Renormalization group study of the Chern-Simons field coupled to scalar matter in a modified BPHZ subtraction scheme

L. C. de Albuquerque,* M. Gomes,† and A. J. da Silva‡
Instituto de Física, USP, C.P. 66318 05389-970, São Paulo-SP, Brazil
(Received 1 March 2000; published 20 September 2000)

We apply a soft version of the Bogolubov-Parasiuk-Hepp-Zimmermann subtraction scheme to the computation of two-loop corrections from an Abelian Chern-Simons field coupled to (massive) scalar matter with a $\lambda(\Phi^\dagger\Phi)^2$ and $\nu(\Phi^\dagger\Phi)^3$ self-interactions. The two-loop renormalization group functions are calculated. We compare our results with those in the literature.

PACS number(s): 11.10.Gh, 11.10.Hi, 11.10.Kk

Field theories with the Chern-Simons (CS) term in $2+1$ dimensions [1,2] are among the best studied models in the past two decades. This is due not only to their potential applications but also because of some subtle conceptual and technical aspects. Indeed, the quantization of these theories raises interesting questions, some of which have not been answered satisfactorily up to now. Of particular interest is the setup of a renormalization or regularization scheme simple and reliable to provide a consistent framework for complex calculations. Many proposals have appeared in the literature [2–7], each presenting advantages as well as disadvantages. For instance, Pauli-Villars regularization explicitly breaks parity, analytic regularization is not gauge invariant, and Slavnov regularization becomes rather intricate beyond one-loop calculations.

It is of course desirable that the regularization scheme preserves as much of the models symmetries as possible. In this respect, the popular dimensional regularization appears to be suited for the task. However, the topological nature of the CS term introduces extra complications. The Levi-Civita symbol does not admit a simple extension to complex dimensions D and to overcome this problem some modifications have to be done [8].

If one insists on keeping the Levi-Civita symbol in three dimensions, other approaches are possible. First, there is the so-called consistent dimensional regularization [9,10] in which $\epsilon_{\mu\nu\lambda}$ is treated as essentially three dimensional, but one has to introduce a Maxwell or Yang-Mills kinetic term as a supplementary regularization. In another proposal, called dimensional reduction [6,11–13] the tensor algebra is done in three dimensions and afterwards the Feynman integrals are promoted to D dimensions. This method may introduce ambiguities in the finite parts of the amplitudes and also in the divergent parts in high order corrections.

Now, it is possible to renormalize divergent integrals without introducing an specific regularization. One of the most efficient and rigorous method is the Bogolubov-Parasiuk-Hepp-Zimmermann (BPHZ) subtraction scheme which has been applied in a variety of situations [14]. One class of this method, the soft BPHZ schemes, is especially adequate for the study of massless theories [15].

In a soft BPHZ scheme one introduces subtraction operators in the external momenta and in the masses of the theory. In order to have a better control of the infrared divergences, some of the subtractions have zero mass whereas others have mass equal to μ (the renormalization scale). In $D=4$ this scheme is in many circumstances equivalent to the dimensional regularization with a minimal subtraction prescription, and leads to a mass-independent renormalization group equation [16].

In this work we use a soft BPHZ scheme in an Abelian CS theory coupled with scalar matter to compute two-loop renormalization group functions. Since this method does not involve analytic continuation in the space-time dimension (i.e., we stay in the physical dimension $D=3$), we evade the problems aforementioned. Although we will be dealing mainly with massive particles the scheme allows, for nonexceptional momenta, a smooth zero mass limit if all superrenormalizable interactions are deleted. The paper is organized as follows: In Sec. II we introduce the model, generalize the soft BPHZ scheme to the CS theory with scalar matter, and show that the soft BPHZ scheme respects the Ward identity. In Sec. III we compute the renormalization group functions up to two-loops. Finally, we draw some conclusions and comments. Our results for the renormalization group functions extend the ones recently computed through the use of the dimensional reduction [13], in the sense that the results agree whenever a comparison is possible. The mentioned agreement indicates that dimensional reduction is a consistent scheme, at least up two-loops, as far as renormalization group functions are concerned. However, differently from Ref. [13] the model that we study includes the most general, renormalizable and U(1) invariant, self-interaction of the scalar particles.

I. THE MODEL AND THE SOFT BPHZ SCHEME

The Lagrangian describing the model reads [the metric has signature $(+,-,-)$ and $\epsilon^{012}=1$]

$$\mathcal{L}=(D_\mu\Phi)^\dagger D^\mu\Phi-m^2\Phi^\dagger\Phi-\frac{\lambda}{6}(\Phi^\dagger\Phi)^2 - \frac{\nu}{36}(\Phi^\dagger\Phi)^3+\frac{1}{4}\epsilon^{\mu\nu\lambda}F_{\mu\nu}A_\lambda-\frac{1}{2\xi}(\partial\cdot A)^2, \quad (1)$$

*Email address: claudio@fma.if.usp.br
†Email address: mgomes@fma.if.usp.br
‡Email address: ajsilva@fma.if.usp.br

where $D_\mu=\partial_\mu-ieA_\mu$ is the covariant derivative and $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$. The canonical operator dimensions are as follows (in unity of mass $[m]=1$): $[\Phi]=1/2$, $[A_\mu]=[\lambda]=[\xi]=1$, $[\nu]=[e]=0$. The ultraviolet degree of superficial divergence of a graph Γ is

$$d(\Gamma)=3-V_{\phi^4}-\frac{1}{2}N_\phi-N_A, \tag{2}$$

where V_{ϕ^4} is the number of insertions of the superrenormalizable vertex $\lambda(\Phi^\dagger\Phi)^2$, and $N_\phi(N_A)$ is the number of scalar (vector) external lines in Γ. Nonetheless, we must stress that, due to the contractions of indexes, this degree is lowered by the number of external trilinear vertices having attached one internal CS line. In this work we will use the Landau gauge, formally obtained by letting $\xi\to 0$.

