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Traversable wormhole

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A class of static Lorentzian wormholes with arbitrarily wide throats is presented in which the source of the weak energy condition violations required by the Einstein equations is the vacuum stress-energy tensor of the neutrino, electromagnetic, or massless scalar field.

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I. INTRODUCTION

A wormhole is a ''tunnel'' connecting a part of the Universe with another part sufficiently remote, or even unconnected (but for the tunnel) with the former one. A wormhole through which a signal can be transmitted is called traversable.[1] Initially traversable wormholes (TWs) were presented just as a funny tool for teaching general relativity [1], but soon it became clear that they play a large role in at least two (allied) fields each of considerable interest.

Time machines. It is (or was before Ref. [1]) popular opinion that time machines are impossible. Intensive ten-year discussion showed that at present this is just a matter of belief—decisive arguments were found neither for nor against time machines. If, however, TWs exist the idea of chronology protection becomes almost untenable.

''Faster-than-light'' travel. As was shown in Ref. [2] local causality does not prevent one from modifying the metric of one's world so that to return from a trip sooner than a photon (in the unmodified world) would have done it. Much as with time machines wormholes are not *necessary* for faster-than-light travel, but it seems to be much more feasible if TWs exist.

At present we do not know whether TWs exist in nature. On the one hand, it is not impossible that wormholes are most common things. In the absence of (restrictive enough) observational bounds (see Sec. VII) we may well speculate that they are 10 (or, say, 10^6) times as abundant as stars. On the other hand, the possibility of their existence has been doubted on theoretical grounds. The point is that to be traversable a wormhole must satisfy at least the following requirements: (I) It must be sufficiently long-lived to be passed by a causal curve; (II) it must be *macroscopic*. Wormholes are often discussed (see Ref. [3], for example) with the radius of the throat of order of the Plank length. Such a wormhole might be observable (in particular, owing to its gravitational field), but it is not obvious (and it is a long way from being obvious, since the analysis would inevitably involve quantum gravity) that any signal at all can be transmitted through such a tunnel.

Problems arise if we want a TW to be a solution of the Einstein equations since the geometrical thus far condition (I) becomes then a restriction on the properties of the matter filling the TW. The matter obeying this restriction is called *exotic* [4]. Strong arguments suggest that the exotic matter must violate the weak energy condition (WEC) [1] and most likely (see the next section, though) the averaged null energy condition (ANEC) [5]. Therefore it is generally believed that the realistic classical matter cannot be exotic. A possible way out [1] is to invoke quantum effects to maintain a wormhole. In particular, in semiclassical gravity the contribution of a quantum field to the right-hand side of the Einstein equations is commonly taken (see Ref. [6] for discussion and references) to be $T^Q_{ij} \equiv \langle \Psi | \hat{T}^Q_{ij} | \Psi \rangle^{\mathrm{ren}}$, where $|\Psi\rangle$ is the quantum state of the field and $\hat{T}^Q_{ij}$ is an operator depending on the background metric. It is known that T^Q_{ij} violates the Weak energy condition in some situations. Wormholes are a most suitable place for seeking such situations and so an elegant idea appeared [7] to look for such a wormhole that its metric g is just the solution of the Einstein equations with $T^Q_{ij}[g]$ as a source (a ''self-maintained wormhole''). A wormhole of this type was found, indeed [3].[2] However, its throat turned out to be of the Planck scale, i.e., nontraversable. This result coupled with the arguments from Refs. [6,9] may give the impression that conditions (I) and (II) are incompatible—the quantum effects can produce the exotic matter but only in microscopic amounts insufficient for supporting a macroscopic wormhole. In the present paper we argue that this is not the case: we present such a class of static wormholes with arbitrarily large throats that all necessary violations of the WEC (and the ANEC) are produced by the vacuum fluctuations of the neutrino, electromagnetic, or massless conformally coupled scalar fields.

II. GEOMETRY OF THE WORMHOLE

The ''definition'' of a wormhole given in the introduction is too vague for our purposes and now we have to make it somewhat more specific (surprisingly, there is no commonly accepted rigorous definition of a wormhole yet).

The space around us is more or less flat. The easiest way to reconcile this with the presumed existence of a wormhole is to require that the gravitational field of the wormhole falls off with distance (*no matter how fast*) and that we just live sufficiently far from it. It is convenient to incorporate this requirement into the definition of a wormhole [10] and to

[1]This definition is slightly less restrictive than that in Ref. [1].

[2]Though the numerical method applied there is disputable [8].

formulate it as follows: a wormhole is a spacetime containing two increasingly flat regions (note that by a wormhole the *whole* spacetime is meant now, not only the tunnel).

Remark 1. Wormhole-type objects such as those considered in Ref. [11] are not wormholes in this sense. Nor, strictly speaking, are the spacetimes with tunnels connecting distant regions of a ''single'' universe.

Remark 2. In addition to being flat the real space is more or less empty. So it seems reasonable to require that the increasingly flat regions be also ''increasingly empty.'' The specific formulation as applied to our case will be given below [see item (iii) in Sec. IV].

