D0- and D1-branes with κ_- and κ_+ extended symmetry

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D0-brane (D-particle) and D1-brane actions possess first and second class constraints that result in local κ symmetry. The κ symmetry of the D-particle and the D1-brane is extended here into a larger symmetry (κ_- and κ_+) in a larger phase space by turning second class constraints into first class constraints. Different gauge fixings of these symmetries result in different presentations of these systems while a ''unitary'' gauge fixing of the new κ_+ symmetry retrieves the original action with $\kappa_- = \kappa$ symmetry. For a D1-brane our extended phase space makes all constraints into first class constraints in the case of a vanishing world sheet electric field [namely, (0,1) string].

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I. INTRODUCTION

An important ingredient in the study of D-brane [1] dynamics is their local fermionic symmetry on the worldvolume, the κ symmetry. The history of this symmetry goes back to the superparticle action [2] where it was identified [3,4] and applied to the superstring [5]. It was used also in the study of super p-branes [6] in different dimensions. The role of the κ symmetry was further emphasized in the study of the D-branes embedded in flat 10D space-time in Refs. [7,8]. The symmetry is generated by 16 irreducible first class fermionic constraints. These constraints are accompanied by another set of 16 second class fermionic constraints which do not correspond to any local symmetry. The covariant separation of the two types of constraints in the brane action was emphasized in Refs. [9,10] and enabled the covariant quantization of the D0- and D1-branes.

It has been found difficult to quantize covariantly the massless superparticle, as is the situation also with the Green-Schwarz formulation of the superstring [5], since in both systems first and second class constraints cannot be separated in a covariant manner. This is a long lasting problem and many attempts have been made to solve it [11,12]. In the massive superparticle action the κ symmetry is explicitly broken. Its first class constraints are replaced now by solvable second class constraints and the system can be quantized covariantly by means of Dirac brackets since all its constraints are second class. Since the massive superparticle can be quantized covariantly, one may be tempted to consider the massless limit of the massive case as a substitute for the covariant quantization of the massless superparticle. However, the Dirac brackets become singular in the $p^2 = m^2 \to 0$ limit. The restoration of the broken κ symmetry of the massive system in an extended phase space [13,14] by adding extra fermionic degrees of freedom was considered in Ref. [15]. Another possibility to restore the κ symmetry is to include a proper Wess-Zumino term in the action, as is the case with the D0-brane [7–10]. This is physically more interesting, but contains in addition to the first class constraints, second class constraints that correspond to the restored κ symmetry. When considering the massless limit, one finds the need to avoid these second class constraints since also here the Dirac brackets become singular in the massless limit. The restoration of symmetry with no second class constraints, gives the full advantage of working within a system with local symmetry. In particular a covariant wave function can be formulated also in the massless limit [15]. For this purpose, it is usually useful to turn the second class constraints into first class. This formulation offers a flexibility to allow various gauge fixings which are physically equivalent. At the same time, the newly introduced first class constraints generate a gauge symmetry which may give more insight into the geometrical structure of the system which is interesting in its own right.

Several other different approaches to this issue share in common the idea of adding extra dynamical degrees of freedom while extending the symmetry of the system in different manners. In the geometrical-superembedding approach, superbrane dynamics are manifestly supersymmetric on the worldvolume as well as in target superspace [16] and the auxiliary commuting spinors superpartners have twistorlike and Lorentz harmonics properties. This approach, which has a wide range of applications in several physical systems, has been developed for super p-branes and D-branes as well. Other treatments of second class constraints include extended phase space variables in Ref. [17] and, more recently, auxiliary commuting twistorlike spinor variables and tensorial central charge coordinates were used in Ref. [18]. Introducing Liouville mode while solving the second class constraints left a final action with only first class constraints in Ref. [19]. Other related approaches can be found in Refs. [20,21].

In the first part of this paper we suggest a new symmetric system for the D-particle in which the second class con-

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straints are turned into first class in an extended phase space which includes extra fermionic degrees of freedom. We define a system that contains $\theta_\alpha, \pi_\alpha$, the original fermionic degrees of freedom of the D0-brane to which extra fermionic degrees of freedom $\zeta_\alpha, \rho_\alpha$ are added ($\zeta_\alpha, \rho_\alpha$ are Majorana-Weyl spinors while $\theta_\alpha, \pi_\alpha$ are only Majorana). The new system has, in addition to the original $\kappa = \kappa_-$ symmetry a new local κ_+ symmetry. The system can be gauge fixed in many different ways while one of these gauge fixings (''unitary'' gauge) retrieves the original D0-brane. The rest of the paper presents, along the same lines, the D1-brane with an extended κ_- and κ_+ local symmetry. We consider the case of a vanishing electric field in the Born-Infeld-Nambu-Goto action.

II. SUPERPARTICLE AND D-PARTICLE

The $N=1$ massless superparticle action in $d=10$ space-time dimensions ([2,5]):

$$S = \int_{\tau_i}^{\tau_f} \mathcal{L}(\tau) d\tau = -\frac{1}{2}\int_{\tau_i}^{\tau_f} d\tau \frac{1}{e}(\dot{x}^\mu - i\overline{\theta}_+ \Gamma^\mu \dot{\theta}_+)^2 \qquad (1)$$

is invariant under the local κ symmetry:

$$\delta x_\mu = i\overline{\theta}_+ \Gamma_\mu \delta\theta_+, \quad \delta\theta_+ = (\dot{x}^\nu - i\overline{\theta}_+ \Gamma^\nu \dot{\theta}_+)\Gamma_\nu \kappa_-,$$

$$\delta e = 4ie\overleftarrow{\overline{\theta}}_+ \kappa_- \qquad (2)$$

$x^\mu (\mu = 0,1,\ldots,9)$ are space-time coordinates and θ_+ is a Majorana-Weyl spinor with positive (or negative) chirality. The spinor κ_- has the opposite chirality of θ_+ and $e(\tau)$ is the ''einbein'' of local reparametrization symmetry. The $32 \times 32 \Gamma^\mu$ matrices ($\mu = 0,1,2 \ldots,9$) are built out of the conventional spin (8) matrices[1] and satisfy $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$ and $\eta^{\mu\nu} = diag\{-+++ \ldots\}$.

The system has eight fermionic first class constraints and eight fermionic second class constraints and thus its phase space has $(32 - 2\times 8 - 8)$ eight independent fermionic degrees of freedom.

Local κ symmetry is explicitly broken in the $N=1$ massive superparticle action in $d=10$ dimensions [15]:

$$S = \int_{\tau_i}^{\tau_f} \mathcal{L}(\tau) d\tau = \int_{\tau_i}^{\tau_f} d\tau \left\{ -\frac{1}{2e}(\dot{x}^\mu - i\overline{\theta}_+ \Gamma^\mu \dot{\theta}_+)^2 + \frac{1}{2}em^2 \right\}. \qquad (3)$$

Here, using Eq. (2), one finds $\delta\mathcal{L} = 2iem^2 \overleftarrow{\overline{\theta}}_+ \kappa_- \neq 0$. All 16 constraints are second class and its phase space has $(32 - 16 =)16$ independent fermionic degrees of freedom.