In a soft BPHZ approach the divergent quantum amplitudes, described by Feynman integrals in perturbation theory, are made finite by the application of subtraction operators $\tau^{d(\gamma)}$ arranged in accord with the forest formula [14].

The subtraction operator τ^n has degree n in the derivatives and uses derivatives with respect to m^2 (each of them having degree 2) if the integrand depends only on the square of this mass. All infrared finite subtractions are made at $m^2=0$ but in the would be infrared divergent terms (possibly occurring in the last subtractions) we make $m^2=\mu^2$. As a rule, we make the minimum number of subtractions necessary to render finite a Feynman amplitude. Explicitly, the first three subtraction operators are defined as

$$\tau^0\,\mathcal{G}(p_i,m)=\mathcal{G}(0,\mu),$$

$$\tau^1\,\mathcal{G}(p_i,m)=\mathcal{G}(0,0)+p_i^\mu\frac{\partial\mathcal{G}}{\partial p_i^\mu}\bigg|_{p_i=0,m=\mu}, \tag{3}$$

$$\tau^2\,\mathcal{G}(p_i,m)=\mathcal{G}(0,0)+p_i^\mu\frac{\partial\mathcal{G}}{\partial p_i^\mu}\bigg|_{p_i=0,m=0}+\frac{1}{2}p_i^\mu p_j^\nu\frac{\partial^2\mathcal{G}}{\partial p_i^\mu\partial p_j^\nu}\bigg|_{p_i=0,m=\mu}+m^2\frac{\partial\mathcal{G}}{\partial m^2}\bigg|_{p_i=0,m=\mu},$$

$$\tau^3\,\mathcal{G}(p_i,m)=\mathcal{G}(0,0)+p_i^\mu\frac{\partial\mathcal{G}}{\partial p_i^\mu}\bigg|_{p_i=0,m=0}+\frac{1}{2}p_i^\mu p_j^\nu\frac{\partial^2\mathcal{G}}{\partial p_i^\mu\partial p_j^\nu}\bigg|_{p_i=0,m=0}+m^2\frac{\partial\mathcal{G}}{\partial m^2}\bigg|_{p_i=0,m=0}+\frac{1}{6}p_i^\mu p_j^\nu p_k^\lambda\frac{\partial^3\mathcal{G}}{\partial p_i^\mu\partial p_j^\nu\partial p_k^\lambda}\bigg|_{p_i=0,m=\mu}+m^2p_i^\mu\frac{\partial^2\mathcal{G}}{\partial p_i^\mu\partial m^2}\bigg|_{p_i=0,m=\mu}.$$

It is apparent that our scheme does not introduce infrared divergences in graphs having at least one internal scalar line. For ultraviolet divergent graphs without internal scalar lines, special care has to be taken. To be precise, for ultraviolet divergent graphs without internal scalar lines, we will use the regularized CS field propagator

$$\Delta_{\mu\nu}(p)=\epsilon_{\mu\nu\rho}\frac{p^\rho}{p^2-M^2+i\epsilon}, \tag{4}$$

where M is a regularization mass which, in the subtraction terms, is to be treated in the same way as m and has to be put equal to zero after all the subtractions are performed.

To see how the method works, in the following we will compute the vacuum polarization at one-loop in the theory specified by Eq. (1). The vacuum polarization is given by the sum of two terms, namely,

$$\pi^{(A)}_{\rho\lambda}=2ie^2g_{\rho\lambda}\int[dq]\,\Delta(q), \tag{5}$$

$$\pi^{(B)}_{\rho\lambda}(p)=-e^2\int[dq]\,(2q+p)_\rho(2q+p)_\lambda\,\Delta(q)\,\Delta(q+p)\equiv-e^2\int[dq]\,I_{\rho\lambda}(p,q;m), \tag{6}$$

where $\Delta(k)=i/(k^2-m^2+i\epsilon)$ is the scalar propagator, and $[dq]=d^3q/(2\pi)^3$. Both $\pi^{(A)}_{\rho\lambda}$ and $\pi^{(B)}_{\rho\lambda}$ are linearly divergents. For the renormalized quantities we obtain

$$\pi^{(A)}_{\rho\lambda}|_R=2ie^2g_{\rho\lambda}\int[dq](1-\tau^1)\,\Delta(q)=-2e^2g_{\rho\lambda}\int[dq]\left[\frac{1}{q^2-m^2}-\frac{1}{q^2}\right], \tag{7}$$

$$\pi^{(B)}_{\rho\lambda}|_R(p)=-e^2\int[dq](1-\tau^1)\,I_{\rho\lambda}(p,q;m)=e^2\int[dq]\left[\frac{(2q+p)_\rho(2q+p)_\lambda}{(q^2-m^2)((q+p)^2-m^2)}-4\frac{q_\rho q_\lambda}{(q^2)^2}-p^\alpha\frac{\partial I_{\rho\lambda}}{\partial p^\alpha}\bigg|_{m=\mu,p=0}\right]. \tag{8}$$