To see what is meant by ''increasingly flat'' consider the Morris-Thorne (MT) wormhole [1] which has the metric

$$g_{\mathrm{MT}}:\ ds^2=-e^{2\Phi(r)}dt^2+(1-b(r)/r)^{-1}dr^2 + r^2(d\theta^2+\sin^2\theta d\varphi^2),$$

where Φ, b/r and all their derivatives$\to 0$ at $r\to\pm\infty$. When $r\to\pm\infty$ this metric tends (component-wise) to the Minkowski metric, all curvature invariants *and* all gravitational forces [as measured by their action on a test particle in a system resting with respect to the system (t,r,θ,φ)] tend to zero. So, it seems that whatever experiment one performs in a cube $Q_a\equiv(x_0^i<x^i<x_0^i+a,\ x^i=t,r,r\theta,r\varphi\sin\theta)$ the difference between the results of this experiment and that in the case of $\Phi=b=0$ (the flat space) will tend to zero as r_0 grows (with a constant). These properties justify the name ''increasingly flat'' for the MT wormhole. As for the meaning of this term in the general case we note the following:

(1) We discriminate ''increasingly flat'' from ''asymptotically flat'' if the latter is taken to mean ''asymptotically simple and empty'' [12]. Among other things, asymptotical flatness implies some restrictions (apparently unjustified in the case at hand) on *how* a spacetime becomes flat. Consider, for example, the metric

$$g_F:\ ds^2=[1+F(r)]^2[-dt^2+dr^2+r^2(d\theta^2+\sin^2\theta d\varphi^2)].$$

If $F=1/\sqrt{r}$ at large r, g_F becomes there just a variety of g_{MT}. So, we wish to call this spacetime increasingly flat. However, it is not asymptotically flat[3] (it is even not asymptotically simple) since $\Omega\notin C^2(\overline{M})$ (see Ref. [12] for notation). We could relax the requirements on smoothness of Ω so that to incorporate this case, but if we recognize that spacetime as increasingly flat why should not we do so with, say, $F=\sin r/r$. But in this latter case even $\Omega\notin C^1(\overline{M})$ and so the condition $\nabla\Omega|_{\partial M}\neq 0$ fails.

Note that the proof in Ref. [5] of the topology censorship theorem relies on asymptotical flatness of the spacetime (specifically, on the structure of its conformal infinity) and so a wormhole is conceivable for which this theorem is inapplicable.

(2) A criterion for increasing flatness must not involve increasingly large portions of the spacetime (e.g., the edge a of the above mentioned cube must not grow with r). Even increasingly weak gravitational forces, when integrated over increasingly large regions, can give a nondecreasing result.

(3) On the other hand, it is hard, if possible, to formulate a relevant point-wise criterion. Given a point it is easy to say whether or not a space is *flat* there, but in the pseudoriemannian case it is unclear what space can be called ''nearly flat in the point.'' Examples are known [13] when *all* curvature scalars vanish in a point P even though the spacetime is not flat in it. Moreover, for any given ε, $\mathcal{E}$ two orthonormal bases can be found in this point, such that all components of the Riemann tensor are bounded by ϵ in one of them, while in the other some of them are greater than $\mathcal{E}$.

The spacetime (M,g) considered in the present article is $\mathbb{R}^2\times S^2$ with the metric

$$g:\ ds^2=\Omega^2(\xi)[-d\tau^2+d\xi^2+K^2(\xi)(d\theta^2+\sin^2\theta d\varphi^2)], \tag{1}$$

where Ω,K are smooth positive even functions. When Ω behaves appropriately at $\xi\to\infty$ the spacetime (M,g) is a wormhole. To see this consider the following specific case:

$$K(\xi>\Xi)=K_0,\quad \Omega(\xi>\Xi)=\Omega_0e^{B\xi} \tag{2}$$

(Ξ is a positive constant). Introduce the coordinates r,t

$$r\equiv\Omega_0B^{-1}e^{B|\xi|}=B^{-1}\Omega,\ \ t\equiv B\tau r$$

in the neighborhood $|t|<T$ [T is an arbitrary constant smaller than $r(\Xi)$] of the surface [$\tau=t=0$, $r>r(\Xi)$]. In these new coordinates the metric (*within the neighborhood*) takes the form

$$ds^2=-dt^2+2t/r\,dt\,dr+[1-(t/r)^2]dr^2+(BKr)^2 \times(d\theta^2+\sin^2\theta d\varphi^2).$$

It exhibits now all the nice properties (as the cube Q_a moves to larger r, the metric inside it (written in appropriate coordinates) uniformly tends component-wise to the Minkowski metric, etc.) that inspired us to call the MT metric increasingly flat. And since the metric (1) is static the same is true for a vicinity of any surface ($\tau=$const) foliating M. Therefore, if Eq. (2) holds we consider the whole region $\xi>\Xi$ as increasingly flat and the spacetime (M,g) as a wormhole.

