiative corrections. This spectrum is shown in Fig. 3. The striking feature of this distribution is that the average D_s momentum is above 2.0 GeV/c with about 75% of the D_s mesons above that limit. Obviously this fraction depends on the theoretical parameter p_F and therefore we have varied p_F in the reasonable range

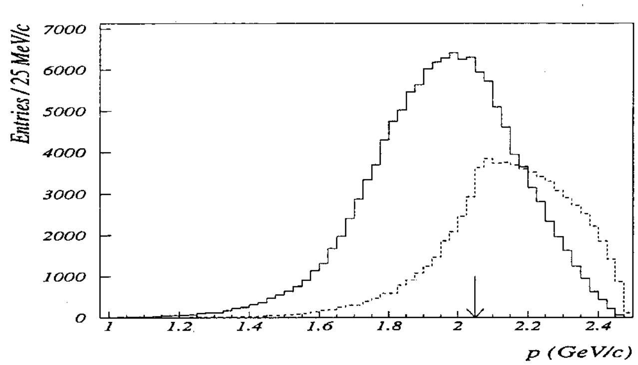

FIG. 3. Expected momentum spectrum for D_s^+ mesons produced from the reaction $\overline{b}\to D_s^+\overline{u}$ (i.e., upper vertex). The dashed line is for direct D_s^+ while the solid line is for D_s^+ mesons coming from direct D_s^{*+} decays.

TABLE II. Efficiencies (in %) for a cut on the D_s momentum at 2, 2.05, and 2.1 GeV/c for four sets of values for the parameters of the ACCMM model.

m_u m_d (MeV)	p_F (MeV)	$p_{D_s}>2$ GeV/c			$p_{D_s}>2.05$ GeV/c			$p_{D_s}>2.1$ GeV/c		
		D_s	D_s^*	all	D_s	D_s^*	all	D_s	D_s^*	all
150	300	76	36	50	65	27	39	53	19	30
10	200	89	50	63	82	39	54	71	29	43
10	300	82	44	57	74	34	48	63	25	38
10	400	74	38	51	65	30	42	55	22	33

(200 MeV/$c<p_F<$400 MeV/c) to evaluate the possible systematic uncertainties related to that parameter. Table II shows the sensitivity of the fraction of D_s for various cuts on p_{D_s}, assuming different values for p_F and the mass of the spectator quark. Should it be possible to measure the recoiling mass to the $D_s^{(*)}$, the value of the cut on p_{D_s} could be reduced, thus increasing the efficiency.

IV. HADRONIC UNCERTAINTIES: DISCUSSION OF THE FACTORIZATION HYPOTHESIS

The method that we propose, being purely hadronic, presents a number of difficulties related to the strong interactions. We will enumerate these effects in the following way:

(1) *Short distance QCD corrections.* These corrections amount to the perturbative calculation of the coefficients c_1 and c_2 (7) or the combination a_1 (3). These corrections do not break factorization by themselves, the factorization hypothesis appearing at the level of calculating the matrix elements of the operators O_1 and O_2 according to the recipe (2). The combination $a_1=c_1+c_2/N_c$ computed in this way depends on the renormalization scale μ and on the renormalisation scheme ('t Hooft-Veltman or naive dimensional regularization). The Rome [14] and Munich [15] groups have computed these coefficients beyond the leading order, obtaining, for $N_c=3$

$$a_1(m_b)=c_1(m_b)+\frac{c_2(m_b)}{3}=1.01\pm0.02. \tag{22}$$

The error comes out to be rather small.

(2) *Other diagrams that do not have the spectator topology.* These are diagrams that can lead to the same final states like exchange diagram, annihilation diagram, etc. These will be discussed in detail in Secs. V and VI.

(3) *Higher order QCD radiative corrections on the quark legs in $b\to D_s^{(*)}q$, that have the same topology as in semileptonic decays.* At lowest order, these are the corrections discussed in the preceding section and given in the Appendix. At higher orders, partial resummations in the semileptonic case have been done [11], and there is no difficulty to extend these methods to the processes under consideration.

A number of remarks is in order here. First, from the relation at one loop between the pole mass and the running modified minimal subtraction scheme ($\overline{\rm MS}$) mass at first order in α_s,

$$m=\overline{m}(\overline{m})\left[1+\frac{4}{3}\frac{\alpha_s(\overline{m})}{\pi}\right] \tag{23}$$

and from Eq. (14) one obtains

$$\Gamma(b\to cl\overline{\nu_l})=\frac{G^2[\overline{m}_b(\overline{m}_b)]^5}{192\pi^3}|V_{cb}|^2 f_{PS}(r) \times\left\{1+\frac{4}{3}\frac{\alpha_s(\overline{m}_b)}{\pi}[5+f_{RC}(r)]\right\} \tag{24}$$

and in formula (12) when $[\overline{m}_b(\overline{m}_b)]^3$ is substituted to m_b^3, a term $\frac{4}{3}(\alpha_s/\pi)\times 3$ has to be added. Taking $\alpha_s(\overline{m}_b)=0.22$ we get from Eq. (23)