From a practical standpoint, the finite integrals in Eqs. (7) and (8) may be most easily computed using an intermediate regularization (cutoff, dimensional, Pauli-Villars, etc.), the final result being, of course, independent of the regularization employed. Note that the last term of the Eq. (8) vanishes upon symmetric integration. We obtain

$$\pi^{(A)}_{\rho\lambda}|_R=-i\frac{e^2m}{2\pi}g_{\rho\lambda}, \tag{9}$$

$$\pi^{(B)}_{\rho\lambda}(p)|_R = i\frac{e^2}{8\pi}\left[2m\left(g_{\rho\lambda}+\frac{p_\rho p_\lambda}{p^2}\right) - \frac{p^2-4m^2}{\sqrt{p^2}}\sinh^{-1}\left(\frac{\sqrt{p^2}}{\sqrt{4m^2-p^2}}\right)\mathcal{T}_{\rho\lambda}\right], \tag{10}$$

where $\mathcal{T}_{\rho\lambda}=g_{\rho\lambda}-p_\rho p_\lambda/p^2$. Using that $\pi_{\rho\lambda}(p)=\pi^{(A)}_{\rho\lambda}+\pi^{(B)}_{\rho\lambda}(p)$, we finally get [note that $\pi(p^2\to 0)=0$ as it should]

$$\pi_{\rho\lambda}(p)|_R = \pi(p)\mathcal{T}_{\rho\lambda} = -i\frac{e^2}{8\pi}\left[2m+\frac{p^2-4m^2}{\sqrt{p^2}} \times\sinh^{-1}\left(\frac{\sqrt{p^2}}{\sqrt{4m^2-p^2}}\right)\right]\mathcal{T}_{\rho\lambda}. \tag{11}$$

The Lagrangian density in Eq. (1) is invariant under global $U(1)$ transformations. At the classical level, this demands the conservation of the current $J_\mu(x)=ie[\Phi^\dagger\overleftrightarrow{\partial}_\mu\Phi]+2e^2\Phi^\dagger\Phi A_\mu$. At the quantum level this current will be quantized with normal product of minimum degree, i.e., two. The corresponding Ward identity for the Green functions reads

$$\begin{aligned}\partial_\mu\langle\, T\,N_2[J^\mu](x)\,X_{\{\nu_j\}}(x_k,y_j)\,\rangle &= \langle\, T\,N_3[\partial^\mu J_\mu](x)\,X_{\{\nu_j\}}(x_k,y_j)\,\rangle \\ &= e\left[\sum_{j=1}^{N}\delta(x-x_j)-\sum_{j=N+1}^{2N}\delta(x-x_j)\right] \times\langle\, T\,X_{\{\nu_j\}}(x_k,y_j)\,\rangle,\end{aligned} \tag{12}$$

where

$$X_{\{\nu_j\}}(x_k,y_j)=\prod_{k=1}^{N}\Phi(x_k)\prod_{i=N+1}^{2N}\Phi^\dagger(x_i)\prod_{l=1}^{L}A_{\nu_l}(y_l),$$

follows from the use of the normal product algorithm which turns out to be valid in our scheme. In the first step Lowenstein's differentiation rule [14] was used to get the partial derivative inside the normal product (which can be done if one increases the degree of the normal product by one unity); in the second step one uses the equations of motion for the bilinears $\Phi^\dagger\Box\Phi$ and $\Phi\Box\Phi^\dagger$, which in our case have the classical form.

The Ward identity for the proper vertex functions has the same form as in Eq. (12) except for a minus sign on its right hand side. Denoting by $\Gamma^{(2N,L)}(p_1,\ldots,p_{2N};k_1,\ldots,k_L)$ the proper vertex function of equal numbers (N) of Φ and $\Phi^\dagger$ fields and L gauge fields, one can verify that, in momentum space

$$q^\mu\Gamma^{(2,1)}_\mu(p,p';q)=-e[\Gamma^{(2)}(p')-\Gamma^{(2)}(p)], \tag{13}$$

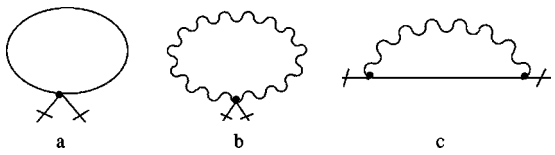

FIG. 1. $\Gamma^{(2)}$ at one loop.

$$q^\mu\Gamma^{(2,2)}_{\mu\nu}(p,p';q,k)=-e[\Gamma^{(2,1)}_\nu(p+q,p';k)-\Gamma^{(2,1)}_\nu(p,p'-q;k)]. \tag{14}$$

Here and in the following we adopt the simplified notation $\Gamma^{(n)}\equiv\Gamma^{(n,0)}$.

II. RENORMALIZATION GROUP FUNCTIONS

The renormalized vertex functions introduced in the previous section satisfy the renormalization group equation

$$\left[\mu\frac{\partial}{\partial\mu}+\beta_{m^2}\frac{\partial}{\partial m^2}+\beta_\lambda\frac{\partial}{\partial\lambda}+\beta_\nu\frac{\partial}{\partial\nu}+\frac{1}{2}\beta_{e^2}\frac{\partial}{\partial e^2} -N\gamma_\Phi-L\gamma_A\right]\Gamma^{(N,L)}=0, \tag{15}$$

where β_{m^2}, β_λ, β_ν, β_{e^2}, γ_Φ, and γ_A are power series in the coupling constants e^2, ν, and λ. We note that $\beta_{e^2}=e^2\gamma_A=0$ as a consequence of the Coleman-Hill theorem [17].

We shall fix now the other functions. We begin by proving that they do not have one-loop contributions. Indeed, in the computation of the renormalization group functions the relevant contributions come through the μ dependence of the subtraction terms. As mentioned before, these subtractions are those which are potentially infrared logarithmically divergent. We shall now examine these possible contributions. We will use a graphical notation in which each diagram represents a set of Feynman graphs differing by the orientation of the external lines. Moreover, to facilitate the discussion, one may use an auxiliary regularization so that each subtraction can be analyzed individually.

