Two parameter texture of nearly bimaximal neutrino mixing

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We propose a texture of a three generation Majorana-type neutrino mass matrix in terms of only two parameters which gives rise to nearly bimaximal mixing angles. We also demonstrate an explicit realization of such a type of neutrino mass-matrix in the context of an $SU(2)_L \times U(1)_Y$ model due to higher-dimensional mass terms through the inclusion of discrete $Z_3 \times Z_4$ symmetry and two extra singlet Higgs fields.

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I. INTRODUCTION

Evidence in favor of neutrino oscillation (as well as neutrino mass) has been provided by the Super-Kamiokande (SK) atmospheric neutrino experiment [1] through the measurement of magnitude and angular distribution of the ν_μ flux produced in the atmosphere due to cosmic ray interactions. Observed depletion of ν_μ flux in earth has been interpreted as the oscillation of ν_μ to some other species of neutrino. In a two flavor neutrino oscillation scenario, oscillation between ν_μ-ν_τ, the experimental result leads to maximal mixing between two species $\sin^2 2\theta \geqslant 0.82$ with a mass-squared difference $\Delta m^2_{\rm atm} \sim (5\times 10^{-4} - 6\times 10^{-3})\ {\rm eV}^2$. Furthermore, recent result of SuperKamiokande experiment disfavors any large mixing between purely ν_μ and ν_s (sterile neutrino) at 99% C.L. [2]. The solar neutrino experimental results [3] are also in concordance with the interpretation of atmospheric neutrino experimental result and the data provide the following values as $\Delta m^2_{e\mu} \sim (0.8-2) \times 10^{-5}\ {\rm eV}^2$, $\sin^2 2\theta \sim 1$ [large angle Mikheyev-Smirnov-Wolfenstein (MSW) solution] or $\Delta m^2_{e\mu} \sim (0.5-6) \times 10^{-10}\ {\rm eV}^2$, $\sin^2 2\theta \sim 1$ (vacuum oscillation solution). Furthermore, the CHOOZ experimental result [4] gives the value of $\Delta m^2_{eX} < 10^{-3}\ {\rm eV}^2$ or $\sin^2 2\theta_{eX} < 0.2$. In order to reconcile with the solar and atmospheric neutrino experimental results, a possible explanation known as bimaximal neutrino mixing is advocated [5], in which $\theta_{12} = \theta_{23} = 45°$, and if, the CHOOZ experimental result is interpreted in terms of ν_e-ν_τ oscillation, then $\theta_{31} < 13°$. Another scenario could still be possible if the solar neutrino experimental result is explained in terms of small angle MSW solution, however, we have not addressed this scenario in the present work. In the present work, we propose a texture of Majorana-type neutrino mass matrix in terms of only two parameters considering only three generations of neutrinos. Two parameter texture of neutrino mass matrix has also been discussed earlier [6–8]. In Ref. [6], with three light neutrinos, different zeroth order textures of both neutrino and charged lepton mass matrices has been proposed in view of the solar and atmospheric neutrino experimental results advocating the implication of flavor symmetry. A detailed analysis is found in Ref. [7] where the implication of L_e-L_μ-L_τ symmetry has been discussed to realize light neutrino mass both via seesaw mechanism and low-energy effective theory. An investigation in this path has also been done in Ref. [8] through the introduction of a partially conserved chiral $U(1)_{f_1} \times U(1)_{f_2}$ symmetry with the standard model gauge group to generate both quark and lepton mass matrices. Apart from the successful description of quark and lepton mass matrices, however, in this model a large value of Higgs coupling of the term of dimension greater than 4 is needed to avoid the conflict between the minimization condition of the Higgs potential and the choice of low value of the vacuum expectation value (VEV) of an $SU(2)_L$ triplet Higgs field when vacuum oscillation solution of solar neutrino problem is considered in addition with the atmospheric neutrino experimental result. This problem is avoided in the present model by discarding any hard (dim $\geqslant 4$) discrete symmetry violating term in the scalar potential.