$$\overline{m}_b(\overline{m}_b)=4.43\ \ \text{GeV}. \tag{25}$$

The values of the b pole mass (15) and of the ($\overline{\rm MS}$) running mass (25) are, respectively, smaller and larger than the values recently quoted in the literature from the analysis of semileptonic b decay, the reason being that a partial resummation of higher order diagrams is made that enhances the radiative corrections by roughly a factor 2 (see, for example, Ref. [11]). The interest of considering the ($\overline{\rm MS}$) mass is that the series is Borel summable, while using the pole mass, there is a renormalon ambiguity, cancelled by another renormalon ambiguity in the pole mass. Ball *et al.* [11] quote, as central values, $m_b=5.05$ GeV, $m_c=1.62$ GeV, and $\overline{m}_b(\overline{m}_b)=4.23$ GeV, $\overline{m}_c(\overline{m}_c)=1.29$ GeV, leading to consistent results for the semileptonic rate in both the ($\overline{\rm MS}$) and on-shell schemes.

Concerning the radiative corrections at higher orders in the processes $b\to D_s^{(*)-}c$, for this part of the radiative corrections that has the same topology than in semileptonic decays (simply at a given value of q^2 instead of integrating over q^2), we expect that they would be similarly enhanced as in the semileptonic decay. This is by the way what happens at order α_s: the correction that we obtain for $b\to D_s^{(*)-}c$ is very close to the one obtained at the same order in the semileptonic case [10]. This would imply, e.g., in the on-shell renormalization scheme, a larger radiative correction by roughly a factor 2 [11], but consistently also a larger $m_b=5.05$ GeV, leading grosso modo to the same results (19) and (20).

(4) *Soft gluon corrections that break factorization.* We must keep in mind, however, that, already at second order in α_s, we could have another type of corrections, absent in the semileptonic decay, that break factorization (e.g., two or more gluons linking the D_s to the quark legs), and the corresponding Bremsstrahlung diagrams. We can only hope that these corrections are small. Actually we have some hint that these corrections are presumably small from the phenomenological analysis of nonleptonic two-body decays, as done in a recent analysis by Neubert and Stech [16]. They find the effective a_1 coefficients

$$a_1(B\to Dh)=1.08\pm 0.04 \quad [0.98\pm 0.04], \tag{26}$$

$$a_1(B\to DD_s^{(*)})=1.10\pm 0.18 \quad [1.05\pm 0.17], \tag{27}$$

where h (light meson) and D_s are emitted. Both sets of figures correspond to two different models for the form factors. We must point out that these parameters are, *unlike* the perturbative $a_1(\mu)$, renormalization scale and renormalization scheme independent. They include soft gluons and subleading $1/N_c$ corrections that change according to the considered decay mode, i.e., in particular violations of factorization.

We see that a_1 is consistent with 1 although with large errors in $B\to DD_s^{(*)}$. An important point for our error analysis of Sec. VIII is that these errors come essentially from the uncertainty on $f_{D_s^{(*)}}$ [16].

Sticking to exclusive two-body modes, we would need another type of modes, namely, those of the type $B\to\pi(\rho)D_s$, where the D_s is emitted, since these modes would have the same topology than the inclusive $B\to X_uD_s$ decay we are interested in. However, their branching ratios have not still been measured, and only upper limits presently exist, of the order of few 10^{-4} for each mode [12].

But even if the effective a_1 for these modes is known some day, the corrections to factorization in the inclusive decay $B\to X_uD_s$ could be different. A *possible test* of factorization could be, following the work of Neubert and Stech for exclusive modes [16], to measure the ratios

$$R^{(*)}=\frac{\Gamma(\overline{B}^0\to X^+D_s^{(*)-})}{d\Gamma(\overline{B}^0\to X^+l^-\overline{\nu})/dq^2|_{q^2=m_{D_s}^2}} =6\pi^2|V_{cs}|^2 f_{D_s^{(*)}}^2 a_1^2 X_q^{(*)}. \tag{28}$$

We find, for the coefficients $X_q^{(*)}$:

$$X_q=\frac{(m_b^2-m_q^2)^2-m_{D_s}^2(m_b^2+m_q^2)}{(m_b^2-m_q^2)^2+m_{D_s}^2(m_b^2+m_q^2-2m_{D_s}^2)}, \quad X_q^*=1. \tag{29}$$

However, $d\Gamma(\overline{B}^0\to X^+l^-\overline{\nu})/dq^2$ is not available experimentally at present, even for X_c. Then, this test will have to wait for future experimental progress. Notice also that although factorization has been proved for $B\to\pi\pi,\ldots,$ in the $m_b\to\infty$ limit (up to calculable corrections) [17], this is of no help in our case where it is not a *light meson* that is *emitted*, but a D_s.