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[1]Our conventions are $\Gamma^m = \sigma^1 \otimes \gamma^m$, $m = 1,2,\ldots 9$, $\Gamma^0 = -i\sigma^2 \otimes \mathcal{I}$, $\Gamma^{11} = \sigma^3 \otimes \mathcal{I}$, $\{\Gamma^{11}, \Gamma^\mu\} = 0$, $\Gamma^\mu = \begin{pmatrix} 0 & \overline{\gamma}^\mu \\ \gamma^\mu & 0 \end{pmatrix}$, $\mu = 0,1,2,\ldots 9$ $\overline{\gamma}^\mu = \{-1, \gamma^l\}$, $\gamma^\mu = \{1, \gamma^l\}$, $l = 1,2,\ldots 9$ γ^k $\{k = 1,2,\ldots 8\}$ are 16×16 spin(8) matrices, $\gamma^9 = \Pi_{k=1}^{k=8} \gamma^k$, $\{\gamma^m, \gamma^n\} = 2\delta^{m,n}$, $m,n = 1,2,\ldots 9$ $\overline{\gamma}^\mu \gamma^\nu + \overline{\gamma}^\nu \gamma^\mu = \gamma^\mu \overline{\gamma}^\nu + \gamma^\nu \overline{\gamma}^\mu = 2\eta^{\mu\nu}, \mu, \nu = 0,1,2,\ldots 9$.

One possible modification by which the local κ_- symmetry can be restored is extending its phase space to $N=2$ while adding an appropriate Wess-Zumino term:

$$\delta x_\mu = i\overline{\theta}\Gamma_\mu \delta\theta, \quad \delta\theta_+ = (\dot{x}^\nu - i\overline{\theta}\Gamma^\nu \dot{\theta})\Gamma_\nu \kappa_-,$$

$$\delta e = 4ie\overleftarrow{\overline{\theta}}_+ \kappa_- \qquad (4)$$

$$\mathcal{L} = -\frac{1}{2e}(\dot{x}^\mu - i\overline{\theta}\Gamma^\mu \dot{\theta})^2 + \tfrac{1}{2}em^2 + \mathcal{L}_2. \qquad (5)$$

Here $\theta = \theta_+ + \theta_-$ (θ is a Majorana spinor and θ_+ and θ_- are Majorana-Weyl spinors of opposite chirality) and $\delta\theta_-$ and $\mathcal{L}_2$ are to be determined below. From Eq. (4) one finds

$$\delta\mathcal{L} = \frac{2i}{e}(\dot{x}^\mu - i\overline{\theta}\Gamma^\mu \dot{\theta})^2 \overleftarrow{\overline{\theta}}_+ \kappa_- - \frac{2i}{e}(\dot{x}^\mu - i\overline{\theta}\Gamma_\mu \dot{\theta})$$

$$\times \overleftarrow{\overline{\theta}}\Gamma^\mu(\delta\theta_+ + \delta\theta_-) + 2iem^2 \overleftarrow{\overline{\theta}}_+ \kappa_- + \delta\mathcal{L}_2. \qquad (6)$$

$\delta\mathcal{L} = 0$ for a properly chosen $\mathcal{L}_2$. A possible solution of the form

$$\delta\mathcal{L}_2 = A_+ \delta\theta_- + B_- \delta\theta_+ \qquad (7)$$

gives $A_+ = -2im\overleftarrow{\overline{\theta}}_+$, $B_- = 2im\overleftarrow{\overline{\theta}}_-$ and $\delta\theta_- = em\kappa_-$ (up to a rescaling $A_+ \to A_+/\alpha$, $B_- \to B_- \alpha$ and $\delta\theta_- \to \delta\theta_- \alpha$)

$$\delta\mathcal{L}_2 = im\,\delta(\overleftarrow{\overline{\theta}}\Gamma^{11}\theta) - im\frac{d}{dt}(\delta\overline{\theta}\Gamma^{11}\theta) \qquad (8)$$

where

$$\Gamma^{11} = \sigma^3 * \mathcal{I} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \{\Gamma^{11}, \Gamma^\mu\} = 0$$

($\mathcal{I}$ is the 16×16 identity matrix). Thus,

$$\mathcal{L}(\tau) = -\tfrac{1}{2}e^{-1}(\dot{x}^\mu - i\overline{\theta}\Gamma^\mu \dot{\theta})^2 + \tfrac{1}{2}em^2 - im\overline{\theta}\Gamma^{11}\dot{\theta} \qquad (9)$$

has a restored κ_- symmetry. The system has now not only 16 first class constraints but also 16 second class constraints and the number of independent degrees of freedom in phase space is the same as the $N=1$ massive superparticle $(64 - 2\times 16 - 16 = 16)$. Indeed, when compared to the massive $N=1$ superparticle action in Eq. (3), the added negative chirality θ_- degrees of freedom (32 degrees of freedom in phase space; θ_- and their canonical conjugate $\overline{\pi}_+$) can be gauged away once the restored κ_- symmetry is gauge fixed $(\theta_- = 0)$. One is left, after gauge fixing, back with $\mathcal{L}(\tau)$ of the massive $N=1$ superparticle in Eq. (3).

A very appealing point of view on $\mathcal{L}(\tau)$ of Eq. (9) is obtained when one starts with the massless superparticle action in $d=11$ dimensions which is given by ([2–5])

$$S = \int_{\tau_i}^{\tau_f} \mathcal{L}(\tau) d\tau = -\frac{1}{2}\int_{\tau_i}^{\tau_f} d\tau e^{-1}(\dot{x}^{\hat{m}} - i\overline{\theta}\Gamma^{\hat{m}} \dot{\theta})^2 \qquad (10)$$

where $x^{\hat{m}}$ ($\hat{m}=0,1 \ldots 10$) are the space-time coordinates and $\theta_\alpha=\theta_{+\alpha}+\theta_{-\alpha}(\alpha=1,2, \ldots ,32)$ are the corresponding fermionic coordinates which can be regarded as two Majorana-Weyl spinors of opposite chiralities, if viewed as spinors in ten dimensions.

When one of the space directions is compactified [23] to a radius of $R=m^{-1}=Z^{-1}$, the $d=11$ massless superparticle action results [9,10] in the D0 brane action:

$$S=\int_{\tau_i}^{\tau_f}\mathcal{L}(\tau)d\tau$$
$$=\int_{\tau_i}^{\tau_f}d\tau\left\{-\frac{1}{2e}(\dot{x}^\mu-i\bar{\theta}\Gamma^\mu\dot{\theta})^2+\frac{1}{2}eZ^2-iZ\bar{\theta}\Gamma^{11}\dot{\theta}\right\}$$
$$+Z[x_{10}(\tau_f)-x_{10}(\tau_i)], \tag{11}$$

where p_{10} was set to $p_{10}=m=Z$, $\Gamma^{\widehat{10}}$ is defined as Γ^{11} and $\mu=0,1, \ldots 9$.

