

Baryon magnetic moments in the QCD string approach

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Magnetic moments of baryons composed of light and strange quarks are computed for the first time through the only parameter of the model—the string tension σ . For example, $\mu_p = m_p/c\sqrt{\sigma}$, $\mu_{\Omega^-} = -\mu_p[1 + \frac{3}{4}(m_s^2/c^2\sigma) - \frac{15}{32}(m_s^4/c^4\sigma^2)]^{-1}$, where m_p is the proton mass, m_s is the strange quark current mass, and $c = 0.957$, a constant which is calculated in the paper. Resulting theoretical values differ from the experimental ones typically by about 10%.

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I. INTRODUCTION

Recently in a stimulating analysis of the hyperon static properties [1] Lipkin displayed remarkably successful relations connecting strange and nonstrange baryon magnetic moments and the corresponding quark masses ν_u and ν_s (in order to avoid similar notation for masses and magnetic moments we denote current quark masses by m_k and dynamical, or constituent, quark masses by ν_k). To understand why these relations agree so well with experiment, and to get insight into the problems encountered in semileptonic decays of baryons [1], one needs a dynamical approach which would enable one to express the constituent masses of the quarks and baryon magnetic moments in terms of a single QCD scale parameter. It is the purpose of the present paper to express the magnetic moments of baryons and constituent masses of the corresponding quarks in terms of only one parameter—the string tension, and to demonstrate that our results are in line with the relations of Lipkin.

Theoretical investigation of baryon magnetic moments (BMMs) has a long history [2]. In the constituent quark model (CQM) BMMs are expressed through the values of constituent quark masses, which are input parameters [3] (see also [4] for discussion). Among other approaches to the problem, mention should be made of different versions of the bag model [5], lattice calculations [6], chiral perturbation theory [7], and the QCD sum rules [8]. In the latter the BMMs are connected to the values of chiral and gluonic condensates and to the quartic quark correlator. Although a lot of efforts have been undertaken along different lines, the theoretical predictions still differ from the experimental values (by 10–15% in the worst case [9]).

In all models however theoretical predictions are somewhat biased by the introduction of supplementary parameters in addition to the only one pertinent to QCD—the overall scale of the theory, which should be specified to make the QCD complete. In the final, ideal case this role is played by Λ_{QCD} ; in our treatment, as well as in lattice QCD calculations, we take as this universal parameter the QCD string tension σ , fixed in nature by the meson and baryon Regge slopes.

The purpose of our paper is to calculate BMMs through this single parameter, σ , in the simplest possible approximation within the nonperturbative QCD approach, developed in [10–15].

II. RELATIVISTIC $3q$ GREEN'S FUNCTION AND EFFECTIVE HAMILTONIAN

The starting point of the approach is the Feynman-Schwinger (world-line) representation of the $3q$ Green's function [10], where the role of “time” parameter along the path $z_\mu^{(i)}(s_i)$ of i th quark is the Fock-Schwinger proper time s_i , $i = 1, 2, 3$. One has [10,12]

$$G^{(3q)}(X, Y) = \int \prod_{i=1}^3 ds_i D z_\mu^{(i)} e^{-K} \langle W_3(X, Y) \rangle \quad (1)$$

where $X; Y = x^{(1)}, x^{(2)}, x^{(3)}; y^{(1)}, y^{(2)}, y^{(3)}$,

$$K = \sum_{i=1}^3 \left[m_i^2 s_i + \frac{1}{4} \int_0^{s_i} \left(\frac{dz_\mu^{(i)}}{d\tau_i} \right)^2 d\tau_i \right]. \quad (2)$$

Here m_i is the current quark mass and the three-lobes Wilson loop is a product of three parallel transporters:

$$\langle W_3(X, Y) \rangle = \left\langle \prod_{i=1}^3 \Phi_{a_i b_i}^{(i)}(x^{(i)}, y^{(i)}) \right\rangle e_{a_1 a_2 a_3} e_{b_1 b_2 b_3}. \quad (3)$$

The standard approximation in the QCD string approach is the minimal area law for Eq. (3), which will be used in what follows:

$$\langle W_3 \rangle = \exp \left(-\sigma \sum_{i=1}^3 S_i \right), \quad (4)$$

where S_i is the minimal area of one loop.