V. BACKGROUNDS WITH TAGGED EVENTS

Other production sources of D_s are shown in Fig. 4. In the following, we discuss these various D_s production mechanisms, evaluate their rate and propose means to reject the ones involving $\overline{b}\to\overline{c}$ transitions or correct for the others. We

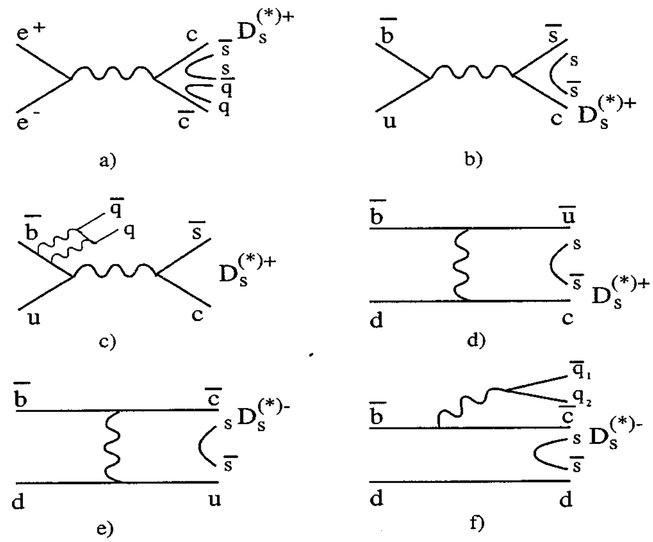

FIG. 4. Diagrams leading to the production of D_s mesons.

should distinguish between the background that concerns tagged or untagged events. Let us begin here with tagged events. In $e^+e^-\to\Upsilon(4S)\to B\bar{B}$, assume that the $\bar{B}$ is identified through its semileptonic decay. Then, the right sign D_s^+ can be produced, besides the main mechanism of Fig. 1, by mechanisms of Figs. 4(a)–4(d).

The $c\bar{c}$ *continuum* background [Fig. 4(a)] has a large cross section, $\sim$1.1 nb. However, these events tend to have a jetlike structure and therefore can be rejected to a large extend by topological cuts. Furthermore, since a D_s meson has to be produced, the creation of a $s\bar{s}$ pair is required, reducing the rate by about an order of magnitude. In addition, the momentum spectrum of the D_s meson produced in the continuum has a mean value larger than $m_B/2$ reducing further this background by more that a factor 3. Finally, it is possible to substract the remaining background by taking data just below the threshold for $B\bar{B}$ production.

The *annihilation* diagram in Fig. 4(b) is obtained from the calculation of the inclusive rate $B^+\to c\bar{s}$ using

$$J_{\mu q}=\bar{s}\gamma_\mu(1-\gamma_5)c, \tag{30}$$

$$\langle B^+|A^\mu|0\rangle=-if_B p_B^\mu \tag{31}$$

that gives [18]

$$\Gamma^{(0)}(B^+\to c\bar{s})=\frac{N_cG_F^2}{8\pi}|V_{ub}^*V_{cs}|^2f_B^2m_B(m_c^2+m_s^2)\times\left(1-\frac{(m_c^2-m_s^2)^2}{m_B^2(m_c^2+m_s^2)}\right)\times\sqrt{1-2\frac{(m_c^2+m_s^2)^2}{m_B^2}+\frac{(m_c^2-m_s^2)^2}{m_B^4}}a_1^2. \tag{32}$$

Neglecting the s quark mass, one gets

$$\Gamma^{(0)}(B^+\to c\bar{s})\simeq\frac{N_cG_F^2}{8\pi}|V_{ub}^*V_{cs}|^2f_B^2m_Bm_c^2\left(1-\frac{m_c^2}{m_B^2}\right)^2a_1^2. \tag{33}$$

Taking into account that one needs to create a $s\bar{s}$ pair in order to obtain a $D_s^{(*)}$ meson, one can assume

$$\Gamma^{(0)}(B^+\to D_s^{(*)+}X)\leqslant\frac{1}{3}\Gamma^{(0)}(B^+\to c\bar{s}). \tag{34}$$

Since $m_B^2f_B\cong m_D^2f_D$ in the heavy quark limit the suppression factor of this mechanism relative to the spectator quark model (8) will be of the order or smaller than

$$\frac{N_c}{3}\left(\frac{m_c}{m_b}\right)^3\sim3\times10^{-2}. \tag{35}$$

This branching fraction is small compared to the one deduced from Fig. 1 and would represent a small correction. The contribution from the diagram in Fig. 4(c) requiring the coupling via 2 gluons is expected to be much smaller and can be neglected.