Notation. Below we use circles to mark quantities related to the metric $\mathring{g}\equiv\Omega^{-2}g$ and hats to indicate components of tensors in the normalized coordinate basis.

III. RESTRICTIONS IMPOSED BY THE WEC

As mentioned above the vacuum expectation T^Q of the stress-energy tensor of a quantum field need not obey the WEC. However, for a given metric T^Q is not arbitrary (we can vary only $|\Psi\rangle$). So, the requirement that T^Q be the only source of the WEC violations still imposes (when coupled with the Einstein equations) restrictions on the possible form of Ω. We claim that these restrictions do not prevent the metric from being of the desired type. To prove this we

[3]In contrast, say, to g_F with $F=1/r$.

express these restrictions in the form of inequalities and in the subsequent sections show that they have appropriate solutions even for Ω and K corresponding to a wormhole.

Let us write down the Einstein equations separating out the term T^Q in the total stress-energy tensor and neglecting the interaction between our field and the other matter:

$$\frac{1}{8\pi}G_{ij}=T^Q_{ij}+T^C_{ij}.$$

As we do not require the wormhole to be self-maintained, T^C need not be zero. It should however satisfy the WEC (describing thus the conventional classic matter). So, in an orthonormal basis diagonalizing T^C_{ij} the following inequalities must hold:

$$\frac{1}{8\pi}G_{00}-T^Q_{00}>0,\quad \frac{1}{8\pi}(G_{00}+G_{jj})-(T^Q_{00}+T^Q_{jj})>0,\qquad j=1,2,3. \tag{3}$$

Now let us specify the quantum state $|\Psi\rangle$, which is necessary for finding T^Q. Let $|\mathring{\Psi}\rangle$ be a vacuum state in the (unphysical) spacetime $(M,\mathring{g})$. It does not matter exactly what vacuum we choose, we only require that [in agreement with the symmetries of $(M,\mathring{g})$] $\mathring{T}^Q_{\hat{i}\hat{j}}=\mathrm{diag}(T_0,T_1,T_2,T_2)$ where T_i are some bounded functions.[4] Let us choose $|\Psi\rangle$ to be the state [in the physical spacetime (M,g)] conformally related to $|\mathring{\Psi}\rangle$. Then the following relation holds [14] for the neutrino, electromagnetic (in dimensional regularization), and massless scalar (conformally coupled) fields:

$$\begin{aligned}T^Q_{\hat{i}\hat{j}}=&\ \Omega^{-4}\mathring{T}^Q_{\hat{i}\hat{j}}-8\alpha\left[(C^{a}{}_{\hat{i}b\hat{j}}\ln\Omega)^{;b}_{;a}+\frac{1}{2}R^{ab}C_{a\hat{i}b\hat{j}}\ln\Omega\right]\\&+\beta[(4R^{ab}C_{a\hat{i}b\hat{j}}-2H_{\hat{i}\hat{j}})-\Omega^{-4}(4\mathring{R}^{ab}\mathring{C}_{a\hat{i}b\hat{j}}-2\mathring{H}_{\hat{i}\hat{j}})]\\&-\frac{1}{6}\gamma[I_{\hat{i}\hat{j}}-\Omega^{-4}\mathring{I}_{\hat{i}\hat{j}}],\end{aligned} \tag{4}$$

where

$$H_{ij}\equiv-R^a_iR_{aj}+\frac{2}{3}RR_{ij}+\left(\frac{1}{2}R^b_aR^a_b-\frac{1}{4}R^2\right)g_{ij},$$

$$I_{ij}\equiv 2R_{;ij}-2RR_{ij}+\left(\frac{1}{2}R^2-2R_{;a}{}^{a}\right)g_{ij},$$

and α, β, and γ are constants characterizing the field. We shall restrict them only by requiring that $\gamma>0$, which holds for all the fields listed above.

[4]They are related by the conformal anomaly, but we shall not use this fact.

Substituting Eq. (4) in Eq. (3) and expressing the geometrical quantities in terms of Ω, and K yields[5] (due to the spherical symmetry the inequalities for $\hat{j}=2$ and $\hat{j}=3$ coincide)

$$\frac{1}{8\pi}\Omega^2(K^{-2}-2\mu-\omega^2-3\varkappa^2-4\varkappa\omega-2\nu)-2\gamma[\mu''+\mu'(4\varkappa-\omega)]-T_0-L_1\ln\Omega-P_1>0, \tag{5}$$

$$\frac{1}{4\pi}\Omega^2(\omega^2-\mu-\nu-\varkappa^2)-2\gamma[\mu''+\mu'(2\varkappa-4\omega)+2\mu^2]-(T_0+T_1)+L_2\ln\Omega+P_2>0, \tag{6}$$

$$\frac{1}{8\pi}\Omega^2(K^{-2}-\nu-2\varkappa^2-2\varkappa\omega)-2\gamma\varkappa\mu'-(T_0+T_2)+L_3\ln\Omega-P_3>0, \tag{7}$$

where

$$\varkappa\equiv K'/K,\quad \omega\equiv\Omega'/\Omega,\quad \nu\equiv\varkappa',\quad \mu\equiv\omega',$$