The divergent graphs contributing to $\Gamma^{(2)}$ at one-loop are shown in Fig. 1. Figure 1(a) is linearly divergent but, as it does not depend on the external momentum, it does not lead to a logarithmic term. The graphs in Figs. 1(b) and 1(c) vanish upon contraction of indexes. More generally, the subtractions for any one-loop graph contributing to $\Gamma^{(N)}$, with an odd number of CS lines, vanishes in the Landau gauge. This can be used to eliminate the possible contributions to $\Gamma^{(6)}$ coming from Fig. 2. Finally, Fig. 3 shows divergent contributions to $\Gamma^{(4)}$ at one-loop. Figure 3(a) is linearly divergent, with no logarithmic corrections. Figure 3(b) is linearly diver-

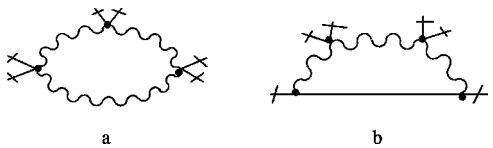

FIG. 2. Divergent contributions to $\Gamma^{(6)}$ at one loop.

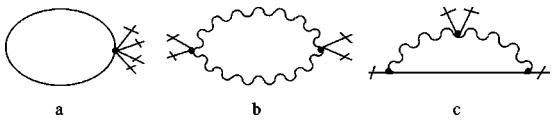

FIG. 3. Divergent contributions to $\Gamma^{(4)}$ at one loop.

gent. Using τ_M^1 in Eq. (3), and, in accord with Eq. (4), taking the limit $M\to 0$ at the end, we found that it does not depend on μ. Figure 3(c) is actually finite due to the observations made after Eq. (2). A similar analysis can be done for $\Gamma^{(2,1)}$ and $\Gamma^{(2,2)}$ leading to the result mentioned in the beginning of this section.

Let $\Sigma(p)$ be the self-energy function defined by $\Gamma^{(2)}(p) = i[p^2 - m^2 - \Sigma(p)]$. We have

$$\left[\mu\frac{\partial}{\partial\mu} + \beta_{m^2}\frac{\partial}{\partial m^2} + \beta_\lambda\frac{\partial}{\partial\lambda} + \beta_\nu\frac{\partial}{\partial\nu} - 2\gamma_\Phi\right]\Sigma(p) + \beta_{m^2} + 2\gamma_\Phi(p^2 - m^2) = 0. \tag{16}$$

It is then easily verified that

$$\mu\frac{\partial}{\partial\mu}\Sigma^{[1]}(p) + \beta_{m^2}^{[1]} + 2\gamma_\Phi^{[1]}(p^2 - m^2) = 0, \tag{17}$$

$$\gamma_\Phi^{[1]} = -\frac{1}{4}\frac{p_\mu p_\nu}{p^2}\frac{\partial^2}{\partial p^\mu \partial p^\nu}\left(\mu\frac{\partial}{\partial\mu}\Sigma^{[1]}\right)\Bigg|_{p=0}, \tag{18}$$

$$\beta_{m^2}^{[1]} = 2m^2\gamma_\Phi^{[1]} - \mu\frac{\partial}{\partial\mu}\Sigma^{[1]}\Bigg|_{p=0}, \tag{19}$$

where the superscript in parenthesis designates the order of the corresponding quantity in the loop expansion. From Eqs. (18) and (19) we obtain that $\gamma_\Phi^{[1]} = \beta_{m^2}^{[1]} = 0$, as there is no μ dependence at one-loop. From Eq. (15) follows then that $\beta_\nu^{[1]} = \beta_\lambda^{[1]} = 0$.

Let us now proceed to the two-loop calculation of the renormalization group functions. It can be easily verified that $\gamma_\Phi^{[2]}$ and $\beta_{m^2}^{[2]}$ are given by Eqs. (18) and (19), with the superscript $[1]$ replaced by $[2]$. Besides that, using the one-loop results, and writing the renormalization group equations for $\Gamma^{(6)}$ and $\Gamma^{(4)}$ in a loop expansion, we obtain at two-loops the following relations:

$$\beta_\nu^{[2]} = 6\nu\gamma_\Phi^{[2]} + i\mu\frac{\partial}{\partial\mu}\Gamma^{(6)[2]}(0), \tag{20}$$

$$\beta_\lambda^{[2]} = 4\lambda\gamma_\Phi^{[2]} + i\mu\frac{\partial}{\partial\mu}\Gamma^{(4)[2]}(0). \tag{21}$$

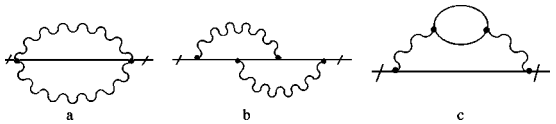

FIG. 4. Contributions to γ_Φ.

We have used the analytical continuation in the number of dimensions as intermediate regularization. It turns out that all integrals needed for the computations may be expressed in terms of [n-dimensional Euclidean space, with $\{dq\} = \sigma^\epsilon d^n q/(2\pi)^n$, where σ is a mass scale, and $\epsilon = 3 - n$]

$$I_1^{(n)}(M_a, M_b, M_c) \equiv \int \{dk\}\{dq\} \times \frac{1}{(k^2 + M_a^2)(q^2 + M_b^2)[(k+q)^2 + M_c^2]} = \frac{1}{32\pi^2}\left[\frac{1}{\epsilon} - \gamma + 1 - \ln\left(\frac{(M_a + M_b + M_c)^2}{4\pi\sigma^2}\right)\right], \tag{22}$$

$$I_2^{(n)}(M_a, M_b) \equiv \int \{dk\}\{dq\}\frac{1}{(k^2 + M_a^2)[(k+q)^2 + M_b^2]} = \frac{1}{16\pi^2}M_a M_b. \tag{23}$$