The *exchange* diagram shown in Fig. 4(d) is evaluated in the same way than the annihilation one using

$$J_{\mu q}=\bar{q}_2\gamma_\mu(1-\gamma_5)q_1, \tag{36}$$

$$\langle B^0|A^\mu|0\rangle=-if_Bp_B^\mu \tag{37}$$

and replacing a_1 with $a_2=c_2+c_1/N_c$ (color-suppressed process)

$$\Gamma^{(0)}(B^o\to q_1\bar{q}_2)\simeq\frac{N_cG_F^2}{8\pi}|V_{q_2b}^*V_{q_1d}|^2f_B^2m_Bm_c^2\left(1-\frac{m_c^2}{m_B^2}\right)^2a_2^2, \tag{38}$$

where $q_1\bar{q}_2$ can either be $c\bar{u}$ or $u\bar{c}$. One should keep in mind that in this case the factorization ansatz is on much weaker ground. Obviously, the case with $q_1=c$ and $\bar{q}_2=\bar{u}$ is suppressed since the Cabibbo-Kobayashi-Maskawa (CKM) factors are $|V_{ub}^*V_{cd}|$. This means that this mechanism in the case of tagging is Cabibbo suppressed and color suppressed relatively to the main mechanism of Fig. 1. Comparing Eq. (8) to Eq. (38), the reduction factor is of the order

$$\tan^2\theta_c\frac{N_c}{3}\left(\frac{m_c}{m_b}\right)^3\left(\frac{a_2}{a_1}\right)^2\lesssim10^{-4} \tag{39}$$

and we can safely neglect this mechanism. The conclusion is that, if there is tagging, the mechanisms that can compete with the interesting process of Fig. 1 either can be discarded by kinematical cuts or are smaller by a factor of the order 10^{-2}. The method seems therefore safe if there is tagging.

VI. BACKGROUNDS WITH UNTAGGED EVENTS

If in $e^+e^-\to B\bar{B}$ we assume no tagging, besides the additional mechanisms of Figs. 4(a)–4(d), we can have also the processes in Figs. 4(e)–4(f), that lead to a wrong sign D_s. First, one must remark that the *continuum* background can also lead to a wrong sign D_s [the lower D in Fig. 4(a)], but we know that one can dispose off of these events by topological cuts. Also, the *exchange* process in Fig. 4(e), that corresponds to replacing in Fig. 4(d) $\overline{u}\to\overline{c}$ and $c\to u$, can lead to a wrong sign D_s. Unlike the case with $q_1=u$ and $\overline{q_2}=\overline{c}$ this process is in principle enhanced because the CKM factors are $|V_{cb}^* V_{ud}|$. Similarly a $s\overline{s}$ pair is required to get a $D_s^{(*)+}$ meson and therefore with $|a_2|=0.2$ one obtains a naive suppression factor smaller than

$$\frac{1}{3}\left|\frac{V_{cb}}{V_{ub}}\right|^2 N_c\left(\frac{m_c}{m_b}\right)^3\left(\frac{a_2}{a_1}\right)^2 \cong 0.20. \tag{40}$$

Although this source is only present for neutral B decays and is smaller than the spectator diagram in Fig. 1, the corresponding branching fraction could be non negligible, and its possible suppression relies on dynamical assumptions that are not very reliable. This branching fraction and Eq. (39) as well may further be enhanced by the emission of gluon from the initial light quark. In this case [19,20], the most important changes relative to Eq. (38) are the absence of the m_c^2/m_B^2 dependence due to helicity and the presence of the factor f_B^2/m_d^2 instead of f_B^2/m_B^2 due to the gluon radiation from the initial light quark. Therefore, gluonic emission may enhance the rate of the exchange diagram by one order of magnitude if one uses $m_d=300$ Mev/c^2 since the d quark must be interpreted as a constituent quark in this process. However, as pointed out in Ref. [21], the presence of the infrared sensitive parameter $1/m_d^2$ makes problematic a rigorous perturbative estimation of this contribution. Furthermore, in the full inclusive decay, according to heavy quark theory, this type of contributions should be suppressed by a factor $1/m_b^3$ relative to the main spectator diagram. Indeed Aglietti has proved that the power enhancements of the form $(m_b/m)^2$ and m_b/m cancel in the totally inclusive width [22]. However, our case does not correspond to this totally inclusive situation, since a D_s is detected. On the other hand, present limits (for example, $\mathrm{Br}(B^o\to D_s^- K^+)<2.4\times10^{-4}$ [23]) tend to disfavor a large enhancement. The same conclusion can be reached using D lifetime measurements [24]. It is nevertheless important to find a way to either measure it or to eliminate it. One possibility could be to observe some of these final states, for example $D_s^{(*)-}K^{(*)+}$ and evaluate their contribution.

The CLEO Collaboration has measured the rate $\overline{b}\to D_s^- X$ [25] due to Figs. 4(e)–4(f), although other sources exist (see next subsection). The total rate was found to be (2.1 ± 1.0) %. However the momentum spectrum of those D_s is expected to be rather soft with less than 0.5% of those having a momentum greater than 2.0 GeV/c. This leads to an effective branching fraction $\mathrm{Br}(B\to D_s^- X[p_{D_s}>2.0\ \mathrm{GeV}/c])<1.5\times10^{-4}$ at 90% .

It is also possible to produce $D_s^{(*)}$ mesons of the wrong sign in *multibody B decays* such as the one shown in Fig. 4(f). The decay rate of this type of modes is potentially large. However, one should note several important points.

The production of D^{**} with orbital excitation L=1 would not lead to $D_s^{(*)}$ as this meson needs to be accompanied by a kaon and the total mass $D_s^{(*)}K$ is larger than the D^{**} mass.

