Hidden dimensions of the large scale universe and isotropy of the cosmic microwave background radiation

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(Received 28 February 2000; published 25 September 2000)

It is suggested to resolve the contradiction between the two main cosmological observations of the high isotropy of the cosmic microwave background and fractal structure of the large-scale universe by consideration of a hidden support dimension of the multifractal space-luminous distribution of visible matter in the Perseus-Pisces redshift survey. It is shown that while a simple set given by the galaxy space positions has a support dimension $D_0 \simeq 2$ inclusion of the galaxy mass (luminosity) leads to a multifractal distribution which can be characterized by two different *support* dimensions. One of them (corresponding to comparatively rare visible matter) is close to two, whereas the second (corresponding to comparatively dense visible matter) is close to three. The crossover between these two states can be considered as a morphological phase transition.

PACS number(s): 98.80.Cq, 98.70.Vc, 98.80.Hw

I. INTRODUCTION

The high isotropy of the cosmic microwave background radiation (CMBR) and fractal nature of galaxy distribution in the large scale universe are among the main facts in modern cosmology from an experimental point of view. For some recent three-dimensional catalogues the space distribution of galaxies and clusters demonstrates stable value of the space support dimension $D \simeq 2$ up to the largest space scales (see for a review Ref. [1]). Since the basic space properties of the cosmic microwave backgrownd radiation (CMBR) are related to those of galaxy distribution in the large scales (see for instance Refs. [2,3], and references therein) we have a significant contradiction between these two main cosmological observations. Hypothetic dark matter could be a means to resolve this contradiction. One can merely assume that the dark matter is uniformly distributed in the universe [1]. However, before using this ''panacea'' one may try to find a solution in the properties of visible matter itself. In fact, the observed value $D \simeq 2$ corresponds to a simple set given by the galaxy space positions. It is clear, however, that interaction between visible matter and CMBR depends not only on the galaxy space positions but also on the galaxy masses. It is recently shown [1] that the inclusion of the galaxy mass (luminosity) leads to a distribution with well defined *multifractal* properties. It will be shown below that while the space *support* dimension of the multifractal space-luminosity measure, D_0, is still close to two, there exists a hidden *support* dimension D_h (corresponding to high dense visible matter), which is approximately equal to three.

While the high value of the hidden *support* dimension corresponds to high dense visible matter or (in multifractal terms) to high order generalized dimensions, the high order generalized dimensions themselves are comparatively small (D_q decreases with q, see Fig. 1). The highest fluctuations are supposed to be located in the largest clusters. This is actually what happens for elliptical galaxies which are located in the cores of the largest clusters. Given this situation one expects that elliptical galaxies are more ''strongly clustered'' than field galaxies (or spirals). Such a situation corresponds to small ''fractal dimensions'' for elliptical galaxies than for field galaxies. Therefore the question is: What dimension: the effective (hidden) support dimension or corresponding D_q dimensions, represents the high dense visible matter in its interaction with the CMBR? This rather nontrivial and crucial question inquires a physical theory describing interaction of the CMBR with visible matter, while the above described result is obtained using only analytic properties of the observed space-luminosity measure (see also the Appendix). Nevertheless we hope that the observation represented in Fig. 1 can be considered as a step toward the matching of the two crucial experimental facts. Moreover, coexistence of the two support dimensions could be a reason of the weak anisotropy observed in the CMBR [4,5].