$L_i=L_i(K^{-2},\varkappa,\nu^{(l)})$, and $P_i=P_i(K^{-2},\mu,\omega,\varkappa,\nu^{(l)})$ are some polynomials of their arguments ($\nu^{(l)}$ are the derivatives of ν: $l=0,1,2$). Each term of these polynomials is a product of a constant (α, β, or γ) and a factor (such as $\mu K^{-2},\nu''$, etc.) of dimension ξ^{-4}. It is important in what follows that L_2 and P_2 do not contain the terms proportional to K^{-4} and to μ^2, respectively.

IV. MATHEMATIZATION

Now we are in position to formulate mathematically the physical problem in discussion. Namely, we shall consider the existence of traversable wormholes possible so far as we prove the existence of the functions $\Omega(\xi),K(\xi)$ such that the following is true.

(i) They are smooth, even, positive, and asymptotically $K\sim$const, $\Omega\sim Ae^{|B\xi|}$ [so that the metric (1) describes a wormhole].

(ii) The quantity min(ΩK), which is the radius of the wormhole's throat, is large and thus the wormhole is macroscopic (what should be regarded as ''large'' is a matter of taste; we shall demonstrate that it can be made *arbitrarily* large).

(iii) The functions $T^Q_{\hat{i}\hat{j}}$ defined by Eq. (4) tend to zero when $\xi\to\infty$. This condition is to rule out the situation (conceivable due to the WEC violation) when neither T^Q, nor T^C fall off, they only compensate each other better and better.

[5]We omit the relevant straightforward calculations, since they are very tiresome (the work can be considerably lightened by the use of the software package GRTENSORII [15]).

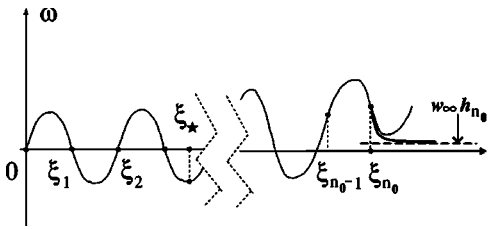

FIG. 1. ξ_n is the abscissa of the nth point of inflection of $\omega(\xi)$. At a point ξ_* near $\xi_{n_0}, \omega(\xi)$ corresponding to y (shown by the thick line) deviates from that corresponding to $\tilde{y}$ (see Sec. VI).

Such a spacetime could hardly describe a wormhole at least a wormhole of the noncosmological size (see remark 2).

(iv) The inequalities (5)–(7) hold.

The remainder of the paper is just the solution of this *mathematical* problem. It is more or less easy to find an Ω with the desired properties near the throat [in fact, just a sinusoid will do for $\omega(\xi)$ here] or at large $|\xi|$ (the asymptotic regions). The hard part is to find Ω satisfying the above conditions over the whole range of ξ *including* the intermediate region, where the wormhole ''flares out'' (see Ref. [1]).

In the next section we consider a particular solution of Eq. (6) on the segment $(-1,1)$. Later, in Sec. VI we shall deform such a solution at large ξ so that to satisfy all the requirements formulated above. To make this deformation possible it is crucial that some fact [see Eq. (42)] takes place in the intermediate region $\xi \sim 1$. So, we prove (this takes up the bulk of the next section) that K can be chosen so that Eq. (42) holds indeed.

Remark 3. The inequalities (5)–(7) contain a few dozen terms each. To handle such formidable expressions we shall, first, combine the initial values $\Omega(0), \Omega''(0)$ into some ϵ_0 and regard ϵ_0 as small parameter (that is to prove anything it will suffice to prove it for sufficiently small ϵ_0). Second, instead of examining $\Omega(\xi)$ we shall mostly consider $y(m)$, where (up to some constant factors) y is $\ln''' \Omega$ and m is $\ln'' \Omega$. These two means lighten the analysis considerably though at the cost of possibility of finding an explicit expression for the thus found $\Omega(\xi)$.

V. THE TUNNEL OF THE WORMHOLE

Before proceeding to examination of the solution of Eq. (6) mentioned above let us introduce some new functions, more convenient in handling inequalities (5)–(7) than Ω, $\varkappa$, etc. Denote by ξ_n the nth zero of μ' (see Fig. 1). For each n such that $\mu_n \equiv \mu(\xi_n) \neq 0$ (for example, this will hold for $n=0$) let

$$h_n \equiv \frac{\sqrt{8\pi\gamma}|\mu_n|}{\Omega(\xi_n)}, \quad \epsilon_n \equiv \frac{8\pi\gamma|\mu_n|}{\Omega^2(\xi_n)}$$

and define the following set of dimensionless functions:

$$w_{(n)} = h_n^{-1}\omega, \quad \lambda_{(n)} \equiv h_n^{-2}(\nu + \varkappa^2), \quad k_{(n)} \equiv h_n^{-1}\varkappa,$$