In this work, we propose an explicit pattern of two parameter texture of neutrino mass matrix which gives rise to nearly bimaximal neutrino mixing and also can accommodate the required mass-squared differences to explain the solar (by large angle MSW solution or by vacuum oscillation) and atmospheric neutrino experimental results. Next, we demonstrate an explicit realization of the proposed texture within the framework of an $SU(2)_L \times U(1)_Y$ model with an extended Higgs sector and discrete symmetry. The plan of the paper is as follows. Section II contains the proposed neutrino mass-matrix and its phenomenology. A model accomplishes the proposed mass matrix is presented in Sec. III. Section IV contains a summary of the present work.

II. NEUTRINO MASS MATRIX

Before going into the details, first of all, we consider the charged lepton mass matrix is diagonal in flavor space. Consider now the following Majorana-type neutrino mass matrix with the basis of the leptonic fields (l_{1L}, l_{2L}, l_{3L}) [where l_{iL} have (2,1) quantum numbers under $SU(2)_L \times U(1)_Y$ gauge group, i is the generation index]

$$M_\nu = \begin{pmatrix} 0 & a & a \\ a & 0 & b \\ a & b & 0 \end{pmatrix}, \tag{1}$$

where a and b are two real model independent parameters

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and we consider $a\neq b$ so that M_ν contains at least two parameters. Also it is to be noted that the absence of $\nu_e\nu_e$ mass term in the above neutrino mass matrix evades the bound on the Majorana-type neutrino due to $\beta\beta_{0\nu}$ decay. Moreover, the above texture admits no observable *CP* violating effect in the leptonic sector as the number of parameters is only 2. The phases of a and b could easily be rotated away by redefining the leptonic fields. The elements of M_ν can be generated either by radiative mechanism or by nonrenormalizable mass operators. We have not addressed here the see-saw-type mass generation because in that case a judicious choice of Dirac-type neutrino mass matrix is necessary. Diagonalizing the neutrino mass matrix M_ν by an orthogonal transformation as $O^T M_\nu O = M_D = \mathrm{diag}(-m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$, where

$$O=\begin{pmatrix} c_{31}c_{12} & c_{31}s_{12} & s_{31} \\ -c_{23}s_{12}-s_{23}s_{31}c_{12} & c_{12}c_{23}-s_{23}s_{31}s_{12} & s_{23}c_{31} \\ s_{23}s_{12}-c_{23}s_{31}c_{12} & -s_{23}c_{12}-s_{31}s_{12}c_{23} & c_{23}c_{31} \end{pmatrix}, \tag{2}$$

we obtain the following values of the mixing angles:

$$\theta_{23}^{\nu}=-\frac{\pi}{4},\quad \theta_{31}^{\nu}=0,\ \tan^2\theta_{12}^{\nu}=\frac{m_{\nu_1}}{m_{\nu_2}} \tag{3}$$

and the eigenvalues of the above mass matrix comes out as

$$-m_{\nu_1}=\frac{b-x}{2},$$

$$m_{\nu_2}=\frac{b+x}{2},$$

$$-m_{\nu_3}=b, \tag{4}$$

where $x=\sqrt{b^2+8a^2}$. The sign of m_{ν_1} and m_{ν_2} can be made positive by redefining lepton doublet fields. Furthermore, in terms of the three eigenvalues m_{ν_1}, m_{ν_2} and m_{ν_3}, the mixing matrix O can be written as

$$O=\begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$=\begin{pmatrix} \sqrt{\frac{m_{\nu_2}}{m_{\nu_1}+m_{\nu_2}}} & \sqrt{\frac{m_{\nu_1}}{m_{\nu_1}+m_{\nu_2}}} & 0 \\ -\frac{1}{\sqrt{2}}\sqrt{\frac{m_{\nu_1}}{m_{\nu_1}+m_{\nu_2}}} & \frac{1}{\sqrt{2}}\sqrt{\frac{m_{\nu_2}}{m_{\nu_1}+m_{\nu_2}}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}\sqrt{\frac{m_{\nu_1}}{m_{\nu_1}+m_{\nu_2}}} & \frac{1}{\sqrt{2}}\sqrt{\frac{m_{\nu_2}}{m_{\nu_1}+m_{\nu_2}}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{5}$$

