Diquarks in cold dense QCD with two flavors

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We derive and analyze the Bethe-Salpeter equations for spin zero diquarks in the color superconducting phase of cold dense QCD with two massless flavors. The spectrum of diquarks contains an infinite number of massive excitations and five (nearly) massless pseudoscalars. The former are singlets while the latter include a doublet, an antidoublet and a singlet with respect to the unbroken $SU(2)_c$. Because of approximate parity doubling at a large chemical potential, all massive states come in pairs. The decay constants, as well as the velocities of the (nearly) massless pseudoscalars, are derived. The different role of the Meissner effect for tightly bound states and quasiclassical bound states is revealed.

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I. INTRODUCTION

With continuing advances in modern nuclear and high energy experiments, it has become feasible to produce deconfined quark matter in the laboratory. Not surprisingly, this has stimulated many theoretical studies of quark matter at high densities and/or temperatures (for recent reviews see, for example, Refs. [1–4]).

Of special interest is the cold quark matter at high densities, i.e., at densities which are at least a few times larger than that of a nucleon, $n_0 \simeq 0.17\,\mathrm{fm}^{-3}$. It has been known for a long time that such matter could be a color superconductor [5,6]. Nevertheless, until recently our understanding of the color superconducting state has remained very poor. The new developments started with the ground breaking estimates of the color superconducting order parameter in Refs. [7,8]. Within a phenomenological (instanton liquid) model, it was shown there that the order parameter could be as large as 100 MeV. These estimates revived the hope of producing and detecting the color superconducting phase either in experiments or in natural systems such as neutron (quark) stars. Being motivated by the potential possibility of observing the color superconducting phase at moderate densities, the subject resulted in numerous studies and new discoveries.

At first, it was not clear at all that the microscopic theory, quantum chromodynamics (QCD), would lead to the same (or, at least, the same order of magnitude) estimates for the superconducting gap as the phenomenological models. It was suggested in Refs. [9,10] that the screening effects of gluons should play a crucial role in the analysis. In particular, while the electric gluon modes are subject to Debye screening at already the scales of order $l_D \sim 1/g_s\mu$, where μ is the chemical potential and g_s is the running coupling related to the scale μ, the magnetic modes are subject only to Landau damping, which does not completely eliminate the long range interaction [10]. Subsequent studies of the gap equation in QCD confirmed that proper treatment of the gluon screening effects is crucial in deriving the estimates for the superconducting gap [11–17].

Also, it was revealed that the ground state of quark matter with three light flavors is given by the so-called color flavor locked (CFL) phase [18]. It is remarkable that the chiral symmetry in such a phase is spontaneously broken and most of the quantum numbers of physical states coincide with those in the hadronic phase. It was tempting, therefore, to suggest that there might exist some kind of continuity between the two phases [19]. Another interesting feature of three flavor QCD was pointed out in Ref. [20], where the possibility of gapless color superconductivity (a metastable phase) was proposed. In addition, many interesting patterns of symmetry breaking were revealed in models with the number of flavors larger than 3 [17], as well as in two-color QCD with quarks in the fundamental representation and in any-color QCD with quarks in the adjoint representation [21]. The anomaly matching conditions were analyzed in Ref. [22].

The low energy dynamics of the color superconducting phase could be efficiently studied by using effective actions whose general structure is fixed by symmetries [23–26]. The finite set of parameters in such theories could be either taken from an experiment (when available) or sometimes derived from QCD (for example, in the limit of the asymptotically large chemical potential). Because of the nature of such an approach, at best it could probe the properties of the pseudo Nambu-Goldstone (NG) bosons, but not the detailed spectrum of the diquark bound states (mesons). It was argued in Ref. [27], however, that, because of long-range interactions mediated by gluons of the magnetic type [9,10], the presence of an infinite tower of massive diquark states could be the key signature of the color superconducting phase of dense quark matter.

In this paper, we consider the problem of spin zero bound

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states in the two flavor color superconductor using the Bethe-Salpeter (BS) equations. (A brief outline of our results was given in Ref. [28].) We find that the spectrum contains five (nearly) massless states and an infinite tower of massive singlets with respect to the unbroken $SU(2)_c$ subgroup. Furthermore, the following mass formula is derived for the singlets:

$$M_n^2 \simeq 4|\Delta_0^-|^2\left(1-\frac{\alpha_s^2\kappa}{(2n+1)^4}\right), \quad n=1,2,\ldots, \tag{1}$$

where κ is a constant of order 1 (we find that $\kappa \simeq 0.27$), $|\Delta_0^-|$ is the dynamical Majorana mass of quarks in the color superconducting phase, and $\alpha_s = g_s^2/4\pi$.

The Meissner effect plays a crucial role in obtaining this result. In particular, the important point is that while the Meissner effect is essentially irrelevant for tightly bound states, it is crucial for the dynamics of quasiclassical bound states (whose binding energy is small).

At a large chemical potential, we also notice an approximate degeneracy between scalar and pseudoscalar channels. As a result of this parity doubling, the massive diquark states come in pairs. In addition, there also exist five massless scalars and five (nearly) massless pseudoscalars [a doublet, an antidoublet and a singlet under $SU(2)_c$]. While the scalars are removed from the spectrum of physical particles by the Higgs mechanism, the pseudoscalars remain in the spectrum, and they are the relevant degrees of freedom of the infrared dynamics. At high density, the massive and (nearly) massless states are narrow resonances.

This paper is organized as follows. In Sec. II, we describe the model and introduce the notation. Then, further developing our notation in Sec. III, we briefly review the approach of the Schwinger-Dyson equation in the color superconducting phase of $N_f=2$ QCD. In Sec. IV, we derive the Ward identities for the quark-gluon vertex functions, corresponding to the broken generators of the color symmetry. These identities are going to be very helpful in the rest of the paper. We outline the general derivation of the Bethe-Salpeter equations for the diquark states in Sec. V. The detailed analysis of the Bethe-Salpeter equations for the NG bosons and the massive diquarks is presented in Secs. VI and VII, respectively. Appendix A contains some useful formulas that we use throughout the paper. In Appendix B, we estimate the effect of the correction to the Schwinger-Dyson equation that comes from the non-perturbative contribution to the vertex function. At last, in Appendix C, we present the approximate analytical solutions to the BS equations.

II. MODEL AND NOTATION

In the case of two flavor dense QCD, the original gauge symmetry $SU(3)_c$ breaks down to $SU(2)_c$ by the Higgs mechanism. The flavor $SU(2)_L \times SU(2)_R$ group remains intact. The appropriate order parameter is given by the vacuum expectation value of the diquark (antidiquark) field that is an antitriplet (triplet) in color and a singlet in flavor. Without loss of generality, we assume that the order parameter points in the third direction of the color space,

$$\varphi = \langle 0|\varepsilon^{ij}\varepsilon_{3ab}(\bar{\Psi}_D)_i^a \gamma^5 (\Psi_D^C)_j^b|0\rangle, \tag{2}$$

where Ψ_D and $\Psi_D^C = C\bar{\Psi}_D^T$ are the Dirac spinor and its charge conjugate spinor, and C is a unitary matrix that satisfies $C^{-1}\gamma_\mu C = -\gamma_\mu^T$ and $C=-C^T$. Here and in what follows, we explicitly display the flavor $(i,j=1,2)$ and color $(a,b=1,2)$ indices of the spinor fields. It is also appropriate to mention that the subscript and superscript indices correspond to complex conjugate representations.

The order parameter in Eq. (2) is even under parity. Such a choice is dictated by the instanton induced interactions [7,8] which, despite being vanishingly small at a large chemical potential, could be sufficiently strong for picking up the right vacuum. In addition, any bare Dirac masses of quarks (which are non-zero in nature) should also favor the parity-even condensate [29,30].

With the choice of the order parameter orientation as in Eq. (2), it is very convenient to introduce the following Majorana spinors:

$$\Psi_a^i = \psi_a^i + \varepsilon_{3ab}\varepsilon^{ij}(\psi^C)_j^b, \quad a=1,2, \tag{3}$$

$$\Phi_a^i = \phi_a^i - \varepsilon_{3ab}\varepsilon^{ij}(\phi^C)_j^b, \quad a=1,2, \tag{4}$$

which are built of the Weyl spinors of the first two colors:

$$\psi_a^i = \mathcal{P}_+(\Psi_D)_a^i, \quad (\psi^C)_j^b = \mathcal{P}_-(\Psi_D^C)_j^b, \tag{5}$$

$$\phi_a^i = \mathcal{P}_-(\Psi_D)_a^i, \quad (\phi^C)_j^b = \mathcal{P}_+(\Psi_D^C)_j^b. \tag{6}$$

Here $\mathcal{P}_\pm = (1\pm\gamma^5)/2$ are the left- and right-handed projectors. The new spinors in Eqs. (3) and (4), as is easy to check from their definition, satisfy the following generalized Majorana conditions:

$$(\Psi^C)_i^a = \varepsilon^{3ab}\varepsilon_{ij}\Psi_b^j, \tag{7}$$

$$(\Phi^C)_i^a = -\varepsilon^{3ab}\varepsilon_{ij}\Phi_b^j. \tag{8}$$

In the color superconducting phase of QCD in which quarks are known to acquire a dynamical (Majorana) mass, the use of four-component Majorana spinors, built of Weyl spinors, is most natural. Of course, when quarks are massive and chiral symmetry is explicitly broken, it would be more appropriate to consider the eight-component Majorana spinors, made of Dirac ones.

With our choice of the order parameter that points in the third direction of the color space, only quarks of the first two colors take part in the condensation. Quarks of the third color do not participate in the color condensate. It is more convenient, therefore, to use the left and right Weyl spinors,

$$\psi^i = \mathcal{P}_+(\Psi_D)_3^i, \quad (\psi^C)_j = \mathcal{P}_-(\Psi_D^C)_j^3, \tag{9}$$

$$\phi^i = \mathcal{P}_-(\Psi_D)_3^i, \quad (\phi^C)_j = \mathcal{P}_+(\Psi_D^C)_j^3, \tag{10}$$

for their description. Notice that the color index ''3'' is omitted in the definition of ψ^i and ϕ^i.

In the color superconducting phase with a parity even condensate (2), parity is a good symmetry. Then, all the quantum states of the Hilbert space, including those in the diquark channel, could be chosen so that they are either parity even or parity odd. In order to construct such states explicitly, we would need to know the following parity transformation properties of the spinors:

$$\psi^i(x)\to\gamma^0\phi^i(x'),\quad \phi^i(x)\to\gamma^0\psi^i(x'),\tag{11}$$

$$\psi_j^C(x)\to-\gamma^0\phi_j^C(x'),\quad \phi_j^C(x)\to-\gamma^0\psi_j^C(x'),\tag{12}$$

$$\Psi_a^i(x)\to\gamma^0\Phi_a^i(x'),\quad \Phi_a^i(x)\to\gamma^0\Psi_a^i(x'),\tag{13}$$

where $x=(x_0,\vec{x})$ and $x'=(x_0,-\vec{x})$.

Before concluding this section, let us rewrite the order parameter (2) in terms of the Majorana spinors:

$$\varphi=-\langle 0|\bar{\Psi}_i^a\mathcal{P}_-\Psi_a^i+\bar{\Phi}_i^a\mathcal{P}_+\Phi_a^i|0\rangle.\tag{14}$$

This representation is explicitly $SU(2)_L\times SU(2)_R\times SU(2)_c$ invariant, and so it is very convenient. By making use of the transformation properties in Eq. (13), we also easily check that φ is even under parity, as it should be.

III. SCHWINGER-DYSON EQUATION

In order to have a self-contained discussion, in this section we briefly review the Schwinger-Dyson (SD) equation using our new notation. This would also serve us as a convenient reference point when we discuss more complicated BS equations.

To start with, let us introduce the multi-component spinor

$$\begin{pmatrix}\Psi_a^i\\ \psi^i\\ \psi_i^C\end{pmatrix},\tag{15}$$

built of left fields alone. Similarly, we could introduce a multi-spinor made of right fields. In our analysis, restricted only to the (hard dense loop improved) rainbow approximation, the left and right sectors of the theory completely decouple. Then, without loss of generality, it is sufficient to study the SD equation only in one of the sectors.

With the notation in Eq. (15), the inverse full propagator of quarks takes a particularly simple block-diagonal form

$$G_p^{-1}=\mathrm{diag}(S_p^{-1}\delta_a^b\delta_j^i,\quad s_p^{-1}\delta_j^i,\quad \overline{s_p}^{-1}\delta_i^j),\tag{16}$$

where, upon neglecting the wave function renormalization effects of quarks [10–16],

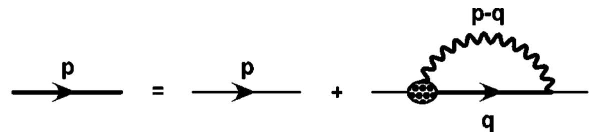

FIG. 1. The diagrammatic representation of the SD equation.

$$\begin{aligned}S_p^{-1}&=-i(\not p+\mu\gamma^0\gamma^5+\Delta_p\mathcal{P}_-+\tilde{\Delta}_p\mathcal{P}_+)\\&=-i[(p_0-\epsilon_p^-)\gamma^0\Lambda_p^++(\Delta_p^-)^*\Lambda_p^++(p_0+\epsilon_p^+)\gamma^0\Lambda_p^-\\&\quad+(\Delta_p^+)^*\Lambda_p^-]\mathcal{P}_+-i[(p_0-\epsilon_p^+)\gamma^0\Lambda_p^++\Delta_p^+\Lambda_p^+\\&\quad+(p_0+\epsilon_p^-)\gamma^0\Lambda_p^-+\Delta_p^-\Lambda_p^-]\mathcal{P}_-,\end{aligned}\tag{17}$$

$$\begin{aligned}s_p^{-1}&=-i(\not p+\mu\gamma^0)\mathcal{P}_+\\&=-i\gamma^0[(p_0-\epsilon_p^-)\Lambda_p^++(p_0+\epsilon_p^+)\Lambda_p^-]\mathcal{P}_+,\end{aligned}\tag{18}$$

$$\begin{aligned}\overline{s_p}^{-1}&=-i(\not p-\mu\gamma^0)\mathcal{P}_-\\&=-i\gamma^0[(p_0-\epsilon_p^+)\Lambda_p^++(p_0+\epsilon_p^-)\Lambda_p^-]\mathcal{P}_-,\end{aligned}\tag{19}$$

with $\epsilon_p^\pm=|\vec{p}|\pm\mu$. The notation for the gap function, $\Delta_p=\Delta_p^+\Lambda_p^++\Delta_p^-\Lambda_p^-$ and $\tilde{\Delta}_p=\gamma^0\Delta_p^\dagger\gamma^0$, as well as the ''on-shell'' projectors of quarks,

$$\Lambda_p^\pm=\frac{1}{2}\left(1\pm\frac{\vec{\alpha}\cdot\vec{p}}{|\vec{p}|}\right),\quad \vec{\alpha}=\gamma^0\vec{\gamma},\tag{20}$$

is the same as in Ref. [11].

Now, it is straightforward to derive the matrix form of the SD equation:

$$\begin{aligned}G_p^{-1}&=(G_p^0)^{-1}+4\pi\alpha_s\\&\quad\times\int\frac{d^4q}{(2\pi)^4}\gamma^{A\mu}G_q\Gamma^{A\nu}(q,p)\mathcal{D}_{\mu\nu}(q-p),\end{aligned}\tag{21}$$

where $\gamma^{A\mu}$ and $\Gamma^{A\mu}$ are the bare and the full vertices, respectively. This equation is diagrammatically presented in Fig. 1. The thin and bold solid lines correspond to the bare and full quark propagators, respectively. The wavy line stands for the full gluon propagator.

The only complication of using the multi-component spinor (15) appears due to the more involved structure of the quark-gluon interaction vertex. Indeed, the explicit form of the bare vertex reads

$$\gamma^{A\mu}=\gamma^{\mu}\begin{pmatrix} [T_a^{Ab}-2\delta_8^A T_a^{8b}\mathcal{P}_-]\delta_j^i & T_a^{A3}\mathcal{P}_+\delta_j^i & -\hat{\varepsilon}_{ac}^{ij}T_3^{Ac}\mathcal{P}_- \\ T_3^{Ab}\mathcal{P}_+\delta_j^i & T_3^{A3}\mathcal{P}_+\delta_j^i & 0 \\ -T_c^{A3}\hat{\varepsilon}_{ij}^{cb}\mathcal{P}_- & 0 & -T_3^{A3}\mathcal{P}_-\delta_i^j \end{pmatrix}, \tag{22}$$

where $\hat{\varepsilon}_{ac}^{ij}\equiv\varepsilon^{ij}\varepsilon_{3ac}$ and T^A are the $SU(3)_c$ generators in the fundamental representation $[\mathrm{tr}(T^AT^B)=\delta^{AB}/2]$. The relatively complicated structure of the bare vertex might suggest that our notation is somewhat unnatural. As we shall see in Sec. IV, because of the breakdown of the $SU(3)_c$ symmetry, this structure, on the contrary, is quite natural, and it is especially so in the case of the full quark-gluon vertex function.

The gluon propagator in the SD equation is the same as in Ref. [11]. When the Meissner effect is neglected, the propagator in the Euclidean space ($k_0=ik_4$) reads

$$\mathcal{D}_{\mu\nu}^{AB}(ik_4,\vec{k})\equiv\delta^{AB}\mathcal{D}_{\mu\nu}(ik_4,\vec{k})\simeq i\delta^{AB}\frac{|\vec{k}|}{|\vec{k}|^3+\pi M^2|k_4|/2}O_{\mu\nu}^{(1)} + i\delta^{AB}\frac{1}{k_4^2+|\vec{k}|^2+2M^2}O_{\mu\nu}^{(2)} + i\delta^{AB}\frac{d}{k_4^2+|\vec{k}|^2}O_{\mu\nu}^{(3)}, \tag{23}$$

where $M^2=N_f\alpha_s\mu^2/\pi$ (with $N_f=2$), and $O_{\mu\nu}^{(i)}$ are the projection operators of three different types of gluons (magnetic, electric and longitudinal, respectively); see Ref. [11]. The Meissner effect could be qualitatively taken into account by the following replacement of the magnetic term [11]:

$$i\delta^{AB}\frac{|\vec{k}|}{|\vec{k}|^3+\pi M^2|k_4|/2}O_{\mu\nu}^{(1)} \to i\delta^{AB}\frac{|\vec{k}|}{|\vec{k}|^3+\pi M^2(|k_4|+c|\Delta_0^-|)/2}O_{\mu\nu}^{(1)} \tag{24}$$

in the propagators of those five gluons that correspond to the broken color generators ($A,B=4,\ldots,8$). In this last expression, $c=O(1)$ is a constant of order 1.

By inverting the expression in Eq. (16), we obtain the following representation for the quark propagator:

$$G_p=\mathrm{diag}(S_p\delta_a^b\delta_j^i, s_p\delta_j^i, \overline{s_p}\delta_i^j), \tag{25}$$

$$S_p=i\frac{\gamma^0(p_0+\epsilon_p^+)-\Delta_p^+}{p_0^2-(\epsilon_p^+)^2-|\Delta_p^+|^2}\Lambda_p^-\mathcal{P}_+ + i\frac{\gamma^0(p_0-\epsilon_p^+)-(\Delta_p^+)^*}{p_0^2-(\epsilon_p^+)^2-|\Delta_p^+|^2}\Lambda_p^+\mathcal{P}_- + i\frac{\gamma^0(p_0-\epsilon_p^-)-\Delta_p^-}{p_0^2-(\epsilon_p^-)^2-|\Delta_p^-|^2}\Lambda_p^+\mathcal{P}_+ + i\frac{\gamma^0(p_0+\epsilon_p^-)-(\Delta_p^-)^*}{p_0^2-(\epsilon_p^-)^2-|\Delta_p^-|^2}\Lambda_p^-\mathcal{P}_-, \tag{26}$$

$$s_p=i\frac{\gamma^0\Lambda_p^+\mathcal{P}_-}{p_0+\epsilon_p^+}+i\frac{\gamma^0\Lambda_p^-\mathcal{P}_-}{p_0-\epsilon_p^-}, \tag{27}$$

$$\overline{s_p}\equiv Cs_{-p}^TC^\dagger=i\frac{\gamma^0\Lambda_p^+\mathcal{P}_+}{p_0+\epsilon_p^-}+i\frac{\gamma^0\Lambda_p^-\mathcal{P}_+}{p_0-\epsilon_p^+}. \tag{28}$$

The bare propagator in Eq. (21) is similar but with zero value of the gap.