$$m_{(n)} \equiv h_n^{-2}\epsilon_n\mu, \quad E_{(n)} \equiv (8\pi\gamma)^{-1}h_n^{-2}\epsilon_n^2\Omega^2,$$

$$y_{(n)} \equiv h_n^{-3}\epsilon_n^2\mu'.$$

Here and subsequently we use an index in parentheses (n) to mark functions and an index without parentheses to mark constants, as a rule we shall write z_n for $z_{(n)}(\xi_n)$ (from here on by z we mean ''any of the dimensionless functions above,'' that is $z = w, k, y,$ etc.). Note that $E_{(n)}(\xi) = [\Omega/\Omega(\xi_n)]^2$ and $m_{(n)}(\xi) = \mu/|\mu_n|$ and thus

$$m_n \equiv m_{(n)}(\xi_n) = \pm 1, \quad E_n \equiv E_{(n)}(\xi_n) = 1.$$

Remark 4. All these indexed functions change just by a constant factor when the index changes. Such a great number of like functions will, however, be convenient later, when we consider each segment (ξ_n, ξ_{n+1}) separately. On each such interval we shall use only the functions with the corresponding indexes—$z_{(n)}$ and $z_{(n+1)}$.

In new notation inequality (6) can be rewritten as follows:[6]

$$\begin{aligned} -\epsilon_0 h_0^{-1} y'_{(0)} - Em + \epsilon_0[E(\omega^2 - \lambda) + y_{(0)}(4w - 2k) - 2m^2] \\ - \epsilon_0^3 h_0^{-4}[T_1 - T_0 + L_2(\underline{K^{-2}}, \varkappa, \nu^{(l)}) \ln \Omega \\ + P_2(K^{-2}, \underline{\epsilon_0^{-1} m}, h_0 w, \varkappa, \nu^{(l)})] > 0. \end{aligned} \tag{8}$$

The polynomials P_2, L_2 do not contain the terms generated by the underlined arguments.

Consider the following equation:

$$\begin{aligned} -\epsilon_0 h_0^{-1} y'_{(0)} - Em + \epsilon_0[E(w^2 - \lambda) + y_{(0)}(4w - 2k) - 2m^2] \\ - \epsilon_0^{3/2}\chi_0^2 = 0 \end{aligned} \tag{9a}$$

$$\Omega(0) = \Omega_0, \quad \Omega'(0) = \Omega'''(0) = 0, \quad \Omega''(0) = \Omega_0\mu_0 \neq 0, \tag{9b}$$

where χ_0 is a nonzero constant. Though written in terms of y Eq. (9) is in fact an ordinary differential equation on $\Omega(\xi)$ (with initial conditions chosen so that Ω is a smooth[7] even function). A solution y of Eq. (9a) together with h, ϵ, determines [when the initial data are fixed by Eq. (9b)] Ω via the equation

$$(\ln \Omega)''' = h^3 \epsilon^{-2} y. \tag{10}$$

The left hand side of Eq. (9a) differs from that of Eq. (8) only by that the term in the lower line is replaced with the term $\epsilon_0^{3/2}\chi_0^2$. So, it is clear that if $y(\xi, \epsilon_0)$ satisfies Eq. (9a) and the corresponding m,w are uniformly bounded (we shall see that this is the case), then $y(\xi, \epsilon_0)$ with sufficiently small ϵ_0, (or, more precisely, with

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[6]We omitted indexes (0) in many terms of this inequality, as will often do below. To avoid confusion note that all indexed terms in any expression have the same index (with or without parentheses) unless otherwise is explicitly indicated.

[7]At least until $\Omega \neq 0$, which holds everywhere below.

$$\epsilon_0^{3/2} h_0^{-4} \quad \text{sufficiently small,}$$

since h_0 will also be treated as a small parameter) satisfies Eq. (8) as well, which means that Ω satisfies Eq. (6). This fact will enable us to take a solution of Eq. (9) to be the desired conformal factor at $\xi \in (0,1)$. Our method of extending the latter to larger ξ leans (as was already mentioned) upon some property [see Eq. (42) below] of solutions of Eq. (9) and it is essentially the proof of this property that constitutes the remainder of the section.

First let us change to new coordinates. We want to consider all functions $z_{(n)}$ as functions of $m_{(n)}$. Of course this cannot be done globally (since $dm_{(n)}/d\xi$ vanishes in each ξ_k), and so we shall do it only for the two intervals (ξ_{n-1}, ξ_n) and (ξ_n, ξ_{n+1}) surrounding ξ_n. Thus for each $z_{(n)}$ we define two new functions:

$$^{-}z_{(n)}(m) \equiv z_{(n)}[\xi(m_{(n)})] \quad \text{at} \ \ \xi \in (\xi_{n-1}, \xi_n),$$

$$^{+}z_{(n)}(m) \equiv z_{(n)}[\xi(m_{(n)})] \quad \text{at} \ \ \xi \in (\xi_n, \xi_{n+1}).$$

These two definitions look similar, but recall that $\xi(m_{(n)})$ in the upper line is not the same as in the lower.