The next step is basic for our approach, and it allows finally to calculate the quark constituent masses ν_i in terms of the quark current masses m_i , defined at the scale of 1 GeV. In this step one connects proper and real times (in the baryon c.m. system):

$$ds_i = \frac{dt}{2\nu_i(t)}, \quad (5)$$

where $t = z_4^{(i)}(s_i)$, $0 \leq t \leq T$, is a common c.m. time on the hypersurface $t = \text{const}$. The new entity $\nu_i(t)$, as will be seen, plays the role of the quark constituent mass and will be calculated through σ (and α_s when perturbative exchanges are taken into account).

Considering the exponent in Eq. (3) as an action one can define the Hamiltonian and go over to the representation [11,14]

$$G^{(3q)} = \int \prod_{i=1}^3 D\nu_i(t) D^3 z^{(i)}(t) e^{-A} \quad (6)$$

with

$$A = \sum_{i=1}^3 \int_0^T dt \left(\frac{m_i^2}{2\nu_i} + \frac{\nu_i}{2} + \frac{(\dot{\mathbf{z}}^{(i)}(t))^2}{2\nu_i} \right) + \sigma S_i. \quad (7)$$

The final step in the approach [12–14] is the derivation of the c.m. Hamiltonian containing ν_i as parameters to be found from the condition of the Hamiltonian minimum. It has the following form:

$$H = \sum_{k=1}^3 \left(\frac{m_k^2}{2\nu_k} + \frac{\nu_k}{2} \right) + \frac{1}{2m} \left(-\frac{\partial^2}{\partial \xi^2} - \frac{\partial^2}{\partial \eta^2} \right) + \sigma \sum_{k=1}^3 |\mathbf{r}^{(k)}|. \quad (8)$$

Here ξ, η are Jacobi coordinates defined as in [13], and $\mathbf{r}^{(k)}$ is the distance from the k th quark to the string-junction position which we take below for simplicity coinciding with the c.m. point. In addition Eq. (8) contains an arbitrary mass parameter m introduced to ensure correct dimensions; this parameter drops out from final expressions. Leaving technical details to the Appendixes, we now treat the Hamiltonian (8) using the hyperspherical formalism [16].

III. EVALUATION OF QUARKS CONSTITUENT MASSES

Considering three quarks with equal masses and introducing the hyperradius $\rho^2 = \xi^2 + \eta^2$, one has in the approximation of the lowest hyperspherical harmonic (which is known [17] to yield accuracy of the eigenvalue E_n around one percent)

$$\frac{d^2 \chi(\rho)}{d\rho^2} + 2\nu \{E_n - W(\rho)\} \chi(\rho) = 0, \quad (9)$$

$$W(\rho) = b\rho + \frac{d}{2\nu\rho^2}, \quad b = \sigma \sqrt{\frac{2}{3}} \frac{32}{5\pi}, \quad d = 15/4. \quad (10)$$

The baryon mass $M_n(\nu)$ is equal to (for equal quark masses)

$$M_n(\nu) = \frac{3m^2}{2\nu} + \frac{3}{2}\nu + E_n(\nu). \quad (11)$$

The crucial point is now the calculation of ν , which is to be found from the minimum of $M_n(\nu)$, as it is prescribed in the QCD string approach [10–15].

At this point it is important to stress that we have changed from $\nu(t)$ depending on t on the trajectory in the path integral (6) to the operator ν to be found from momenta and coordinates in Eq. (8) as in [14] and finally to the constant ν to be found from the minimum of the mass M , as it was

suggested in [11,12,15]. The accuracy of this replacement was tested recently in [18] to be around 5% or better for lowest levels.