In the improved ladder (rainbow) approximation, both vertices in the SD equation are bare. By making use of the propagators in Eqs. (23) and (25), along with the vertex in Eq. (22), we derive the well-known gap equation [11–16]

$$\Delta_p^-=\frac{4}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{\Delta_q^-\,\mathrm{tr}(\gamma^\mu\Lambda_q^+\gamma^\nu\Lambda_p^-)}{q_0^2-(\epsilon_q^-)^2-|\Delta_q^-|^2}\mathcal{D}_{\mu\nu}(q-p). \tag{29}$$

After calculating the trace and performing the angular integration (see Appendix A), this equation considerably simplifies. Then, by assuming that the dependence of the gap on the spatial component of the momentum is irrelevant in the vicinity of the Fermi surface, one arrives at the following approximate equation [10–15]:

$$\Delta^-(p_4)\simeq\frac{2\alpha_s}{9\pi}\int_0^\Lambda\frac{dq_4\Delta^-(q_4)}{\sqrt{q_4^2+|\Delta_0^-|^2}}\ln\frac{\Lambda}{|p_4-q_4|}, \tag{30}$$

where $\Lambda=(4\pi)^{3/2}\mu/\alpha_s^{5/2}$. The analytical solution to this equation is relatively easy to obtain [11]:

$$\Delta^-(p_4)\simeq|\Delta_0^-|J_0\left(\nu\sqrt{\frac{p_4}{|\Delta_0^-|}}\right),\quad p_4\leqslant|\Delta_0^-|, \tag{31a}$$

$$\Delta^-(p_4)\simeq|\Delta_0^-|\sqrt{J_0^2(\nu)+J_1^2(\nu)}\times\sin\left(\frac{\nu}{2}\ln\frac{\Lambda}{p_4}\right),\quad p_4\geqslant|\Delta_0^-|, \tag{31b}$$

where $J_i(z)$ are the Bessel functions and $\nu=\sqrt{8\alpha_s/9\pi}$. The corresponding result for the value of the gap reads

$$|\Delta_0^-| \simeq \frac{(4\pi)^{3/2}\,\mathrm{e}\mu}{\alpha_s^{5/2}} \exp\left(-\frac{3\pi^{3/2}}{2^{3/2}\sqrt{\alpha_s}}\right), \tag{32}$$

where $\mathrm{e}=2.718\ldots$. Most of the existing studies [10–16] seem to agree upon the dependence of this result on the coupling constant. The issue of the overall constant factor, however, is still not settled. The analysis of Ref. [15], for example, suggests that the wave function renormalization effects of quarks give an extra factor of order 1.[1] Another source of corrections might be due to the running of the coupling constant [31]. In addition to these, as we argue in Appendix B using the Ward identities, there also exists at least one non-perturbative correction that could modify the constant factor in the expression for the gap.

IV. WARD IDENTITIES

As in any other gauge theory, in order to preserve the gauge invariance in QCD, one has to make sure that some exact relations (Ward identities) between Green functions are satisfied. In this section, we consider the simplest Ward identities that relate the vertex functions and the quark propagators. In addition to establishing the longitudinal part of the full vertex function, these identities will play a very important role in our analysis of the BS equations for the NG and pseudo NG bosons.

In general, the structure of Ward identities in non-Abelian gauge theories (the Slavnov-Taylor identities) is much more complicated than in Abelian ones: they include contributions of the Faddeev-Popov ghosts. Fortunately in the (hard dense loop improved) ladder approximation, used in this paper, the situation simplifies. Indeed, since the direct interactions between gluons are neglected in this approximation, the Ward identities have an Abelian-like structure.

To start with, let us rewrite the conserved currents (related to the color symmetry) in terms of the Majorana fields, defined in Eqs. (3) and (4), and the Weyl spinors of the third color. By making use of their definitions, it is straightforward to obtain the following representation for the currents:

$$\begin{aligned} j_\mu^A(x) &= \overline{\Psi}_D(x)\gamma_\mu T^A \Psi_D(x) \\ &= \frac{1}{2\sqrt{3}}\delta_8^A \overline{\Psi}_i^a(x)\gamma_\mu \mathcal{P}_+ \Psi_a^i(x) + \overline{\psi}_i(x)\gamma_\mu T_3^{Aa}\mathcal{P}_+\Psi_a^i(x) \\ &\quad + \overline{\Psi}_i^a(x)\mathcal{P}_-\gamma_\mu T_a^{A3}\psi^i(x) + \overline{\psi}_i(x)\gamma_\mu T_3^{A3}\psi^i(x) \\ &\quad + \frac{1}{2\sqrt{3}}\delta_8^A \overline{\Phi}_i^a(x)\gamma_\mu \mathcal{P}_- \Phi_a^i(x) \\ &\quad + \overline{\phi}_i(x)\gamma_\mu T_3^{Aa}\mathcal{P}_-\Phi_a^i(x) + \overline{\Phi}_i^a(x)\mathcal{P}_+\gamma_\mu T_a^{A3}\phi^i(x) \\ &\quad + \overline{\phi}_i(x)\gamma_\mu T_3^{A3}\phi^i(x), \end{aligned} \tag{33}$$

where, in agreement with the remark made above, the irrelevant (in this approximation) contributions of gluons and ghosts are omitted. Here and in the rest of this section, we assume that $A=4,\ldots,8$; i.e., we do not consider the currents which correspond to the generators of the unbroken $SU(2)_c$ subgroup.

As we mentioned earlier, we are interested in the Ward identities that relate the quark-gluon vertices to the propagators of quarks. Therefore, let us introduce the following (non-amputated) vertex functions:

$$\mathbf{\Gamma}_{aj,\mu}^{A,i}(x,y) = \langle 0|Tj_\mu^A(0)\Psi_a^i(x)\overline{\psi}_j(y)|0\rangle, \tag{34a}$$

$$\mathbf{\Gamma}_{j,\mu}^{A,ai}(x,y) = \langle 0|Tj_\mu^A(0)\psi^i(x)\overline{\Psi}_j^a(y)|0\rangle, \tag{34b}$$

$$\mathbf{\Gamma}_{j,\mu}^{A,i}(x,y) = \langle 0|Tj_\mu^A(0)\Psi_a^i(x)\overline{\Psi}_j^a(y)|0\rangle, \tag{34c}$$

$$\tilde{\mathbf{\Gamma}}_{aj,\mu}^{A,i}(x,y) = \langle 0|Tj_\mu^A(0)\Phi_a^i(x)\overline{\phi}_j(y)|0\rangle, \tag{34d}$$

$$\tilde{\mathbf{\Gamma}}_{j,\mu}^{A,ai}(x,y) = \langle 0|Tj_\mu^A(0)\phi^i(x)\overline{\Phi}_j^a(y)|0\rangle, \tag{34e}$$

$$\tilde{\mathbf{\Gamma}}_{j,\mu}^{A,i}(x,y) = \langle 0|Tj_\mu^A(0)\Phi_a^i(x)\overline{\Phi}_j^a(y)|0\rangle. \tag{34f}$$

Besides the operator of the conserved current, the first three vertices include only left-handed quark fields, while the other three vertices contain only right-handed fields. Because of the invariance under the parity, all the mixed, left-right vertices are trivial. For this reason, they are of no special interest here, and we do not consider them.

As usual, in order to derive the Ward identities, one needs to know the transformation properties of the quark fields. The color symmetry transformations of the Dirac spinors are well known. By making use of them, it is straightforward to derive the following infinitesimal transformations for the spinors of interest:

$$\delta\psi^i = i\omega^A(T_3^{Aa}\mathcal{P}_+\Psi_a^i + T_3^{A3}\psi^i), \tag{35}$$

$$\delta\overline{\psi}_i = -i\omega^A(\overline{\Psi}_i^a\mathcal{P}_-T_a^{A3} + \overline{\psi}_i T_3^{A3}), \tag{36}$$

$$\delta\Psi_a^i = i\omega^A\left(T_a^{Ab}\mathcal{P}_+\Psi_b^i + T_a^{A3}\psi^i - \varepsilon_{3ab}\varepsilon^{ij}T_3^{Ab}\psi_j^C - \delta_8^A\frac{1}{2\sqrt{3}}\mathcal{P}_-\Psi_a^i\right), \tag{37}$$

[1] Note that the argument of Ref. [15] is somewhat incomplete, since the calculation is performed for the critical temperature rather than the order parameter itself. The celebrated BCS relation between the critical temperature and the gap might not be satisfied after the Meissner effect is carefully taken into account.

$$\delta\overline{\Psi}_i^a=-i\omega^A\bigg(\overline{\Psi}_i^b\mathcal{P}_-T_b^{Aa}+\overline{\psi}_iT_3^{Aa}-\varepsilon^{3ab}\varepsilon_{ij}\overline{\psi}^{Cj}T_b^{A3}-\delta_8^A\frac{1}{2\sqrt{3}}\overline{\Psi}_i^a\mathcal{P}_+\bigg), \tag{38}$$

for the left-handed fields, and

$$\delta\phi^i=i\omega^A(T_3^{Aa}\mathcal{P}_-\Phi_a^i+T_3^{A3}\phi^i), \tag{39}$$

$$\delta\overline{\phi}_i=-i\omega^A(\overline{\Phi}_i^a\mathcal{P}_+T_a^{A3}+\overline{\phi}_iT_3^{A3}), \tag{40}$$

$$\delta\Phi_a^i=i\omega^A\bigg(T_a^{Ab}\mathcal{P}_-\Phi_b^i+T_a^{A3}\phi^i-\varepsilon_{3ab}\varepsilon^{ij}T_3^{Ab}\phi_j^C-\delta_8^A\frac{1}{2\sqrt{3}}\mathcal{P}_+\Phi_a^i\bigg), \tag{41}$$

$$\delta\overline{\Phi}_i^a=-i\omega^A\bigg(\overline{\Phi}_i^b\mathcal{P}_+T_b^{Aa}+\overline{\phi}_iT_3^{Aa}-\varepsilon^{3ab}\varepsilon_{ij}\overline{\phi}^{Cj}T_b^{A3}-\delta_8^A\frac{1}{2\sqrt{3}}\overline{\Phi}_i^a\mathcal{P}_-\bigg), \tag{42}$$

for the right-handed fields. In all these expressions, ω^A are small parameters, parametrizing the transformations of the $SU(3)_c$ group.

In a standard way, by making use of the current conservation as well as the definition of the vertices in Eq. (34), we obtain the following Ward identities for the non-amputated vertices:

$$P^\mu\Gamma_{aj,\mu}^{A,i}(k+P,k)=iT_a^{A3}\delta_j^i[s_k-S_{k+P}]\mathcal{P}_-, \tag{43a}$$

$$P^\mu\Gamma_{j,\mu}^{A,ai}(k+P,k)=iT_3^{Aa}\delta_j^i\mathcal{P}_+[S_k-s_{k+P}], \tag{43b}$$

$$P^\mu\boldsymbol{\Gamma}_{j,\mu}^{A,i}(k+P,k)=\frac{i}{2\sqrt{3}}\delta_8^A\delta_j^i[2\mathcal{P}_+S_k-\mathcal{P}_-S_k-2S_{k+P}\mathcal{P}_-+S_{k+P}\mathcal{P}_+], \tag{43c}$$

$$P^\mu\tilde{\Gamma}_{aj,\mu}^{A,i}(k+P,k)=iT_a^{A3}\delta_j^i[\tilde{s}_k-\tilde{S}_{k+P}]\mathcal{P}_+, \tag{43d}$$

$$P^\mu\tilde{\Gamma}_{j,\mu}^{A,ai}(k+P,k)=iT_3^{Aa}\delta_j^i\mathcal{P}_-[\tilde{S}_k-\tilde{s}_{k+P}], \tag{43e}$$

$$P^\mu\tilde{\boldsymbol{\Gamma}}_{j,\mu}^{A,i}(k+P,k)=\frac{i}{2\sqrt{3}}\delta_8^A\delta_j^i[2\mathcal{P}_-\tilde{S}_k-\mathcal{P}_+\tilde{S}_k-2\tilde{S}_{k+P}\mathcal{P}_++\tilde{S}_{k+P}\mathcal{P}_-], \tag{43f}$$

where S_k, s_k, and $\tilde{S}_k$, $\tilde{s}_k$ are the Fourier transforms of the quark propagators in the left and right sectors, respectively,

$$S(x-y)\delta_a^b\delta_j^i=\langle 0|T\Psi_a^i(x)\overline{\Psi}_j^b(y)|0\rangle, \tag{44}$$

$$s(x-y)\delta_j^i=\langle 0|T\psi^i(x)\overline{\psi}_j(y)|0\rangle, \tag{45}$$

$$\tilde{S}(x-y)\delta_a^b\delta_j^i=\langle 0|T\Phi_a^i(x)\overline{\Phi}_j^b(y)|0\rangle, \tag{46}$$

$$\tilde{s}(x-y)\delta_j^i=\langle 0|T\phi^i(x)\overline{\phi}_j(y)|0\rangle. \tag{47}$$

As we discussed in Sec. III, in the approximation with no wave function renormalization effects, the explicit form of the momentum space propagators for the left-handed fields is given in Eqs. (26) and (27). For the completeness of our presentation, we also mention that the right-handed propagators are the same, except that the projectors $\mathcal{P}_-$ and $\mathcal{P}_+$ interchange.

At this point, let us note that the use of non-amputated vertices in this section is not accidental. In fact, it is crucial for a quick derivation of the Ward identities. Other than that, non-amputated vertices are not very convenient to work with. In fact, it is amputated rather than non-amputated vertices that are usually used in Feynman diagrams. For example, both the bare and full vertices in Fig. 1 are amputated ones. Similarly, it is amputated vertices that appear in the BS equation in Sec. V. The formal definitions of amputated vertices read

$$\Gamma_{aj,\mu}^{A,i}(k+P,k)=S_{k+P}^{-1}\boldsymbol{\Gamma}_{aj,\mu}^{A,i}(k+P,k)s_k^{-1}, \tag{48a}$$

$$\Gamma_{j,\mu}^{A,ai}(k+P,k)=s_{k+P}^{-1}\boldsymbol{\Gamma}_{j,\mu}^{A,ai}(k+P,k)S_k^{-1}, \tag{48b}$$

$$\Gamma_{j,\mu}^{A,i}(k+P,k)=S_{k+P}^{-1}\boldsymbol{\Gamma}_{j,\mu}^{A,i}(k+P,k)S_k^{-1}, \tag{48c}$$

$$\tilde{\Gamma}_{aj,\mu}^{A,i}(k+P,k)=\tilde{S}_{k+P}^{-1}\tilde{\boldsymbol{\Gamma}}_{aj,\mu}^{A,i}(k+P,k)\tilde{s}_k^{-1}, \tag{48d}$$

$$\tilde{\Gamma}_{j,\mu}^{A,ai}(k+P,k)=\tilde{s}_{k+P}^{-1}\tilde{\boldsymbol{\Gamma}}_{j,\mu}^{A,ai}(k+P,k)\tilde{S}_k^{-1}, \tag{48e}$$

$$\tilde{\Gamma}_{j,\mu}^{A,i}(k+P,k)=\tilde{S}_{k+P}^{-1}\tilde{\boldsymbol{\Gamma}}_{j,\mu}^{A,i}(k+P,k)\tilde{S}_k^{-1}. \tag{48f}$$

These, as is clear from our discussion above, are directly related to quark-gluon interactions. As is clear from Eq. (43), they satisfy the following identities of their own:

$$P^\mu\Gamma_{aj,\mu}^{A,i}(k+P,k)=iT_a^{A3}\delta_j^i[S_{k+P}^{-1}-s_k^{-1}]\mathcal{P}_+, \tag{49a}$$

$$P^\mu\Gamma_{j,\mu}^{A,ai}(k+P,k)=iT_3^{Aa}\delta_j^i\mathcal{P}_-[s_{k+P}^{-1}-S_k^{-1}], \tag{49b}$$

$$P^\mu\Gamma_{j,\mu}^{A,i}(k+P,k)=\frac{i}{2\sqrt{3}}\delta_8^A\delta_j^i[2S_{k+P}^{-1}\mathcal{P}_+-S_{k+P}^{-1}\mathcal{P}_--2\mathcal{P}_-S_k^{-1}+\mathcal{P}_+S_k^{-1}], \tag{49c}$$

$$P^\mu\Gamma_{aj,\mu}^{A,i}(k+P,k)=iT_a^{A3}\delta_j^i[\tilde{S}_{k+P}^{-1}-\tilde{s}_k^{-1}]\mathcal{P}_-, \tag{49d}$$

$$P^\mu\Gamma_{j,\mu}^{A,ai}(k+P,k)=iT_3^{Aa}\delta_j^i\mathcal{P}_+[\tilde{s}_{k+P}^{-1}-\tilde{S}_k^{-1}], \tag{49e}$$

$$P^{\mu}\Gamma^{A,i}_{j,\mu}(k+P,k)=\frac{i}{2\sqrt{3}}\delta^{A}_{8}\delta^{i}_{j}[2\tilde{S}^{-1}_{k+P}\mathcal{P}_{-}-\tilde{S}^{-1}_{k+P}\mathcal{P}_{+}-2\mathcal{P}_{+}\tilde{S}^{-1}_{k}+\mathcal{P}_{-}\tilde{S}^{-1}_{k}]. \tag{49f}$$

In the rest of the paper, we are going to use these Ward identities a number times. Because of the relatively simple structure of the inverse quark propagators, this last form of the identities will be particularly convenient.

In connection with the Ward identities, it is appropriate to mention here the complementary analysis of Ref. [32]. The authors of that paper consider the contribution to the Ward identity that is directly related to the wave function renormalization of quarks.

V. DERIVATION OF THE BS EQUATION

In quantum field theory, bound states and resonances reveal themselves through the appearance of poles in Green functions. These latter satisfy some general BS equations which usually are rather complicated.

To consider the problem of diquark bound states in cold dense QCD, one has to introduce a four-point Green function that describes the two particle scattering in the diquark channel of interest. The residue at the pole of the Green function is related to the BS wave function of the bound state. By starting from the (inhomogeneous) BS equation for the four-point Green function, it is straightforward to derive the so-called homogeneous BS equation for the wave function.