The D0 brane action in Eq. (11) is the same action obtained in Eq. (9) and its Wess-Zumino term $Z\bar{\theta}\Gamma^{11}\dot{\theta}$ establishes the local κ_- symmetry, which is the original symmetry of the $d=11$ massless superparticle action. Thus, instead of 32 second class constraints as in the $N=2$, $d=10$ massive superparticle action, the D0 has 16 first class constraints and 16 second class constraints which is the same number of constraints as the massless $N=2$, $d=10$ superparticle and here too the 16 first class constraints result in κ_- symmetry. An important difference between the D0 action and the massless superparticle is the fact that in the D0 case the first and second class constraints can be separated in a covariant manner [9,10]; this cannot be done for the massless $N=2$, $d=10$ superparticle.

We would like to treat now the D0 system in a more symmetrical manner by turning also its remaining 16 second class constraints into first class. The resulting system will have in addition to the original κ_- symmetry, a κ_+ symmetry generated by the new first class constraints. Among all possible different gauge fixing of the new κ_+ symmetry, one should also be able to retrieve the original D0 system, by appropriately gauge fixing (''unitary'' gauge fixing) the extended symmetric system.

After implementing the κ_+ extended symmetry into the system, the number of independent degrees of freedom should not change. Thus, one has to extend the phase space of the new, symmetric system by adding extra fermionic degrees of freedom to account for the increase of symmetry. In the following we define and summarize the properties of the κ_+,κ_- symmetric system. From Eq. (11) (ignoring the boundary term) or from

$$S=-Z\int_{\tau_i}^{\tau_f}d\tau\{\sqrt{-(\dot{x}^\mu-i\bar{\theta}\Gamma^\mu\dot{\theta})^2}+i\bar{\theta}\Gamma^{11}\dot{\theta}\} \tag{12}$$

one finds the constraints

$$\bar{T}_\alpha=\bar{\pi}_\alpha+i(\bar{\theta}\not{p})_\alpha+iZ(\bar{\theta}\Gamma^{11})_\alpha=0, \quad p^2+Z^2=0 \tag{13}$$

where $\bar{\pi}_\alpha$ is the momentum, canonical conjugate of θ_α (right handed derivatives are used when taking a derivative with respect to $\dot{\theta}_\alpha$).

The momentum is

$$p_\mu=Z\left\{\frac{\dot{x}_\mu-i\bar{\theta}\Gamma_\mu\dot{\theta}}{\sqrt{-(\dot{x}_\nu-i\bar{\theta}\Gamma_\nu\dot{\theta})^2}}\right\};$$

the Hamiltonian is $H_0=0$. Using the Poisson brackets

$$[x_\mu,p^\nu]=\delta_\mu^\nu, \quad [\theta^\alpha,\bar{\pi}_\beta]=\delta_\beta^\alpha \quad \text{all others}=0. \tag{14}$$

One finds

$$[\bar{T}_\alpha,\bar{T}_\beta]=2i(\Gamma^0(\not{p}+Z\Gamma^{11}))_{\alpha\beta} \tag{15}$$

and we have

$$\not{p}+Z\Gamma^{11}=\begin{pmatrix} Z & \bar{\not{p}} \\ \not{p} & -Z\end{pmatrix}, \quad (\not{p}+Z\Gamma^{11})^2=(p^2+Z^2)*\mathcal{I} \tag{16}$$

(here, $\mathcal{I}$ is the 32×32 identity matrix)

$$\det[\Gamma^0(\not{p}+Z\Gamma^{11})]=(p^2+Z^2)^{16}=0. \tag{17}$$

In the 32×32 matrix $\Gamma^0(\not{p}+Z\Gamma^{11})$, each of its 16×16 blocks has a nonzero determinant, and $\Gamma^0(\not{p}+Z\Gamma^{11})$ has rank 16. The first and second class constraints can be covariantly separated by defining [9,10]

$$\bar{T}_1=\bar{T}(\not{p}+Z\Gamma^{11})\left(\frac{1-\Gamma^{11}}{2}\right)=\bar{\pi}_-\not{p}-Z\bar{\pi}_++i\bar{\theta}_+(p^2+Z^2)$$

and

$$\bar{T}_2=\bar{T}\left(\frac{1+\Gamma^{11}}{2}\right)=\bar{\pi}_-+i\bar{\theta}_+\not{p}+iZ\bar{\theta}_- \tag{18}$$

as seen from the following Poisson brackets relations:

$$[\bar{T}_{1\alpha},\bar{T}_{1\beta}]=-2i(p^2+Z^2)\left(\Gamma^0\frac{1+\Gamma^{11}}{2}\not{p}\right)_{\alpha\beta},$$
$$[\bar{T}_{2\alpha},\bar{T}_{2\beta}]=2i\left(\Gamma^0\frac{1-\Gamma^{11}}{2}\not{p}\right)_{\alpha\beta}$$
$$[\bar{T}_{1\alpha},\bar{T}_{2\beta}]=-2i(p^2+Z^2)\left(\Gamma^0\frac{1+\Gamma^{11}}{2}\right)_{\alpha\beta} \tag{19}$$

where we used

$$[\theta_{\pm\alpha},\bar{\pi}_{\mp\beta}]=\left(\frac{1\pm\Gamma^{11}}{2}\right)_{\alpha\beta}. \tag{20}$$

The generator of κ symmetry and reparametrization is given in terms of the parameters κ_- and ϵ_p by

$$G=\tfrac{1}{2}\epsilon_p(p^2+Z^2)+\bar{T}_1\kappa_-. \tag{21}$$

As mentioned above, the D0-brane has a total of 16 independent fermionic degrees of freedom in phase space $[32\times 2-(2\times 16+16)]$ as reflected by the 16 first class and 16 second class fermionic constraints in Eq. (19).

In an extended phase space where the system is described by extra degrees of freedom, second class constraints can be turned into first class [13,14]. One denotes the second class constraints Poisson bracket by

$$[\overline{T}_{2\alpha},\overline{T}_{2\beta}]=2i\left(\Gamma^0\frac{1-\Gamma^{11}}{2}\not p\right)_{\alpha\beta}=V_{\alpha\gamma}\omega_{\gamma\delta}V_{\delta\beta}. \quad (22)$$

$V_{\alpha\beta}$ constructs the Becchi-Rouet-Stora-Tyotin (BRST) operator in the extended symmetric system and $\omega_{\gamma\delta}$ is used in order to define a linear combination of extra 32 fermionic degrees of freedom in phase space. We have [up to similarity transformations of ω in the symplectic structure of Eq. (22)]:

$$V_{\alpha\beta}=\left(\Gamma^0\frac{1-\Gamma^{11}}{2}\not p\right)_{\alpha\beta},\quad \omega_{\alpha\beta}=-\frac{2i}{p^2}\left[\left(\frac{1+\Gamma^{11}}{2}\right)\not p\Gamma^0\right]_{\alpha\beta}. \quad (23)$$

We define the linear combination:

$$\overline{\Phi}_{-\alpha}=-\tfrac{1}{2}\overline{\rho}_{-\alpha}+\widetilde{\omega}_{\alpha\beta}\zeta_{+\beta}=-\tfrac{1}{2}\overline{\rho}_{-\alpha}-i\tfrac{1}{2}(\overline{\zeta}_{+}\not p)_{\alpha} \quad (24)$$

where we used

$$\widetilde{\omega}_{\alpha\beta}=-i\frac{1}{2}\left[\Gamma^0\left(\frac{1-\Gamma^{11}}{2}\right)\not p\right]_{\alpha\beta},\quad \omega_{\alpha\gamma}\widetilde{\omega}_{\gamma\beta}=\left(\frac{1+\Gamma^{11}}{2}\right)_{\alpha\beta}. \quad (25)$$