The equation defining the stationary points of M_n as function of ν for equal masses is

$$\left. \frac{\partial M_n}{\partial \nu} \right|_{\nu=\nu_{(0)}} = 0. \quad (12)$$

The generalization for baryon made of three quarks with different masses is straightforward. The perturbative gluon exchanges and spin-dependent terms can be self-consistently included in the above picture [15]. Including the Coulomb term and passing to dimensionless quantities x , ε_n , and λ defined as

$$\begin{aligned} x &= (2\nu b)^{1/3} \rho, \\ \varepsilon_n &= \frac{2\nu E_n}{(2\nu b)^{2/3}}, \\ \lambda &= \alpha_s \frac{8}{3} \left(\frac{10\sqrt{3}\nu^2}{\pi^2 \sigma} \right)^{1/3}, \end{aligned} \quad (13)$$

where α_s is the strong coupling constant, one arrives at the following reduced equation:

$$\left\{ -\frac{d^2}{dx^2} + x + \frac{d}{x^2} - \frac{\lambda}{x} - \varepsilon_n(\lambda) \right\} \chi(x) = 0. \quad (14)$$

It is now a simple task to find eigenvalues $\varepsilon_n(\lambda)$ of Eq. (14) either numerically, or analytically (see below). Then Eq. (12) would yield the following equation defining the quark dynamical mass ν :

$$\begin{aligned} \varepsilon_n(\lambda) \left(\frac{\sigma}{\nu^2} \right)^{2/3} \left\{ 1 + \frac{2\lambda}{\varepsilon_n(\lambda)} \left| \frac{d\varepsilon_n}{d\lambda} \right| \right\} \\ + \frac{9}{16} \left(\frac{75\pi^2}{2} \right)^{1/3} \left(\frac{m^2}{\nu^2} - 1 \right) = 0. \end{aligned} \quad (15)$$

It turns out that numerical solution of Eq. (14) may be reproduced analytically with the accuracy of (1–2)% provided one replaces the potential $W(x) = x + d/x^2 - \lambda/x$ in Eq. (14) by oscillator potential near the stationary point $W'(x_0) = 0$ —see Appendix A. Equation (15) applied to the nucleon ($m=0$) yields the dynamical mass ν_u of the light quark, and applied to Ω^- ($m=m_s$) gives the strange quark mass ν_s . Before presenting these solutions we remind the reader about spin-spin forces responsible, e.g., for $N-\Delta$ splitting. Contrary to what might be naively expected the inclusion of spin-spin interaction considerably simplifies the problem due to remarkable cancellation of Coulomb and spin-spin contributions into the dynamical (constituent) quark mass—see Appendix A. Therefore these terms should be kept only if one wishes to calculate the BMMs with the accuracy much higher than 10%, in which case one should also take into

TABLE I. Magnetic moments of baryons (in nuclear magnetons) computed using Eqs. (12), (20), and (21) in comparison with experimental data from PDG [27].

Baryon	p	n	Λ	Σ^-	Σ^0	Σ^+	Ξ^-	Ξ^0	Ω^-
Present work	2.54	-1.69	-0.69	-0.90	0.80	2.48	-0.63	-1.49	-2.04
Experiment	2.79	-1.91	-0.61	-1.16		2.46	-0.65	-1.25	-2.02

account pion corrections, higher hyperspherical harmonics, etc. which is out of the scope of the present paper.

Thus in order to determine the quark masses and eventually the baryon magnetic moments one needs only two parameters: the string tension σ and the strange quark current mass m_s (u, d current quark masses are set to zero). Present calculations were performed for

$$\sigma = 0.15 \text{ GeV}^2, \quad m_s = 0.245 \text{ GeV}. \quad (16)$$

The string tension value (16) which is smaller than in the meson case is in line with baryon calculations by Capstick and Isgur [19]. A similar smaller value of σ is implied by recent lattice calculations by Bali [20]. Since the values of the above parameters are allowed to vary within certain limits [15,20,21] one can in principle formulate the inverse problem, namely express the BMMs in line with the present work and then fit their experimental values to determine the optimal choice of σ and m_s .