Remark 5. We write $^{+}z_{(n)}(m)$ instead of $^{+}z_{(n)}(m_{(n)})$, because we regard $m_{(n)}$ with different n as functions mapping ξ into *the same* target space. This, in particular, allows us to draw pictures similar to Fig. 2 and to write formulas such as Eq. (27).

It is easy to write down E and w as functions of m (we omit the superscripts $^+$ and $^-$ when all terms in an expression have the same superscripts and it does not matter which):

$$E_{(n)}(m) = \exp\left\{ 2\epsilon \int_{m_n}^{m} \frac{w_{(n)} dm}{y_{(n)}} \right\}, \quad w_{(n)}(m) = w_n + \int_{m_n}^{m} \frac{m dm}{y_{(n)}}, \tag{11}$$

where $w_n \equiv w_{(n)}(\xi_n)$. Similarly, for each $\xi \in (\xi_{n-1}, \xi_{n+1})$

$$\xi(m) = \xi_n + \epsilon_n h_n^{-1} \int_{m_n}^{m} \frac{dm}{y(n)}. \tag{12}$$

Since $\epsilon h^{-1} y' = y y_{,m}$ Eq. (9a) can be equivalently rewritten as the following set of equations in $y_{(n)}(m)$:

$$-y y_{,m} - Em + \epsilon[E(w^2 - \lambda) + y(4w - 2k) - 2m^2] - \epsilon^{3/2}\chi^2 = 0, \ \ \forall n, \tag{13}$$

where for brevity we write y for $y_{(n)}(m), \epsilon$ for ϵ_n, etc., and where $\chi_n \equiv \chi_0(\epsilon_n/\epsilon_0)^{3/2}(h_n/h_0)^{-4}$. To make the system (13) complete and equivalent to Eq. (9) we must fix the initial data for $n=0$ so that Eq. (9b) would hold, and for each $n \neq 0$ so that to make the resulting Ω smooth. We shall do it as follows. Consider a point $\xi_\star \in (\xi_n, \xi_{n+1})$ such that $m_n(\xi_\star) = m_{n+1}(\xi_\star) = 0$. Ω and its derivatives in $\xi_\star$ can be written in terms of quantities $^{+}z_{\star n} \equiv {}^{+}z_{(n)}(0)$ as well as in terms of $^{-}z_{\star n+1} \equiv {}^{-}z_{(n+1)}(0)$. Thus the requirement that Ω should be smooth can be presented in the form of the following relations:

$$^{+}E_{\star n} h_n^2 \epsilon_n^{-2} = \frac{\Omega^2(\xi_\star)}{8\pi\gamma} = {}^{-}E_{\star n+1} h_{n+1}^2 \epsilon_{n+1}^{-2}, \tag{14a}$$

$$^{+}w_{\star n} h_n = \omega(\xi_\star) = {}^{-}w_{\star n+1} h_{n+1}, \tag{14b}$$

$$^{+}y_{\star n} h_n^3 \epsilon_n^{-2} = \mu'(\xi_\star) = {}^{-}y_{\star n+1} h_{n+1}^3 \epsilon_{n+1}^{-2}. \tag{14c}$$

Now given initial data for $n=0$, from Eq. (14) we can find them for all other n.

It is easy to solve Eq. (13) for $\epsilon = 0$:

$$y = \vartheta\sqrt{1-m^2}, \quad \text{where} \quad \vartheta \equiv \operatorname{sgn} y \tag{15}$$

[i.e., $y(m)$ is just a semicircle] and

$$w = w_n + \vartheta \int_{m_n}^{m} \frac{m}{\sqrt{1-m^2}} dm = w_n - \vartheta\sqrt{1-m^2}. \tag{16}$$

In what follows, however, we shall be interested in behavior of w at $\xi \sim 1$, where corrections due to nonzero (though small) ϵ may not be small. To find these corrections we shall employ a perturbational scheme.

Let us introduce the function

$$f(m) \equiv \frac{y^2}{1-m^2} - 1. \tag{17}$$

Now Eq. (13) can be rewritten as

$$f = T[f] \equiv \frac{2}{1-m^2} \int_{m_n}^{m} \sum_{i=1}^{4} A_i[f] dm. \tag{18}$$

Here the operators A_i are defined by

$$A_1[\phi] \equiv -m(E[\phi] - 1),$$

$$A_2[\phi] \equiv \epsilon(w^2[\phi]E[\phi] - \lambda[\phi]E[\phi] - 2m^2),$$

$$A_3[\phi] \equiv \epsilon\vartheta\sqrt{(\phi+1)(1-m^2)}(4w[\phi] - 2k[\phi]),$$

$$A_4[\phi] \equiv -\epsilon^{3/2}\chi^2,$$

and

$$E[\phi] \equiv \exp\left\{ 2\epsilon\vartheta \int_{m_n}^{m} \frac{w[\phi]}{\sqrt{(\phi+1)(1-m^2)}} dm \right\}, \tag{19}$$

$$w[\phi] \equiv w_n + \vartheta \int_{m_n}^{m} \frac{m}{\sqrt{(\phi+1)(1-m^2)}} dm. \tag{20}$$