At two-loops, the nonvanishing contributions to γ_Φ come from the three (quadratically divergent) diagrams in Fig. 4. Figure 4(a) may be written as

$$\Sigma_A^{[2]}(p) = -2ie^4\int [dk][dq]\,\Delta(k+q-p) \times \Delta^{\mu\nu}(k)\,\Delta_{\mu\nu}(q)\big|_{SBPHZ}. \tag{24}$$

This diagram has three divergent subgraphs and eight forests, but the relevant term comes from the subtraction associated to the diagram as a whole. An application of the forest formula leads to the following contribution to $\gamma_\Phi^{[2]}$:

$$\gamma_{\Phi\,A}^{[2]} = -\frac{e^4}{48\pi^2}. \tag{25}$$

The second graph, Fig. 4(b), corresponds to the unsubtracted integral

$$\Sigma_B^{[2]}(p) = ie^4\int [dk][dq]\,(2p+k)^\mu(2p+2k+q)^\alpha \times (2p+k+2q)^\nu(2p+q)^\beta\,\Delta(p+k)\Delta \times (p+k+q)\,\Delta(p+q)\,\Delta_{\mu\nu}(k)\,\Delta_{\alpha\beta}(q)\big|_{SBPHZ}. \tag{26}$$

In this case there is no divergent subgraph and the calculation of $\gamma_{\Phi\,B}^{[2]}$ is reduced to just the contribution from the forest corresponding to the graph as a whole. We then get

$$\gamma_{\Phi\,B}^{[2]} = -\frac{e^4}{12\pi^2}. \tag{27}$$

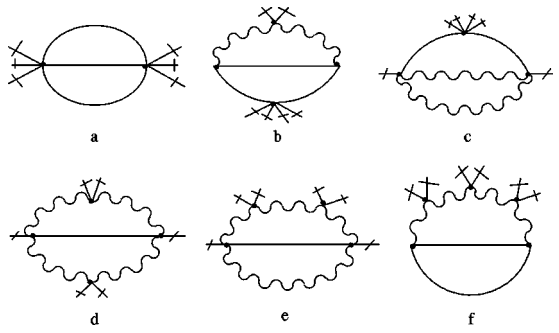

FIG. 5. Two-loop contributions to $\Gamma^{(6)}$.

The graph in Fig. 4(c) is written as

$$\Sigma_C^{[2]}(p)=ie^4\int [dk][dq](2p-k)^\mu(2q-k)^\alpha(2q-k)^\beta \times(2p-k)^\nu\Delta(k-p)\Delta(q)\,\Delta \times(q-k)\Delta_{\mu\alpha}(k)\,\Delta_{\beta\nu}(k)|_{SBPHZ}. \quad (28)$$

The calculation now is more involved and details will be provided in the Appendix. That analysis produces the result

$$\gamma_{\Phi\,C}^{[2]}=-\frac{e^4}{24\pi^2}. \quad (29)$$

Adding the results in Eqs. (25), (27), and (29), we obtain

$$\gamma_\Phi^{[2]}=-\frac{7e^4}{48\pi^2}, \quad (30)$$

which is in accord with the result obtained in Ref. [13] using dimensional reduction.

The diagrams that contribute with a logarithmic correction to $\Gamma^{(6)[2]}$ are shown in Fig. 5; each one of them is superficially logarithmically divergent. Concerning these graphs we make the following comments. Figures 5(a) and 5(d) do not have divergent subgraphs. The other graphs, Figs. 5(b),5(c) and 5(e),5(f) have just one divergent subgraph. However, in all cases the contribution coming from the subgraph vanishes. For Fig. 5(b) this happens after the contraction of indexes whereas for Figs. 5(e) and 5(f) this results from a cancellation between different forests; also, the possible contribution arising from the subgraph of Fig. 5(c) is just Fig. 3(b) whose μ dependent subtractions vanish under symmetric integration. The conclusion is that in each case one has to compute only the contribution of the forest containing just the graph as a whole. The results of these calculations are summarized in Table I. Summing those contributions and using Eq. (20) we obtain

$$\beta_\nu^{[2]}=\frac{7}{24\pi^2}\nu^2-\frac{5}{\pi^2}e^4\nu+\frac{72}{\pi^2}e^8. \quad (31)$$

This result coincides with that of Ref. [13]. To obtain $\beta_\lambda^{[2]}$ we need to compute logarithmic contributions arising from $\Gamma^{(4)[2]}$. The relevant graphs are listed in Fig. 6. Figure 6(a)

TABLE I. Contributions to $\Gamma^{(6)[2]}$ arising from the graphs in Fig. 5.

Diagram	Contribution
5(a)	$-\frac{7i}{24\pi^2}\nu^2\ln\left(\frac{m}{\mu}\right)$
5(b)	$\frac{9i}{4\pi^2}e^4\nu\ln\left(\frac{m}{\mu}\right)$
5(c)	$\frac{15i}{4\pi^2}e^4\nu\ln\left(\frac{m}{\mu}\right)$
5(d)	$-\frac{27i}{\pi^2}e^8\ln\left(\frac{m}{\mu}\right)$
5(e)	$-\frac{9i}{\pi^2}e^8\ln\left(\frac{m}{\mu}\right)$
5(f)	$-\frac{9i}{\pi^2}e^8\ln\left(\frac{m}{\mu}\right)$

has $d(\Gamma)=0$, and does not contain a divergent subdiagram. A straightforward calculation gives

$$\Gamma_A^{(4)[2]}=-\frac{i}{8\pi^2}\lambda\nu\ln\left(\frac{m}{\mu}\right). \quad (32)$$