In the problem at hand, we could construct quite a few different diquark states. Not all of them could actually be bound states. For example, one would not expect from a two particle state to form a bound state unless there is some attraction in the corresponding channel. Now, in dense QCD, the dominant interaction between quarks is given by one-gluon exchange. As we know, this interaction is attractive only in antisymmetric diquark channels. Therefore, without loss of generality, it is sufficient to consider only the following bound states:

$$\boldsymbol{\chi}^{(\tilde{b})}_{a}(p,P)=\delta^{\tilde{b}}_{a}\boldsymbol{\chi}(p,P)=\langle 0|T\Psi^{i}_{a}(p+P/2)\overline{\psi}_{i}(p-P/2)|P;\tilde{b}\rangle_{L},\quad \tilde{b}=1,2, \tag{50a}$$

$$\boldsymbol{\lambda}^{b}_{(\tilde{a})}(p,P)=\delta^{b}_{\tilde{a}}\boldsymbol{\lambda}(p,P)=\langle 0|T\psi^{i}(p+P/2)\overline{\Psi}^{b}_{i}(p-P/2)|P;\tilde{a}\rangle_{L},\quad \tilde{a}=1,2, \tag{50b}$$

$$\boldsymbol{\eta}(p,P)=\langle 0|T\Psi^{i}_{a}(p+P/2)\overline{\Psi}^{a}_{i}(p-P/2)|P\rangle_{L}, \tag{50c}$$

$$\boldsymbol{\sigma}(p,P)=\langle 0|T\psi^{i}(p+P/2)\overline{\psi}_{i}(p-P/2)|P\rangle_{L}, \tag{50d}$$

plus the states made out of the right handed fields,

$$\tilde{\boldsymbol{\chi}}^{(\tilde{b})}_{a}(p,P)=\delta^{\tilde{b}}_{a}\tilde{\boldsymbol{\chi}}(p,P)=\langle 0|T\Phi^{i}_{a}(p+P/2)\overline{\phi}_{i}(p-P/2)|P;\tilde{b}\rangle_{R},\quad \tilde{b}=1,2, \tag{51a}$$

$$\tilde{\boldsymbol{\lambda}}^{b}_{(\tilde{a})}(p,P)=\delta^{b}_{\tilde{a}}\tilde{\boldsymbol{\lambda}}(p,P)=\langle 0|T\phi^{i}(p+P/2)\overline{\Phi}^{b}_{i}(p-P/2)|P;\tilde{a}\rangle_{R},\quad \tilde{a}=1,2, \tag{51b}$$

$$\tilde{\boldsymbol{\eta}}(p,P)=\langle 0|T\Phi^{i}_{a}(p+P/2)\overline{\Phi}^{a}_{i}(p-P/2)|P\rangle_{R}, \tag{51c}$$

$$\tilde{\boldsymbol{\sigma}}(p,P)=\langle 0|T\phi^{i}(p+P/2)\overline{\phi}_{i}(p-P/2)|P\rangle_{R}. \tag{51d}$$

Notice that analogous states containing charge conjugate fields of the third color, ψ^{C}_{i} and ϕ^{C}_{i}, are not independent. Because of the property in Eqs. (7) and (8), they are related to those already introduced.

For completeness, let us note that the only other diquark channel that we do not consider here is a triplet under $SU(2)_c$. It is however clear that repulsion dominates in such a channel because this triplet comes from the $SU(3)_c$ sextet. Notice that, although one does not expect the appearance of a $\boldsymbol{\sigma}$ bound state, we keep the $\boldsymbol{\sigma}$ wave function in the analysis. This is because the equations for the BS wave functions of the two singlets, $\boldsymbol{\sigma}$ and $\boldsymbol{\eta}$, may not decouple. Notice also that the doublet, antidoublet and singlets coming from the $SU(3)_c$ triplet and antitriplet can mix with the doublet, antidoublet and singlets coming from the $SU(3)_c$ nonet (octet plus singlet).

Before proceeding further with the analysis of the bound states, let us recall that parity is not broken in dense QCD with two flavors; see Sec. II. Then, all the bound states can be chosen in such a way that they are either parity even or parity odd. Clearly, the states in Eqs. (50) and (51) do not share this property. In order to fix this, we could have constructed the following scalars and pseudoscalars:

$$|P;n\rangle_{s}=\frac{1}{\sqrt{2}}(|P;n\rangle_{L}+|P;n\rangle_{R}), \tag{52}$$

$$|P;n\rangle_{p}=\frac{1}{\sqrt{2}}(|P;n\rangle_{L}-|P;n\rangle_{R}), \tag{53}$$

where n denotes the appropriate state.

In our analysis, however, we find it more convenient to work with the bound states constructed of either left-handed or right-handed fields separately. This is because, in the (hard dense loop improved) ladder approximation, the two sectors of the theory stay completely decoupled. Besides that, the dynamics of the left and right fields are identical in the approximation used. Under these conditions, the degeneracy of the left and right sectors is equivalent to the degeneracy of the parity-even and parity-odd ones. In this way, we reveal the parity doubling property of the spectrum of bound states in QCD at an asymptotically high density of quark matter.[2]

[2]Notice that there are some subtleties in applying this parity doubling argument to the case of the (pseudo-)NG bound states; see Sec. VI.

Equations for non-amputated wave functions

In order to derive the BS equations, we use the method developed in Ref. [33] for the case of a zero chemical potential (for a review, see Ref. [34]). To this end, we need to know the quark propagators and the quark-gluon interactions in the color superconducting phase. From the analysis of the SD equation (see Sec. III), we got the structure of the quark propagator. We also know that the approximation with no wave function renormalization effects is quite reliable, at least in the leading order. By combining these facts together, we arrive at the following effective Lagrangian of quarks:

$$\begin{aligned}\mathcal{L}_{eff}=&\overline{\Psi}^a_i(\not p+\mu\gamma^0\gamma^5+\Delta\mathcal{P}_-+\tilde{\Delta}\mathcal{P}_+)\Psi^i_a+\overline{\Psi}^a_i\not A^B[T^{Bb}_a-2\delta^B_8T^{8b}_a\mathcal{P}_-]\Psi^i_b+\overline{\psi}_i\not A_3{}^b\mathcal{P}_+\Psi^i_b-\overline{\psi}^{Ci}\not A_a{}^3\hat{\varepsilon}^{ab}_{ij}\mathcal{P}_-\Psi^j_b\\&+\overline{\psi}_i(\not p+\mu\gamma^0)\mathcal{P}_+\psi^i+\overline{\psi}^{Ci}(\not p-\mu\gamma^0)\mathcal{P}_-\psi^C_i+\overline{\psi}_i\not A_3{}^3\mathcal{P}_+\psi^i-\overline{\psi}^{Ci}\not A_3{}^3\mathcal{P}_-\psi^C_i+\overline{\Psi}^a_i\not A_a{}^3\mathcal{P}_+\psi^i-\overline{\Psi}^a_i\hat{\varepsilon}^{ij}_{ab}\not A_3{}^b\mathcal{P}_-\psi^C_j\\&+\overline{\Phi}^a_i(\not p-\mu\gamma^0\gamma^5+\Delta\mathcal{P}_++\tilde{\Delta}\mathcal{P}_-)\Phi^i_a+\overline{\Phi}^a_i\not A^B[T^{Bb}_a-2\delta^B_8T^{8b}_a\mathcal{P}_+]\Phi^i_b+\overline{\phi}_i\not A_3{}^b\mathcal{P}_-\Phi^i_b-\overline{\phi}^{Ci}\not A_a{}^3\hat{\varepsilon}^{ab}_{ij}\mathcal{P}_+\Phi^j_b\\&+\overline{\phi}_i(\not p+\mu\gamma^0)\mathcal{P}_-\phi^i+\overline{\phi}^{Ci}(\not p-\mu\gamma^0)\mathcal{P}_+\phi^C_i+\overline{\phi}_i\not A_3{}^3\mathcal{P}_-\phi^i-\overline{\phi}^{Ci}\not A_3{}^3\mathcal{P}_+\phi^C_i+\overline{\Phi}^a_i\not A_a{}^3\mathcal{P}_-\phi^i-\overline{\Phi}^a_i\hat{\varepsilon}^{ij}_{ab}\not A_3{}^b\mathcal{P}_+\phi^C_j,\end{aligned}\tag{54}$$

where, by definition, $\Delta=\Delta^+_p\Lambda^+_p+\Delta^-_p\Lambda^-_p$, $\tilde{\Delta}=\gamma^0\Delta^\dagger\gamma^0$, and $\hat{\varepsilon}^{ab}_{ij}=\varepsilon^{3ab}\varepsilon_{ij}$. The choice of Δ, as is easy to check, corresponds to the case of the parity-even Majorana mass.

The effective Lagrangian in Eq. (54) is the starting point in derivation of the BS equations for the wave functions introduced in Eqs. (50) and (51). While using the notation of the multicomponent spinor in Eq. (15), it is natural to combine the (left-handed) wave functions of the bound states into the following matrix:

$$X(p,P)=\begin{pmatrix}\frac{1}{2}\boldsymbol{\eta}(p,P)\delta^b_a\delta^i_j & \boldsymbol{\chi}^{(\tilde{b})}_a(p,P)\delta^i_j & \hat{\varepsilon}^{ij}_{ac}C[\boldsymbol{\lambda}^c_{(\tilde{a})}(-p,P)]^TC^\dagger\\ \boldsymbol{\lambda}^b_{(\tilde{a})}(p,P)\delta^i_j & \boldsymbol{\sigma}(p,P)\delta^i_j & 0\\ C[\boldsymbol{\chi}^{(\tilde{b})}_c(-p,P)]^TC^\dagger\hat{\varepsilon}^{cb}_{ij} & 0 & C\boldsymbol{\sigma}^T(-p,P)C^\dagger\delta^j_i\end{pmatrix},\tag{55}$$

where we took into account the property of Majorana spinors given in Eq. (7). We could also introduce a similar matrix wave function for the right-handed fields. Since, however, in the (hard dense loop improved) ladder approximation the left-handed and right-handed sectors decouple, we study one of them in detail, and only occasionally refer to the other.

In the (hard dense loop improved) ladder approximation, the BS wave function in Eq. (55) satisfies the following matrix equation:

$$G^{-1}\left(p+\frac{P}{2}\right)X(p;P)G^{-1}\left(p-\frac{P}{2}\right)=-4\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^{A\mu}X(q;P)\gamma^{B\nu}\mathcal{D}^{AB}_{\mu\nu}(q-p),\tag{56}$$

where $\mathcal{D}^{AB}_{\mu\nu}(q-p)$ is the gluon propagator and $\gamma^{A\mu}$ is the bare quark-gluon vertex. This approximation has the same status as the rainbow approximation in the SD equation. It assumes that the coupling constant is weak, and the leading perturbative expression for the kernel of the BS equation adequately represents the quark interactions. Schematically, the BS equation (56) is shown in Fig. 2.

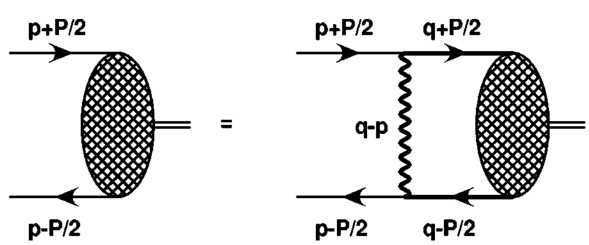

FIG. 2. The diagrammatic representation of the BS equation.

By writing it in components, we arrive at the set of four equations

$$\begin{aligned}S^{-1}_{p+P/2}\boldsymbol{\chi}^{(\tilde{b})}_a(p,P)s^{-1}_{p-P/2}=&-\frac{2}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\{\mathcal{P}_-\boldsymbol{\chi}^{(\tilde{b})}_a(q,P)\mathcal{P}_-\\&+3\mathcal{P}_-C[\boldsymbol{\chi}^{(\tilde{b})}_a(-q,P)]^TC^\dagger\mathcal{P}_-\\&-\mathcal{P}_+\boldsymbol{\chi}^{(\tilde{b})}_a(q,P)\mathcal{P}_-\}\gamma^\nu\mathcal{D}_{\mu\nu}(q-p),\end{aligned}\tag{57}$$

$$\begin{aligned}s^{-1}_{p+P/2}\boldsymbol{\lambda}^b_{(\tilde{a})}(p,P)S^{-1}_{p-P/2}=&-\frac{2}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\{\mathcal{P}_+\boldsymbol{\lambda}^b_{(\tilde{a})}(q,P)\mathcal{P}_+\\&+3\mathcal{P}_+C[\boldsymbol{\lambda}^b_{(\tilde{a})}(-q,P)]^TC^\dagger\mathcal{P}_+\\&-\mathcal{P}_+\boldsymbol{\lambda}^b_{(\tilde{a})}(q,P)\mathcal{P}_-\}\gamma^\nu\mathcal{D}_{\mu\nu}(q-p),\end{aligned}\tag{58}$$

$$S^{-1}_{p+P/2}\boldsymbol{\eta}(p,P)S^{-1}_{p-P/2}$$

$$=-\frac{8}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^{\mu}\bigg[\mathcal{P}_+\boldsymbol{\eta}(q,P)\mathcal{P}_+ + \mathcal{P}_-\boldsymbol{\eta}(q,P)\mathcal{P}_- + \frac{5}{4}\mathcal{P}_-\boldsymbol{\eta}(q,P)\mathcal{P}_+ + \frac{5}{4}\mathcal{P}_+\boldsymbol{\eta}(q,P)\mathcal{P}_- + \frac{3}{2}\mathcal{P}_+\boldsymbol{\sigma}(q,P)\mathcal{P}_- + \frac{3}{2}\mathcal{P}_-C\boldsymbol{\sigma}^T(-q,P)C^{\dagger}\mathcal{P}_+\bigg]\gamma^{\nu}\mathcal{D}_{\mu\nu}(q-p), \tag{59}$$

$$s^{-1}_{p+P/2}\boldsymbol{\sigma}(p,P)s^{-1}_{p-P/2}$$

$$=-\frac{2}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^{\mu}[3\mathcal{P}_+\boldsymbol{\eta}(q,P)\mathcal{P}_- + 2\mathcal{P}_+\boldsymbol{\sigma}(q,P)\mathcal{P}_-]\gamma^{\nu}\mathcal{D}_{\mu\nu}(q-p). \tag{60}$$

The right-handed fields satisfy a similar set of equations.

In order to solve the BS equations, it is important to determine the Dirac structure of the BS wave function. There is the following useful statement. Let us consider a BS wave function of an arbitrary bound state for a *non-zero* chemical potential in the *center of mass frame*. Then, the number of independent terms in its decomposition over the Dirac matrices coincides with the number of the terms in the decomposition of the BS wave function at *zero* chemical potential.

The proof of this statement is simple. The Dirac decomposition is determined by all the space-time tensors characterizing the bound state, e.g., the momenta P^{ν} and p^{ν}, the polarization vector e^{ν} (in the case of a massive spin one bound state), etc. In this respect, the case of a non-zero chemical potential is distinguished by the occurrence of only one additional vector $u^{\nu}=(1,\vec{0})$. But in the center of mass frame, where the total momentum $P^{\nu}=(P_0,\vec{0})$, the vector u^{ν} is proportional to P^{ν} and therefore is not independent. Thus, the number of terms in the Dirac decomposition of a BS wave function in this frame is the same for both zero and non-zero chemical potentials.

Of course, there is an essential difference between these two cases: while for zero chemical potential the number of Dirac structures is the same in all frames, in the case of a non-zero chemical potential, it is different for $\vec{P}=0$ (the center of mass frame) and for $\vec{P}\neq 0$ (all other frames). For example, as we will see for spin zero diquarks, when $\mu\neq 0$, there are four independent terms in the center of mass frame, and there are eight terms in other frames.

Strictly speaking, this statement is valid only for massive bound states. However, in the case of spin zero bound states it is still valid also for massless states (in particular, for NG bosons): the point is that the limit $M\to 0$ is smooth for BS wave functions of spin zero states, and $P^{\nu}\to 0$ is a very useful limit for studying properties of NG bosons.

In the next two sections we study these BS equations for the diquark states in detail. In order to approach the problem, we first need to determine the Dirac structure of the BS wave functions. As we shall see, the Ward identities, derived in Sec. IV, are of great help in dealing with this task. Moreover, in the particular case of the (pseudo-)NG bosons, knowledge of the Ward identities is powerful enough to reveal a complete solution to the BS equations. We consider this important case in the next section.

VI. BS EQUATIONS FOR NG AND PSEUDO-NG BOSONS

In this section we consider massless bound diquark states. The latter should include NG and pseudo-NG bosons. Before proceeding to a detailed analysis of the BS equations, it is instructive to describe the qualitative physical picture in the problem at hand.

Let us start from a simple observation. As we stressed many times, the QCD dynamics at a large chemical potential consists of two essentially decoupled and identical (left-handed and right-handed) sectors. Then, as long as it concerns the diquark paring dynamics, no changes would appear in the model if one enlarges the gauge group of QCD from $SU(3)_c$ to the approximate $SU(3)_{c,L}\times SU(3)_{c,R}$, assuming that the coupling constants of both gauge groups are identical. In the modified theory, the pattern of the symmetry breaking should be $SU(3)_{c,L}\times SU(3)_{c,R}\to SU(2)_{c,L}\times SU(2)_{c,R}$. In this case, ten NG bosons should appear. If the gauge group were $SU(3)_{c,L}\times SU(3)_{c,R}$, all ten NG bosons would be unphysical because of the Higgs mechanism. However, since the true gauge group of QCD is vector-like $SU(3)_c$, only five NG bosons (scalars) are removed from the spectrum of physical particles by the Higgs mechanism. The other five NG bosons (pseudoscalars) should remain in the spectrum. In the complete theory, these latter are the pseudo-NG bosons. They should get non-zero masses due to higher order corrections that are beyond the improved ladder approximation (an example of such corrections is the box diagram in the BS kernel with two intermediate gluons). At the same time, since the theory is weakly coupled at a large chemical potential, it is natural to expect that the masses of the pseudo-NG bosons are small even compared to the value of the dynamical quark mass [35].

For the completeness of our discussion, let us also add that, even though the massless scalars are removed from the physical spectrum, they exist in the theory as some kind of ''ghosts'' [36]. In fact, one cannot completely get rid of them, unless a unitary gauge is found.[3] It is also important to mention that these ghosts play a very important role in getting rid of unphysical poles from on-shell scattering amplitudes [36].

A. Structure of the BS wave functions of (pseudo-)NG bosons

Earlier we mentioned in passing that the use of the Ward identities is crucial for revealing the Dirac structure of the BS wave functions of the (pseudo-)NG bosons. Now let us

[3]Note that, because of the composite (diquark) nature of the order parameter in color superconducting phase of dense QCD, it does not seem to be straightforward to define and to use the unitary gauge there.

elaborate on this point. We start with the definition of the vertices in Eq. (34). By making use of them, one can show that the corresponding Fourier transforms develop poles whenever the total momentum of the incoming quarks, P, satisfies the on-shell condition of a bound state. In particular, as $P\to 0$, we obtain

$$\mathbf{\Gamma}^{A,i}_{aj,\mu}(p+P/2,p-P/2)|_{P\to 0}\simeq \frac{P^{(\chi)}_{\mu}F^{(\chi)}}{P^{\nu}P^{(\chi)}_{\nu}}\sum_{\tilde{a}}\delta^{i}_{j}T^{A3}_{\tilde{a}}\boldsymbol{\chi}^{(\tilde{a})}_{a}(p,0) \equiv \frac{P^{(\chi)}_{\mu}F^{(\chi)}}{P^{\nu}P^{(\chi)}_{\nu}}\delta^{i}_{j}T^{A3}_{a}\boldsymbol{\chi}(p,0), \quad (61a)$$

$$\mathbf{\Gamma}^{A,ai}_{j,\mu}(p+P/2,p-P/2)|_{P\to 0}\simeq \frac{P^{(\lambda)}_{\mu}F^{(\lambda)}}{P^{\nu}P^{(\lambda)}_{\nu}}\sum_{\tilde{a}}\delta^{i}_{j}T^{A\tilde{a}}_{3}\boldsymbol{\lambda}^{a}_{(\tilde{a})}(p,0) \equiv \frac{P^{(\lambda)}_{\mu}F^{(\lambda)}}{P^{\nu}P^{(\lambda)}_{\nu}}\delta^{i}_{j}T^{Aa}_{3}\boldsymbol{\lambda}(p,0), \quad (61b)$$

$$\mathbf{\Gamma}^{A,i}_{j,\mu}(p+P/2,p-P/2)|_{P\to 0}\simeq \frac{P^{(\eta)}_{\mu}F^{(\eta)}}{P^{\nu}P^{(\eta)}_{\nu}}\frac{1}{2}\delta^{i}_{j}\delta^{A}_{8}\boldsymbol{\eta}(p,0), \quad (61c)$$

$$\tilde{\mathbf{\Gamma}}^{A,i}_{aj,\mu}(p+P/2,p-P/2)|_{P\to 0}\simeq \frac{P^{(\chi)}_{\mu}\tilde{F}^{(\chi)}}{P^{\nu}P^{(\chi)}_{\nu}}\sum_{\tilde{a}}\delta^{i}_{j}T^{A3}_{\tilde{a}}\tilde{\boldsymbol{\chi}}^{(\tilde{a})}_{a}(p,0) \equiv \frac{P^{(\chi)}_{\mu}\tilde{F}^{(\chi)}}{P^{\nu}P^{(\chi)}_{\nu}}\delta^{i}_{j}T^{A3}_{a}\tilde{\boldsymbol{\chi}}(p,0), \quad (61d)$$

$$\tilde{\mathbf{\Gamma}}^{A,ai}_{j,\mu}(p+P/2,p-P/2)|_{P\to 0}\simeq \frac{P^{(\lambda)}_{\mu}\tilde{F}^{(\lambda)}}{P^{\nu}P^{(\lambda)}_{\nu}}\sum_{\tilde{a}}\delta^{i}_{j}T^{A\tilde{a}}_{3}\tilde{\boldsymbol{\lambda}}^{a}_{(\tilde{a})}(p,0) \equiv \frac{P^{(\lambda)}_{\mu}\tilde{F}^{(\lambda)}}{P^{\nu}P^{(\lambda)}_{\nu}}\delta^{i}_{j}T^{Aa}_{3}\tilde{\boldsymbol{\lambda}}(p,0), \quad (61e)$$

$$\tilde{\mathbf{\Gamma}}^{A,i}_{j,\mu}(p+P/2,p-P/2)|_{P\to 0}\simeq \frac{P^{(\eta)}_{\mu}\tilde{F}^{(\eta)}}{P^{\nu}P^{(\eta)}_{\nu}}\frac{1}{2}\delta^{i}_{j}\delta^{A}_{8}\tilde{\boldsymbol{\eta}}(p,0), \quad (61f)$$

where $F^{(x)}$ and $\tilde{F}^{(x)}$ (with x being λ, χ, or η) are the decay constants of the (pseudo-)NG bosons. The rigorous definition and calculation of their values will be given in Sec. VI E. For our purposes here, it is sufficient to know that they are constants expressed through the parameters of the theory. Since the Lorentz symmetry is explicitly broken by the chemical potential, the dispersion relations of the (pseudo-)NG bosons respect only the spatial rotation symmetry. In order to take this into account, we introduced the following four-momentum notation: $P^{(x)}_{\mu}=(P^{0},-c_{x}^{2}\vec{P})$ where $c_x<1$ is the velocity of the appropriate (pseudo-)NG boson.