Consider first the case of a nucleon made of three quarks with zero current masses and equal dynamical masses ν_u . Keeping in mind cancellation of the Coulomb and spin-spin terms and thus setting in Eqs. (14) and (15) $\lambda = 0$ and making use of the oscillator approximation described in Appendix A, one finds, from Eq. (15),

$$\begin{aligned} \nu_u &= 2 \sqrt{\frac{2\sigma}{\pi}} \left[\frac{2}{3 \times 5^{1/3}} \left(1 + \frac{2}{3\sqrt{5}} \right) \right]^{3/4} \\ &\equiv c \sqrt{\sigma} \approx 0.957 \sqrt{\sigma} = 0.37 \text{ GeV}. \end{aligned} \quad (17)$$

This result agrees with the exact solution of Eq. (14) at $\lambda = 0$ with the accuracy better than 1%. A similar procedure applied to the Ω^- baryon yields the strange quark dynamical mass ν_s . From Eqs. (15) and (17) one gets

$$\nu_s \approx c \sqrt{\sigma} \left(1 + \frac{3}{4} \frac{m_s^2}{c^2 \sigma} - \frac{15}{32} \frac{m_s^4}{c^4 \sigma^2} \right) = 0.46 \text{ GeV} \quad (18)$$

for $m_s = 0.245 \text{ GeV}$, and where the constant c is defined in Eq. (17). Now we turn to baryon magnetic moments.

IV. BARYON MAGNETIC MOMENTS

The form (1) for $G^{(3q)}$ does not take into account spins of quarks. When those are inserted, a new additive term appears in the exponent of Eq. (6), proportional to the external magnetic field \mathbf{B} , namely A acquires the following term [22,23]:

$$\delta A = \sum_{k=1}^3 \int_0^{s_k} d\tau_k e_k \boldsymbol{\sigma}^{(k)} \cdot \mathbf{B} = \sum_{k=1}^3 \int_0^{s_k} \frac{T e_k \boldsymbol{\sigma}^{(k)} \cdot \mathbf{B}}{2\nu_k} dt, \quad (19)$$

where e_k is the electric charge of the quark, $\boldsymbol{\sigma}^{(k)}$ is the corresponding spin operator, and the definition (5) of the constituent mass was used.

Introducing the z component of the magnetic moment operator

$$\mu_z = \sum_{k=1}^3 \frac{e_k \sigma_z^{(k)}}{2\nu_k}, \quad (20)$$

one can write the BMMs as matrix elements

$$\mu_B \equiv \langle \Psi_B | \mu_z | \Psi_B \rangle, \quad (21)$$

where Ψ_B is the eigenfunction of Eq. (8), and ν_k is taken at the stationary point, given by Eq. (12).

For the baryon wave function we shall take here the simplest approximation, namely

$$\Psi_B = \Psi^{\text{symm}}(r) \psi^{\text{symm}}(\sigma, f) \psi^a(\text{color}), \quad (22)$$

where $\psi(\sigma, f)$ is the spin-flavor part of the wave function. The form (22) neglects the nonsymmetric components in the coordinate $\psi(r)$ and spin-flavor parts of the wave function, which appear in the higher approximation of the hyperspherical formalism [16], and for lowest states contribute only few percent to the normalization [16]. The spin-flavor functions for different baryons were known for a long time [3], and are briefly outlined in Appendix B. Using these functions it is a simple task to calculate the matrix element (21), e.g., the proton and neutron magnetic moments are given by

$$\begin{aligned} \mu_p &= \frac{m_p}{\nu_u} = \frac{1}{2} \sqrt{\frac{\pi}{2\sigma}} \left[\frac{2}{3 \times 5^{1/3}} \left(1 + \frac{2}{3\sqrt{5}} \right) \right]^{-3/4} \approx 2.54 \mu_N, \\ \mu_n &= -\frac{2}{3} \mu_p \approx -1.69 \mu_N. \end{aligned} \quad (23)$$

Magnetic moments of other baryons as well as new relations between them are obtained from Eqs. (15), (17), and (18).