In the case of nonresonant $D_s^{(*)}K$ production from the lower vertex, the energy is shared between the final three or more particles and therefore the momentum spectrum of the $D_s^{(*)}$ is softer and barely reaches the range where $B\to D_s^{(*)+}X_u$ is expected. As discussed in the above subsection, CLEO measurements indicate that this type of decay should not be a problem.

The CLEO measurement mentioned in the previous section shows that this background should not be large.

VII. FEASIBILITY STUDY USING THE BABAR DETECTOR

The feasibility of this new method for measuring $|V_{ub}|$ has been verified for the BaBar detector at the SLAC B factory PEP-II. We have used the full detector simulation and the reconstruction program [26] to generate 5000 events with the following decay of one B meson $B\to D^{(*)}D_s^{(*)}$, where D_s decays to the $\phi\pi$ final state, and $\phi\to K^+K^-$. This gives a D_s spectrum peaked at 1.7 GeV/c in the center of mass system, therefore only slightly below the expected signal of D_s coming from V_{ub} transitions. Generic $B\overline{B}$ decays were used to measure the background level.

We have studied two crucial points for this analysis: the reconstruction efficiency for the D_s and the momentum resolution.

The analysis to isolate the D_s signal proceeds as follows: $K^\pm$ are identified using the combined information coming from the Silicon Vertex Tracker, the Drift Chamber and the DIRC detector and then selected if their invariant mass is in the 1020 ± 10 MeV/c^2 interval. A third track, assumed to be a pion, is then selected. A cut on $\cos\psi$, $|\cos\psi|>0.4$, where ψ is the angle between one Kaon and the D_s momentum in the ϕ rest frame, is then applied. The resolution on the D_s mass is 6.4 ± 0.3 MeV/c^2 [Fig. 5(a)].

The reconstruction efficiency is 39 ± 2 % and the momentum resolution is 6.7 ± 0.3 MeV/c [Fig. 5(b)]. The latter result insures that there will be no leaking from the lower to higher momenta. This excellent resolution is due to the fact that we implicitly reject mismeasured tracks: these will not give a D_s candidate with the right invariant mass.

Using these results, we can compute the number of reconstructed signal events that we expect. We have for untagged events

$$n_{\mathrm{rec}}=2\ n_{B\overline{B}}\mathrm{Br}(B\to D_s^{(*)}X_u)\mathrm{Br}(D_s^+\to\phi\pi^+)\times\mathrm{Br}(\phi\to K^+K^-)\epsilon_{\mathrm{rec}}=143$$

per 30 fb^{-1}, the nominal integrated luminosity for one year of data taking at BaBar. We took $\mathrm{Br}(B\to D_s^{(*)}X_u)=6.8\times10^{-4}$ from Eq. (20), $\mathrm{Br}(D_s^+\to\phi\pi^+)=3.5\%$, and ϵ_{rec} takes into account also the cut on D_s momentum at

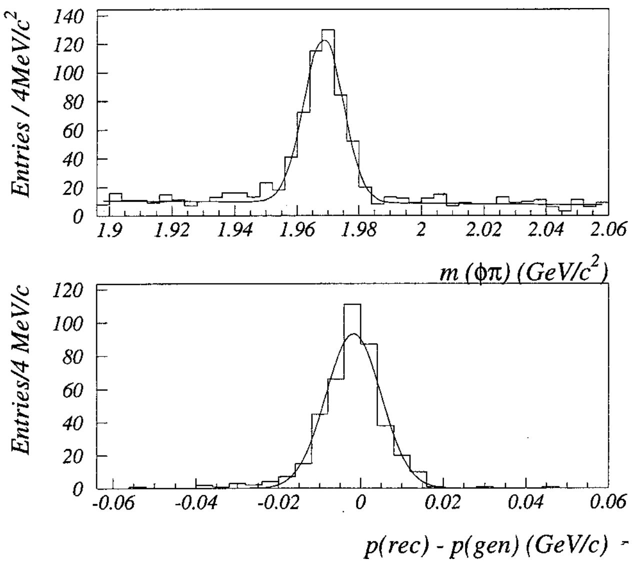

FIG. 5. Mass and momentum resolution for D_s mesons using the full BaBar detector simulation and reconstruction programs.

2.05 GeV/c. Therefore we can conclude that the number of reconstructed events will be sufficient to measure $|V_{ub}|$ with a good statistical precision. This number can be improved by reconstructing the D_s meson in other modes.

As we have pointed out above, there are unwanted sources of D_s beyond the kinematical limit for $B\to D_s^+ X_c$, and it is suitable to be able to reject them experimentally. As we have emphasized, one way to do this is to tag the flavor of the recoil B meson in the event, for instance by considering its semileptonic decay. The correlation between the sign of the lepton and the sign of the D_s meson is opposite for $B\to D_s^+ X_u$ and for the transitions due to the exchange diagrams of Fig. 4(e) [as well as for multibody B decays as in Fig. 4(f)].