By recalling that parity is preserved in the color superconducting phase of dense QCD, we conclude that the decay constants of the left- and right-handed composites should be equal, i.e., $\tilde{F}^{(\lambda)}=F^{(\lambda)}$, $\tilde{F}^{(\chi)}=F^{(\chi)}$, and $\tilde{F}^{(\eta)}=F^{(\eta)}$.

The existence of poles in the full vertex functions as $P\to 0$ is also required by the Ward identities, discussed in Sec. IV. Moreover, the Ward identities alone allow us to establish the explicit form of the poles. Indeed, by making use of the relations for the amputated vertices in Eq. (49) as well as the explicit form of the quark propagators in Eqs. (26), (27) and (28), we obtain

$$\Gamma^{A,i}_{aj,\mu}(k+P,k)|_{P\to 0}\simeq \frac{P^{(\chi)}_{\mu}}{P^{\nu}P^{(\chi)}_{\nu}}T^{A3}_{a}\delta^{i}_{j}\tilde{\Delta}_{k}\mathcal{P}_{+}, \quad (62a)$$

$$\Gamma^{A,ai}_{j,\mu}(k+P,k)|_{P\to 0}\simeq -\frac{P^{(\lambda)}_{\mu}}{P^{\nu}P^{(\lambda)}_{\nu}}T^{Aa}_{3}\delta^{i}_{j}\Delta_{k}\mathcal{P}_{-}, \quad (62b)$$

$$\Gamma^{A,i}_{j,\mu}(k+P,k)|_{P\to 0}\simeq \frac{P^{(\eta)}_{\mu}}{P^{\nu}P^{(\eta)}_{\nu}}\delta^{A}_{8}\delta^{i}_{j}\frac{\sqrt{3}}{2}[\tilde{\Delta}_{k}\mathcal{P}_{+}-\Delta_{k}\mathcal{P}_{-}], \quad (62c)$$

$$\tilde{\Gamma}^{A,i}_{aj,\mu}(k+P,k)|_{P\to 0}\simeq \frac{P^{(\chi)}_{\mu}}{P^{\nu}P^{(\chi)}_{\nu}}T^{A3}_{a}\delta^{i}_{j}\tilde{\Delta}_{k}\mathcal{P}_{-}, \quad (62d)$$

$$\tilde{\Gamma}^{A,ai}_{j,\mu}(k+P,k)|_{P\to 0}\simeq -\frac{P^{(\lambda)}_{\mu}}{P^{\nu}P^{(\lambda)}_{\nu}}T^{Aa}_{3}\delta^{i}_{j}\Delta_{k}\mathcal{P}_{+}, \quad (62e)$$

$$\tilde{\Gamma}^{A,i}_{j,\mu}(k+P,k)|_{P\to 0}\simeq \frac{P^{(\eta)}_{\mu}}{P^{\nu}P^{(\eta)}_{\nu}}\delta^{A}_{8}\delta^{i}_{j}\frac{\sqrt{3}}{2}\times[\tilde{\Delta}_{k}\mathcal{P}_{-}-\Delta_{k}\mathcal{P}_{+}]. \quad (62f)$$

Now, by taking into account the definition of the amputated vertices in Eq. (48) and comparing the pole residues in Eq. (61) with those in Eq. (62), we unambiguously deduce the Dirac structure of the amputated (as well as non-amputated) BS wave functions of the (pseudo-)NG bosons:

$$\chi(p,0)=S_{p}^{-1}\boldsymbol{\chi}(p,0)s_{p}^{-1}=\frac{\tilde{\Delta}_{p}}{F^{(\chi)}}\mathcal{P}_{+}, \quad (63a)$$

$$\lambda(p,0)=s_{p}^{-1}\boldsymbol{\lambda}(p,0)(p,0)S_{p}^{-1}=-\frac{\Delta_{p}}{F^{(\lambda)}}\mathcal{P}_{-}, \quad (63b)$$

$$\eta(p,0)=S_{p}^{-1}\boldsymbol{\eta}(p,0)S_{p}^{-1}=\frac{\sqrt{3}}{F^{(\eta)}}(\tilde{\Delta}_{p}\mathcal{P}_{+}-\Delta_{p}\mathcal{P}_{-}), \quad (63c)$$

$$\tilde{\chi}(p,0)=\tilde{S}_p^{-1}\tilde{\boldsymbol{\chi}}(p,0)\tilde{s}_p^{-1}=\frac{\tilde{\Delta}_p}{F^{(\chi)}}\mathcal{P}_-\,, \tag{63d}$$

$$\tilde{\lambda}(p,0)=\tilde{s}_p^{-1}\tilde{\boldsymbol{\lambda}}(p,0)\tilde{S}_p^{-1}=-\frac{\Delta_p}{F^{(\lambda)}}\mathcal{P}_+\,, \tag{63e}$$

$$\tilde{\eta}(p,0)=\tilde{S}_p^{-1}\tilde{\boldsymbol{\eta}}(p,0)\tilde{S}_p^{-1}=\frac{\sqrt{3}}{F^{(\eta)}}(\tilde{\Delta}_p\mathcal{P}_- -\Delta_p\mathcal{P}_+). \tag{63f}$$

This concludes our derivation. Before concluding this subsection, we would like to emphasize that the arguments used here cannot be generalized for the case of massive diquarks. The reason is that the corresponding on-shell pole contributions to the vertex functions [compare with Eq. (61)] must appear at a non-vanishing momentum P. Obviously, the structure of such poles cannot be clarified by utilizing the Ward identities alone.

B. NG doublet $\chi_a^{(\tilde{b})}$

Now, let us consider the BS equation for the massless χ doublet; see Eq. (57). As soon as the color symmetry is spontaneously broken in the model at hand, a non-trivial solution to this equation should exist. In order to verify the self-consistency of our approach, we have to check that this is the case.

The most general Dirac structure of the amputated BS wave function, $\chi_a^{(\tilde{b})}(p,P)=S_{p+P/2}^{-1}\boldsymbol{\chi}_a^{(\tilde{b})}(p,P)s_{p-P/2}^{-1}$, that is allowed by the space-time symmetries is given by

$$\begin{aligned}\chi_a^{(\tilde{b})}(p,P)=\delta_a^{\tilde{b}}[&\chi_1^-\Lambda_p^+ +\chi_1^+\Lambda_p^- +(p_0-\epsilon_p^-)\chi_2^-\gamma^0\Lambda_p^+\\&+(p_0+\epsilon_p^+)\chi_2^+\gamma^0\Lambda_p^- +\chi_3(\vec{\gamma}\cdot\vec{P})+\chi_4(\vec{\alpha}\cdot\vec{P})\\&+\chi_5\sigma^{nm}p^nP^m+\chi_6\gamma^0\sigma^{nm}p^nP^m]\mathcal{P}_+\,,\end{aligned} \tag{64}$$

where $n,m=1,2,3$ are space indices, $\sigma^{nm}=i/2[\gamma^n,\gamma^m]$, and the factors $(p_0-\epsilon_p^-)$ and $(p_0+\epsilon_p^+)$ are introduced here for convenience. It is of great advantage to notice that four out of eight independent functions in this expression become irrelevant in the limit $\vec{P}\to 0$. This agrees with the general statement made in Sec. V A (indeed, there are four independent Dirac structures in the BS wave functions of spin zero states at zero chemical potential [34]). We will consider only this limit (which, in the case of NG bosons, implies that the total momentum $P\to 0$).

After multiplying both sides of the BS equation (57) by the appropriate quark propagators on the left and on the right, we obtain the equation for the amputated BS wave function. This latter splits into the following set of two equations:

$$\begin{aligned}&\mathcal{P}_+\chi_a^{(\tilde{b})}(p,0)\mathcal{P}_+\\&=-\frac{2}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\mathcal{P}_-\{S_q\chi_a^{(\tilde{b})}(q,0)s_q+3\overline{s_q}C\\&\quad\times[\chi_a^{(\tilde{b})}(-q,0)]^TC^\dagger S_q\}\mathcal{P}_-\gamma^\nu\mathcal{D}_{\mu\nu}(q-p),\end{aligned} \tag{65}$$

$$\begin{aligned}&\mathcal{P}_-\chi_a^{(\tilde{b})}(p,0)\mathcal{P}_+\\&=\frac{2}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\mathcal{P}_+S_q\chi_a^{(\tilde{b})}(q,0)s_q\mathcal{P}_-\gamma^\nu\mathcal{D}_{\mu\nu}(q-p).\end{aligned} \tag{66}$$

The quark propagators S_q and s_q are given in Eqs. (26) and (27). As we can see from those explicit representations, not all of the terms are equally important. While some of them develop large contributions in the vicinity of the Fermi surface, the others are suppressed by powers of μ. These latter could be safely neglected in the leading order of the theory. Using expression (23) for the gluon propagator, we arrive at the following approximate form of the BS equations:

$$\begin{aligned}&\mathcal{P}_+\chi_a^{(\tilde{b})}(p,0)\mathcal{P}_+\\&\simeq\frac{2}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\mathcal{P}_-\Lambda_q^-\Bigg(\frac{\gamma^0(q_0-\epsilon_q^-)-(\Delta_q^-)^*}{q_0^2-(\epsilon_q^-)^2-|\Delta_q^-|^2}\\&\quad\times\chi_a^{(\tilde{b})}(q,0)\frac{\gamma^0}{q_0-\epsilon_q^-}+\frac{3\gamma^0}{q_0+\epsilon_q^-}[\chi_a^{(\tilde{b})}(-q,0)]^T\\&\quad\times\frac{\gamma^0(q_0+\epsilon_q^-)-(\Delta_q^-)^*}{q_0^2-(\epsilon_q^-)^2-|\Delta_q^-|^2}\Bigg)\Lambda_q^-\mathcal{P}_-\gamma^\nu\mathcal{D}_{\mu\nu}(q-p),\end{aligned} \tag{67}$$

$$\begin{aligned}&\mathcal{P}_-\chi_a^{(\tilde{b})}(p,0)\mathcal{P}_+\\&\simeq-\frac{2}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\mathcal{P}_+\Lambda_q^+\frac{\gamma^0(q_0+\epsilon_q^-)-\Delta_q^-}{q_0^2-(\epsilon_q^-)^2-|\Delta_q^-|^2}\\&\quad\times\chi_a^{(\tilde{b})}(q,0)\frac{\gamma^0\Lambda_q^-\mathcal{P}_-}{q_0-\epsilon_q^-}\gamma^\nu\mathcal{D}_{\mu\nu}(q-p).\end{aligned} \tag{68}$$

In component form, these become

$$\begin{aligned}\chi_1^-(p)\simeq&\frac{1}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\{\chi_1^-(q)+3\chi_1^-(-q)-(\Delta_q^-)^*\\&\times[\chi_2^-(q)+3\chi_2^-(-q)]\}\frac{1}{q_0^2-(\epsilon_q^-)^2-|\Delta_q^-|^2}\\&\times\mathrm{tr}(\gamma^\mu\Lambda_q^-\gamma^\nu\Lambda_p^+)\mathcal{D}_{\mu\nu}(q-p),\end{aligned} \tag{69}$$

$$(p_0-\epsilon_p^-)\chi_2^-(p)$$
$$\simeq -\frac{1}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{[q_0^2-(\epsilon_q^-)^2]\chi_2^-(q)-\Delta_q^-\chi_1^-(q)}{(q_0-\epsilon_p^-)[q_0^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]}$$
$$\times\mathrm{tr}(\gamma^\mu\Lambda_q^+\gamma^0\gamma^\nu\Lambda_p^+\gamma^0)\mathcal{D}_{\mu\nu}(q-p), \quad (70)$$

plus the expressions that define χ_i^+ in terms of χ_i^-. By noting that the equations for the even and odd combinations of the BS wave functions, $\chi_i^-(p)\pm\chi_i^-(-p)$, decouple and satisfy the same kind of equation, we argue that it is sufficient for our purposes to consider only the even combination. Indeed, from the Ward identities, one finds that $\chi_1^-(p)$ is related to the gap Δ_p^-. Then, if $\chi_1^-(p)$ is odd, we must have a non-trivial solution for the gap satisfying $\Delta_0^-=\Delta_p^-|_{p=0}=0$. Analysis of the SD equation shows that no such solution exists. Therefore, without loss of generality, we put $\chi_i^-(-p)=\chi_i^-(p)$, and obtain

$$\chi_1^-(p)\simeq\frac{4}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{\chi_1^-(q)-(\Delta_q^-)^*\chi_2^-(q)}{q_0^2-(\epsilon_q^-)^2-|\Delta_q^-|^2}$$
$$\times\mathrm{tr}(\gamma^\mu\Lambda_q^-\gamma^\nu\Lambda_p^+)\mathcal{D}_{\mu\nu}(q-p), \quad (71)$$

$$(p_0-\epsilon_p^-)\chi_2^-(p)$$
$$\simeq -\frac{1}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{[q_0^2-(\epsilon_q^-)^2]\chi_2^-(q)-\Delta_q^-\chi_1^-(q)}{(q_0-\epsilon_p^-)[q_0^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]}$$
$$\times\mathrm{tr}(\gamma^\mu\Lambda_q^+\gamma^0\gamma^\nu\Lambda_p^+\gamma^0)\mathcal{D}_{\mu\nu}(q-p). \quad (72)$$

By comparing our ansatz for the BS wave function in Eq. (64) with the structure in Eq. (63a) that is required by the Ward identities, we see that $\chi_2^{\pm}(p)$ components should be zero. Direct analysis of the BS equations, on the other hand, shows that these component functions $\chi_2^{\pm}$ cannot be identically zero. It is not hard to pinpoint the origin of the discrepancy. Indeed, in our approximation, we completely neglected the wave function renormalization effects of quarks. Upon taking them into account, the Ward identity (49) would lead to a modified structure of the BS wave function, and all allowed Dirac structures would be non-zero.

Therefore, as in the case of wave function renormalization, we estimate the effect of $\chi_2^-(p)$ perturbatively. To this end, we use $\chi_2^-(p)=0$ in the leading order of the theory. Then, the equation for $\chi_1^-(p)$ reads

$$\chi_1^-(p)\simeq\frac{4}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{\chi_1^-(q)\mathrm{tr}(\gamma^\mu\Lambda_q^-\gamma^\nu\Lambda_p^+)}{q_0^2-(\epsilon_q^-)^2-|\Delta_q^-|^2}\mathcal{D}_{\mu\nu}(q-p). \quad (73)$$

On comparison with the gap equation (29), we see that $\chi_1^-(p)=(\Delta_p^-)^*/F^{(\chi)}$ [as required by the Ward identities; see Eq. (63a)] is the exact solution to the BS equation in the leading order approximation. Here, of course, we assume that Δ_p^- is the solution to the gap equation. By substituting the leading order solution $\chi_1^-(p)$ into Eq. (72), we get the estimate for $\chi_2^-(p)$. In the most important region, $|\Delta_0^-|^2 \lesssim p_4^2+(\epsilon_p^-)^2\lesssim\mu^2$, we find that

$$\chi_2^-(p)\sim\frac{\alpha_s|\Delta_0^-|}{F^{(\chi)}\sqrt{p_4^2+(\epsilon_p^-)^2}}\ln\frac{(2\mu)^2}{p_4^2+(\epsilon_p^-)^2}. \quad (74)$$

Now, we also can check that this function could safely be neglected in the equation for $\chi_1^-(p)$. Indeed, its substitution into Eq. (71) produces a result of order $\alpha_s(\Delta_p^-)^*/F^{(\chi)}$ which is suppressed by a power of α_s compared to $\chi_1^-(p)=(\Delta_p^-)^*/F^{(\chi)}$.

Therefore, both the corrections due to the wave function renormalization of quarks [10–16] and those due to the non-vanishing component functions $\chi_2^{\pm}(p)$ are small in the leading order of the theory. Moreover, consistency with the Ward identities requires that either both effects be taken into account or neither of them.

C. NG antidoublet $\boldsymbol{\lambda^b_{(\tilde{a})}}$

The analysis of the BS equation for the λ antidoublet follows very closely the analysis for the χ doublet. For completeness of the presentation, we still give all the details.