In particular one has

$$\mu_{\Omega^-} \approx -\mu_p \left(1 + \frac{3}{4} \frac{m_s^2}{c^2 \sigma} - \frac{15}{32} \frac{m_s^4}{c^4 \sigma^2} \right)^{-1} = -2.04 \mu_N, \quad (24)$$

$$\mu_{\Sigma^+} (4c^2 \sigma + m_s^2) = -2 \mu_{\Xi^0} (3c^2 \sigma + 2m_s^2), \quad (25)$$

where terms of the order $m_s^4/c^4 \sigma^2$ were omitted in deriving the last relation.

Results on the BMMs are summarized in Table I. As

these results differ from the experimental values typically by only about 10% and are subjected to plentiful corrections (meson exchanges, higher harmonics, etc.), one may conclude that the outlined QCD approach is successful even in its simplest form.

More refined treatment of the Ω^- baryon magnetic moment within the present approach should take into account the fact that for baryons with spin $S=3/2$ Coulomb and spin-spin forces do not cancel each other in the eigenvalue equation (15) as it was explained here for baryons with $S=1/2$. The account of this [24] does not change the results for quark constituent masses, $\nu_u=0.37$ GeV, $\nu_s=0.46$ GeV, and for the baryon magnetic moments presented in Table I, but allows us to reproduce them with a smaller value of the strange quark current mass, namely with $m_s=0.15$ GeV—see [24] for details.

It is important to realize that the QCD string model used above is a fully relativistic string model for light current masses, and the “nonrelativistic” appearance of the Hamiltonian (1) is a consequence of the rigorous einbein formalism [25] which was introduced in the most general form in [26]. The approach enables us to investigate other electromagnetic properties of baryons: transition magnetic moments, polarizabilities [28], etc.

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APPENDIX A: EIGENVALUE EQUATION FOR BARYONS

The eigenvalue equation for baryons in hyperspherical basis [16] in its standard form is given by Eq. (9) and in its reduced form with Coulomb term included, by Eq. (14). Though Eq. (14) can be easily solved numerically, it is instructive to present the analytical solution resorting to the oscillator approximation near the stationary point $W'(x_0)=0$. This approximation is known to yield the accuracy of 1–2% [17]. It will be demonstrated below that the Coulomb term which tends to increase the quark dynamical mass almost exactly cancels with the spin-spin interaction term. Therefore one can consider Eq. (21) at $\lambda=0$. Then the ground state energy is equal to $W(x_0)$ plus the first quantum correction $\omega/2$, where $\omega^2=2W''(x_0)$.

This yields

$$\varepsilon(0) = \frac{3}{2} \left(\frac{15}{2} \right)^{1/3} \left(1 + \frac{2}{3\sqrt{5}} \right). \quad (\text{A1})$$

Substitution of this result into Eqs. (13) and Eqs. (11) and (12) leads to the expression (17) for the light quark current mass ν_u .

Next we demonstrate the cancellation of the Coulomb and spin-spin contributions into the quark mass. Again this conclusion results directly from numerical calculations but it is always preferable to present transparent estimates. The derivative $dE_n/d\nu$ [see Eqs. (11), (12), and (13)] may be written as

$$\frac{dE_n}{d\nu} = \varepsilon(\lambda) \frac{\partial}{\partial \nu} \frac{(2\nu b)^{2/3}}{2\nu} - \frac{(2\nu b)^{2/3}}{2\nu} \frac{\partial \lambda}{\partial \nu} \left| \frac{d\varepsilon}{d\lambda} \right|, \quad (\text{A2})$$

where the $(-)$ sign stems from the fact that $d\varepsilon/d\lambda < 0$. Expanding $\varepsilon(\lambda)$ in Taylor series in λ and keeping only the linear term [the small parameter is $\lambda/\varepsilon(0) \approx 1/4$] one gets

$$\frac{dE_n}{d\nu} \approx - \frac{(2\nu b)^{2/3}}{6\nu^2} \varepsilon(0) \left\{ 1 + \frac{\lambda}{\varepsilon(0)} \left| \frac{d\varepsilon}{d\lambda} \right| \right\}. \quad (\text{A3})$$