This method has already been used by other experiments like CLEO and Argus to study the lepton spectrum in the B semileptonic decays. A cut on the angle between the lepton and the D_s meson allows to reject the pairs due to a D_s and a lepton from the same B meson. The only major problem of this method is the further reduction of the selected sample it implies, which should be no larger than 5–10 % of the number of reconstructed events estimated above. Therefore this is a possibility which is open but it would probably require a big experimental effort to reconstruct the largest possible fraction of D_s mesons to be really viable.

VIII. ESTIMATION OF THE ERRORS ON $|V_{qb}|$ ($q=u,c$) AND $|V_{ub}|/|V_{cb}|$

In the case of tagging, one can calculate the error on $|V_{cb}|$ or $|V_{ub}|$ from the following expression:

$$\frac{\sigma(|V_{qb}|)}{|V_{qb}|}=\frac{\sigma[\mathrm{Br}(B\to D_sX_q)]}{2\times\mathrm{Br}[B\to D_sX_q]}\oplus\frac{3}{2}\frac{\sigma(m_b)}{m_b}\oplus\frac{\sigma(f_{D_s^{(*)}})}{f_{D_s^{(*)}}}\oplus\frac{\sigma(a_1)}{a_1}, \tag{41}$$

where

$$\frac{\sigma[\mathrm{Br}(B\to D_sX_q)]}{\mathrm{Br}(B\to D_sX_q)}=\frac{1}{\sqrt{N}}\sqrt{1+N\left(\frac{\sigma(f_{p>p_{\min}})}{f_{p>p_{\min}}}\right)^2}, \tag{42}$$

$f(p>p_{\min})$ being the fraction of the D_s spectrum above the momentum cut, given in Sec. III. The first error is statistical and would scale as $1/2\sqrt{N}$ at low statistics and as $\sigma(f_{p>p_{\min}})/2f_{p>p_{\min}}$ at large statistics. In the following we will use

$$m_b=4.85\pm0.06, \tag{43}$$

$$f_{D_s}=0.241\pm0.037, \tag{44}$$

$$\frac{f_{D_s^*}}{f_{D_s}}=1.176\pm0.027, \tag{45}$$

$$a_1=1.01\pm0.02. \tag{46}$$

The errors on f_{D_S} and $f_{D_s^*}/f_{D_s}$ are given by lattice QCD in the quenched approximation [27]. We have adopted the error on a_1 given by the perturbative Wilson coefficients, Eq. (22). If we had adopted, e.g., the parameter a_1 from two-body decays, Eq. (27), we would have double counting because the large error in Eq. (27) comes essentially from the uncertainties on $f_{D_s^{(*)}}$ [16]. In this case we would have to use the error on the product $a_1 f_{D_s^{(*)}}$ from Ref. [16], leading to final similar errors. For $|V_{cb}|$ one can use $\mathrm{Br}(B\to D_sX_q)=0.100\pm0.025$ from the Particle Data Group (PDG) [23]. Therefore one gets, adding the errors in quadrature:

$$\begin{aligned}|V_{cb}|&=0.0449\pm0.0056\pm0.0008\pm0.0069\pm0.0009\\&=0.0449\pm0.0090.\end{aligned} \tag{47}$$

For $|V_{ub}|$, $f_{p>2.05\ \mathrm{GeV}}=0.48\pm0.06$ and therefore one would get the systematic error

$$\frac{\sigma(|V_{ub}|)}{|V_{ub}|}=0.17. \tag{48}$$

The main contribution comes from the present error on f_{D_s}, which should improve in the future, and from the uncertainty on $f_{p>2.05\ \mathrm{GeV}}$ due to the Fermi motion. From the measurement of the full D_s momentum spectrum, one can extract the ratio $|V_{ub}|/|V_{cb}|$, for which *most of the systematic errors cancel*. One would obtain

$$\frac{\sigma(|V_{ub}|/|V_{cb}|)}{(|V_{ub}|/|V_{cb}|)}=0.063. \tag{49}$$

If tagging is not assumed, there are hadronic uncertainties, discussed in Sec. VI, that deserve further investigation.

IX. CONCLUSION

In conclusion, we have shown that the process $B\rightarrow D_s^+X_u$ can allow the determination of the CKM matrix element $|V_{ub}|$ in e^+e^- collisions at the $\Upsilon(4S)$, as in the BaBar experiment. If there is tagging of one B meson, the prospects are very good since the backgrounds to the main spectator model mechanism, whose spectrum would allow the determination of $|V_{ub}|$, are either suppressed by a factor of the order of 10^{-2}, or can be disposed off by kinematical cuts. However, the number of events is drastically reduced by tagging.

If tagging is not assumed, other mechanisms can give a large background, but the method could still work if theoretical and experimental studies of these additional processes leading to a wrong sign D_s are performed in the future. It should be noted that these wrong sign backgrounds [Figs. 4(e) and 4(f)] are Cabibbo enhanced but suppressed by color and other dynamical effects (Sec. V). In contrast, in semileptonic decays, even when the hadronic background is studied [2], misidentified direct $b\rightarrow c$ decays are Cabibbo enhanced and difficult to exclude kinematically because of the neutrino. Admittedly, the semileptonic method has the advantage of statistics.