Finally, it should be noted that different methods are used for fractal calculations related to cosmological catalogs and for different ranges of scales in these catalogs (see for a review Ref. [1]). A vigorous discussion about applicability of these methods has taken place in the recent decade. The

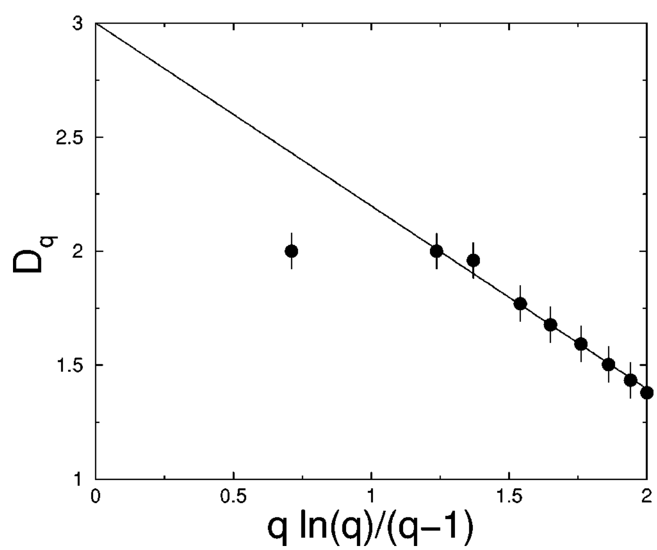

FIG. 1. The generalized dimension D_q against $q \ln q/(q-1)$ for the Perseus-Pisces redshift survey (data taken from Ref. [12]). The solid straight line indicates agreement between the data and representation (20) with $D_h \simeq 3$.

homogeneity is considered by several authors as a criterion of a fair cosmological sample. But, as shown in recent papers discussed in Ref. [1], to consider a sample as a fair one we only need enough number of points to derive some statistical properties unambiguously. Moreover, using of weighting schemes and treatment of boundary conditions can lead to misinterpretation of the data, in particular in respect of homogeneity. In the present paper, we use the data obtained in Ref. [12] for the Perseus-Pisces survey without using any weighting scheme or treatment of boundary conditions, and only a limiting effective depth of the Perseus-Pisces survey (which was estimated in Ref. [12] to be ~ 30 Mpc/h) is a restriction for applicability of this analysis (authors of Ref. [12] eliminated from the statistics the points for which a sphere of radius $r > 30$ Mpc/h is not fully included within the sample boundaries). Tests were also performed to check applicability of this analysis to be considered samples [1,12] and this data can be considered among the best 3D data now available.

II. ANALYTIC PROPERTIES OF MULTIFRACTAL MEASURE

For multifractal distribution, the scaling properties can be different for different regions of the system. Let us introduce the multifractal measure following Ref. [1]. Suppose that the total volume of the sample consists of a cube size L. The density distribution of visible matter is described by

$$\rho(\mathbf{x}) = \sum_{i=1}^{N} m_i \delta(\mathbf{x} - \mathbf{x}_i) \tag{1}$$

where m_i is the mass of the galaxy (proportional to its luminosity), N is the number of points in the sample, and $\delta(\mathbf{x})$ is the Dirac delta function. The dimensionless normalized density function is then defined as

$$\mu(\mathbf{x}) = \sum_{i=1}^{N} \mu_i \delta(\mathbf{x} - \mathbf{x}_i) \tag{2}$$

with $\mu_i = m_i/M$, where M is the total mass of the sample. We divide the volume into boxes of linear size l and label each box by the index i. Then we can construct the function

$$\mu_i(r) = \int_{i\text{th } box} \mu(\mathbf{x}) d\mathbf{x} \tag{3}$$

where dimensionless variable $r = l/L$. The definition of the box-counting fractal dimension α is

$$\lim_{r\to 0} \mu_i(r) \sim r^{\alpha(\mathbf{x})} \tag{4}$$

where $\alpha(\mathbf{x})$ is constant (fractal) dimension D_0 in all the occupied boxes in the case of a simple fractal. This exponent fluctuates widely with the position $\mathbf{x}$ in the case of multifractal. In general we will find several boxes with a measure that scales with the same exponent α. These boxes form a fractal subset with dimension $f = f(\alpha)$. Hence the number of boxes that have a measure μ that scales with exponent in the range $[\alpha, \alpha + d\alpha]$ varies with r as

$$N(\alpha, r) d\alpha = \rho(\alpha) r^{-f(\alpha)} d\alpha \tag{5}$$

where $\rho(\alpha)$ is some function on α but not on r. The function $f(\alpha)$ is usually a single humped function with the maximum at $\max_\alpha f(\alpha) = D_0$, where D_0 is the dimension of the space support. In the case of a single fractal, the function $f(\alpha)$ is reduced to a single point $f(\alpha) = \alpha = D_0$.