ρ_- and ζ_+ are a canonical pair of Majorana-Weyl spinors representing extra 32 fermionic degrees of freedom whose Poisson bracket is

$$[\overline{\rho}_{-\alpha},\zeta_{+\beta}]=\left(\frac{1+\Gamma^{11}}{2}\right)_{\alpha\beta}. \quad (26)$$

The linear combination in Eq. (24) of the extra degrees of freedom $\Phi_{-\alpha}$ have the Poisson bracket

$$[\overline{\Phi}_{-\alpha},\overline{\Phi}_{-\beta}]=-\widetilde{\omega}_{\alpha\beta}=i\frac{1}{2}\left[\Gamma^0\left(\frac{1-\Gamma^{11}}{2}\right)\not p\right]_{\alpha\beta}. \quad (27)$$

One defines now, in the extended phase space, the following constraints, which are first class:

$$\overline{T'}_{-\alpha}=\overline{T}_{2\alpha}+\overline{\Phi}_{-\beta}\omega_{\beta\gamma}V_{\gamma\alpha},\quad [\overline{T'}_{-\alpha},\overline{T'}_{-\beta}]=0. \quad (28)$$

Thus, the dynamics in the extended phase space are defined by the two opposite chirality sets of constraints $\overline{T}_+,\overline{T'}_-$ and their Poisson bracket:

$$\overline{T}_1\equiv\overline{T}_+=\overline{\pi}_-\not p-Z\overline{\pi}_++i\overline{\theta}_+(p^2+Z^2),$$
$$\overline{T'}_-=\overline{\pi}_-+i\overline{\theta}_+\not p+iZ\overline{\theta}_--i\overline{\rho}_-+\overline{\zeta}_+\not p. \quad (29)$$

Using the extended phase space Poisson brackets in Eq. (20) and Eq. (26) one finds the Poisson brackets of two chiral multiplets of first class constraints :

$$[\overline{T}_{+\alpha},\overline{T}_{+\beta}]=-2i(p^2+Z^2)\times\left[\Gamma^0\left(\frac{1+\Gamma^{11}}{2}\right)\not p\right]_{\alpha\beta},\quad [\overline{T'}_{-\alpha},\overline{T'}_{-\beta}]=0$$
$$[\overline{T}_{+\alpha},\overline{T'}_{-\beta}]=-2i(p^2+Z^2)\left[\Gamma^0\left(\frac{1+\Gamma^{11}}{2}\right)\right]_{\alpha\beta}. \quad (30)$$

The total extended phase space Hamiltonian is

$$H_T=H_0+\tfrac{1}{2}\lambda_p(p^2+Z^2)+\overline{T}_+\lambda_-+\overline{T'}_-\lambda_+,$$
$$H_0=-\tfrac{1}{2}e(p^2+Z^2) \quad (31)$$

The generator of κ_- and κ_+ gauge symmetries and reparametrization is

$$G=\epsilon_e\pi_e+\frac{\epsilon_p}{2}(p^2+Z^2)+\{\overline{\pi}_-\not p-Z\overline{\pi}_++i\overline{\theta}_+(p^2+Z^2)\}\kappa_-$$
$$+\{\overline{\pi}_-+i\overline{\theta}_+\not p+iZ\overline{\theta}_--i\overline{\rho}_-+\overline{\zeta}_+\not p\}\kappa_+ \quad (32)$$

and the phase space action is

$$S=\int_{\tau_i}^{\tau_f}d\tau\left\{p_\mu\dot{x}^\mu+\pi_e\dot{e}+\overline{\pi}_+\dot{\theta}_-+\overline{\pi}_-\dot{\theta}_++\overline{\rho}_-\dot{\zeta}_+\right.$$
$$\left.+\frac{e}{2}(p^2+Z^2)-\lambda_e\pi_e-\tfrac{1}{2}\lambda_p(p^2+Z^2)-\overline{T}_+\lambda_--\overline{T'}_-\lambda_+\right\}. \quad (33)$$

The κ_- and κ_+ transformations generated by the generator G in Eq. (32) are given by

$$\delta x_\mu=(\overline{\pi}_-\Gamma_\mu+2ip_\mu\overline{\theta}_+)\kappa_-+(i\overline{\theta}_++\overline{\zeta}_+)\Gamma_\mu\kappa_+,$$
$$\delta p_\mu=0,\quad \delta\theta_+=\not p\kappa_-+\kappa_+,\quad \delta\theta_-=-Z\kappa_-,$$
$$\delta\overline{\pi}_+=-Zi\overline{\kappa}_+,\quad \delta\overline{\pi}_-=-i(p^2+Z^2)\overline{\kappa}_-+i\overline{\kappa}_+\not p,$$
$$\delta\zeta_+=-i\kappa_+,\quad \delta\overline{\rho}_-=\overline{\kappa}_+\not p. \quad (34)$$

The action in Eq. (33) is invariant under these transformations if supplemented also by

$$\delta\lambda_-=\dot{\kappa}_-,\quad \delta\lambda_+=\dot{\kappa}_+,$$
$$\delta\lambda_p=4i(\overline{\kappa}_-\lambda_+-\overline{\kappa}_+\lambda_-+\overline{\kappa}_-\not p\lambda_-)$$

as well as invariant under reparametrization

$$\delta x_\mu=p_\mu\epsilon_p,\quad \delta p_\mu=0,\quad \delta e=\epsilon_e,\quad \delta\pi_e=0,$$
$$\delta\lambda_e=\dot{\epsilon}_e,\quad \delta\lambda_p=\epsilon_e+\dot{\epsilon}_p,\quad \epsilon_p(\tau_i)=\epsilon_p(\tau_f)=0. \quad (35)$$

In Eq. (33) the bosonic (λ_e,λ_p) Lagrange multipliers and the Majorana-Weyl (λ_-,λ_+) Lagrange multipliers are associated with the bosonic and fermionic first class constraints $\pi_e=p^2+Z^2=0,\overline{T}_-=\overline{T}_+=0$.