In the limit $b\to 0$, $\theta_{12}^{\nu}\to\pi/4$, the two eigenvalues m_{ν_1} and m_{ν_2} become degenerate and we can achieve the exact bimaximal neutrino mixing. In this situation, although we obtain the exact bimaximal neutrino mixing, however, the obtained eigenvalues $m_{\nu_1}=m_{\nu_2}$ and $m_{\nu_3}=0$, can be fitted with either the solar or the atmospheric neutrino experimental result. Removal of degeneracy between the two eigenvalues require further higher order corrections. For our analysis, we set the value of $\Delta m_{21}^2=\Delta m_{\mathrm{sol}}^2$ which in turn sets the value of θ_{12}^{ν}. The value of x depends on the hierarchical relation between a and b parameters which is manifested from the values of

$$\Delta m_{21}^2=bx \tag{6}$$

and

$$\Delta m_{23}^2=\frac{1}{4}(3b+x)(x-b). \tag{7}$$

Now, if, $b^2\gg 8a^2$, then the value of x comes out as $x\simeq b$ and $\Delta m_{23}^2\simeq 0$, $\Delta m_{21}^2\simeq b^2$, hence, in this case it is not possible to accommodate both the results of solar and atmospheric neutrino experiments. Thus, for a phenomenologically viable model, we have to consider the hierarchy $8a^2\gg b^2$ and in this case m_{ν_1} has also become positive. The pattern of neutrino mass is presented in Fig. 1. In this situation, we obtain $\Delta m_{21}^2\simeq 2\sqrt{2}ab$, $\Delta m_{23}^2\simeq 2a^2$. For a typical value of $\Delta m_{23}^2 \simeq 4\times 10^{-3}\ \mathrm{eV}^2$ which can explain the atmospheric neutrino deficits, we obtain $2a^2\simeq 4\times 10^{-3}\ \mathrm{eV}^2$. For a typical value of $\Delta m_{21}^2\simeq 4\times 10^{-10}\ \mathrm{eV}^2$ which can explain the solar neutrino deficits due to vacuum oscillation, the value of b^2 comes out as $b^2\sim 10^{-17}\ \mathrm{eV}^2$ whereas for the large angle MSW solution

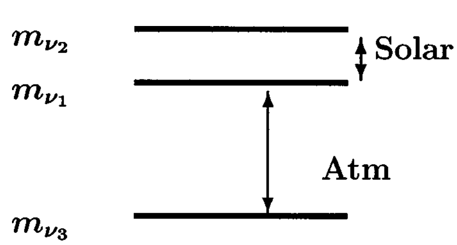

FIG. 1. Neutrino mass spectrum in the present model.

a typical value of $\Delta m_{21}^2 \sim 10^{-5}\,\mathrm{eV}^2$ the value of b^2 comes out of the order of $10^{-9}\,\mathrm{eV}^2$. The mixing angle θ_{12}^{ν} comes out as $\tan^2\theta_{12}^{\nu} \simeq (2a\sqrt{2}-b)/(2a\sqrt{2}+b)$ and since $a \gg b$, $\theta_{12}^{\nu} \to 45°$, and, hence, there is no conflict to satisfy the value of θ_{12}^{ν} well within the allowed range of the experimental value.