We are aware that our study is a preliminary survey of the possibility of measuring $|V_{ub}|$ with a new method. Work remains to be done. For the elementary processes $\overline{b}\rightarrow D_s^{(*)+}\overline{q}$ $(\overline{q}=\overline{u},\overline{c})$, one would need to compute the spectrum taking into account the radiative corrections and comparison with the spectrum $B\rightarrow D_s^{\pm}X$ measured by CLEO [12] needs to be done as a check. Up to now, only the integrated corrected rate has been computed [9]. On the other hand, theoretical or phenomenological work needs to be done to further constrain the sources of background and of hadronic uncertainties in the case of tagged and also, hopefully, although it is more difficult, for untagged events.

We insist that the method proposed here, having very different systematic errors than the semileptonic one, would provide an irreplaceable check of $|V_{ub}|$. We should emphasize that the errors that we find on $|V_{ub}|$ and mostly on $|V_{ub}|/|V_{cb}|$ are very encouraging.

ACKNOWLEDGMENTS

The authors acknowledge useful discussions with J. Charles, lattice data from D. Becirevic, and partial support from the EEC-TMR Program, Contract No. CT 98-0169. Laboratoire de Physique Theorique is Unité Mixte de Recherche CNRS-UMR 8627.

APPENDIX

In the on-shell renormalization scheme and dimensional regularization we find the following two-body $b\rightarrow qD_s^{(*)}$ and Bremsstrahlung $b\rightarrow qD_s^{(*)}g$ rates. For D_s we obtain, for $r\rightarrow 0$ $(r=m_q/m_b)$,

$$\Gamma_{D_s}^{\text{two-body}}=\Gamma_{D_s}^{(0)}\left\{1+\frac{4}{3}\frac{\alpha_s}{\pi}[[D(r,\xi)]_{r\rightarrow 0}+[F_V(r,\xi)]_{r\rightarrow 0}]\right\}. \tag{A1}$$

$[D(r,\xi)]_{r\rightarrow 0}$ contains the $1/(D-4)$ and the divergent terms as $m_q\rightarrow 0$,

$$\begin{aligned}[D(r,\xi)]_{r\rightarrow 0}=&\left[2-2\log\left(\frac{1-\xi}{r}\right)\right]\\ &\times\left[\frac{1}{D-4}-2\log(2\pi)+\gamma+\log\left(\frac{\pi m_b^2}{\mu^2}\right)\right]\\ &+\log(r)\left[-\frac{5}{2}+2\log(1-\xi)\right]+\log^2(r)\end{aligned} \tag{A2}$$

and $[F_V(r,\xi)]_{r\rightarrow 0}$ is the surviving finite piece

$$\begin{aligned}[F_V(r,\xi)]_{r\rightarrow 0}=&-3-\frac{\pi^2}{6}-3\log^2(1-\xi)\\ &+4\log(1-\xi)+\frac{1}{\xi}\log(1-\xi)\\ &+Sp(1-\xi)+\log\xi\log(1-\xi),\end{aligned} \tag{A3}$$

where $Sp(z)$ is the Spence function. Analogously, we obtain

$$\Gamma_{D_s}^{\text{Brem}}=-\Gamma_{D_s}^{(0)}\frac{4}{3}\frac{\alpha_s}{\pi}\{[D(r,\xi)]_{r\rightarrow 0}+[F_B(r,\xi)]_{r\rightarrow 0}\}, \tag{A4}$$

$$\begin{aligned}[F_B(r,\xi)]_{r\rightarrow 0}=&-\frac{21}{4}+\frac{\pi^2}{3}+\frac{13}{2}\log(1-\xi)\\ &+\log(\xi)\log(1-\xi)\\ &+Sp(\xi)-3\log^2(1-\xi)+\frac{\xi}{1-\xi}\log(\xi).\end{aligned} \tag{A5}$$

We observe also that the singular terms in $1/(D-4)$, $\log(r)$ and $\log^2(r)$ contained in $[D(r,\xi)]_{r\rightarrow 0}$ cancel among the two-body and bremsstrahlung rates.

Then, it follows, for $m_q\rightarrow 0$, the total rate:

$$\begin{aligned}&\Gamma_{D_s}^{\text{two-body}}+\Gamma_{D_s}^{\text{Brem}}\\ &=\Gamma_{D_s}^{(0)}\left\{1+\frac{4}{3}\frac{\alpha_s}{\pi}\left[\frac{9}{4}-\frac{\pi^2}{3}-2Sp(\xi)\right.\right.\\ &\quad-\log(\xi)\log(1-\xi)+\frac{1}{\xi}\log(1-\xi)\\ &\quad\left.\left.-\frac{5}{2}\log(1-\xi)-\frac{\xi}{1-\xi}\log(\xi)\right]\right\}\end{aligned} \tag{A6}$$

that gives, at $q^2=0$, i.e., $\xi=0$, formulas (12) and (13). For the D_s^* the cancellations occur in an analogous manner and we find the final finite result

$$\Gamma_{D_s^*}^{\text{two-body}}+\Gamma_{D_s^*}^{\text{Brem}} = \Gamma_{D_s^*}^{(0)}\left\{1+\frac{4}{3}\frac{\alpha_s}{\pi}\left[2-\frac{\pi^2}{3}-2Sp(\xi) -\log(\xi)\log(1-\xi)-\frac{5+4\xi}{2(1+2\xi)}\log(1-\xi) -\frac{3-\xi-10\xi^2}{4(1-\xi)(1+2\xi)}-\frac{\xi(1-\xi-2\xi^2)}{(1-\xi)^2(1+2\xi)}\log(\xi)\right]\right\}. \tag{A7}$$