One notices that in the new phase space action of Eq. (33) only the linear combination $-i\overline{\rho}_-+\overline{\zeta}_+\not{p}$ of new fermionic degrees of freedom appears. The orthogonal combination does not appear in the action and is thus decoupled from any dynamics of the system. This ''Batalin-Fradkin decoupling'' (see Refs. [13–15]) assures that the correct independent degrees of freedom defines the extended symmetric system. Namely, we started with $(64-16\times2-16=)16$ fermionic degrees of freedom in phase space, 32 degrees of freedom were added and the κ_+ symmetry was introduced. We have now $(64+32-16\times2-16\times2=16+16)16$ independent degrees of freedom as in the original system while the other 16 are the ''Batalin-Fradkin decoupled'' degrees of freedom. In the extended symmetry system, in addition to the possible gauge fixing (e.g., [9,10]) that eliminates the θ_- degrees of freedom by fixing the κ_- gauge, other gauge fixings are acceptable as well. Clearly, as seen in Eq. (34), a properly chosen gauge fixing (''unitary'' gauge fixing) of the new κ_+ symmetry will eliminate the linear combination of the new fermionic degrees of freedom $-i\overline{\rho}_-+\overline{\zeta}_+\not{p}$. For example a possible unitary gauge fixing is

$$\theta_-=0 \quad \text{and} \quad -i\overline{\rho}_-+\overline{\zeta}_+\not{p}=0. \tag{36}$$

This results in the same gauge fixed system that was used in Ref. [9]. A different and interesting gauge fixing that eliminates the old degrees of freedom and leaves only the new 16 degrees of freedom is simply,

$$\theta_-=0 \quad \text{and} \quad \theta_+=0. \tag{37}$$

The gauge fixed D0 system is given in this gauge in terms of $-i\overline{\rho}_-+\overline{\zeta}_+\not{p}$ only. As in the case of the unitary gauge in Eq. (36), the Poisson bracket matrix $[\overline{T}_{\pm\alpha},\chi_{\pm\beta}]$ between the constraints $\overline{T}_{+\alpha},\overline{T}'_{-\alpha}$ and the gauge fixing conditions $\chi_-=\theta_-,\chi_+=\theta_+$ is not singular since $p^2+Z^2=0$. Of course, other combinations of κ_- and κ_+ gauge fixings are also possible.

An interesting set of constraints is defined by

$$\overline{T}'_{-\alpha}=\overline{T}_{2\alpha}+\overline{\Phi}_{-\beta}\omega_{\beta\gamma}V_{\gamma\alpha}=\overline{\pi}_-+i\overline{\theta}_+\not{p}+iZ\overline{\theta}_--i\overline{\rho}_-+\overline{\zeta}_+\not{p}$$

$$\begin{aligned}\overline{T}'_{+\alpha}&=\overline{T}_++i2\left(\frac{p^2+Z^2}{p^2}\right)\left[\overline{\Phi}\left(\frac{1+\Gamma^{11}}{2}\right)\not{p}\right]_\alpha\\&=\overline{\pi}_-\not{p}-Z\overline{\pi}_++i\overline{\theta}_+(p^2+Z^2)+\overline{\zeta}_+(p^2+Z^2)\\&\quad-i\overline{\rho}_-\not{p}\left(\frac{p^2+Z^2}{p^2}\right).\end{aligned} \tag{38}$$

These constraints satisfy the following Poisson bracket relations:

$$\begin{gathered}[\overline{T}'_{+\alpha},\overline{T}'_{+\beta}]=-2i\left(\frac{Z^2}{p^2}\right)(p^2+Z^2)\left[\Gamma^0\left(\frac{1+\Gamma^{11}}{2}\right)\not{p}\right]_{\alpha\beta},\\ [\overline{T}'_{-\alpha},\overline{T}'_{-\beta}]=0,\quad [\overline{T}'_{+\alpha},\overline{T}'_{-\beta}]=0.\end{gathered} \tag{39}$$

We note in the $p^2\gg Z^2$ limit, $\overline{T}'_{+\alpha}$ and $\overline{T}'_{-\alpha}$ are functions of (π_+,θ_-) and $(\pi_--i\rho_-,\theta_+-i\zeta_+)$ only. It is expected, in this limit, that the system behaves as the $N=2$ massless superparticle—a system with 16 independent fermionic degrees of freedom in its phase space, as seen also directly from the action in Eq. (10). Indeed, one notes that not only $\overline{\rho}_-$ and ζ_+ appear only in the linear combinations $\overline{\rho}_-+i\overline{\zeta}_+\not{p}$ but now also $\overline{\pi}_-+i\overline{\theta}_+\not{p}$ is the only linear combination of $\overline{\pi}_-$ and θ_+ that appears in the constraints. Thus, after taking into account the decoupling of their orthogonal linear combination and the fact that the fermionic degrees of freedom in phase space are now constrained by 16 first class constraints $(\overline{T}'_{-\alpha})$ while $(\overline{T}'_{+\alpha})$ are now second class only (since $p^2+Z^2\neq0$), one finds indeed in the $p^2\gg Z^2$ limit only 16 independent fermionic degrees of freedom as for the $N=2$ massless superparticle. Namely, $64+32-16\times2-16=16+16+16$ where the last $16+16$ fermionic degrees of freedom are decoupled in the same sense as the ''Batalin-Fradkin decoupling'' (do not appear in the constraints or in the Hamiltonian of the extended system).

The path integral formulation [22] of the system in Eqs. (30) and (31) with κ_- and κ_+ symmetry which has only first class constraints is given by

$$\begin{aligned}S=\int_{\tau_i}^{\tau_f}d\tau\Big\{&p_\mu\dot{x}^\mu+\overline{\pi}_+\dot{\theta}_-+\overline{\pi}_-\dot{\theta}_++\overline{\rho}_-\dot{\zeta}_++\overline{\pi}_{\lambda+}\dot{\lambda}_-+\overline{\pi}_{\lambda-}\dot{\lambda}_++\pi_p\dot{\lambda}_p+\tilde{\mathcal{C}}\dot{\mathcal{P}}+\tilde{\mathcal{P}}\dot{\mathcal{C}}+\overline{\tilde{\mathcal{C}}}_+\dot{\mathcal{P}}_-+\overline{\tilde{\mathcal{P}}}_+\dot{\mathcal{C}}_-+\overline{\tilde{\mathcal{C}}}_-\dot{\mathcal{P}}_+\\&+\overline{\tilde{\mathcal{P}}}_-\dot{\mathcal{C}}_+-H_0+\pi_p\chi+\overline{\pi}_{\lambda-}\chi_++\overline{\pi}_{\lambda+}\chi_--\frac{\lambda_p}{2}(p^2+Z^2)-\overline{T}_+\lambda_--\overline{T}_-\lambda_++\overline{\tilde{\mathcal{C}}}_+[\chi_-,\overline{T}_+]\mathcal{C}_-\\&+\overline{\tilde{\mathcal{C}}}_-[\chi_+,\overline{T}_-]\mathcal{C}_+-\tilde{\mathcal{P}}\mathcal{P}-\tilde{\mathcal{P}}_+\mathcal{P}_--\tilde{\mathcal{P}}_-\mathcal{P}_+-4\tilde{\mathcal{P}}\overline{\lambda}_-\not{p}\mathcal{C}_-+4Z\tilde{\mathcal{P}}(\overline{\lambda}_-\mathcal{C}_+-\overline{\lambda}_+\mathcal{C}_-)\Big\}.\end{aligned} \tag{40}$$

Here, $\mathcal{C}_{\pm}$ and $\tilde{\mathcal{P}}_{\mp}$ are canonical pairs of bosonic ghosts and $\mathcal{P}_{\pm}$ and $\tilde{\mathcal{C}}_{\mp}$ are canonical pairs of bosonic antighosts, associated with the fermionic constraints T_+ and T_-. The Majorana-Weyl $\pi_{\lambda+},\pi_{\lambda-}$ are the canonical conjugates of the Lagrange multipliers λ_-,λ_+. The bosonic π_p is the canonical conjugate of the Lagrange multiplier λ_p associated with the constraint $p^2+Z^2=0$ and χ_+,χ_+ are gauge fixings. The fermionic ghost and its canonical conjugate are denoted by $\mathcal{C}$ and $\tilde{\mathcal{P}}$, and the canonical pairs of fermionic anti-ghosts as $\mathcal{P},\tilde{\mathcal{C}}$.