The most general BS wave function of this antidoublet is given by

$$\lambda^b_{(\tilde{a})}(p,P)=\delta^b_{\tilde{a}}\mathcal{P}_-[\lambda_1^+\Lambda_p^++\lambda_1^-\Lambda_p^-+(p_0-\epsilon_p^-)\lambda_2^-\gamma^0\Lambda_p^+$$
$$+(p_0+\epsilon_p^+)\lambda_2^+\gamma^0\Lambda_p^-+\lambda_3(\vec{\gamma}\cdot\vec{P})+\lambda_4(\vec{\alpha}\cdot\vec{P})$$
$$+\lambda_5\sigma^{nm}p^nP^m+\lambda_6\gamma^0\sigma^{nm}p^nP^m]. \quad (75)$$

As in the case of the χ doublet, to simplify the analysis we restrict ourselves to the case of the vanishing total momentum $P\to 0$. Then, the equations for two projections of the wave function, $\mathcal{P}_-\lambda^b_{(\tilde{a})}(p,0)\mathcal{P}_-$ and $\mathcal{P}_-\lambda^b_{(\tilde{a})}(p,0)\mathcal{P}_+$, read

$$\mathcal{P}_-\lambda^b_{(\tilde{a})}(p,0)\mathcal{P}_-$$
$$=-\frac{2}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\mathcal{P}_+\{s_q\lambda^b_{(\tilde{a})}(q,0)S_q+3S_qC$$
$$\times[\lambda^b_{(\tilde{a})}(-q,0)]^TC^\dagger\overline{s_q}\}\mathcal{P}_+\gamma^\nu\mathcal{D}_{\mu\nu}(q-p), \quad (76)$$

$$\mathcal{P}_-\lambda^b_{(\tilde{a})}(p,0)\mathcal{P}_+$$
$$=\frac{2}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\mathcal{P}_+s_q\lambda^b_{(\tilde{a})}(q,0)S_q\mathcal{P}_-\gamma^\nu\mathcal{D}_{\mu\nu}(q-p). \quad (77)$$

After extracting the most significant of the Fermi surface contributions in the vicinity from the quark propagators S_q and s_q, we arrive at

$$\mathcal{P}_-\lambda^b_{(\tilde{a})}(p,0)\mathcal{P}_-$$
$$\simeq\frac{2}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\mathcal{P}_+\Lambda^+_q\Bigg(\frac{\gamma^0}{q_0-\epsilon^-_q}\lambda^b_{(\tilde{a})}(q,0)\times\frac{\gamma^0(q_0-\epsilon^-_q)-\Delta^-_q}{q_0^2-(\epsilon^-_q)^2-|\Delta^-_q|^2}+\frac{3\gamma^0(q_0+\epsilon^-_q)-\Delta^-_q}{q_0^2-(\epsilon^-_q)^2-|\Delta^-_q|^2}\times[\lambda^b_{(\tilde{a})}(-q,0)]^T\frac{\gamma^0}{q_0+\epsilon^-_q}\Bigg)\Lambda^+_q\mathcal{P}_+\gamma^\nu\mathcal{D}_{\mu\nu}(q-p), \quad (78)$$

$$\mathcal{P}_-\lambda^b_{(\tilde{a})}(p,0)\mathcal{P}_+$$
$$\simeq-\frac{2}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\frac{\mathcal{P}_+\Lambda^+_q\gamma^0}{q_0-\epsilon^-_q}\lambda^b_{(\tilde{a})}(q,0)\times\frac{\gamma^0(q_0+\epsilon^-_q)-(\Delta^-_q)^*}{q_0^2-(\epsilon^-_q)^2-|\Delta^-_q|^2}\mathcal{P}_-\Lambda^-_q\gamma^\nu\mathcal{D}_{\mu\nu}(q-p). \quad (79)$$

Finally, rewriting this in components, we get

$$\lambda^-_1(p)\simeq\frac{4}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{\lambda^-_1(q)-\Delta^-_q\lambda^-_2(q)}{q_0^2-(\epsilon^-_q)^2-|\Delta^-_q|^2}\times\mathrm{tr}(\gamma^\mu\Lambda^+_q\gamma^\nu\Lambda^-_p)\mathcal{D}_{\mu\nu}(q-p), \quad (80)$$

$$(p_0-\epsilon^-_p)\lambda^-_2(p)$$
$$\simeq-\frac{1}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\times\frac{[q_0^2-(\epsilon^-_q)^2]\lambda^-_2(q)-(\Delta^-_q)^*\lambda^-_1(q)}{(q_0-\epsilon^-_p)[q_0^2-(\epsilon^-_q)^2-|\Delta^-_q|^2]}\times\mathrm{tr}(\gamma^\mu\Lambda^+_q\gamma^0\gamma^\nu\Lambda^+_p\gamma^0)\mathcal{D}_{\mu\nu}(q-p), \quad (81)$$

where again, without loss of generality, we assumed that $\lambda^-_i(p)$ are even functions of momenta.

By repeating the arguments of the previous subsection, we would find that $\lambda^-_2(p)$ should be zero in a consistent approximation when the wave function renormalizations of quarks are neglected. As in the case of the χ doublet, the equation for the $\lambda^-_1(p)$ component has the solution $\lambda^-_1(p)=-\Delta^-_p/F^{(\lambda)}$, which is consistent with the Ward identities; see Eq. (63b).

D. NG singlets η

The case of massless singlets is very special. This is already seen from the fact that the BS equations for the η and σ singlets are coupled in general. This might appear somewhat puzzling if one traces back the origin of the singlets. While the η singlet contains the antisymmetric tensor product of two fundamental representations of $SU(3)$, the σ singlet comes from the product of the fundamental and the antifundamental representations. Based on this observation, one might have concluded that the bound state should form only in the η channel. As we shall see below, this argument is not completely groundless, although the real situation is slightly different. We would like to point out that there is no symmetry in the color superconducting phase of dense QCD which could prevent the coupling between the two singlet channels.

Let us consider the equation for the amputated BS wave functions of singlets, $\eta(p,0)$ and $\sigma(p,0)$ (we consider only the case of $P\to0$ below). The most general structure of the wave functions is

$$\eta(p,0)=[\eta^-_1\Lambda^+_p+\eta^+_1\Lambda^-_p+(p_0-\epsilon^-_p)\eta^-_2\gamma^0\Lambda^+_p+(p_0+\epsilon^+_p)\eta^+_2\gamma^0\Lambda^-_p]\mathcal{P}_++[\eta^+_3\Lambda^+_p+\eta^-_3\Lambda^-_p+(p_0+\epsilon^-_p)\eta^-_4\gamma^0\Lambda^-_p+(p_0-\epsilon^+_p)\eta^+_4\gamma^0\Lambda^+_p]\mathcal{P}_-, \quad (82)$$

$$\sigma(p,0)=\mathcal{P}_-\gamma^0[(p_0-\epsilon^-_p)\sigma^-\Lambda^+_p+(p_0+\epsilon^+_p)\sigma^+\Lambda^-_p]\mathcal{P}_+. \quad (83)$$

Before proceeding any further, we would like to recall the definition of the η-singlet wave function in Eq. (50c). Unlike other diquarks, it is built of only Majorana spinors. By making use of the generalized Majorana property in Eq. (3), we observe that the BS wave function $\eta(p,P)$ should satisfy the following constraint[4]:

$$C\eta^T(-p,P)C^\dagger=\eta(p,P). \quad (84)$$

While rewritten in components, this restriction is satisfied when the odd components $\eta^{\pm}_{1,3}(p)$ are even functions of momenta and when $\eta^{\pm}_2(-p)=\eta^{\pm}_4(p)$.

The equations for different chiral projections of the amputated BS wave functions, $\eta(p,0)=S^{-1}_p\boldsymbol{\eta}(p,0)S^{-1}_p$ and $\sigma(p,0)=s^{-1}_p\boldsymbol{\sigma}(p,0)s^{-1}_p$, read

$$\mathcal{P}_+\eta(p,0)\mathcal{P}_+$$
$$=-\frac{8}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\mathcal{P}_-S_q\eta(q,0)S_q\mathcal{P}_-\gamma^\nu\mathcal{D}_{\mu\nu}(q-p), \quad (85)$$

$$\mathcal{P}_-\eta(p,0)\mathcal{P}_-$$
$$=-\frac{8}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\gamma^\mu\mathcal{P}_+S_q\eta(q,0)S_q\mathcal{P}_+\gamma^\nu\mathcal{D}_{\mu\nu}(q-p), \quad (86)$$

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[4]Strictly speaking, from Eq. (50c) one derives a relation for the non-amputated BS wave function. Assuming that the gap is an even function of the momentum, it is straightforward to show that the same relation holds for the amputated wave function η.

$$\mathcal{P}_- \eta(p,0)\mathcal{P}_+ = -\frac{2}{3}\pi\alpha_s \int \frac{d^4q}{(2\pi)^4}\gamma^\mu \mathcal{P}_+[5S_q\eta(q,0)S_q + 6s_q\sigma(q,0)s_q] \times \mathcal{P}_-\gamma^\nu \mathcal{D}_{\mu\nu}(q-p), \tag{87}$$

$$\mathcal{P}_+ \eta(p,0)\mathcal{P}_- = -\frac{2}{3}\pi\alpha_s \int \frac{d^4q}{(2\pi)^4}\gamma^\mu \mathcal{P}_-\{5S_q\eta(q,0)S_q + 6\overline{s_q} \times [\sigma(-q,0)]^T\overline{s_q}\}\mathcal{P}_+\gamma^\nu \mathcal{D}_{\mu\nu}(q-p), \tag{88}$$

$$\mathcal{P}_- \sigma(p,0)\mathcal{P}_+ = -\frac{2}{3}\pi\alpha_s \int \frac{d^4q}{(2\pi)^4}\gamma^\mu \mathcal{P}_+[3S_q\eta(q,0)S_q + 2s_q\sigma(q,0)s_q]\mathcal{P}_-\gamma^\nu \mathcal{D}_{\mu\nu}(q-p). \tag{89}$$

In components, these become

$$\eta_1^-(p) \simeq \frac{4}{3}\pi\alpha_s \int \frac{d^4q}{(2\pi)^4} \frac{[q_0^2-(\epsilon_q^-)^2]\eta_1^-(q) + (\Delta_q^{-*})^2\eta_3^-(q) - 2(\Delta_q^-)^*[q_0^2-(\epsilon_q^-)^2]\eta_2^-(q)}{[q_0^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]^2} \times \mathrm{tr}(\gamma^\mu\Lambda_q^-\gamma^\nu\Lambda_p^+)\mathcal{D}_{\mu\nu}(q-p), \tag{90}$$

$$\eta_3^-(p) \simeq \frac{4}{3}\pi\alpha_s \int \frac{d^4q}{(2\pi)^4} \frac{[q_0^2-(\epsilon_q^-)^2]\eta_3^-(q) + (\Delta_q^-)^2\eta_1^-(q) - 2\Delta_q^-[q_0^2-(\epsilon_q^-)^2]\eta_2^-(q)}{[q_0^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]^2} \mathrm{tr}(\gamma^\mu\Lambda_q^+\gamma^\nu\Lambda_p^-)\mathcal{D}_{\mu\nu}(q-p), \tag{91}$$

$$(p_0-\epsilon_p^-)\eta_2^-(p) \simeq \frac{1}{3}\pi\alpha_s \int \frac{d^4q}{(2\pi)^4}\left(5\frac{[q_0^2-(\epsilon_q^-)^2]\eta_2^-(q) + |\Delta_q^-|^2\eta_4^-(q) - \Delta_q^-\eta_1^-(q) - (\Delta_q^-)^*\eta_3^-(q)}{[q_0^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]^2} + \frac{6\sigma^-(q)}{q_0^2-(\epsilon_q^-)^2}\right) \times (q_0-\epsilon_q^-)\mathrm{tr}(\gamma^\mu\Lambda_q^+\gamma^0\gamma^\nu\Lambda_p^+\gamma^0)\mathcal{D}_{\mu\nu}(q-p), \tag{92}$$

$$(p_0-\epsilon_p^-)\sigma^-(p) \simeq \frac{1}{3}\pi\alpha_s \int \frac{d^4q}{(2\pi)^4}\left(3\frac{[q_0^2-(\epsilon_q^-)^2]\eta_2^-(q) + |\Delta_q^-|^2\eta_4^-(q) - \Delta_q^-\eta_1^-(q) - (\Delta_q^-)^*\eta_3^-(q)}{[q_0^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]^2} + \frac{2\sigma^-(q)}{q_0^2-(\epsilon_q^-)^2}\right) \times (q_0-\epsilon_q^-)\mathrm{tr}(\gamma^\mu\Lambda_q^+\gamma^0\gamma^\nu\Lambda_p^+\gamma^0)\mathcal{D}_{\mu\nu}(q-p), \tag{93}$$

along with the expressions for the plus components irrelevant for our analysis. Note that we did not write down the equation for the $\eta_4^-(p)$ component since it is related to $\eta_2^-(p)$ as we argued above.

Now, let us analyze the BS equations for the singlets. By repeating the argument involving the Ward identities, we see that the component functions $\eta_{2,4}^{\pm}(p)$ should be *exactly* zero. As opposed to the case of an (anti-)doublet, a crucial difference appears in the case of singlets. As we discussed in Sec. VI A, the Dirac structure of the BS wave function of the η singlet is determined by the pole structure of the vertex in Eq. (61c). The explicit form of the latter is determined by the Ward identity in Eq. (49c), and the result is presented in Eq. (62c). The remarkable property of this result is that it does not get any corrections even after the wave function renormalization effects of quarks are taken into account. To see this, one should note that the mentioned Ward identity (49c) is given in terms of a single propagator, S_p. Because of this, all wave function renormalization effects always cancel from the leading pole contribution to the vertex of interest.

It is very rewarding, therefore, to check that $\eta_1^-(p) = -\sqrt{3}(\Delta_p^-)^*/F^{(\eta)}$, $\eta_3^-(p) = \sqrt{3}\Delta_p^-/F^{(\eta)}$, and $\eta_{2,4}^-(p) = \sigma^-(p) = 0$ is an exact solution to the BS equations,[5] assuming that Δ_p is the solution to the gap equation. It is interesting to notice that no admixture of the σ singlet appears in this solution for the (pseudo-)NG boson.

E. Decay constants

In order to define the decay constants of $\chi_a^{(\tilde{b})}$, $\lambda_{(\tilde{a})}^b$, η (pseudo-)NG bosons (as well as their counterparts built of the right-handed fields), it is convenient to introduce the following combinations of the currents:

$$j_{(a)\mu} = \sum_{A=4}^{7} T_a^{A3} j_\mu^A, \quad a = 1,2, \tag{94}$$

[5] We believe that this is the only non-trivial solution to the BS equation, although we were unable to rigorously prove that no other solutions exist.

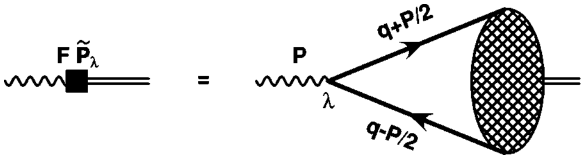

FIG. 3. The definition of the decay constant.

$$j_\mu^{(b)}=\sum_{A=4}^{7} T_3^{Ab} j_\mu^A \quad b=1,2, \tag{95}$$

$$j_\mu=j_\mu^8, \tag{96}$$

where j_μ^A for $A=4,\ldots,8$ is defined in Eq. (33). It is easy to check that the doublet $\chi_a^{(\tilde{b})}$ couples only to $j_{(a)\mu}$, while the anti-doublet $\lambda_{(\tilde{a})}^{b}$ couples only to $j_\mu^{(b)}$. The singlet couples only to j_μ.

We will consider the low energy limit when the energy of the diquark (pseudo-)NG bosons $P_0\to 0$. Then, we define their decay constants as follows:

$$\langle 0|j_{(a)\mu}(0)|P,\tilde{b}\rangle_L=i\delta_a^{\tilde{b}}P_\mu^{(\chi)}F^{(\chi)}, \tag{97}$$

$$\langle 0|j_\mu^{(b)}(0)|P,\tilde{a}\rangle_L=i\delta_{\tilde{a}}^{b}P_\mu^{(\lambda)}F^{(\lambda)}, \tag{98}$$

$$\langle 0|j_\mu(0)|P\rangle_L=iP_\mu^{(\eta)}F^{(\eta)}, \tag{99}$$

where $P_\mu^{(x)}=(P_0,-c_x^2\vec{P})$ and c_x is the velocity of (pseudo-)NG bosons.

From the definition of the current in Eq. (33) in terms of quark fields and from the definition of the BS wave functions in Eq. (50), we obtain

$$P_\mu^{(\chi)}F^{(\chi)}=\frac{i}{2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}[\gamma_\mu\mathcal{P}_+\boldsymbol{\chi}(q,P)]$$
$$=\frac{i}{2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}[\gamma_\mu\mathcal{P}_+S_{q+P/2}\chi(q,P)s_{q-P/2}], \tag{100}$$

$$P_\mu^{(\lambda)}F^{(\lambda)}=\frac{i}{2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}[\mathcal{P}_-\gamma_\mu\boldsymbol{\lambda}(q,P)]$$
$$=\frac{i}{2}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}[\mathcal{P}_-\gamma_\mu s_{q+P/2}\lambda(q,P)S_{q-P/2}], \tag{101}$$

$$P_\mu^{(\eta)}F^{(\eta)}=\frac{i}{2\sqrt{3}}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}[\gamma_\mu\mathcal{P}_+\boldsymbol{\eta}(q,P)]$$
$$=\frac{i}{2\sqrt{3}}\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}[\gamma_\mu\mathcal{P}_+S_{q+P/2}\eta(q,P)S_{q-P/2}], \tag{102}$$

where the doublet and the anti-doublet BS wave functions are defined so that $\boldsymbol{\chi}_a^{(\tilde{b})}=\delta_a^{\tilde{b}}\boldsymbol{\chi}$ and $\boldsymbol{\lambda}_{(\tilde{a})}^{b}=\delta_{\tilde{a}}^{b}\boldsymbol{\lambda}$. The generic definition is diagrammatically presented in Fig. 3.

The definitions of the decay constants above are exact. The problem is, however, that a solution for the BS wave functions $\chi(q,P)$, $\lambda(q,P)$, and $\eta(q,P)$ at $P\neq 0$ is very hard to obtain. In order to get estimates of the decay constants and velocities, we will use the analogue of the Pagels-Stokar approximation [37] (for a review see Ref. [38]). In this approximation, the wave functions at $P\neq 0$ are substituted by their values at $P=0$, i.e.,

$$\chi(q,P)\simeq\chi(q,0)=\frac{\tilde{\Delta}_q}{F^{(\chi)}}\mathcal{P}_+, \tag{103}$$

$$\lambda(q,P)\simeq\lambda(q,0)=-\frac{\Delta_q}{F^{(\lambda)}}\mathcal{P}_-, \tag{104}$$

$$\eta(q,P)\simeq\eta(q,0)=\frac{\sqrt{3}}{F^{(\eta)}}(\tilde{\Delta}_q\mathcal{P}_+-\Delta_q\mathcal{P}_-), \tag{105}$$

where the right hand sides are fixed by the Ward identities. By making use of this approximation and the explicit form of the quark propagators in Eqs. (26) and (27), we derive, in the weak coupling limit,

$$(F^{(\chi)})^2\begin{Bmatrix}P_0\\ c_\chi^2\vec{P}\end{Bmatrix}\simeq\frac{\mu^2}{16\pi^3}\int_0^1 x\,dx\int_0^\pi d\theta\sin\theta\begin{Bmatrix}P_0\\ \vec{P}\cos^2\theta\end{Bmatrix}$$
$$\times\int\frac{dq_4\,d\epsilon_q^-|\Delta_q^-|^2}{[q_4^2+(\epsilon_q^-)^2+x|\Delta_q^-|^2]^2}$$
$$=\frac{\mu^2}{8\pi^2}\begin{Bmatrix}P_0\\ \frac{1}{3}\vec{P}\end{Bmatrix}, \tag{106}$$

$$(F^{(\lambda)})^2\begin{Bmatrix}P_0\\ c_\lambda^2\vec{P}\end{Bmatrix}\simeq\frac{\mu^2}{16\pi^3}\int_0^1 x\,dx\int_0^\pi d\theta\sin\theta\begin{Bmatrix}P_0\\ \vec{P}\cos^2\theta\end{Bmatrix}$$
$$\times\int\frac{dq_4\,d\epsilon_q^-|\Delta_q^-|^2}{[q_4^2+(\epsilon_q^-)^2+x|\Delta_q^-|^2]^2}$$
$$=\frac{\mu^2}{8\pi^2}\begin{Bmatrix}P_0\\ \frac{1}{3}\vec{P}\end{Bmatrix}, \tag{107}$$

$$(F^{(\eta)})^2\left\{\begin{matrix} P_0 \\ c_\eta^2\vec{P}\end{matrix}\right\} \simeq \frac{\mu^2}{8\pi^3}\int_0^\pi d\theta \sin\theta \left\{\begin{matrix} P_0 \\ \vec{P}\cos^2\theta\end{matrix}\right\} \times \int \frac{dq_4 d\epsilon_q^- |\Delta_q^-|^2}{[q_4^2+(\epsilon_q^-)^2+|\Delta_q^-|^2]^2} = \frac{\mu^2}{8\pi^2}\left\{\begin{matrix} P_0 \\ \frac{1}{3}\vec{P}\end{matrix}\right\}. \tag{108}$$

We observe that the decay constants of all NG bosons are of order μ, and all the velocities are equal to $1/\sqrt{3}$. This agrees with similar results for NG bosons in three flavor dense QCD [23–26], as well as with the studies of Ref. [39], dealing with two flavor QCD in the framework of the effective theory approach. Notice that these estimates of the decay constants imply that the infrared masses of the five gluons are of order $g_s\mu$. (For a discussion of subtleties concerning the generation of the mass of the eighth gluon, see Ref. [40].)