Let $\mathfrak{B}_a$ be the space

$$\{\phi \in C^\infty[a, m_n], \ \|\phi\| \leq 1/2\}, \quad \text{where}$$

$$\|\phi\| = \sup_{[a, m_n]} |\phi|, \ \ a \in (-m_n, m_n). \tag{21}$$

It can be shown that $\mathfrak{B}_a$ is a complete metric space (with respect to the metric induced by the norm $\|\ \|$) and when ϵ is sufficiently small T is a contraction operator in $\mathfrak{B}_a$ with $T(\mathfrak{B}_a)\subset\mathfrak{B}_a$. So, when $\epsilon\to 0$, f uniformly tends to $T[0]$ and thus

$$f=\frac{2}{1-m^2}\int_{m_n}^{m}\sum_{i=1}^{3} B_i dm+o(\epsilon), \tag{22}$$

where B_i are the linear (in ϵ) parts of A_i:

$$B_1=2\epsilon\left[m^2-|m|-\vartheta w_n\left(m \arcsin m-\frac{\pi}{2}|m|\right)\right],$$

$$B_2=\epsilon(w^2[0]-\lambda_n-2m^2),$$

$$B_3=\epsilon\vartheta\sqrt{1-m^2}(4w[0]-2k_n)$$

[as usual $\lambda_n\equiv\lambda_{(n)}(\xi_n)$, $k_n\equiv k_{(n)}(\xi_n)$]. Thus [see Eq. (17)]

$$y=\vartheta\sqrt{1-m^2}+\frac{f}{2}\vartheta\sqrt{1-m^2}+o(\epsilon). \tag{23}$$

It can be proven[8] that when ϵ_n is small ϵ_{n+1} is also small. More specifically [see Eq. (31) below] $\epsilon_{n+1}=\epsilon_n+O(\epsilon_n^2)$. This means that by choosing small ϵ_0 one can make ϵ_n small and Eqs. (23),(22) valid for all n at once[9] (and so we shall sometimes speak of just ''small ϵ'').

An important consequence of Eq. (15) is that

$$m_{n+1}=-m_n. \tag{24}$$

Also

$$\xi_{n+1}=\xi_n-\pi\epsilon h^{-1}\vartheta m_n+o(\epsilon) \tag{25}$$

and hence

$$\vartheta=-m_n \quad \text{for } \xi\in(\xi_n,\xi_{n+1}). \tag{26}$$

Equations (23), (24), and (26) show that as long as ϵ_n remains small a point moving with increasing ξ rotates clockwise in an approximately circle path on the plane (y,m) as depicted in Fig. 2.

Our next concern is the behavior of the quantity w_n when n increases. Let us introduce the symbol δ (for quantities both with and without superscripts):

$$\forall z \quad \delta z\equiv {}^{(-)}z_{n+1}-{}^{(+)}z_n.$$

$\delta E_\star$ can be found from Eqs. (16),(19)

$$\delta E_\star\approx-2\epsilon\vartheta\int_{m_n}^{m_{n+1}}\frac{w[0]}{\sqrt{1-m^2}}dm=-2\epsilon m_n(2-\vartheta\pi w_n), \tag{27}$$

[8]By first taking a in Eq. (21) to be close to $-m_n$ and second noting that on $(-m_n,a)$ by Eq. (13) $|y^{2'}|>1+O(\epsilon)$.

[9]At least as far as we deal with $n<N=O(\epsilon_0^{-1})$.

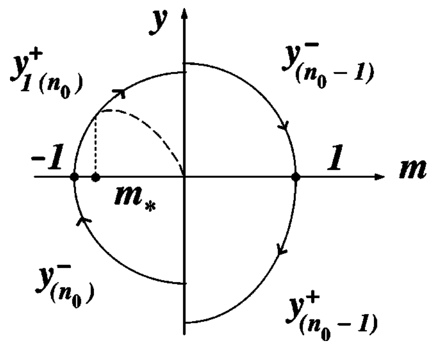

FIG. 2. The arrows are directed in the sense of increasing ξ. The dashed line depicts ${}^{+}y_{(n_0)}$ (see Sec. VI).

where $\approx$ means ''is equal up to terms of order $o(\epsilon)$.'' From Eq. (22)

$$\delta f_\star\approx-2\int_{m_n}^{m_{n+1}}\sum_{i=1} B_i dm$$
$$=-2\epsilon m_n\left[6+2\lambda-\vartheta\left(\frac{3\pi}{2}w_n-\pi k\right)-2w_n^2\right]. \tag{28}$$