The last three lines in Eq. (40) are given by $-[\Psi,\Omega]$ where the BRST operator Ω is given by

$$\Omega=\mathcal{P}\pi_p+\overline{\mathcal{P}}_+\pi_{\lambda-}+\overline{\mathcal{P}}_-\pi_{\lambda+}+\overline{T}_+\mathcal{C}_-+\overline{T}_-\mathcal{C}_++\frac{\mathcal{C}}{2}(p^2+Z^2)$$
$$+2\tilde{\mathcal{P}}\overline{\mathcal{C}}_-\not{p}\mathcal{C}_-+2Z\tilde{\mathcal{P}}(\overline{\mathcal{C}}_-\mathcal{C}_+-\overline{\mathcal{C}}_+\mathcal{C}_-) \tag{41}$$

and the gauge fixing Ψ is given by

$$\Psi=-\tilde{\mathcal{P}}\lambda-\overline{\tilde{\mathcal{P}}}_+\lambda_--\overline{\tilde{\mathcal{P}}}_-\lambda_++\tilde{\mathcal{C}}\chi+\overline{\tilde{\mathcal{C}}}_+\chi_-+\overline{\tilde{\mathcal{C}}}_-\chi_+. \tag{42}$$

The above κ_-,κ_+ symmetric D0 defined in the extended phase space $(\theta_{\pm},\pi_{\mp},\zeta_+,\rho_-)$ is physically equivalent to the ordinary D0 with κ_- symmetry of Eq. (11). This, as mentioned, is demonstrated by choosing the ''unitary'' gauge fixing $\chi_{\pm}$ in Eq. (36) that sets the extended phase space variables $\overline{\rho_-+i\zeta_+\not{p}}$ to zero. On the other hand the above symmetric system accepts many different gauge fixings $\chi_{\pm}$ giving different presentations of the D0 brane [for example, Eq. (37)].

III. D1-BRANE WITH κ_- AND κ_+ EXTENDED SYMMETRY

Following along similar lines we present now the extension of this derivation to the case of a D1-brane. It results in a system with κ_- and κ_+ symmetry which will be discussed below.

The action of the D1-brane consists of the Born-Infeld-Nambu-Goto term and the Chern-Simons two form Ω_2 term [7]

$$S=\int\mathcal{L}(\sigma)d^2\sigma=-T\left\{\int d^2\sigma\sqrt{-\det(G_{\mu\nu}+\mathcal{F}_{\mu\nu})}+\int\Omega_2\right\} \tag{43}$$

where $G_{\mu\nu}$ is the supersymmetric induced world-volume metric and $\mathcal{F}_{\mu\nu}$ is the supersymmetric Born-Infeld field strength:

$$G_{\mu\nu}=\Pi^m_\mu\Pi_{\nu m},\quad \Pi^m_\mu=\partial_\mu x^m-\overline{\theta}\Gamma^m\partial_\mu\theta,\quad \mu,\nu=0,1;$$
$$m=0,1,2,\dots,9 \tag{44}$$

$$\mathcal{F}_{01}=F_{01}-b_{01}(\tau_3),\quad F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu, \tag{45}$$

$$b_{01}(\tau_k)=-\overline{\theta}\Gamma_m\tau_k\{\partial_0\theta\Pi^m_1-\partial_1\theta\Pi^m_0+\tfrac{1}{2}[\partial_0\theta(\overline{\theta}\Gamma^m\partial_1\theta)$$
$$-\partial_1\theta(\overline{\theta}\Gamma^m\partial_0\theta)]\} \tag{46}$$

where θ^A_α, $\alpha=1,2,\dots,32$ are two Majorana-Weyl spinors with the same chirality, and τ_k are Pauli matrices acting on indices $A=1,2$. The Lagrangian can be rewritten as

$$\mathcal{L}(\sigma)=-T\{\sqrt{G_{01}^2-G_{00}G_{11}-\mathcal{F}_{01}^2}+b_{01}(\tau_1)\}. \tag{47}$$

The canonical momenta for the world sheet gauge field is given by the electric field E^μ

$$E^0=\frac{\partial\mathcal{L}}{\partial\dot{A}_0}=0,\quad E^1=\frac{\partial\mathcal{L}}{\partial\dot{A}_1}=\frac{T\mathcal{F}_{01}}{\sqrt{G_{01}^2-G_{00}G_{11}-\mathcal{F}_{01}^2}}. \tag{48}$$

The canonical momenta $\overline{\pi}_\alpha$ and p_m are defined for θ_α and x^m, respectively:

$$p_m=\tilde{p}_m-\overline{\theta}\Gamma_m T_E\partial_1\theta,\quad \tilde{p}_m=T\frac{G_{11}\Pi_{0m}-G_{01}\Pi_{1m}}{\sqrt{G_{01}^2-G_{00}G_{11}-\mathcal{F}_{01}^2}},$$
$$\overline{\pi}=\overline{\theta}\not{\Pi}_1T_E-\overline{\theta}\tilde{\not{p}}+(\overline{\theta}\Gamma^m\partial_1\theta)(\overline{\theta}\Gamma_mT_E),\quad T_E=E^1\tau_3+T\tau_1. \tag{49}$$

We will suppress the indices $A=1,2$ of θ^A_α when it is easily recognized. From Eq. (47) one finds the fermionic constraints $\overline{\Phi}^A_\alpha$

$$\overline{\Phi}_\alpha=\overline{\pi}_\alpha+(\overline{\theta}\not{p})_\alpha-(\overline{\theta}\Gamma^mT_E)_\alpha(\partial_1x_m)+(\overline{\theta}\Gamma^m\partial_1\theta)(\overline{\theta}\Gamma_mT_E)_\alpha$$
$$=0 \tag{50}$$

which satisfy the Poisson bracket relations:

$$[\overline{\Phi}_\alpha(\sigma),\overline{\Phi}_\beta(\sigma')]=2[(\Gamma^0\tilde{\not{p}})_{\alpha\beta}-(\Gamma^0\not{\Pi}_1T_E)_{\alpha\beta}]\delta(\sigma-\sigma'). \tag{51}$$

In addition to the fermionic constraints in Eq. (50) one finds from Eq. (47) also the bosonic first class constraints :

$$\tilde{p}^2+G_{11}(E_1^2+T^2)=0,\quad \tilde{p}_m\Pi^m_1=0. \tag{52}$$

The constraints in Eq. (50) can be separated covariantly into first class and second class constraints [9,10]:

$$\overline{T}_{1\alpha}=\left[\overline{\Phi}(\tilde{\not{p}}-\not{\Pi}T_E)\left(\frac{1+\tau_3}{2}\right)\right]_\alpha,\quad \overline{T}_{2\alpha}=\left[\overline{\Phi}\left(\frac{1-\tau_3}{2}\right)\right]_\alpha. \tag{53}$$

The Poisson bracket $[T_{1\alpha},T_{1\beta}]$ vanishes on the constraints hyperplane.