Figure 6(b) also has $d(\Gamma)=0$, and no divergent subdiagram. It leads to

$$\Gamma_B^{(4)[2]}=\frac{i}{\pi^2}\lambda e^4\ln\left(\frac{m}{\mu}\right). \quad (33)$$

Figure 6(c) has Fig. 3(b) as a divergent subdiagram. We already know that this subgraph does not contribute. The calculation of the forest containing just the overall diagram gives

$$\Gamma_C^{(4)[2]}=\frac{i}{2\pi^2}\lambda e^4\ln\left(\frac{m}{\mu}\right). \quad (34)$$

Taking the above results and using Eq. (21) we obtain

$$\beta_\lambda^{[2]}=\frac{1}{8\pi^2}\lambda\nu-\frac{25}{12\pi^2}e^4\lambda. \quad (35)$$

We turn now to the computation of $\beta_{m^2}^{[2]}$. According to Eq. (19), we still need to compute the graphs shown in Fig. 7. Figure 7(a) is the same as that in Fig. 4(a), now with $p=0$. There is a cancellation between forests. After some manipulations, we obtain

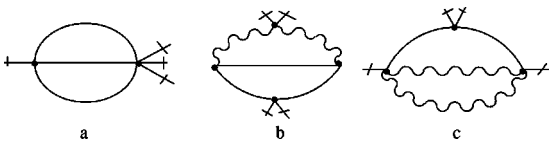

FIG. 6. Contributions to $\Gamma^{(4)}$ at two loop.

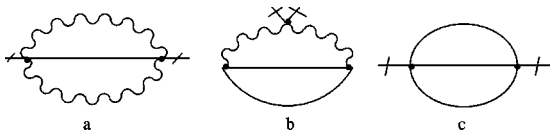

FIG. 7. Contributions to $\Sigma^{[2]}(p=0)$.

$$\Sigma_A^{[2]}(0) = -\frac{1}{8\pi^2} m^2 e^4 \ln\left(\frac{m}{\mu}\right). \tag{36}$$

The diagram in Fig. 7(b) has $d(\Gamma)=2$, and three divergent subgraphs. However, due to cancellations the end result is

$$\Sigma_B^{[2]}(0) = -\frac{1}{4\pi^2} m^2 e^4 \ln\left(\frac{m}{\mu}\right). \tag{37}$$

Finally, Fig. 7(c) has $d(\Gamma)=0$ and no divergent subdiagram. The result is

$$\Sigma_C^{[2]}(0) = \frac{1}{32\pi^2} \lambda^2 \ln\left(\frac{m}{\mu}\right). \tag{38}$$

Collecting the partial results $\Sigma^{[2]}(0)=\Sigma_A^{[2]}(0)+\Sigma_B^{[2]}(0)+\Sigma_C^{[2]}(0)$ from Eqs. (36)–(38), and using Eq. (19) with $n=2$, we obtain

$$\beta_{m^2}^{[2]} = \frac{1}{32\pi^2}\lambda^2 - \frac{2}{3\pi^2} m^2 e^4. \tag{39}$$

For $\lambda=0$ this agrees with the anomalous dimension of the composite operator $\Phi^\dagger\Phi$ as computed in Ref. [13], as it should.

To discuss the fixed points structure of the model we introduce a dimensionless coupling by $\lambda \equiv \hat{\lambda}\mu$. Up to two-loops the *beta* functions are given by Eq. (31),

$$\beta_{\hat{\lambda}} = -\hat{\lambda} + \frac{1}{8\pi^2}\hat{\lambda}\nu - \frac{25}{12\pi^2} e^4 \hat{\lambda}, \tag{40}$$

and by Eq. (39) with λ replaced by $\mu\hat{\lambda}$. If m and $\hat{\lambda}$ are zero there is no induction of $\Phi^\dagger\Phi$ and $(\Phi^\dagger\Phi)^2$ counterterms as $\beta_{\hat{\lambda}}$ and β_{m^2} both vanish. The case $e=0$ has been analyzed in the literature [18] unveiling an interesting tricritical behavior. Near the trivial fixed point, for small momenta the running couplings $\hat{\lambda}_{ef}$ and ν_{ef} are driven away and approach the origin, respectively. For large momenta the opposite happens. As mentioned in Ref. [13], for $e\neq 0$ there are no other fixed points since β_ν never vanishes in the perturbative region. In addition, the behavior of $\hat{\lambda}_{ef}$ near the origin is not sensible to the introduction of e.

III. CONCLUSIONS

In this paper we have shown that it is possible to define a consistent, gauge invariant subtraction scheme with a soft behavior in the infrared regime (soft BPHZ) for an Abelian Chern-Simons theory coupled to a massive scalar matter with $\lambda(\Phi^\dagger\Phi)^2$ and $\nu(\Phi^\dagger\Phi)^3$ self-interactions. Within the soft BPHZ approach we circumvent the problems associated with the analytical continuation of the Levi-Civita tensor. Hence, there is no need to deal with a complicated consistent dimensional regularization and neither it is necessary to introduce a Maxwell term. However, there is a price for this simplification, since we have to deal with various forests with the consequent increase in the number of Feynman integrals. In the process of calculation, however, we have found that these integrals can be reduced to a few primitive ones. In all cases that we studied we explicitly verified the finiteness of the subtracted integrals.

We have done a two-loop calculation of the renormalization group functions. Analogous models were studied in Refs. [6,11,13,19]. Our $\gamma_\Phi^{[2]}$ agrees with the one computed in Ref. [6] (Abelian case). We found only a qualitative agreement with Ref. [11] but a more close comparison seems infeasible due to the lack of details in Ref. [11].