III. A MODEL

In this section, we demonstrate an explicit realization of the above neutrino mass matrix as well as a flavor diagonal charged lepton mass matrix within the framework of an $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ model with two singlet Higgs fields and discrete $Z_3 \times Z_4$ symmetry. The charged masses are arising in a similar way to the standard model (SM) whereas neutrino masses are generated through nonrenormalizable operators. We have also discussed the situation when the mixing is exactly bimaximal. Instead of three almost degenerate neutrinos [9,10], we obtain a hierarchical pattern of neutrino masses. To obtain a realistic low-energy phenomenological model, several attempts have been made through the inclusion of discrete symmetry [11]. Recently, it has been shown [12] that non-Abelian discrete groups (such as dihedral groups D_n, dicyclic groups Q_{2n}) plays an attractive role to obtain required mixing pattern in the fermionic sector. A recent work in this path has been done [13] through the inclusion of $\mathrm{U}(1)\times Z_2$ symmetry in the flavor space to explain both the quark and leptonic sector mixing angles. Although the question of embedding such symmetries under a large symmetry is still open, nevertheless, to understand from the low-energy point of view, inclusion of discrete symmetry and extra matter fields is an attractive way. The discrete $Z_3 \times Z_4$ symmetry prohibits unwanted mass terms in the charged lepton and neutrino mass matrices in the present model. We consider soft discrete symmetry breaking terms in the scalar potential, which are also responsible to obtain nonzero values of the VEV's of the Higgs fields upon minimization of the scalar potential. It is to be noted that in order to avoid conflict between the choice of VEV's of the Higgs fields (ρ and ξ) with the minimization condition of the Higgs potential, we discard any hard discrete symmetry breaking term in the Higgs potential. Discrete symmetry invariant soft or hard terms will not cause hierarchical problem as addressed in Ref. [8]. The Majorana neutrino masses are obtained due to explicit breaking of lepton number through higher-dimensional terms. The representation content of the leptonic fields and Higgs fields considered in the model is given in Table I. Apart from the standard model doublet h Higgs field, we introduced another two singlet Higgs ξ and ρ fields to obtain two independent parameters for the neutrino sector.

The most general lepton-Higgs Yukawa interaction in the present model generating Majorana neutrino masses is given by

$$L_Y^{\nu} = \frac{(l_{1L}l_{2L})hh\rho}{M_f^2} + \frac{(l_{1L}l_{3L})hh\rho^*}{M_f^2} + \frac{(l_{2L}l_{3L})hh\xi^2}{M_f^3} \qquad (8)$$

TABLE I. Representation content of the lepton and Higgs fields considered in the present model. The generators of Z_3 and Z_4 groups are ω and i, respectively.

Fields	$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	Z_3	Z_4
leptons			
l_{1L}	$(2,-1)$	1	1
l_{2L}	$(2,-1)$	ω	$-i$
l_{3L}	$(2,-1)$	ω	i
e_R	$(1,-2)$	$\omega^{\star}$	1
μ_R	$(1,-2)$	1	$-i$
τ_R	$(1,-2)$	1	i
Higgs			
h	$(2,1)$	ω	1
ρ	$(1,0)$	1	i
ξ	$(1,0)$	ω	-1

and the Yukawa interaction which is responsible for generation of charged lepton masses is given by

$$L_Y^E = f_1 l_{1L}^- e_R h + f_2 l_{2L}^- \mu_R h + f_3 l_{3L}^- \tau_R h + \mathrm{H.c.} \qquad (9)$$

We consider ρ is a complex scalar field whereas ξ is a real scalar field. The present model contains a large mass scale M_f, and for our analysis we set $M_f \sim M_{\mathrm{GUT}}$ which is the highest scale considered in the present model. The VEV's, $\langle\xi\rangle$ and $\langle\rho\rangle$ are constrained by the solar and atmospheric neutrino experimental results.