These 16 first class constraints $T_{1\alpha}$ generate the local κ symmetry of the D1-brane. On the other hand

$$[\overline{T}_{2\alpha},\overline{T}_{2\beta}]=2(\Gamma^0\not{P}\tau_-)_{\alpha\beta}\delta(\sigma-\sigma') \tag{54}$$

where

$$P_m=\tilde{p}_m+E^1\Pi_{1m}=p_m+\overline{\theta}\Gamma_mT_E\partial_1\theta+E^1(\partial_1x_m-\overline{\theta}\Gamma_m\partial_1\theta).$$

Since $P^2=\tilde{p}^2+2E_1(\tilde{p}\Pi_1)+E_1^2G_{11}=-T^2G_{11}$ on the constraints hyperplane, we obtain a nonvanishing $\det[\overline{T}_{2\alpha},\overline{T}_{2\beta}]$ (apart from the case $G_{11}=0$) implying that $\overline{T}_{2\alpha}$ are 16 second class constraints. The condition $G_{11}\neq 0$ is essential for separating the first and second class constraints and the covariant quantization of the D1 system. In Ref. [9]

it has been emphasized that in the static gauge (where $x^\mu = \sigma^\mu$ for $\mu = 0,1$) indeed $G_{11} \neq 0$. The implications of this fact on the ground state spectrum and on the relation to the work of Ref. [7] on the type IIB fundamental string have been cleared there. Both Refs. [7] and [9] discuss the properties of the static gauge and elucidate its physics content. Since the static gauge is a natural gauge for D1, we follow this point of view.

We define now a new system in an extended phase space that includes in addition to the 64 fermionic degrees of freedom θ^A_α and π^A_α extra fermionic 32 degrees of freedom [13,14] that satisfy

$$[\overline{\rho}^A_\alpha(\sigma), \zeta^B_\beta(\sigma')] = \delta(\sigma - \sigma')\tau_-^{AB}\delta_{\alpha\beta}. \tag{55}$$

The constraints of the new system $\overline{T}'^A_\alpha(x,p,\theta,\pi,\zeta,\rho)$ are obtained from the constraints in Eq. (53) in a similar way the constraints in the extended phase space in Eq. (28) were obtained for the D-particle. Namely, $\overline{T}'^A_{1\alpha}(x,p,\theta,\pi,\zeta,\rho) = \overline{T}^A_{1\alpha}(x,p,\theta,\pi)$ is left unchanged and does not depend on (ζ,ρ), whereas the other constraint $T^A_{2\alpha}$ is modified as

$$\overline{T}'^A_{2\alpha}(x,p,\theta,\pi,\zeta,\rho) = \overline{T}^A_{2\alpha}(x,p,\theta,\pi) - \overline{\rho}^A_\alpha + (\overline{\zeta}^B \not{P})_\alpha \tau_-^{BA} \tag{56}$$

which depends on (ζ,ρ) and satisfies the Poisson bracket relation[2]:

$$\begin{aligned}[\overline{T}'_{2\alpha}, \overline{T}'_{2\beta}] = & -2E^1 \delta(\sigma - \sigma')[2(\Gamma^0\Gamma^m)_{\alpha\beta}(\overline{\zeta}\Gamma_m \tau_- \partial_1 \theta) \\ & - (\overline{\zeta}\Gamma^m)_\alpha(\partial_1 \overline{\zeta}\Gamma_m \tau_-)_\beta] \\ & - 2E^1 \frac{\partial \delta(\sigma - \sigma')}{\partial \sigma'}(\overline{\zeta}\Gamma^m)_\alpha(\overline{\zeta}\Gamma_m \tau_-)_\beta. \end{aligned} \tag{57}$$

In the case of $E^1 = 0$ the new system has only first class constraints and local symmetries κ_1 and κ_2 generated by $T_{1\alpha}$ and by $T'_{2\alpha}$, respectively. The symmetric system phase space is given by the coordinates $\theta^A_\alpha(\sigma), \pi^A_\alpha(\sigma), \rho^A_\alpha(\sigma)$, and $\overline{\zeta}^B_\beta(\sigma)$ where the number of independent fermionic degrees of freedom has not been changed. Namely, we started with $2 \times 32 - 2 \times 16 - 16 = 16$ independent fermionic degrees of freedom in phase space and in the extended phase space we have $3 \times 32 - 2 \times 32 = 16 + 16$ (BF) degrees of freedom where the 16 (BF) degrees of freedom are "Batalin-Fradkin decoupled" [14,15] leaving 16 independent fermionic degrees of freedom.

[2]In deriving Eq. (57), the following relations for Majorana λ_i have been used $(\Gamma_0\Gamma^m T\lambda_2)_\alpha(\overline{\lambda}_3\Gamma_m)_\beta + (\Gamma_0\Gamma^m T\lambda_2)_\beta(\overline{\lambda}_3\Gamma_m)_\alpha = (\Gamma_0\Gamma_m)_{\alpha\beta}(\overline{\lambda}_3\Gamma^m T\lambda_2)$ where T is a matrix in the internal space of the Majorana spinors (such as τ_- and T_E). Also $(\overline{\lambda}_2\Gamma_m)_\alpha(\overline{\lambda}_3\Gamma^m T)_\beta = (\overline{\lambda}_2\Gamma_m T)_\alpha(\overline{\lambda}_3\Gamma^m)_\beta$.

We note from Eq. (48) that setting $E^1 = 0$ means also that $\mathcal{F}_{01} = 0$ which results in the Lagrangian of Eq. (47) to be very similar to the Green-Schwarz (GS) string.

The GS string is described by the action [5]

$$S = \int \mathcal{L}(\sigma) d^2\sigma = -\frac{T}{2}\int d^2\sigma \sqrt{h} h^{\alpha\beta} G_{\alpha\beta} + \int \mathcal{L}_2 d^2\sigma, \tag{58}$$

$$\begin{aligned}\mathcal{L}_2 = & -T\epsilon^{\alpha\beta}\partial_\alpha x^m(\overline{\theta^1}\Gamma_m \partial_\beta \theta^1 - \overline{\theta^2}\Gamma_m \partial_\beta \theta^2) \\ & - T\epsilon^{\alpha\beta}(\overline{\theta^1}\Gamma^m \partial_\alpha \theta^1)(\overline{\theta^2}\Gamma_m \partial_\beta \theta^2) \\ = & -T[\partial_0 x^m(\overline{\theta}\Gamma_m \tau_3 \partial_1 \theta) - \partial_1 x^m(\overline{\theta}\Gamma_m \tau_3 \partial_0 \theta)] \\ & - \frac{T}{2}(\overline{\theta}\Gamma^m \tau_3 \partial_0 \theta)(\overline{\theta^A}\Gamma_m \partial_1 \theta^A) + \frac{T}{2}(\overline{\theta}\Gamma^m \tau_3 \partial_1 \theta) \\ & \times(\overline{\theta^A}\Gamma_m \partial_0 \theta^A). \end{aligned} \tag{59}$$