The value of $\varepsilon(0)$ is given by Eq. (A1), the estimate of $d\varepsilon/d\lambda$ in the small λ regime is straightforward, then recalling that according to Eqs. (13) $\lambda \propto (\nu^2/\sigma)^{1/3}$ and solving the simple equations one obtains that due to Coulomb interaction ν_u increases by ≈ 0.03 GeV. This is confirmed by numerical solution of Eq. (14).

The spin-spin interaction in baryons results in the shift of E_n equal to

$$\delta E_n = \frac{16}{9} \alpha_s \sum_{i>j} \frac{\mathbf{s}_i \mathbf{s}_j}{\nu_i \nu_j} \delta(\mathbf{r}_{ij}). \quad (\text{A4})$$

For proton the summation over (i, j) yields a factor $-4/3$; all three delta-functions smeared over infinitesimal regions [17] are equal to each other and scale with ν as $\nu_u^{3/2} \cdot \delta$, where the constant δ for nucleon has been with high accuracy computed by the Green’s function Monte Carlo method in [17]. As a result spin-spin interaction leads to a contribution into $dE_n/d\nu$ proportional to $\nu^{-3/2}$. This in turn results in the decrease of the quark dynamical mass ν_u by 0.035 GeV, i.e., the contributions from the Coulomb and spin-spin interaction into the quark mass almost exactly cancel each other. Thus we are led to a value $\nu_u=0.37$ GeV given by Eq. (17).

APPENDIX B: SPIN-FLAVOR WAVE FUNCTIONS AND BMM IN IMPULSE APPROXIMATION

As stated in the main text we have restricted the basis by considering only totally symmetric components of the baryon wave function in the coordinate space (see [3] for the discussion of corrections to this approximation). Therefore the spin-flavor part of the wave function has to be symmetric too. Then the calculation of the BMMs in “impulse” (additive) approximation proceeds along the well trodden path [3]. For example, the nucleon spin-flavor symmetric wave function entering into Eq. (22) is a combination $\psi^{\text{symm}}(\sigma, f) = (\varphi''(\sigma)\chi''(f) + \varphi'(\sigma)\chi'(f))/\sqrt{2}$, where prime and double prime denote mixed symmetry functions symmetric and antisymmetric with respect to the $1 \rightarrow 2$ permutation. The Σ^- spin-flavor wave function is obtained from that of the neutron one by substitution of the u -quark by the s -one and so on for other baryons. Due to the antisymmetry of the com-

plete wave function (22) the calculation of the matrix element (21) reduces to the averaging of the operator ($\delta_{3u}/\nu_u + \delta_{3s}/\nu_s$) or ($\delta_{3d}/\nu_u + \delta_{3s}/\nu_s$). In this way one arrives at the well-known relations

$$\frac{\mu_n}{\mu_p} = -\frac{2}{3}, \frac{\mu_\Lambda}{\mu_p} = -\frac{\nu_u}{3\nu_s}, \frac{\mu_{\Sigma^+}}{\mu_p} = \frac{8}{9} + \frac{\nu_u}{9\nu_s}, \frac{\mu_{\Sigma^-}}{\mu_p} = -\frac{4}{9} + \frac{\nu_u}{9\nu_s},$$

$$\frac{\mu_{\Sigma^0}}{\mu_p} = \frac{2}{9} + \frac{\nu_u}{9\nu_s}, \frac{\mu_{\Xi^-}}{\mu_p} = \frac{1}{9} - \frac{4}{9} \frac{\nu_u}{\nu_s}, \frac{\mu_{\Xi^0}}{\mu_p} = -\frac{2}{9} - \frac{4}{9} \frac{\nu_u}{\nu_s}. \quad (\text{B1})$$

Various corrections to the impulse approximation have been discussed in the literature [3,4].

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