The comparison with the renormalization group functions computed with the consistent dimensional regularization scheme in Ref. [19] is more difficult. Indeed, the RG functions computed in Ref. [19] contain divergent contributions in the pure CS limit (no Maxwell term) that are absent in our approach. However there is some partial agreement between our results and the finite parts of $\beta_\lambda^{[2]}$, $\beta_{m^2}^{[2]}$, and $\beta_\nu^{[2]}$ of Ref. [19] (some coefficients of the expansion of these functions are identical to ours). Our result for $\gamma_\Phi^{[2]}$ is entirely different from that in Ref. [19] as those authors claim that $\gamma_\Phi^{[2]}=0$. The discrepancy could in principle be attributed to the use of different renormalization schemes. The use of an extra regularization represented by a Maxwell term for the A^μ field brings additional complications in their proposal as some of the coefficients of the renormalization group functions become singular as the regularization is removed. A more careful analysis of this method is still lacking.

Although our main interest resides in the pure CS model, we would like to make a few comments on the possible changes in our scheme if a Maxwell term $-(a/4)F^{\mu\nu}F_{\mu\nu}$ is added to Eq. (1). The propagator for the gauge field is then modified to

$$\Delta_{\mu\nu}(p) = \epsilon_{\mu\nu\rho}\frac{p^\rho}{p^2-M^2} + \frac{1}{1-a^2p^2}(a^2\epsilon_{\mu\nu\rho}p^\rho + ia\mathcal{T}_{\mu\nu}), \tag{41}$$

where M is the auxiliary massive parameter as introduced in Eq. (4) and $\mathcal{T}_{\mu\nu}$ is the transversal projector defined below Eq. (10). Observe that a has dimension -1 in units of mass. The degree of superficial divergence is now given by

$$d(\Gamma) = 3 - \frac{1}{2}N_A - \frac{1}{2}N_\phi - V_{\phi^4} - V_{A^2\phi^2} - \frac{1}{2}V_{A\phi^*\partial\phi}, \tag{42}$$

where $V_{\mathcal{O}}$ denotes the number of vertices associated to $\mathcal{O}$ in Γ. With this new power counting many graphs previously divergent turn out to be convergent. In this situation, the following possibilities can be envisaged.

(i) Instead of Eq. (2) one adopts Eq. (42) to define the graphs to be subtracted. The outcome is a well defined theory as far as a, the coefficient of the Maxwell term, is kept nonvanishing. The result, however, is not analytic in a and the limit $a\to 0$ does not exist.

(ii) One uses the old power counting and the parameter a is not changed in the subtraction terms. This means that many graphs will be oversubtracted. However, such subtractions are needed if, as $a\to 0$, one wants to recover the pure CS model studied in this paper.

Finite one-loop renormalization constants for the non-Abelian CS theory with fermionic matter were computed in Ref. [20] using consistent dimensional renormalization and found to be different from the ones obtained from the dimensional reduction prescription. It should be noticed that, as remarked in Ref. [21], even for finite theories there could exist different families of BRST invariant regularizations leading to distinct results for the one-loop radiative corrections. This may imply that in spite of the numerical differences there are no physical inconsistencies between the two approaches.

The model studied in Ref. [13] is a particular case of Eq. (1), with $\lambda=m^2=0$. We found a complete agreement in $\gamma_\Phi^{[2]}$, $\beta_\nu^{[2]}$, and $\gamma_{\Phi^\dagger\Phi}^{[2]}$. Our results show that the dimensional reduction method is consistent at two-loops, at least, as far as the computation of the RG functions is concerned.

ACKNOWLEDGMENTS

This work was partially supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). L.C.A. would like to thank the Mathematical Physics Department for their hospitality.

APPENDIX

In this appendix we shall present details of the calculation of the contribution of Fig. 4(c) to $\gamma_\Phi^{[2]}$. The graph (Γ) is logarithmically divergent and has just one divergent subgraph which is the same as the one associated to the $\pi^B_{\rho\lambda}$ contribution to the CS vacuum polarization. We will denote this subgraph by γ. There are four forests: $\varnothing$ (the empty forest), Γ, γ, and $\{\Gamma,\gamma\}$. Thus, the application of the *forest formula* to Eq. (28) leads to

$$\Sigma_C^{[2]}(p)=16ie^4p^\mu p^\nu\int[dk][dq]\{q^\alpha q^\beta\Delta_{\mu\alpha}(k)\Delta_{\beta\nu}(k)\times[\Delta(k-p,m)\Delta(q,m)\Delta(q-k,m)-\Delta(k,\mu)\Delta(q,\mu)\Delta(q-k,\mu)]-q^\alpha q^\beta\Delta_{\mu\alpha}(k)\Delta_{\beta\nu}(k)\Delta(k-p,m)[\Delta^2(q,0)-ik\cdot q\Delta^3(q,\mu)]+q^\alpha q^\beta\Delta_{\mu\alpha}(k)\Delta_{\beta\nu}(k)\Delta(k,\mu)\times[\Delta^2(q,0)-ik\cdot q\Delta^3(q,\mu)]\}. \tag{A1}$$

The divergent parts of the terms containing the $k\cdot q$ factor cancel between themselves; the finite parts associated to them are odd and vanish under symmetrical integration. We can therefore rewrite the above expression as

$$\Sigma_C^{[2]}(p)=16e^4p^\mu p^\nu\epsilon_{\mu\alpha\lambda}\epsilon_{\beta\nu\sigma}\int[dk][dq]k^\lambda k^\sigma\times\left\{\frac{1}{[(k-p)^2-m^2][q^2-m^2][(q-k)^2-m^2]}-\frac{1}{[k^2-\mu^2][q^2-\mu^2][(q-k)^2-\mu^2]}-\frac{1}{[(k-p)^2-m^2](q^2)^2}+\frac{1}{[k^2-\mu^2](q^2)^2}\right\}. \tag{A2}$$