For $\xi=0$ we also recover Eqs. (12) and (13), because the calculation of the D_s^* rate follows along the same lines, as the D_s one with the replacement

$$f_D^2 p_D^\mu p_D^\nu \rightarrow f_{D*}^2 m_{D*}^2 \sum_\lambda \varepsilon^{(\lambda)\mu}\varepsilon^{*(\lambda)\nu} = f_{D*}^2(p_{D*}^\mu p_{D*}^\nu - m_{D*}^2 g^{\nu\nu}). \tag{A8}$$

The results for the radiative corrections for $r\neq 0$ and arbitrary ξ are given analytically and numerically in Ref. [9].

[1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **42**, 652 (1973).

[2] In the determination of $|V_{ub}|$ in the semileptonic decay, an enormous theoretical effort has been developed, namely through the analysis of the charged lepton end spectrum, pioneered by G. Altarelli *et al.*, Nucl. Phys. **B208**, 365 (1982), or through the hadronic recoil mass spectrum, as proposed, for example, by I. Bigi, R. D. Bikelman, and N. Uraltsev, Eur. Phys. J. C **4**, 453 (1998); M. Battaglia, P. Klint, and E. Piotto, paper submitted to the ICHEP '98 Conference, Vancouver, DELPHI 98-97 CONF 165 (1998); ALEPH Collaboration, R. Barate *et al.*, Eur. Phys. J. C **6**, 55 (1999).

[3] M. Beneke, G. Buchalla, and I. Dunietz, Phys. Lett. B **393**, 132 (1997); A. F. Falk and A. A. Petrov, Phys. Rev. D **61**, 034020 (2000).

[4] B. Grinstein and R. F. Lebed, Phys. Rev. D **60**, 031302 (1999); D. H. Evans, B. Grinstein, and D. R. Nolte, Phys. Rev. Lett. **83**, 4947 (1999).

[5] M. K. Gaillard and B. Lee, Phys. Rev. Lett. **33**, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. **52B**, 351 (1974).

[6] W. F. Palmer and B. Stech, Phys. Rev. D **48**, 4174 (1993).

[7] A. M. Shifman and M. Voloshin, Sov. J. Nucl. Phys. **47**, 511 (1988); A. M. Shifman, Nucl. Phys. **B388**, 246 (1992).

[8] R. Aleksan, A. Le Yaouanc, L. Oliver, O. Pène, and J.C. Raynal, Phys. Lett. B **316**, 567 (1993); A. Le Yaouanc *et al.*, Phys. Rev. D **52**, 2813 (1995).

[9] R. Aleksan, M. Zito, A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Report No. LPT-Orsay 99-36, DAPNIA/SPP 99-19, hep-ph/9906505.

[10] N. Cabibbo and L. Maiani, Phys. Lett. **79B**, 109 (1978); Q. Hokim and X.-Y. Pham, Ann. Phys. (N.Y.) **155**, 202 (1984); Y. Nir, Phys. Lett. B **221**, 184 (1989).

[11] P. Ball, M. Beneke, and V. M. Braun, Phys. Rev. D **52**, 3929 (1995).

[12] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).

[13] G. Altarelli *et al.*, Nucl. Phys. **B208**, 365 (1982).

[14] M. Ciuchini, E. Franco, G. Martinelli, L. Reina, and L. Silvestrini, Z. Phys. C **68**, 239 (1995).

[15] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[16] M. Neubert and B. Stech, in ''Heavy Flavors II,'' CERN-TH-97-099, 1997, hep-ph/9705292.

[17] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999).

[18] S. Hagelin, Nucl. Phys. **B193**, 123 (1981).

[19] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **44**, 7 (1980); **44**, 962(E) (1980).

[20] H. Fritzsch and P. Minkowski, Phys. Lett. **90B**, 455 (1980).

[21] G. Altarelli and S. Petrarca, Phys. Lett. B **261**, 303 (1991).

[22] U. Aglietti, Phys. Lett. B **325**, 473 (1994).

[23] Particle Data Group, R. M. Barnett, *et al.*, Phys. Rev. D **54**, 1 (1996).

[24] I. Bigi, Z. Phys. C **5**, 313 (1980).

[25] CLEO Collaoration, X. Fu *et al.*, Report No. CLEO CONF95-11.

[26] BaBar Simulation and reconstruction software.

[27] D. Becirevic *et al.*, Phys. Rev. D **60**, 074501 (1999); D. Becirevic (private communication).