For completeness of our presentation, let us mention that the above expressions for the decay constants in the Pagels-Stokar approximation also contain the following subleading derivative term corrections:

$$\delta[(F^{(\chi)})^2] = \delta[(F^{(\lambda)})^2] = \frac{\mu^2}{4\pi^3}\int_0^1 x(1-x)dx \int \frac{dq_4 d\epsilon |\Delta_q^-|^2 q_4 \partial_{q_4}|\Delta_q^-|^2}{[q_4^2+(\epsilon_q^-)^2+x|\Delta_q^-|^2]^3} - \frac{\mu^2}{8\pi^3}\int_0^1 dx \int \frac{dq_4 d\epsilon q_4 (3/4-x)\partial_{q_4}|\Delta_q^-|^2}{[q_4^2+(\epsilon_q^-)^2+x|\Delta_q^-|^2]^2}, \tag{109}$$

$$\delta[c_\chi^2(F^{(\chi)})^2] = \delta[c_\lambda^2(F^{(\lambda)})^2] = \frac{\mu^2}{12\pi^3}\int_0^1 x(1-x)dx \times \int \frac{dq_4 d\epsilon |\Delta_q^-|^2 \epsilon \partial_\epsilon |\Delta_q^-|^2}{[q_4^2+(\epsilon_q^-)^2+x|\Delta_q^-|^2]^3} - \frac{\mu^2}{24\pi^3}\int_0^1 dx \times \int \frac{dq_4 d\epsilon \epsilon (3/4-x)\partial_\epsilon |\Delta_q^-|^2}{[q_4^2+(\epsilon_q^-)^2+x|\Delta_q^-|^2]^2}, \tag{110}$$

$$\delta[(F^{(\eta)})^2] = -\frac{\mu^2}{16\pi^3}\int \frac{dq_4 d\epsilon q_4 \partial_{q_4}|\Delta_q^-|^2}{[q_4^2+(\epsilon_q^-)^2+|\Delta_q^-|^2]^2}, \tag{111}$$

$$\delta[c_\eta^2(F^{(\eta)})^2] = -\frac{\mu^2}{48\pi^3}\int \frac{dq_4 d\epsilon \epsilon \partial_\epsilon |\Delta_q^-|^2}{[q_4^2+(\epsilon_q^-)^2+|\Delta_q^-|^2]^2}, \tag{112}$$

where $\epsilon \equiv \epsilon_q = |\vec{q}| - \mu$. In the calculation, we also assumed that the gap is an even function of q_4 and ϵ. By making use of the explicit solution to the gap equation (see Sec. III and Appendix B in Ref. [11]), we could check that these corrections are suppressed by a power of the coupling constant α_s compared to the leading order results in Eqs. (106), (107) and (108).

Here we worked out the decay constants of the left-handed diquark composites. It is straightforward to repeat an analogous analysis for the right-handed diquarks. As it should be, the results would be the same, in agreement with the invariance under parity.

It is worthwhile to note that the decay constants in the two sectors of the theory could have been equivalently defined through the matrix elements of the left- and right-handed color current. As we discussed earlier, both such currents are approximately conserved at large chemical potential. In other words, the latter means that the corresponding vector and axial vector currents are also approximately conserved. Now, by making use of the definition of scalar and pseudoscalar diquarks in Eqs. (52) and (53), it is easy to show that their decay constants are equal, up to a factor of $\sqrt{2}$, to those of the left- and right-handed states. Of course, here we assume that the definition of the decay constants of scalars and pseudoscalars are given in terms of the vector and axial vector currents, respectively.

While the NG scalars (a doublet, an antidoublet and a singlet) are not physical particles because of the Higgs mechanism, the five pseudoscalars remain in the physical spectrum. Since the latter are (nearly) massless in dense QCD, they should be relevant degrees of freedom in the infrared dynamics. The decay constants of these pseudoscalars are the physical observables that could be measured in an experiment. The most likely decay products of these pseudoscalars should be gluons of the unbroken $SU(2)_c$ and the massless quarks of the third color [which might eventually get a small mass too if another (non-scalar) condensate is generated [7,41]].

VII. BS EQUATION FOR MASSIVE COMPOSITES

The essential property of the quark pairing dynamics in $N_f=2$ dense QCD is the long range interaction mediated by gluons of the magnetic type [9,10]. Of course, the Meissner effect in the color superconducting phase produces masses for five out of the total eight magnetic modes. Nevertheless, there are still three modes that remain long ranged. This simple feature has many interesting consequences. One of them was conjectured in Ref. [27] where it was suggested that there should exist an infinite tower of massive radial excitations in the diquark channels with the quantum numbers of the (pseudo-)NG bosons. This conclusion was reached by making use of an indirect argument based on special properties of the effective potential in the color superconductor.

In this section, we study the problem of massive radial excitations by using the rigorous approach of the BS equation, derived in Sec. V. As we shall see, the conjecture of Ref. [27] is essentially correct. At the same time, it will turn out that some details of the pairing dynamics are rather sensitive to the specific details (such as the Meissner effect) and could not have been anticipated, based on the qualitative arguments of Ref. [27].

A. Bound states and the Meissner effect

In this subsection, we would like to clarify the role of the Meissner effect in the dynamics of diquark bound states. The main point we want to emphasize is the existence of two classes of bound states, for which the role of the Meissner effect is very different. The first class consists of light bound states with masses $M \ll |\Delta_0^-|$. The binding energy of these states is large (tightly bound states). The second class includes quasiclassical states with the masses close to their threshold $k|\Delta_0^-|$ where $k=1$ for diquark doublets, built of one massive and one massless quark, and $k=2$ for diquark singlets built of two massive quarks. The binding energy of the quasiclassical states is small, i.e.,

$$\frac{k|\Delta_0^-|-M}{k|\Delta_0^-|} \ll 1. \tag{113}$$

It is clear that the quasiclassical bound states are sensitive to the details of the infrared dynamics. Indeed, for long range potentials, the quasiclassical part of the spectrum is almost completely determined by the behavior of the potential at large distances. In the particular case of cold dense QCD, the interaction between quarks is long ranged in the (imaginary) time direction and essentially short ranged in the spatial ones [10,27]. Because of that, the region with $|k_0|<|\Delta_0^-| \lesssim |\vec{k}|$ is particularly important for the pairing dynamics of the quasiclassical diquark states. This implies that the inclusion of the Meissner effect is crucial for extracting the properties of the states from this second class (see Appendix C).

On the other hand, the Meissner effect is essentially irrelevant for the light bound states. This point can be illustrated by the BS equations for the lightest diquarks, the massless NG bosons. As was shown in Sec. VI, the BS equations for them are essentially equivalent to the gap equation. And we know from the experience of solving the gap equation [11–16] that the most important region of momenta in the equation is given by $|\Delta_0^-| \lesssim |k_0| \ll |\vec{k}| \lesssim \mu$. In this particular region, the two kernels, with and without the Meissner effect taken into account [compare Eqs. (23) and (24)], are practically indistinguishable. Obviously, the same should be true for all very light bound states with $M \ll |\Delta_0^-|$.

In the rest of this section, we show that, because of the Meissner effect, an infinite tower of (quasiclassical) massive diquarks occurs only in the singlet channel. In the doublet channel, the only bound states are the (tightly bound) NG bosons. This is connected with the fact that the interaction in the doublet channel is mediated exclusively by the gluons subject to the Meissner effect. In the singlet channel, on the other hand, the interaction is partly due to the unscreened gluons of the unbroken $SU(2)_c$ subgroup and, therefore, the formation of massive (quasiclassical) states is not prohibited.

B. Massive doublet $\chi_a^{(\tilde{b})}$

Let us start from the analysis of the BS equation for the massive χ doublets. By choosing the spatial component of the center of mass momentum of the bound state zero, $P=(M_\chi,0,0,0)$, we find that, similarly to Eq. (64), the most general structure of the BS wave function in the center of mass frame reads

$$\begin{aligned}\chi_a^{(\tilde{b})}(p,M_\chi) = \delta_a^{\tilde{b}}[&\chi_1^-\Lambda_p^+ + \chi_1^+\Lambda_p^- + (p_0-\epsilon_p^- \\ &+M_\chi/2)\chi_2^-\gamma^0\Lambda_p^+ + (p_0+\epsilon_p^+ \\ &+M_\chi/2)\chi_2^+\gamma^0\Lambda_p^-]\mathcal{P}_+ ,\end{aligned} \tag{114}$$

where, for convenience, we introduced here the factors $(p_0-\epsilon_p^-+M_\chi/2)$ and $(p_0+\epsilon_p^++M_\chi/2)$. In contrast to the case of (pseudo-)NG bosons, the structure of the wave functions of massive states cannot be established from the Ward identities. Although the vertex functions would also have the poles, corresponding to the massive intermediate states, the Ward identities are insufficient for extracting the structure of the residues unambiguously.

Be repeating the analysis similar to that in Sec. VI B, we arrive at the following set of equations for the components of the BS wave function:

$$\begin{aligned}\chi_1^-(p) = \frac{4}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}&\frac{(q_0-\epsilon_q^-+M_\chi/2)[\chi_1^-(q)-(\Delta_q^-)^*\chi_2^-(q)]}{(q_0-\epsilon_q^--M_\chi/2)[(q_0+M_\chi/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]}\\ &\times\mathrm{tr}[\gamma^\mu\Lambda_q^-\gamma^\nu\Lambda_p^+]\mathcal{D}_{\mu\nu}(q-p),\end{aligned} \tag{115}$$

$$\begin{aligned}(p_0-\epsilon_p^-+M_\chi/2)\chi_2^-(p) = -\frac{1}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}&\frac{[(q_0+M_\chi/2)^2-(\epsilon_q^-)^2]\chi_2^-(q)-\Delta_q^-\chi_1^-(q)}{(q_0-\epsilon_q^--M_\chi/2)[(q_0+M_\chi/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]}\\ &\times\mathrm{tr}[\gamma^\mu\gamma^0\Lambda_q^-\gamma^\nu\Lambda_p^+\gamma^0]\mathcal{D}_{\mu\nu}(q-p).\end{aligned} \tag{116}$$

The other two equations express $\chi^{+}_{1,2}(p)$ in terms of $\chi^{-}_{1,2}(p)$ and, therefore, they are irrelevant. As is easy to check by tracing back the derivation of the BS equations, the interaction in this doublet channel is provided exclusively by gluons that are subject to the Meissner effect in the color superconducting phase. This fact could be taken into account qualitatively by replacing the propagators of the magnetic modes according to the qualitative rule in Eq. (24). In accordance with the discussion in Sec. VII A, while such a modification was irrelevant in the gap equation and in the BS equations for (pseudo-)NG bosons, it is going to play a very important role in the analysis of the quasiclassical massive states.

In order to proceed with the analysis of the BS equation, we will use the approximation with $\chi^{\pm}_{2}=0$. Recall that such an approximation was completely justified in the case of (pseudo-)NG bosons. It is certain that it gives a very good approximation for light bound states, with $M \ll |\Delta^{-}_{0}|$, in general. Our *conjecture* is that the ansatz with $\chi^{\pm}_{2}=0$ yields a reasonable approximation even in the case of quasiclassical states. In order to justify this approximation one would need to prove that the (perturbative) correction due to nonzero $\chi^{\pm}_{2}$ is small. By repeating the arguments used for (pseudo-)NG bosons, we could show again that $\chi^{\pm}_{2}$ is irrelevant in the ultraviolet region $|\Delta^{-}_{0}| \lesssim p_0 \lesssim \mu$. While this observation is promising, it is not sufficient yet because the infrared region $0<p_0<|\Delta^{-}_{0}|$ is also important for the pairing dynamics of the quasiclassical radial excitations. Therefore, for such states, it should be considered as a (reasonable) conjecture.

Now, we drop $\chi^{\pm}_{2}$ from the analysis and assume that the component functions $\chi^{\pm}_{1}$ depend only on the time component of the momentum (compare with the analysis of the gap equation in Refs. [10–16]). Then, we arrive at the following equation:

$$\chi^{-}_{1}(p_4)=\frac{2\alpha_s}{9\pi}\int_0^{\Lambda} dq_4 K^{(\chi)}(q_4)\chi^{-}_{1}(q_4)\ln\frac{\Lambda}{|q_4-p_4|+c|\Delta^{-}_{0}|}, \tag{117}$$

where $\Lambda=(4\pi)^{3/2}\mu/\alpha_s^{5/2}$ and the kernel reads

$$K^{(\chi)}(q_4)=\frac{M^2_{\chi}(q_4+\sqrt{q_4^2+|\Delta^{-}_{0}|^2})^2-|\Delta^{-}_{0}|^4}{\sqrt{q_4^2+|\Delta^{-}_{0}|^2}[4M^2_{\chi}q_4^2-(|\Delta^{-}_{0}|^2-M^2_{\chi})^2]}. \tag{118}$$

To analyze the BS equation, we could convert it into a differential equation, using the same approach as in the case of the gap equation in Ref. [11] (see Appendix C). It is straightforward to show then that, in the ultraviolet region $|\Delta^{-}_{0}|<p_4<\Lambda$, the BS wave function of the massive doublet, $\chi^{-}_{1}(p_4)$, has the same behavior as the BS wave function of the NG boson in the χ channel (which, as we know from the Ward identities, is proportional to the gap function). The deviations might appear only in the infrared region, $0<p_4<|\Delta^{-}_{0}|$. Note, however, that the BS wave function in that region is essentially constant. To see this, one should notice that the dependence on p_4 on the right hand side of Eq. (117) comes through the argument of the logarithm, in which either large q_4 or $c|\Delta^{-}_{0}|$ dominates.

By matching the infrared and the ultraviolet asymptotes of the solution, we find that the non-trivial wave function $\chi^{-}_{1}(p_4)$ cannot have zeros in the whole range of momenta, $0<p_4<\Lambda$. This means that the only solution to the integral equation (117) corresponds to the NG boson with $M_{\chi}=0$. Indeed, a solution that describes a massive radial excitation of the NG boson must have at least one zero. Because of the Meissner effect, no such solutions exist.

From physical point of view, massive gluons cannot provide a sufficiently strong interaction to form massive radial excitations of the NG bosons in the doublet channel. To understand this better, it might be instructive to point out that, in absence of the Meissner effect, an infinite tower of (quasiclassical) massive states would appear (see Appendix C). However, the binding energy of all these states would be very small compared to the value of the superconducting gap. This indicates that it is the long range dynamics that is primarily responsible for the formation of such quasiclassical bound states. Then, in agreement with the discussion in Sec. VII A, even relatively small screening effects for gluons in the infrared region are sufficient to prevent binding of quarks in the doublet channel.

C. Massive antidoublet $\lambda^{b}_{(\tilde{a})}$

The analysis for the $\lambda^{b}_{(\tilde{a})}$ antidoublet resembles a lot the analysis in the previous subsection, so we outline only the main points, omitting the unnecessary details.

The general structure of the BS wave function is given as

$$\begin{aligned}\lambda^{b}_{(\tilde{a})}(p,M_{\lambda})=\delta^{b}_{\tilde{a}}\mathcal{P}_{-}[&\lambda^{+}_{1}\Lambda^{+}_{p}+\lambda^{-}_{1}\Lambda^{-}_{p}+(p_0-\epsilon^{-}_{p}\\&+M_{\lambda}/2)\lambda^{-}_{2}\gamma^0\Lambda^{+}_{p}+(p_0+\epsilon^{+}_{p}\\&+M_{\lambda}/2)\lambda^{+}_{2}\gamma^0\Lambda^{-}_{p}].\end{aligned} \tag{119}$$

The equations for the components of the BS wave functions of the antidoublet are almost the same as those for the doublet. The only difference is that M_{χ} is replaced by $-M_{\lambda}$. By repeating all the arguments of the previous subsection, we again conclude that, after the Meissner effect is taken into consideration, the gluon interaction is not strong enough to provide binding of the massive radial excitations in the $\lambda^{b}_{(\tilde{a})}$ antidoublet channel.

D. Massive singlet η

Let us consider massive singlet diquark with $M_{\eta}\neq 0$. Since the equations for η and σ do not decouple, the massive radial excitation of the η NG boson would have a non-zero admixture of σ. The general structures of the BS wave functions in the center of mass frame, $P=(M_{\eta},0,0,0)$, are

$$\eta(p,M_\eta)=[\eta_1^-\Lambda_p^+ +\eta_1^+\Lambda_p^- +(p_0-\epsilon_p^- +M_\eta/2)\,\eta_2^-\gamma^0\Lambda_p^+ +(p_0+\epsilon_p^+ +M_\eta/2)\,\eta_2^+\gamma^0\Lambda_p^-]\mathcal{P}_+ \\ +[\eta_3^+\Lambda_p^+ +\eta_3^-\Lambda_p^- +(p_0+\epsilon_p^- -M_\eta/2)\,\eta_4^-\gamma^0\Lambda_p^- +(p_0-\epsilon_p^+ -M_\eta/2)\,\eta_4^+\gamma^0\Lambda_p^+]\mathcal{P}_-, \tag{120}$$

$$\sigma(p,M_\eta)=\mathcal{P}_-\gamma^0[(p_0-\epsilon_p^- +M_\eta/2)\sigma^-\Lambda_p^+ +(p_0+\epsilon_p^+ +M_\eta/2)\sigma^+\Lambda_p^-]\mathcal{P}_+ . \tag{121}$$

The components satisfy the following set of equations:

$$\eta_1^-(p)=\frac{4}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{1}{[(q_0-M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2][(q_0+M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]}\left\{\left[q_0^2-\left(\epsilon_q^- -\frac{M_\eta}{2}\right)^2\right]\eta_1^-(q) \right. \\ \left. +(\Delta_q^{-*})^2\eta_3^-(q)-(\Delta_q^-)^*\left[q_0^2-\left(\epsilon_q^- -\frac{M_\eta}{2}\right)^2\right][\eta_2^-(q)+\eta_4^-(q)]\right\}\mathrm{tr}[\gamma^\mu\Lambda_q^-\gamma^\nu\Lambda_p^+]\mathcal{D}_{\mu\nu}(q-p), \tag{122}$$

$$\left(p_0+\frac{M_\eta}{2}-\epsilon_p^-\right)\eta_2^-(p)$$

$$=\frac{5}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{1}{[(q_0-M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2][(q_0+M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]}\left(\left(q_0+\frac{M_\eta}{2}+\epsilon_q^-\right)\right. \\ \times\left\{\left[q_0^2-\left(\epsilon_q^- -\frac{M_\eta}{2}\right)^2\right]\eta_2^-(q)-(\Delta_q^-)^*\eta_3^-(q)\right\}+\left(q_0-\frac{M_\eta}{2}+\epsilon_q^-\right)[|\Delta_q^-|^2\eta_4^-(q)-\Delta_q^-\eta_1^-(q)]\Big) \\ \times\mathrm{tr}[\gamma^0\gamma^\mu\Lambda_q^+\gamma^0\gamma^\nu\Lambda_p^+]\mathcal{D}_{\mu\nu}(q-p)+2\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{\sigma^-}{q_0-M_\eta/2-\epsilon_q^-}\mathrm{tr}[\gamma^0\gamma^\mu\Lambda_q^+\gamma^0\gamma^\nu\Lambda_p^+]\mathcal{D}_{\mu\nu}(q-p), \tag{123}$$

$$\eta_3^-(p)=\frac{4}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{1}{[(q_0-M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2][(q_0+M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]}\left\{\left[q_0^2-\left(\epsilon_q^- +\frac{M_\eta}{2}\right)^2\right]\eta_3^-(q)\right. \\ \left. +(\Delta_q^-)^2\eta_1^-(q)-\Delta_q^-\left[\left(q_0+\frac{M_\eta}{2}\right)^2-(\epsilon_q^-)^2\right]\eta_2^-(q)-\Delta_q^-\left[\left(q_0-\frac{M_\eta}{2}\right)^2-(\epsilon_q^-)^2\right]\eta_4^-(q)\right\} \\ \times\mathrm{tr}[\gamma^\mu\Lambda_q^+\gamma^\nu\Lambda_p^-]\mathcal{D}_{\mu\nu}(q-p), \tag{124}$$

$$\left(p_0-\frac{M_\eta}{2}+\epsilon_p^-\right)\eta_4^-(p)$$

$$=\frac{5}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{1}{[(q_0-M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2][(q_0+M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]}\left(\left(q_0-\frac{M_\eta}{2}-\epsilon_q^-\right)\right. \\ \times\left\{\left[q_0^2-\left(\epsilon_q^- -\frac{M_\eta}{2}\right)^2\right]\eta_4^-(q)-(\Delta_q^-)^*\eta_3^-(q)\right\}+\left(q_0+\frac{M_\eta}{2}-\epsilon_q^-\right)[|\Delta_q^-|^2\eta_2^-(q)-\Delta_q^-\eta_1^-(q)]\Big) \\ \times\mathrm{tr}[\gamma^0\gamma^\mu\Lambda_q^-\gamma^0\gamma^\nu\Lambda_p^-]\mathcal{D}_{\mu\nu}(q-p)+2\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{\sigma^-(-q)}{q_0+M_\eta/2+\epsilon_q^-}\mathrm{tr}[\gamma^0\gamma^\mu\Lambda_q^-\gamma^0\gamma^\nu\Lambda_p^-]\mathcal{D}_{\mu\nu}(q-p), \tag{125}$$

$$\left(p_0+\frac{M_\eta}{2}-\epsilon_p^-\right)\sigma^-(p)$$
$$=\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{1}{[(q_0-M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2][(q_0+M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]}\left(\left(q_0+\frac{M_\eta}{2}+\epsilon_q^-\right)\right.$$
$$\times\left\{\left[q_0^2-\left(\epsilon_q^--\frac{M_\eta}{2}\right)^2\right]\eta_2^-(q)-(\Delta_q^-)^*\eta_3^-(q)\right\}+\left(q_0-\frac{M_\eta}{2}+\epsilon_q^-\right)[|\Delta_q^-|^2\eta_4^-(q)-\Delta_q^-\eta_1^-(q)]\Big)$$
$$\times\mathrm{tr}[\gamma^0\gamma^\mu\Lambda_q^+\gamma^0\gamma^\nu\Lambda_p^+]\mathcal{D}_{\mu\nu}(q-p)+\frac{2}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{\sigma^-}{q_0-M_\eta/2-\epsilon_q^-}\mathrm{tr}[\gamma^0\gamma^\mu\Lambda_q^+\gamma^0\gamma^\nu\Lambda_p^+]\mathcal{D}_{\mu\nu}(q-p), \tag{126}$$