One can also introduce the partition function [6]

$$Z_q = \sum_{i=1}^{N} \mu_i^q . \tag{6}$$

It follows from Eqs. (3)–(6) that

$$Z_q = \int \rho(\alpha) r^{q\alpha - f(q)} d\alpha. \tag{7}$$

In the multifractal case Z_q scales as

$$Z_q \sim r^{\tau(q)}. \tag{8}$$

In the limit $r \to 0$ the sum (7) is dominated by the term $e^{\min_\alpha [q\alpha - f(\alpha)]}$. Then from Eqs. (7) and (8) one obtains

$$\tau(q) = \min_\alpha [q\alpha - f(\alpha)]. \tag{9}$$

Thus, the $\tau(q)$ is obtained by Legendre transforming the $f(\alpha)$. When $f(\alpha)$ and $\tau(q)$ are smooth functions, the relationship (9) can be rewritten in the following way

$$\tau(q) = q\alpha - f(\alpha), \quad \frac{df}{d\alpha} = q. \tag{10}$$

The thermodynamic interpretation of these relationships means [6] that q can be interpreted as an inverse temperature $q = T^{-1}$ and the limit $r \to 0$ can be seen as the thermodynamic limit of infinite volume $[V = \ln(1/r) \to \infty]$. Then by identifying $\alpha_i = \ln \mu_i / \ln(1/r)$ to the energy E_i (per unit volume) of a microstate i, one can rewrite the partition function under the familiar form

$$Z_q = \sum_i \exp(-qE_i). \tag{11}$$

From the definition: $f(\alpha) = \ln N_\alpha(r)/\ln(1/r)$, the function $f(\alpha)$ plays the role of the entropy (per unit volume).

One can use expansion of the multifractal entropy $f[\alpha(q)]$ in power series (the high-temperature expansion)

$$f(q) = f(0) + q\left(\frac{df}{dq}\right)\bigg|_{q=0} + \frac{1}{2} q^2 \left(\frac{d^2 f}{dq^2}\right)\bigg|_{q=0} + \cdots. \tag{12}$$

It is known that entropy can have singularities in the complex temperature plane (see, for instance, Refs. [7–9], and references therein). If the multifractal entropy $f(q)$ has singularities in the complex q-plane, then the radius of convergence of the *real* Maclaurin series expansion (12) is determined by the distance from the point $q = 0$ to a nearest singularity of $f(q)$ in the complex plane. One can then use the standard procedure of analytic continuation to obtain power series expansions beyond the circle of convergence of the expansion (12)

$$f(q)=f(q_0)+(q-q_0)\left(\frac{df}{dq}\right)\bigg|_{q=q_0}+\frac{1}{2}(q-q_0)^2\left(\frac{d^2f}{dq^2}\right)\bigg|_{q=q_0}+\cdots \tag{13}$$

where q_0 is the modulus of the nearest to point $q=0$ complex-temperature singularity.

If one takes finite number of terms in the series (13): N, then one could also rewrite the series in a form similar to (12)

$$f(q)=a_0+a_1q+\cdots+a_Nq^N. \tag{14}$$

The coefficients $a_0,\ a_1,\ldots,a_N$ depend on N. Since the Maclaurin expansion (12) in the interval $0<q<q_0$ is unique, the coefficients a_n must diverge for sufficiently large N. If, however, finite

$$\lim_{n\to\infty}a_0=f(q_0)-q_0\frac{df}{dq}\bigg|_{q=q_0}+\frac{1}{2}q_0^2\frac{d^2f}{dq^2}\bigg|_{q=q_0}+\cdots \tag{15}$$

do exist, then this value ($\lim_{n\to\infty}a_0$) could be interpreted geometrically as a hidden support dimension D_h [cf. Eq. (12) and recall that $f(0)=D_0$]. Moreover, if $q_0>1$, then this hidden support dimension corresponds to the localized areas with *high* density [10] of the visible matter. In the case when finite $\lim_{n\to\infty}a_1=A<\infty$ also exists one can use approximation

$$f(q)\simeq D_h+Aq \tag{16}$$

in some vicinity of q_0.