This can be compared to the $b_{01}(\tau_k)$ of Eq. (46) which can be written also as

$$\begin{aligned} b_{01}(\tau_k) = & [\partial_0 x^m(\overline{\theta}\Gamma_m \tau_k \partial_1 \theta) - \partial_1 x^m(\overline{\theta}\Gamma_m \tau_k \partial_0 \theta)] \\ & + \tfrac{1}{2}(\overline{\theta}\Gamma^m \tau_k \partial_0 \theta)(\overline{\theta^A}\Gamma_m \partial_1 \theta^A) - \tfrac{1}{2}(\overline{\theta}\Gamma^m \tau_k \partial_1 \theta) \\ & \times(\overline{\theta^A}\Gamma_m \partial_0 \theta^A). \end{aligned} \tag{60}$$

Thus $\mathcal{L}_2 = -Tb_{01}(\tau_3)$ compared to $-Tb_{01}(\tau_1)$ in the Wess-Zumino term of the D1-brane. Similarly, using the equation of motion for $h^{\alpha\beta}$ one notices that the D1 action in Eq. (43) with $E_1 = 0$ (namely $\mathcal{F}_{\mu\nu} = 0$) is identical to the Green and Schwarz action when τ_3 is replaced by τ_1 [10]. Since we are using the static gauge as a natural gauge for D1 [7], the massless modes are projected out. This relation between the physics of the type IIB fundamental string and the D1 system in the static gauge has been noted in Ref. [9].

We also note that the electric field E^1 is quantized and represents the number of fundamental string bound to the D1-brane producing (n,m) string [24,25]. Therefore we have succeeded in extending the system where all the second class constraints are turned into first class constraints at least for the case of the (0,1) string, namely the genuine D1-brane without F1 provided the massless modes which are projected out by using, for instance, the static gauge.

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[1] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995).
[2] L. Brink and J.H. Schwarz, Phys. Lett. **100B**, 310 (1981); L. Brink and M.B. Green, *ibid.* **106B**, 393 (1981); R. Casalbuoni, *ibid.* **62B**, 49 (1976); Nuovo Cimento A **33A**, 389 (1976).
[3] J. Azcarraga and J. Lukierski, Phys. Lett. **113B**, 170 (1982); Phys. Rev. D **28**, 1337 (1983).
[4] W. Siegel, Phys. Lett. **128B**, 397 (1983); Class. Quantum Grav. **2**, L95 (1985); Nucl. Phys. **B263**, 93 (1986); *Symposium on Anomalies, Geometry and Topology*, edited by W.A. Bardeen and A.R. White (World Scientific, Singapore, 1985), and Ref. [12].
[5] M.B. Green and J.H. Schwarz, Phys. Lett. **136B**, 367 (1984); Nucl. Phys. **B243**, 285 (1984).
[6] J. Hughes, J. Liu, and J. Polchinski, Phys. Lett. B **180**, 370 (1986); see also the review in Ref. [16].
[7] M. Aganagic, C. Popescu, and J.H. Schwarz, Nucl. Phys. **B495**, 99 (1997); Phys. Lett. B **393**, 311 (1997).
[8] E. Bergshoeff and P.K. Townsend, Nucl. Phys. **B490**, 145 (1997).
[9] R. Kallosh, Phys. Rev. D **56**, 3515 (1997).
[10] M. Hatsuda and K. Kamimura, Nucl. Phys. **B520**, 335 (1998).
[11] T. Hori and K. Kamimura, Prog. Theor. Phys. **73**, 476 (1985); F. Bastianelli, G.W. Delius, and E. Laenen, Phys. Lett. B **229**, 223 (1989); U. Lindström, M. Rocek, W. Siegel, P. van Nieuwenhuizen, and A.E. van de Ven, J. Math. Phys. **31**, 1761 (1990); Phys. Lett. B **224**, 285 (1989); A. Mikovic, M. Rocek, W. Siegel, P. van Nieuwenhuizen, J. Yamron, and A.E. van de Ven, *ibid.* **235**, 106 (1990); F. Essler, M. Hatsuda, E. Laenen, W. Siegel, J.P. Yamron, T. Kimura, and A. Mikovic, Nucl. Phys. **B364**, 67 (1991).
[12] W. Siegel, Phys. Rev. D **52**, 3563 (1995).
[13] L.D. Faddeev, Phys. Lett. B **145**, 81 (1984); L.D. Faddeev, and S.L. Shatashvili, *ibid.* **167**, 225 (1986).
[14] I.A. Batalin and E.S. Fradkin, Nucl. Phys. **B279**, 514 (1987); I.A. Batalin, E.S. Fradkin, and T.E. Fradkina, *ibid.* **B314**, 158 (1989).
[15] J. Feinberg and M. Moshe, Phys. Lett. B **247**, 509 (1990); Ann. Phys. (N.Y.) **206**, 272 (1991); M. Moshe, *Proc. of Johns Hopkins Workshop*, Lanzhou, China, 1987, edited by Yi-shi Duan *et al.* (World Scientific, Singapore, 1987); see also Refs. [17,18] for other applications of [13,14].
[16] D. Sorokin, V. Tkach, and D.V. Volkov, Mod. Phys. Lett. A **4**, 901 (1989); D. Sorokin, Phys. Rep. **329**, 1 (2000).
[17] Y. Eisenberg and S. Solomon, Nucl. Phys. **B309**, 709 (1988).
[18] I. Bandos and J. Lukierski, Mod. Phys. Lett. A **14**, 1257 (1999); I. Bandos, J. Lukierski, and D. Sorokin, Phys. Rev. D **61**, 045002 (2000).
[19] W. Siegel, Nucl. Phys. **B263**, 93 (1985); see also Ref. [12].
[20] M. Tonin, Phys. Lett. B **266**, 312 (1991); I.A. Bandos and A.A. Zheltukhin, *ibid.* **288**, 77 (1992); S. Aoyama, P. Pasti, and M. Tonin, *ibid.* **283**, 213 (1992); N. Berkovits, Nucl. Phys. **B431**, 258 (1994); N. Berkovits, C. Vafa, and E. Witten, J. High Energy Phys. **03**, 18 (1999).
[21] A. Duncan and M. Moshe, Nucl. Phys. **B268**, 706 (1986); E.S. Fradkin and Sh.M. Shvartsman, Mod. Phys. Lett. A **6**, 1977 (1991); A.V. Galajinsky and D.M. Gitman, Nucl. Phys. **B536**, 435 (1999).
[22] E.S. Fradkin and G.A. Vilkovisky, Phys. Lett. **55B**, 224 (1975); I.A. Batalin and E.S. Fradkin, *ibid.* **122B**, 157 (1983); *ibid.* **128B**, 303 (1983).
[23] M.B. Green, M. Gutperle, and H-h. Kwon, J. High Energy Phys. **08**, 012 (1999). In this paper the light-cone superspace quantum mechanics of the massless $d=11$ superparticle and its dimensional reduction with $p_{11}=0$ have been employed.
[24] E. Witten, Nucl. Phys. **B460**, 493 (1996).
[25] J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, England, 1998), Vol. II, Sec. 13.6.