In the case of massless NG bosons, the component functions $\eta_{2,4}^-$ equal zero. Similarly to the case of doublets, we assume that the ansatz with these functions being equal to zero yields a good approximation also for massive diquarks. By substituting $\eta_{2,4}^-=0$ into the BS equations above, we obtain the following simple set of equations:

$$\eta_1^-(p)=\frac{4}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{[q_0^2-(\epsilon_q^--M_\eta/2)^2]\eta_1^-(q)+(\Delta_q^{-*})^2\eta_3^-(q)}{[(q_0-M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2][(q_0+M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]}\mathrm{tr}[\gamma^\mu\Lambda_q^-\gamma^\nu\Lambda_p^+]\mathcal{D}_{\mu\nu}(q-p), \tag{127}$$

$$\eta_3^-(p)=\frac{4}{3}\pi\alpha_s\int\frac{d^4q}{(2\pi)^4}\frac{[q_0^2-(\epsilon_q^-+M_\eta/2)^2]\eta_3^-(q)+(\Delta_q^-)^2\eta_1^-(q)}{[(q_0-M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2][(q_0+M_\eta/2)^2-(\epsilon_q^-)^2-|\Delta_q^-|^2]}\mathrm{tr}[\gamma^\mu\Lambda_q^+\gamma^\nu\Lambda_p^-]\mathcal{D}_{\mu\nu}(q-p), \tag{128}$$

plus the equation for σ^- which does not allow a non-trivial solution for a bound state.

Now, in order to solve the set of equations for η_1^- and η_3^-, we make the following substitution:

$$\eta_1^-(p)=-\frac{(\Delta_p^-)^*}{|\Delta_p^-|}h_1(p), \tag{129}$$

$$\eta_3^-(p)=\frac{\Delta_p^-}{|\Delta_p^-|}h_3(p), \tag{130}$$

and, as in the case of doublets, we assume that the wave functions ($h_{1,3}$) depend only on the time component of the momentum $p_4=ip_0$ (also compare with the analysis of the gap equation in Refs. [10–16]). At the end, we arrive at the following equation for the BS wave function ($h_1=h_3$) of the massive singlet:

$$h_1(p_4)=\frac{\alpha_s}{4\pi}\int_0^\Lambda dq_4K^{(\eta)}(q_4)h_1(q_4)\ln\frac{\Lambda}{|q_4-p_4|}, \tag{131}$$

where $\Lambda=(4\pi)^{3/2}\mu/\alpha_s^{5/2}$, and the kernel reads

$$K^{(\eta)}(q_4)=\frac{\sqrt{q_4^2+|\Delta_0^-|^2}}{q_4^2+|\Delta_0^-|^2-(M_\eta/2)^2}. \tag{132}$$

At this point it is appropriate to emphasize that, as we saw already in the previous two subsections, the Meissner effect plays an important role in the analysis of the massive bound states. Indeed, our analysis indicates that only the long range interaction mediated by the unscreened gluons of the unbroken $SU(2)_c$ is strong enough to produce massive bound states. This is taken into account in Eq. (131) where the effective coupling constant differs by the factor 9/8 from the coupling in the gap equation (30).

In order to get the solution for the BS wave function $h_1(p)$, we use the same method as in the case of the gap equation [11]. In particular, we convert Eq. (131) into the differential equation

$$p_4h_1''(p_4)+h_1'(p_4)+\frac{\alpha_s}{4\pi}K^{(\eta)}(p_4)h_1(p_4)=0, \tag{133}$$

along with the boundary conditions

$$h_1'(0)=0 \quad\text{and}\quad h_1(\Lambda)=0. \tag{134}$$

Now, we solve the differential equation (133) in the following three qualitatively different regions: $0\leqslant p_4\leqslant\sqrt{|\Delta_0^-|^2-(M_\eta/2)^2}$, $\sqrt{|\Delta_0^-|^2-(M_\eta/2)^2}\leqslant p_4\leqslant|\Delta_0^-|$ and $|\Delta_0^-|\leqslant p_4\leqslant\Lambda$. The kernel (132) has a simple behavior in each region, and the BS equation allows the analytical solutions

$$h_1(p_4)=C_0J_0\left(\sqrt{\frac{\alpha_s|\Delta_0^-|p_4}{\pi[|\Delta_0^-|^2-(M_\eta/2)^2]}}\right),$$
$$\text{for}\quad 0\leqslant p_4\leqslant\sqrt{|\Delta_0^-|^2-(M_\eta/2)^2}, \tag{135a}$$

$$h_1(p_4)=C_1J_0\left(\sqrt{\frac{\alpha_s|\Delta_0^-|}{\pi p_4}}\right)+C_2N_0\left(\sqrt{\frac{\alpha_s|\Delta_0^-|}{\pi p_4}}\right),$$
$$\text{for}\quad \sqrt{|\Delta_0^-|^2-(M_\eta/2)^2}\leqslant p_4\leqslant|\Delta_0^-|, \tag{135b}$$

$$h_1(p_4)=C_3\sin\left(\frac{1}{2}\sqrt{\frac{\alpha_s}{\pi}}\ln\frac{\Lambda}{p_4}\right),\quad \text{for}\quad |\Delta_0^-|\leqslant p_4\leqslant\Lambda, \tag{135c}$$

where J_n and N_n are Bessel functions of the first and second types. The solutions are chosen so that the boundary conditions are automatically satisfied. In the above expressions, C_i $(i=0,\ldots,3)$ are the integration constants. To obtain the spectrum of the massive diquark states, we match the logarithmic derivatives of the appropriate pairs of the solutions at $\sqrt{|\Delta_0^-|^2-(M_\eta/2)^2}$ and $|\Delta_0^-|$. After taking into account the equation that determines the value of the gap (see Appendix B in Ref. [11]),

$$\ln\frac{\Lambda}{|\Delta_0^-|}=\frac{2}{\nu}\arctan\left(\frac{J_0(\nu)}{J_1(\nu)}\right),\quad \nu=\sqrt{\frac{8\alpha_s}{9\pi}}, \tag{136}$$

the matching condition reads

$$\frac{J_0(z_0)J_1(z_0)}{J_1(z_0)N_0(z_0)+J_0(z_0)N_1(z_0)}\simeq\frac{\sqrt{\alpha_s\pi}}{4}\cot\left(\frac{3\pi}{4\sqrt{2}}\right), \tag{137}$$

where the coupling is assumed to be small, $\alpha_s\ll 1$, and

$$z_0=\sqrt{\frac{\alpha_s|\Delta_0^-|}{\pi\sqrt{|\Delta_0^-|^2-(M_\eta/2)^2}}}. \tag{138}$$

It is straightforward to check that the left hand side of Eq. (137) is an oscillating function having an infinite number of zeros $(z_0\approx 2.40, 3.83, 5.52,\ldots)$. In the weakly coupled theory, each zero (or rather a nearby point that approaches the zero as $\alpha_s\to 0$) determines a corresponding value of the diquark mass.[6] In the quasiclassical limit, i.e., when $M_\eta\to 2|\Delta_0^-|$ from below, the left hand side of Eq. (137) is approximately given by $\cot(2z_0)$. Then, we derive the following simple estimates for the masses of the η singlets:

$$M_n^2\simeq 4|\Delta_0^-|^2\left(1-\frac{2^8\alpha_s^2}{\pi^6(2n+1)^4}\right),\quad n\gg 1. \tag{139}$$

This agrees with the expression presented in Eq. (1) when $\kappa=2^8/\pi^6\approx 0.27$. Accidentally, one could also check from the position of the zeros on the left hand side of Eq. (137) that the expression in Eq. (139) gives a good approximation even for the low lying states $(n=1,2,\ldots)$. Notice that the state with $n=0$ does not appear.

[6]Note that there is also a zero at $z_0=0$, but Eq. (137) does not have a solution in its vicinity.

VIII. CONCLUSION

In this paper we studied the problem of diquark bound states in the color superconducting phase of $N_f=2$ cold dense QCD. We used the conventional method of BS equations that suits the problem best. We derived the general BS equations, and then analyzed them in spin zero channels.

Our analytical analysis of the BS equations in cold dense QCD shows that the theory contains five (nearly) massless pseudoscalars (pseudo-NG bosons) which transform as a doublet, an antidoublet and a singlet under unbroken $SU(2)_c$. To the best of our knowledge, these pseudoscalar diquarks have not been discussed in the literature before. We estimate the decay constants of these pseudoscalars, and find that their orders of magnitude are the same as that of the chemical potential. The velocities of the pseudoscalars are equal to $1/\sqrt{3}$, and this coincides with the velocity of the NG bosons in three flavor QCD. While being (nearly) massless, the five pseudoscalar diquarks should be the relevant degrees of freedom in the low energy dynamics of $N_f=2$ dense QCD.

The parity-even partners of the pseudoscalar diquarks are the NG bosons which are the ghosts in the theory. Although they are removed from the spectrum of physical particles by the Higgs mechanism, one cannot get rid of them completely, unless a special (unitary) gauge is defined. Since the order parameter is given by a diquark composite field, it does not seem to be straightforward to define and to use the unitary gauge in dense QCD. In all the covariant gauges we use here, NG bosons are always present and they play an important role in removing unphysical poles from physical scattering amplitudes.

We also studied the problem of massive diquarks. In accordance with the conjecture of Ref. [27], there exists an infinite tower of massive bound states which are the radial excitations of the (pseudo-)NG bosons. As a result of the Meissner effect, it appears that massive radial excitations occur only in the singlet channel. This could be understood in the following way. The interaction in the doublet and the antidoublet channels is provided exclusively by the gluons affected by the Meissner effect. Such an interaction is not sufficiently strong to form massive radial excitations in those channels. The important point in this analysis is the different role the Meissner effect plays for tightly bound states and quasiclassical bound states.

As we know, parity is unbroken in the color superconducting phase of two flavor dense QCD. Then by recalling that the left- and right-handed sectors of the theory approximately decouple, we could see that all the massive diquarks come in pairs of degenerate parity-even (scalar) and parity-odd (pseudoscalar) states.

Regarding the nature of the massive diquark states, let us note that they may truly be just resonances in the full theory, since they could decay into pseudo-NG bosons and/or gluons of unbroken $SU(2)_c$. At high density, however, both the running coupling $\alpha_s(\mu)$ and the effective Yukawa coupling $g_Y=|\Delta_0^-|/F^{(x)}\sim|\Delta_0^-|/\mu$ are small, and, therefore, these massive resonances are narrow.

At the end, we would like to add a few words about the higher spin channels that we do not study here. In view of studies in Ref. [13], it would be of great interest to investigate also the case of spin one diquarks, as they might be rather light in the color superconducting phase. The general form of the BS equations for such diquarks are exactly the same as in Eqs. (57)-(60). Of course, the structure of the BS wave functions would differ.

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APPENDIX A: ANGULAR INTEGRATION

We need to calculate the following traces over the Dirac indices:

$$\mathrm{tr}[\gamma^\mu \Lambda_p^{(e)} \gamma^\nu \Lambda_q^{(e')}] = g^{\mu\nu}(1+ee't) - 2ee' g^{\mu 0} g^{\nu 0} t + ee' \frac{\vec{q}^\mu \vec{p}^\nu + \vec{q}^\nu \vec{p}^\mu}{|\vec{q}||\vec{p}|} + \dots, \tag{A1}$$

$$\mathrm{tr}[\gamma^\mu \gamma^0 \Lambda_p^{(e)} \gamma^\nu \gamma^0 \Lambda_q^{(e')}] = -g^{\mu\nu}(1-ee't) + \left(g^{\mu 0} - e\frac{\vec{q}^\mu}{|\vec{q}|}\right)\left(g^{\nu 0} - e'\frac{\vec{p}^\nu}{|\vec{p}|}\right) + \left(g^{\mu 0} - e'\frac{\vec{p}^\mu}{|\vec{p}|}\right)\left(g^{\nu 0} - e\frac{\vec{q}^\nu}{|\vec{q}|}\right), \tag{A2}$$

where $e,e' = \pm 1$, $t = \cos\theta$ is the cosine of the angle between three-vectors $\vec{q}$ and $\vec{p}$, and irrelevant antisymmetric terms are denoted by the ellipsis.

By contracting these traces with the projectors of the magnetic, electric and longitudinal types of gluon modes, we arrive at

$$O_{\mu\nu}^{(1)} \mathrm{tr}[\gamma^\mu \Lambda_p^{(-)} \gamma^\nu \Lambda_q^{(+)}] = 2(1-t)\frac{q^2+p^2+qp(1-t)}{q^2+p^2-2qpt}, \tag{A3}$$

$$O_{\mu\nu}^{(2)} \mathrm{tr}[\gamma^\mu \Lambda_p^{(-)} \gamma^\nu \Lambda_q^{(+)}] = 2(1+t)\frac{q^2+p^2-qp(1+t)}{q^2+p^2-2qpt} - (1+t)\frac{(q-p)^2+(q_4-p_4)^2}{q^2+p^2-2qpt+(q_4-p_4)^2}, \tag{A4}$$

$$O_{\mu\nu}^{(3)} \mathrm{tr}[\gamma^\mu \Lambda_p^{(-)} \gamma^\nu \Lambda_q^{(+)}] = (1+t)\frac{(q-p)^2+(q_4-p_4)^2}{q^2+p^2-2qpt+(q_4-p_4)^2}, \tag{A5}$$

$$O_{\mu\nu}^{(1)} \mathrm{tr}[\gamma^\mu \Lambda_p^{(-)} \gamma^\nu \Lambda_q^{(-)}] = 2(1+t)\frac{q^2+p^2-qp(1+t)}{q^2+p^2-2qpt}, \tag{A6}$$

$$O_{\mu\nu}^{(2)} \mathrm{tr}[\gamma^\mu \Lambda_p^{(-)} \gamma^\nu \Lambda_q^{(-)}] = 2(1-t)\frac{q^2+p^2+qp(1-t)}{q^2+p^2-2qpt} - (1-t)\frac{(q+p)^2+(q_4-p_4)^2}{q^2+p^2-2qpt+(q_4-p_4)^2}, \tag{A7}$$

$$O_{\mu\nu}^{(3)} \mathrm{tr}[\gamma^\mu \Lambda_p^{(-)} \gamma^\nu \Lambda_q^{(-)}] = (1-t)\frac{(q+p)^2+(q_4-p_4)^2}{q^2+p^2-2qpt+(q_4-p_4)^2}, \tag{A8}$$

$$O_{\mu\nu}^{(1)} \mathrm{tr}[\gamma^\mu \gamma^0 \Lambda_p^{(-)} \gamma^\nu \gamma^0 \Lambda_q^{(-)}] = -2(1-t)\frac{q^2+p^2+qp(1-t)}{q^2+p^2-2qpt}, \tag{A9}$$

$$O_{\mu\nu}^{(2)} \mathrm{tr}[\gamma^\mu \gamma^0 \Lambda_p^{(-)} \gamma^\nu \gamma^0 \Lambda_q^{(-)}] = \frac{2qp(1-t^2)}{q^2+p^2-2qpt} + (1+t) \times \frac{(q-p)^2-(q_4-p_4)^2-2i(q_4-p_4)(q-p)}{q^2+p^2-2qpt+(q_4-p_4)^2}, \tag{A10}$$

$$O_{\mu\nu}^{(3)} \mathrm{tr}[\gamma^\mu \gamma^0 \Lambda_p^{(-)} \gamma^\nu \gamma^0 \Lambda_q^{(-)}] = -(1+t)\frac{(q-p)^2-(q_4-p_4)^2-2i(q_4-p_4)(q-p)}{q^2+p^2-2qpt+(q_4-p_4)^2}, \tag{A11}$$

$$O_{\mu\nu}^{(1)} \mathrm{tr}[\gamma^\mu \gamma^0 \Lambda_p^{(-)} \gamma^\nu \gamma^0 \Lambda_q^{(+)}] = -2(1+t)\frac{q^2+p^2-qp(1+t)}{q^2+p^2-2qpt}, \tag{A12}$$

$$O^{(2)}_{\mu\nu}\,\mathrm{tr}[\gamma^\mu\gamma^0\Lambda^{(-)}_p\gamma^\nu\gamma^0\Lambda^{(+)}_q]$$
$$=-\frac{2qp(1-t^2)}{q^2+p^2-2qpt}+(1-t)\frac{(q+p)^2-(q_4-p_4)^2-2i(q_4-p_4)(q+p)}{q^2+p^2-2qpt+(q_4-p_4)^2}, \tag{A13}$$

$$O^{(3)}_{\mu\nu}\,\mathrm{tr}[\gamma^\mu\gamma^0\Lambda^{(-)}_p\gamma^\nu\gamma^0\Lambda^{(+)}_q]$$
$$=-(1-t)\frac{(q+p)^2-(q_4-p_4)^2-2i(q_4-p_4)(q+p)}{q^2+p^2-2qpt+(q_4-p_4)^2}, \tag{A14}$$

where $q\equiv|\vec{q}|$, $p\equiv|\vec{p}|$, $q_4\equiv -iq_0$, and $p_4\equiv -ip_0$.