In order to avoid any zero values of the VEV's of the Higgs fields upon minimization of the scalar potential, we have to consider discrete symmetry breaking terms. Without going into the details of the scalar potential, this feature can be realized in the following way. In general, the scalar potential can be written as (keeping upto dim=4 terms)

$$V = Ay^4 + By^3 + Cy^2 + Dy + E, \qquad (10)$$

where y is the VEV of any Higgs field and A, B, C, D, E are generic couplings of the terms contained in the scalar potential. Minimizing the scalar potential with respect to y, we obtain

$$V' = A'y^3 + B'y^2 + C'y + D. \qquad (11)$$

Equation (10) reflects the fact that as long as $D \neq 0$, and A' or B' or C' is not equal to zero, we will get nonzero solutions for y. Thus, in order to obtain $y \neq 0$ solution, it is necessary to retain the terms with generic coefficients D and A' or B' or C'. In the present model, both the discrete symmetry breaking terms soft and hard, correspond to the term with coefficient D. Discarding hard symmetry breaking terms, we retain soft discrete symmetry breaking terms, and, hence, none of the VEV is zero upon minimization of the scalar potential.

Let us look at the leptonic sector of the present model. Substituting the VEV's of the Higgs fields appearing in Eq. (9), we obtain flavor diagonal charged lepton mass matrix as

$$M_E = \begin{pmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{pmatrix}, \tag{12}$$

where $d = f_1\langle h\rangle$, $e = f_2\langle h\rangle$ and $f = f_3\langle h\rangle$ and substituting the VEV's of ξ, h and ρ Higgs fields in Eq. (8), we get the Majorana-type neutrino mass matrix as follows:

$$M_\nu = \begin{pmatrix} 0 & a & a \\ a & 0 & b \\ a & b & 0 \end{pmatrix}, \tag{13}$$

where[1] $a = \langle h\rangle^2\langle\rho\rangle/M_f^2$, $b = \langle\xi\rangle^2\langle h\rangle^2/M_f^3$. The parameter a can be fitted with the value $\Delta m_{23}^2 \simeq 2a^2 \simeq 4\times10^{-3}\,\mathrm{eV}^2$ which explains atmospheric neutrino experimental data by setting $M_f \sim 10^{12}\,\mathrm{GeV}$, $\langle h\rangle \simeq 174\,\mathrm{GeV}$, and $\langle\rho\rangle \simeq 10^{11}\,\mathrm{GeV}$. Using the same values of M_f and $\langle h\rangle$, it is possible to set the value of b as $b^2 \simeq 10^{-17}\,\mathrm{eV}^2$ through the choice of $\langle\xi\rangle \simeq 10^7\,\mathrm{GeV}$ in order to explain the solar neutrino experimental results due to vacuum oscillation solution. For both the cases, the mixing angle θ_{12}^ν [given in Eq. (3)] comes out as nearly maximal. For large angle MSW solution, a typical value $\Delta m_{21}^2 \sim 10^{-5}\,\mathrm{eV}^2$ gives rise to $b^2 \simeq 10^{-9}\,\mathrm{eV}^2$ for $\langle\xi\rangle \simeq 2\times10^9\,\mathrm{GeV}$.

[1]The discrete symmetry invariant $\nu_e\nu_e$ mass term appears in the present model at M_f^5 order which is naturally vanishingly small.

IV. SUMMARY

In summary, we propose a texture of Majorana-type neutrino mass matrix which gives rise to nearly bimaximal neutrino mixing in a natural way as well as required mass-squared differences in order to explain the solar and atmospheric neutrino experimental results. The elements of the mass matrix could be generated either by radiative mechanism or by the use of nonrenormalizable operators and, thus, those elements are model independent. The proposed neutrino mass matrix gives rise to the eigenvalues of the three neutrino masses as $m_{\nu_1} \simeq m_{\nu_2} \gg m_{\nu_3}$ which ends up to an hierarchy between three neutrino mass-squared differences as $m_{23}^2 \gg m_{21}^2$. We demonstrate an explicit realization of the proposed mass-matrix due to nonrenormalizable mass operators in the context of an $SU(2)_L\times U(1)_Y$ model through the inclusion of two extra singlet Higgs fields and discrete $Z_3\times Z_4$ symmetry. With a suitable choice model parameters the required mass-squared differences can be accommodated in order to expalin the solar (both large angle MSW solution and Vacuum oscillation) and atmospheric neutrino experimental results.

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