The contribution to $\gamma_\Phi^{[2]}$ arising from the above integral can be most easily computed if one employs an intermediate auxiliary regularization. Adopting dimensional regularization we arrive at

$$\gamma_{\Phi C}^{[2]}=-16e^4[X_1-X_2], \tag{A3}$$

where

$$X_1=\int[dk][dq]\mu\frac{\partial}{\partial\mu}\frac{q^2}{k^2(k^2\mu^2)(q^2-\mu^2)[(k+q)^2-\mu^2]}, \tag{A4}$$

$$X_2=\int[dk][dq]\mu\times\frac{\partial}{\partial\mu}\frac{(k\cdot q)^2}{(k^2)^2(k^2-\mu^2)(q^2-\mu^2)[(k+q)^2-\mu^2]} \tag{A5}$$

which, after some simple manipulations, can be calculated with the help of the results (22) and (23). We obtain

$$X_1(\mu)=\frac{1}{960\pi^2}, \tag{A6}$$

$$X_2(\mu)=-\frac{1}{640\pi^2}, \tag{A7}$$

from which we obtain the result quoted in the text.

[1] J. Schonfeld, Nucl. Phys. **B185**, 157 (1981).
[2] S. Deser, R. Jackiw, and S. Templeton, Phys. Rev. Lett. **48**, 975 (1982); Ann. Phys. (N.Y.) **140**, 372 (1982).
[3] L. Alvarez-Gaumè, J. Labastida, and A. Ramallo, Nucl. Phys. **B334**, 103 (1990).
[4] M. Asorey and F. Falceto, Phys. Lett. B **241**, 31 (1990).
[5] B. M. Pimentel, A. T. Suzuki, and J. L. Tomazelli, Int. J. Mod. Phys. A **7**, 5307 (1992).

[6] G. W. Semenoff, P. Sodano, and Y. S. Wu, Phys. Rev. Lett. **62**, 715 (1989); W. Chen, G. W. Semenoff, and Y. S. Wu, Mod. Phys. Lett. A **5**, 1833 (1990); W. Chen, Phys. Lett. B **251**, 415 (1990); W. Chen, G. W. Semenoff, and Y. S. Wu, Phys. Rev. D **46**, 5521 (1992).

[7] J. H. León, C. P. Martin, and F. Ruiz Ruiz, Phys. Lett. B **355**, 531 (1995); Nucl. Phys. **B436**, 545 (1995); M. Asorey and F. Falceto, Phys. Rev. D **54**, 5290 (1996); T. D. Bakeyev and A. A. Slavnov, Mod. Phys. Lett. A **11**, 1539 (1996).

[8] R. Delbourgo and A. B. Waites, Phys. Lett. B **300**, 241 (1993).

[9] C. P. Martin, Phys. Lett. B **241**, 513 (1990); G. Giavarini, C. P. Martin, and F. Ruiz Ruiz, Nucl. Phys. **B381**, 222 (1992).

[10] P. Breitenlohner and D. Maison, Commun. Math. Phys. **52**, 11 (1977).

[11] L. V. Avdeev, G. V. Grigoryev, and D. I. Kazakov, Nucl. Phys. **B382**, 561 (1992).

[12] V. S. Alves, M. Gomes, S. V. L. Pinheiro, and A. J. da Silva, Phys. Rev. D **60**, 027701 (1999); **59**, 045002 (1999).

[13] V. S. Alves, M. Gomes, S. V. L. Pinheiro, and A. J. da Silva, Phys. Rev. D **61**, 065003 (2000).

[14] W. Zimmermann, in *Lectures on Elementary Particles and Quantum Field Theory*, 1970 Brandeis Summer Institute in Theoretical Physics, edited by S. Deser, M. Grisaru, and H. Pendleton (MIT Press, Cambridge, 1970), Vol. 1.; J. H. Lowenstein, in *Renormalization Theory*, Proceedings of the International School ''Ettore Majorana,'' edited by G. Velo and A. S. Wightman (Reidel, Dordrecht, 1976).

[15] M. Gomes and B. Schroer, Phys. Rev. D **10**, 3525 (1974); J. H. Lowenstein, Commun. Math. Phys. **47**, 53 (1976); J. H. Lowenstein and P. K. Mitter, Ann. Phys. (N.Y.) **105**, 138 (1977).

[16] S. Weinberg, Phys. Rev. D **8**, 3497 (1973); G. 't Hooft, Nucl. Phys. **B61**, 455 (1973).

[17] S. Coleman and B. Hill, Phys. Lett. **159B**, 184 (1985).

[18] C. A. Aragão de Carvalho, Nucl. Phys. **B119**, 401 (1977); I. D. Lawrie and S. Sarbach, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. L. Lebowitz (Academic, New York, 1984), Vol. 9.

[19] P.-N. Tan, B. Tekin, and Y. Hosotani, Nucl. Phys. **B502**, 483 (1997); Phys. Lett. B **388**, 611 (1996).

[20] M. Chaichian and W. F. Chen, Phys. Rev. D **58**, 125004 (1998).

[21] M. Asorey, F. Falceto, J. L. López, and G. Luzón, Nucl. Phys. **B429**, 344 (1994); G. Giavarini, C. P. Martin, and F. Ruiz Ruiz, Phys. Lett. B **332**, 345 (1994); K. Nittoh and T. Ebihara, Mod. Phys. Lett. A **13**, 2231 (1998).