Therefore,

$$I_1^{-+}=q^2\int d\Omega\mathcal{D}_{\mu\nu}(q-p)\,\mathrm{tr}[\gamma^\mu\Lambda^{(-)}_p\gamma^\nu\Lambda^{(+)}_q]$$
$$\approx 2i\pi\left[\frac{2}{3}\ln\frac{(2\mu)^3}{|\epsilon_q^-|^3+\pi M^2\omega/2}+\ln\frac{(2\mu)^2}{(\epsilon_q^-)^2+2M^2+\omega^2}+\xi\right], \tag{A15}$$

$$I_1^{--}=q^2\int d\Omega\mathcal{D}_{\mu\nu}(q-p)\,\mathrm{tr}[\gamma^\mu\Lambda^{(-)}_p\gamma^\nu\Lambda^{(-)}_q]$$
$$\approx 2i\pi\left[\frac{2}{3}\ln\frac{(2\mu)^3}{|\epsilon_q^-|^3+\pi M^2\omega/2}-\frac{\alpha_s}{\pi}\ln\frac{(2\mu)^2}{(\epsilon_q^-)^2+2M^2+\omega^2}+\xi\ln\frac{(2\mu)^2}{(\epsilon_q^-)^2+\omega^2}\right], \tag{A16}$$

where $M^2=2\alpha_s\mu^2/\pi$ and $\omega=|q_4-p_4|$, and

$$I_2^{--}=q^2\int d\Omega\mathcal{D}_{\mu\nu}(q-p)\,\mathrm{tr}[\gamma^\mu\gamma^0\Lambda^{(-)}_p\gamma^\nu\gamma^0\Lambda^{(-)}_q]$$
$$\approx 2i\pi\left[-\frac{2}{3}\ln\frac{(2\mu)^3}{|\epsilon_q^-|^3+\pi M^2\omega/2}+\ln\frac{(2\mu)^2}{(\epsilon_q^-)^2+2M^2+\omega^2}-\xi\right], \tag{A17}$$

$$I_2^{-+}=q^2\int d\Omega\mathcal{D}_{\mu\nu}(q-p)\,\mathrm{tr}[\gamma^\mu\gamma^0\Lambda^{(-)}_p\gamma^\nu\gamma^0\Lambda^{(+)}_q]$$
$$\approx 2i\pi\left[-\frac{2}{3}\ln\frac{(2\mu)^3}{|\epsilon_q^-|^3+\pi M^2\omega/2}+\ln\frac{(2\mu)^2}{(\epsilon_q^-)^2+2M^2+\omega^2}-\xi\ln\frac{(2\mu)^2}{(\epsilon_q^-)^2+\omega^2}\right]. \tag{A18}$$

APPENDIX B: A NON-PERTURBATIVE CORRECTION TO THE SD EQUATION

In light of our analysis in Sec. IV, one could argue that the SD equation might get a large non-perturbative contribution, coming from the pole contributions in the full vertex function; see Eq. (62) and Fig. 4. We recall that the pole structure of the vertices is related to the existence of NG and pseudo-NG bosons in the theory (for more on this see Sec. VI).

If one recalls that the SD equation is quite sensitive to the long range dynamics ($|P|\ll\mu$), it would be very natural to ask whether the pole contributions to the vertex function in Eqs. (62) could modify the SD equation and its solution. The revealed non-perturbative contributions could conveniently be combined in matrix form as follows:

$$\delta\Gamma^{A\mu}(q+P,q)|_{P\to 0}=\frac{\tilde{P}^\mu}{P^\nu\tilde{P}_\nu}\begin{pmatrix} 3\delta^i_j\delta^A_8(T^8)^b_a(\Delta_q\mathcal{P}_- - \tilde{\Delta}_q\mathcal{P}_+) & \delta^i_j(T^A)^3_a\tilde{\Delta}_q\mathcal{P}_+ & -\hat{\varepsilon}^{ij}_{ac}(T^A)^c_3\Delta_q\mathcal{P}_- \\ -\delta^i_j(T^A)^b_3\Delta_q\mathcal{P}_- & 0 & 0 \\ \hat{\varepsilon}^{cb}_{ij}(T^A)^3_c\tilde{\Delta}_q\mathcal{P}_+ & 0 & 0\end{pmatrix}. \tag{B1}$$

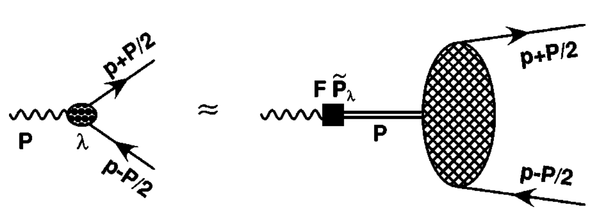

FIG. 4. The pole contribution to the vertex as $P\to 0$.

It is of great importance to notice that this is longitudinal, i.e., $\delta\Gamma^{A\mu}(q+P,q)\sim\tilde{P}^\mu\equiv(P^0,\vec{P}/3)$. As a result of this property, the contraction of this vertex with the transverse (with respect to $\vec{P}$) projector of the magnetic gluon modes is equal to zero. Regarding the other two types of gluon modes (electric and longitudinal), the corresponding contractions are non-zero, and they lead to a finite contribution to the right hand side of the SD equation (21). We stress that the vertex in Eq. (B1) is longitudinal with respect to $\tilde{P}^\mu$ (notice the tilde), while the projector of the electric modes is transverse with respect to P^{μ} (no tilde here). This difference is responsible for a non-zero contraction involving the electric gluon modes. It is still, however, the longitudinal gluon mode that, after being contracted with the vertex in Eq. (B1), gives the most significant contribution to the SD equation. By performing the explicit calculation, we arrive at the following extra term to the right hand side of the gap equation,

$$\delta\Delta_p^- \simeq -\frac{\pi}{2}\alpha_s \int \frac{d^4q}{(2\pi)^4}\mathrm{tr}(\gamma^{\mu}\Lambda_q^+\gamma^0\Lambda_p^-) \times \frac{(q_0+\epsilon_q^-)\Delta_q^-}{q_0^2-(\epsilon_q^-)^2-|\Delta_0^-|^2}\frac{(\tilde{q}-\tilde{p})^{\nu}}{(\tilde{q}-\tilde{p})^{\lambda}(q-p)_{\lambda}}\mathcal{D}_{\mu\nu}(q-p), \tag{B2}$$

which results in the following term to the equations for Δ_p^- :

$$\delta\Delta_p^- \simeq \frac{\alpha_s}{16\pi^2}\int dq_4 dq \frac{\Delta_q^-}{q_4^2+(\epsilon_q^-)^2+|\Delta_0|^2} \times\left[-\xi + O\left(\frac{q_4^2+(\epsilon_q^-)^2}{M^2}\ln\frac{(2\mu)^2}{q_4^2+(\epsilon_q^-)^2}\right)\right], \simeq \frac{\alpha_s}{16\pi}\int \frac{dq_4\Delta_q^-}{\sqrt{q_4^2+|\Delta_0^-|^2}}\left[-\xi+O\left(\frac{q_4^2}{M^2}\right)\right], \tag{B3}$$

This correction is of the same order as the correction from the longitudinal gluon modes in the gap equation (29) when the bare vertices are used [11]. Therefore, such an additional correction could only modify the overall constant factor in the solution for Δ_p^- . The exponential factor and the overall power of the coupling constant in the solution should remain intact.

Of course, as we discussed earlier, the overall constant in the expression for the gap might get other kinds of corrections which have not been analyzed here [15,31]. Sorting out all such corrections is a rather complicated problem that is outside the scope of this paper.

APPENDIX C: THE ANALYTICAL SOLUTIONS TO THE BS EQUATIONS

In this appendix, we present the approximate analytical solutions to the BS equations. In general, our approach here resembles the method commonly used for solving the gap equation [11–16]. One of the purposes of the analysis below is to illustrate that, while the Meissner effect is irrelevant for the pairing dynamics of light bound states with $M \ll |\Delta_0^-|$, it is crucial for the pairing of quasiclassical bound states (see Sec. VII A).

1. NG bosons

Let us start from the BS equation (73) for (pseudo-)NG bosons in the χ-doublet channel. After performing the standard approximations that were extensively discussed in many studies of the gap equation [11–16] (see also Sec. III), we arrive at the following integral equation:

$$\chi(p_4) \simeq \frac{2\alpha_s}{9\pi}\int_0^{\Lambda}\frac{dq_4\chi(q_4)}{\sqrt{q_4^2+|\Delta_0^-|^2}}\ln\frac{\Lambda}{|p_4-q_4|+c|\Delta_0^-|}, \tag{C1}$$

where, for brevity of notation, we use $\chi \equiv \chi_1^-$ and $\Lambda = (4\pi)^{3/2}\mu/\alpha_s^{5/2}$. Notice that, in accordance with Eq. (24), the Meissner effect is taken into account by the term $c|\Delta_0^-|$ in the logarithm. Without loss of generality, we substitute $c=1$ in what follows.[7] The integral equation (C1) could be approximately reduced to the following second order differential equation:

$$(p_4+|\Delta_0^-|)\chi''(p_4)+\chi'(p_4)+\frac{\nu^2}{4}\frac{\chi(p_4)}{\sqrt{p_4^2+|\Delta_0^-|^2}}=0, \quad \text{where} \quad \nu=\sqrt{\frac{8\alpha_s}{9\pi}}, \tag{C2}$$

subject to the infrared and the ultraviolet boundary conditions

$$\chi'(0)=0 \quad \text{and} \quad \chi(\Lambda)-|\Delta_0^-|\chi'(\Lambda)\approx 0. \tag{C3}$$

In order to get an estimate for the solution, we consider the differential equation on the two adjacent intervals, $0 \leqslant p_4 \leqslant |\Delta_0^-|$ and $|\Delta_0^-| \leqslant p_4 \leqslant \Lambda$, separately. The approximate analytical solutions that satisfy the boundary conditions in Eq. (C3) read

$$\chi(p_4)=C_1\exp\left(\frac{p_4}{2|\Delta_0^-|}\right)\left[\sqrt{1-\nu^2}\cosh\left(\frac{p_4\sqrt{1-\nu^2}}{2|\Delta_0^-|}\right) - \sinh\left(\frac{p_4\sqrt{1-\nu^2}}{2|\Delta_0^-|}\right)\right], \quad \text{for} \quad 0\leqslant p_4 \leqslant |\Delta_0^-|, \tag{C4a}$$

$$\chi(p_4)=C_2\sin\left(\frac{\nu}{2}\ln\frac{\Lambda+|\Delta_0^-|}{p_4}\right), \quad \text{for} \quad |\Delta_0^-|\leqslant p_4 \leqslant \Lambda, \tag{C4b}$$

where C_i $(i=1,2)$ are the integration constants. By matching the solutions at $p_4=|\Delta_0^-|$, we arrive at the following condition that determines the value of the gap:

$$\ln\frac{\Lambda+|\Delta_0^-|}{|\Delta_0^-|}=\frac{2}{\nu}\arctan\left(\frac{\sqrt{1-\nu^2}\coth(\sqrt{1-\nu^2}/2)-1}{\nu}\right). \tag{C5}$$

[7] The analysis for $c \neq 1$ is a little more complicated since the two scales, $c|\Delta_0^-|$ and $|\Delta_0^-|$, are different. Despite this, the final result for $c \neq 1$ would remain qualitatively the same as soon as c is a constant of order 1.

This leads to the same (up to an overall constant of order one) expression for the gap as in Eq. (32). Recall that the latter was derived without taking the Meissner effect into account. We conclude, therefore, that the solution for the χ-doublet NG boson is not sensitive to the screening due to the Meissner effect.

The analysis of the BS equation for (pseudo-)NG bosons in the λ-doublet and η-singlet channels is very similar to the analysis for the χ doublet and we do not repeat it here.

2. Massive diquarks

Let us consider the integral equation (117) for the BS wave function of the massive χ doublet. It is instructive to start with the analysis of this equation by neglecting the Meissner effect at first. This is achieved by substituting $c=0$. In this special case, the differential equation reads

$$p_4\chi''(p_4)+\chi'(p_4)+\frac{\nu^2}{4}K^{(\chi)}(p_4)\chi(p_4)=0,$$

$$\text{where}\quad \nu=\sqrt{\frac{8\alpha_s}{9\pi}}, \tag{C6}$$

along with the same boundary conditions as in Eq. (C3). The kernel [compare with Eq. (118)] is approximately given by

$$K^{(\chi)}(p_4)=\begin{cases}|\Delta_0^-|/(|\Delta_0^-|^2-M_\chi^2) & \text{for}\quad 0\leqslant p_4\leqslant z_M^2|\Delta_0^-|,\\ 1/p_4 \quad \text{for} & z_M^2|\Delta_0^-|\leqslant p_4\leqslant\Lambda,\end{cases} \tag{C7}$$

where $z_M=\sqrt{|\Delta_0^-|^2-M_\chi^2}/|\Delta_0^-|$.

The analytical solutions to the differential equation in two qualitatively different regions are given by

$$\chi(p_4)=C_1J_0\left(\frac{\nu}{z_M}\sqrt{\frac{p_4}{|\Delta_0^-|}}\right)\quad \text{for}\quad 0\leqslant p_4\leqslant z_M^2|\Delta_0^-|, \tag{C8a}$$

$$\chi(p_4)=C_2\sin\left(\frac{\nu}{2}\ln\frac{\Lambda}{p_4}\right)\quad \text{for}\quad z_M^2|\Delta_0^-|\leqslant p_4\leqslant\Lambda. \tag{C8b}$$

By matching these two solutions at $p_4=z_M^2|\Delta_0^-|$, we obtain

$$\ln\frac{\Lambda}{z_M^2|\Delta_0^-|}=\frac{2}{\nu}\arctan\left(\frac{J_0(\nu)}{J_1(\nu)}\right)+\frac{2\pi n}{\nu},\quad n=1,2,\ldots\ . \tag{C9}$$

By comparing this with the gap equation (136) (see also Appendix B in Ref. [11]), we derive the following spectrum of massive diquarks in absence of the Meissner effect:

$$M_n=|\Delta_0^-|\sqrt{1-\exp\left(-\frac{2\pi n}{\nu}\right)},\quad n=1,2,\ldots\ . \tag{C10}$$

Below we argue that none of these massive states survive after the Meissner effect is taken into account. In fact, this is almost obvious when we notice that the BS wave functions that correspond to the states with masses in Eq. (C10) have rather rich node structure in the far infrared region $0<p\ll|\Delta_0^-|$. Topologically, the nth wave function has exactly n zeros. These n zeros appear at

$$p_4^{(k)}=|\Delta_0^-|\exp\left[-\frac{2}{\nu}\left(\pi k-\arctan\frac{J_0(\nu)}{J_1(\nu)}\right)\right],\quad k=1,2,\ldots,n. \tag{C11}$$

In the weakly coupled theory, $\nu\ll1$, we find that $p_4^{(k)}\ll|\Delta_0^-|$ for any k. This suggests that after taking the Meissner effect back into consideration, the mentioned structure of the nodes in the BS wave functions of the massive χ diquarks would become impossible due to the strong screening effects of the gluons in the infrared region $0<p_4\lesssim|\Delta_0^-|$.

To substantiate the claim of the previous paragraph, let us now consider the equation where the Meissner effect is qualitatively taken into account. We arrive at the following differential equation:

$$(p_4+|\Delta_0^-|)\chi''(p_4)+\chi'(p_4)+\frac{\nu^2}{4}K^{(\chi)}(p_4)\chi(p_4)=0,$$

$$\text{where}\quad \nu=\sqrt{\frac{8\alpha_s}{9\pi}}. \tag{C12}$$

The wave function should again satisfy the boundary conditions in Eq. (C3). To get the estimate for the solution, we consider the differential equation on the three adjacent intervals, $0\leqslant p_4\leqslant z_M^2|\Delta_0^-|$, $z_M^2|\Delta_0^-|\leqslant p_4\leqslant|\Delta_0^-|$, and $|\Delta_0^-|\leqslant p_4\leqslant\Lambda$, separately. The corresponding analytical solutions read

$$\chi(p_4)=C_0\exp\left(\frac{p_4}{2|\Delta_0^-|}\right)\left[\sqrt{z_M^2-\nu^2}\cosh\left(\frac{p_4\sqrt{z_M^2-\nu^2}}{2z_M|\Delta_0^-|}\right)-z_M\sinh\left(\frac{p_4\sqrt{z_M^2-\nu^2}}{2z_M|\Delta_0^-|}\right)\right]\quad \text{for}\quad 0\leqslant p_4\leqslant z_M^2|\Delta_0^-|, \tag{C13a}$$

$$\chi(p_4)=C_1\left[G_{12}^{20}\left(\frac{p_4}{|\Delta_0^-|}\left|\begin{matrix}1-\frac{\nu^2}{4} & \\ 0 & 1\end{matrix}\right.\right)+C_2\frac{p_4}{|\Delta_0^-|}{}_1F_1\left(1+\frac{\nu^2}{4},2,-\frac{p_4}{|\Delta_0^-|}\right)\right]\quad \text{for}\quad z_M^2|\Delta_0^-|\leqslant p_4\leqslant|\Delta_0^-|, \tag{C13b}$$

$$\chi(p_4)=C_3\sin\left(\frac{\nu}{2}\ln\frac{\Lambda+|\Delta_0^-|}{p_4}\right)\quad \text{for}\quad |\Delta_0^-|\leqslant p_4\leqslant\Lambda, \tag{C13c}$$

where G_{12}^{20} is the Meijer's G function. The solutions in the first and the last regions are chosen so that they satisfy the boundary conditions in Eq. (C3).

We note that the ultraviolet asymptote (C13c) of the BS wave function of a massive doublet coincides with that of the NG boson (C4b). Moreover, this property is shared by all massive states that exist, irrespective of the value of their mass. Now, unlike the wave functions of (pseudo-)NG bosons which have no nodes, the BS wave functions of massive excitations should have at least one zero somewhere in the region $0 \leqslant p_4 < \Lambda$. Since, in agreement with our previous statement, there cannot be any nodes in the ultraviolet region $|\Delta_0^-| < p_4 < \Lambda$, they might occur only in the infrared, $0 \leqslant p_4 \leqslant |\Delta_0^-|$.

By matching the logarithmic derivatives of the solutions at $p_4 = z_M^2 |\Delta_0^-|$ and $p_4 = |\Delta_0^-|$, we obtain the two different expressions for the integration constant C_2:

$$C_2 = \mathcal{F}(z_M) \quad \text{and} \quad C_2 = \mathcal{F}(1), \tag{C14}$$

where the explicit form of function $\mathcal{F}(z)$ reads

$$\frac{\nu^2 z^2 {}_1F_1\left(1+\frac{\nu^2}{4},2,-z^2\right) + z\left[\sqrt{z^2-\nu^2}\coth\left(\frac{z\sqrt{z^2-\nu^2}}{2}\right) - z\right]\left[2\,{}_1F_1\left(1+\frac{\nu^2}{4},2,-z^2\right) - z^2\left(1+\frac{\nu^2}{4}\right){}_1F_1\left(2+\frac{\nu^2}{4},3,-z^2\right)\right]}{2z[\sqrt{z^2-\nu^2}\coth(z\sqrt{z^2-\nu^2}/2) - z]G_{12}^{20}\left(z^2\left|\begin{matrix}-\nu^2/4 & \\ 0 & 0\end{matrix}\right.\right) - \nu^2 G_{12}^{20}\left(z^2\left|\begin{matrix}1-\nu^2/4 & \\ 0 & 1\end{matrix}\right.\right)}. \tag{C15}$$

Notice that we used the gap equation (C5) to derive the second expression in Eq. (C14).

The spectrum of massive excitations (if any) should be determined by the solutions of the equation $\mathcal{F}(z) = \mathcal{F}(1)$ where $z < 1$. Note that the obvious solution $z = 1$ corresponds to the no-node wave function of the NG boson. By studying the equation numerically, we find that there are no solutions which would correspond to wave functions with nodes. However, in addition to the $z = 1$ solution, there is another solution for $z < 1$. This latter also corresponds to a wave function *without* nodes in the whole region of momenta $0 \leqslant p_4 < \Lambda$. In fact, its shape barely differs from the wave function of the NG boson. In the spectral problem at hand, however, one does not expect to have two solutions with the same no-node topology. Therefore, we believe that the extra solution is an artifact of the approximations used. Its appearance apparently results from two *different* splittings of the whole region of momenta into separate intervals for $M = 0$ and $M \neq 0$ cases. This is also supported by the observation that, because of the Meissner effect, the BS wave function in the doublet channel is always almost a constant function (and, therefore, cannot have zeros) in the infrared region $0 \leqslant p_4 < |\Delta_0^-|$. This is seen already from the integral version of the BS equation (117). It is natural that there is only one no-node solution to the BS equation. Since the BS wave function of the NG boson is such a solution, no other nontrivial solutions should exist in the doublet channel.

The analysis of the BS equation for the λ antidoublet is similar and we do not repeat it here. The analysis for the singlet is presented in Sec. VII B in detail. There the Meissner effect is qualitatively taken into account by considering only the interaction that is mediated by the gluons of the unbroken $SU(2)_c$ subgroup.

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