Using Eq. (10) it is easy to show that $\tau(q)$ corresponding to Eq. (16) is

$$\tau(q)\simeq -D_0+(C-A)q+Aq\ln q \tag{17}$$

where C is some constant.

The generalized dimension spectrum D_q can be defined as follows [11]:

$$D_q=\lim_{r\to 0}\frac{\ln Z_q}{(q-1)\ln r}. \tag{18}$$

Then

$$D_q=\frac{\tau_q}{(q-1)} \tag{19}$$

and from Eqs. (17),(19) we obtain

$$D_q\simeq D_h+A\frac{q\ln q}{(q-1)} \tag{20}$$

in some vicinity of $q=q_0$. If $D_0<D_h$ and both of them are integers, then we have a morphological phase transition.

III. COMPARISON WITH 3D DATA AND DISCUSSION

Figure 1 shows the generalized dimensions spectrum D_q against $q\ln q/(q-1)$ computed using Perseus-Pisces redshift survey [12]. The solid straight line indicates agreement between these data and representation (20) for sufficiently large q. Intersection of this line with the vertical axis indicates that $D_h\simeq 3$ for this case. On the other hand, one can see from Fig. 1 that $D_0\simeq 2$ (see also Ref. [1]), and therefore, we can expect the morphological phase transition from the two-dimensional distribution of comparatively rare luminous matter to distribution which has *support* dimension approximately equal to 3 for comparatively dense luminous matter. And indeed, the data behavior in this figure seems to be consistent with this expectation.

Possibility of dynamical character of the space dimension related to mass distribution has been discussed in the string theory (see, for instance, [2,13–15], and references therein). The relation between the background radiation and distribution of the visible matter requires a complex theory [1,2] and the above obtained result can be considered as a step toward reconciliation between these two cosmological observations: high isotropy of the CMBR and the value $D_0\simeq 2$ for the space support dimension. Moreover, the observed weak anisotropy of the CMBR could be related to coexistence of the two different *support* dimensions (2 and 3) for visible matter. The problem with elliptical galaxies discussed in the Introduction shows, however, that significant theoretical and experimental efforts should be made to perform complete reconciliation between these two basic observations.

ACKNOWLEDGMENTS

The author is grateful to D. Stauffer for discussions and encouragement, to the Machanaim Center (Jerusalem), and to the Graduate School of Science and Engineering of the Chuo University (Tokyo) for support.

APPENDIX

To show more clear appearance of the hidden dimension let us consider multifractality of a strange attractor of the baker map for which analytical results are available. This transformation is defined as

$$[x_{n+1},y_{n+1}]=[l_1x_n,y_n/\eta],\quad y_n<\eta, \tag{A1}$$

$$[x_{n+1},y_{n+1}]=[\tfrac{1}{2}+l_2x_n,(y_n-\eta)/(1-\eta)],\quad y_n>\eta. \tag{A2}$$

The attractor of this map consists of an infinite number of lines in the y direction which intersect a horizontal line in two interwoven Cantor sets. These sets are characterized by contraction rates l_1 and l_2, and are visited with probability η and $1-\eta$, respectively. The dimension spectrum D_q of the cross section follows from

$$\frac{\eta^q}{l_1^{(q-1)D_q}}+\frac{(1-\eta)^q}{l_2^{(q-1)D_q}}=1. \tag{A3}$$

If we introduce the following definitions

$$\eta^q = a, \quad (1-\eta)^q = b, \quad l_2^{(q-1)D_q} = G,$$

and $\ln l_1/\ln l_2 = k$, then we can rewrite Eq. (A3) as the following equation:

$$G^k - bG^{(k-1)} - a = 0. \tag{A4}$$

From this equation one obtains

$$\frac{dG}{dq} = \frac{da/dq + (db/dq)G^{(k-1)}}{kG^{(k-1)} - b(k-1)G^{(k-2)}}.$$

Hence dG/dq has a singularity when $G = b(k-1)/k$. Substituting this relationship into Eq. (A4) we obtain value of $q = q_0$ for which dG/dq has a singularity

$$q_0 = \frac{\ln c}{\ln[\eta/(1-\eta)^k]} \tag{A5}$$

where

$$c = -\frac{(k-1)^{(k-1)}}{k^k}. \tag{A6}$$

The constant $c > 0$ only when $(k-1) = -1/n$, where $n = 3,5,7, \ldots$. For these specific values of k the values of q_0 are real numbers and, dG/dq (and, consequently, dD_q/dq) have singularities on the real axis. If we take into account that

$$f(q) = D_q + q(q-1)\frac{dD_q}{dq} \tag{A7}$$

then we can claim that the entropy $f(q)$ of the baker map has real singularities at these specific values of k. In the general case, however, these singularities are complex.

Let us consider an example with concrete values of $\eta = 0.6$, $l_1 = 0.25$, and $l_2 = 0.4$. For this case we obtain from Eqs. (A5) and (A6) a complex value of $q_0 \simeq -1.1 + i3.6$ and, consequently $|q_0| \simeq 3.8$. This is a value of the radius of convergence of the high-temperature Taylor series expansion (12) for the baker map. The corresponding radius of convergence of the finite-temperature Taylor series expansion (13) is $R \simeq 5.8$, so that the interval of applicability of the finite-temperature expansion is approximately $0 < q < 10$. For the first order approximation we should exclude a narrow vicinity of the point $q = 0$ from this interval. Figure 2 shows a set of values D_q for this situation. The axes on this figure are chosen for comparison with the first order approximation of the finite-temperature expansion (20). The straight line in this figure corresponds to this approximation and one can see good agreement between the data and the approximation (20). The hidden support dimension D_h has also been indicated in this figure.

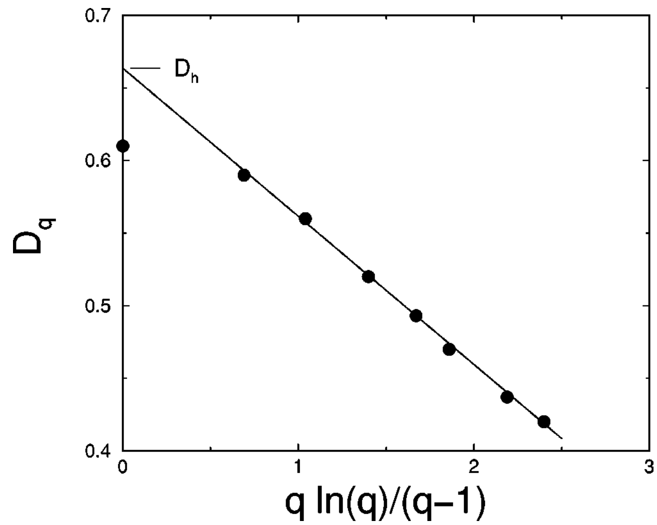

FIG. 2. The generalized dimensions D_q against $q \ln q/(q-1)$ for a strange attractor of the baker map. The solid straight line indicates agreement between the data and representation (20).

It is clear that there can also exist multifractals without complex-temperature singularities, and in each concrete case we can check the presence of the complex-temperature singularities analytically (if it is possible), or by using graphical representation D_q versus $q \ln(q)/(q-1)$ as is shown in Figs. 1 and 2.

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