Finally, by Eqs. (20) and (23)

$$w_\star\approx w_n-\vartheta-\frac{\vartheta}{2}\int_{m_n}^{0}\frac{f m dm}{\sqrt{1-m^2}}$$
$$\approx w_n-\vartheta-\frac{\vartheta}{2}f_\star+\vartheta\int_{m_n}^{0}\frac{1}{\sqrt{1-m^2}}\sum_{i=1} B_i dm \tag{29}$$

and hence

$$\delta w_\star\approx\delta w_n-\frac{\vartheta}{2}\delta f_\star-\vartheta\int_{m_n}^{m_{n+1}}\frac{1}{\sqrt{1-m^2}}\sum_{i=1} B_i dm. \tag{30}$$

It follows from Eq. (14) that

$$2\frac{\delta h}{h}-2\frac{\delta\epsilon}{\epsilon}+\frac{\delta E_\star}{E_\star}\approx 0, \tag{31a}$$

$$\frac{\delta h}{h}+\frac{\delta w_\star}{w_\star}\approx 0, \tag{31b}$$

$$3\frac{\delta h}{h}-2\frac{\delta\epsilon}{\epsilon}+\frac{\delta y_\star}{y_\star}\approx 0, \tag{31c}$$

whence, in particular,

$$\frac{\delta h}{h}\approx\frac{\delta E_\star}{E_\star}-\frac{\delta y_\star}{y_\star}\approx\delta E_\star-\frac{1}{2}\delta f_\star \tag{32}$$

and

$$\delta w_\star\approx-(w_n-\vartheta)\left(\delta E_\star-\frac{1}{2}\delta f_\star\right). \tag{33}$$

Thus [see Eqs. (27) and (28)]

$$\frac{\delta h}{h}\approx\epsilon m_n\left[2+2\lambda+\vartheta\left(\frac{\pi}{2}w_n+\pi k\right)-2w_n^2\right]. \tag{34}$$

Also combining Eq. (30) with Eq. (33) and taking the integrals we get

$$\delta w_n \approx 2\epsilon m_n(2-\vartheta\pi w_n)(w_n-\vartheta)-w_n\epsilon m_n\Big[6+2\lambda -\vartheta\Big(\frac{3\pi}{2}w_n-\pi k\Big)-2w_n^2\Big]-\epsilon m_n\vartheta\Big[-\Big(\frac{3\pi}{2}+4+\pi\lambda\Big) +2\vartheta(\pi w_n-2k)+\pi w_n^2\Big]. \tag{35}$$

Let us introduce the symbol Δ to describe how a quantity changes in one ''period'' (see Fig. 1):

$$\forall z \quad \Delta z \equiv z_{n+2}-z_n.$$

Clearly [see Eqs. (24) and (26)] $\Delta z=-2\times$(the coefficient at $m_n\vartheta$ in δz). So, by Eqs. (34) and (35) we have

$$\Delta h/h \approx -\epsilon\pi(w_n+2k),$$

$$\Delta w_n \approx -\epsilon\pi(3+2\lambda-2w_nk-3w_n^2),$$

and by Eq. (25)

$$\Delta\xi \approx \epsilon\pi\cdot 2h^{-1},$$

which gives

$$2\Delta h/h_n \approx -h_n(w_n+2k_n)\Delta\xi, \tag{36a}$$

$$2\Delta w_n \approx -h_n(3+2\lambda_n-2w_nk_n-3w_n^2)\Delta\xi. \tag{36b}$$

Define smooth functions $h_s(\xi), w_s(\xi)$ to be the solutions of the system

$$2h_s' = -(w_sh_s^2+2\varkappa h_s), \tag{37a}$$

$$2w_s' = -h_s(3+2\lambda_s-2w_sk_s-3w_s^2), \tag{37b}$$

$$h_s(0)=h_0, \quad w_s(0)=0,$$

where $k_s \equiv h_s^{-1}\varkappa$, $\lambda_s \equiv h_s^{-2}(\nu+\varkappa^2)$, and $\xi\in[0,1]$. As we shall see $h_s\neq 0$ on this interval and so for ϵ tending to zero, h_n uniformly tend to $h_s(\xi_n)$ (and the same for w_n and w_s).

The system (37) can be simplified by rewriting in terms of functions $\kappa = Kh_s$, $\omega_s \equiv w_sh_s$:

$$2\kappa' = -\omega_s\kappa, \tag{38a}$$

$$2\omega_s' = 2\omega_s^2-3K^{-2}\kappa^2+2, \tag{38b}$$

$$\kappa(0)=\kappa_0\in K(0)h_0, \quad \omega_s(0)=0,$$

So far we have not specify K in any way. Let us now fix it. We chose K to be a smooth even function with

$$K=\begin{cases} K_0\cos\xi & \text{at } \xi<1,\\ K_0 & \text{at } \xi>2\end{cases} \tag{39}$$

(K_0 is a positive constant). Then for $\xi<1$, $h_0\to 0$ the solutions of Eq. (38) are

$$\omega_s=\tan\xi+o(1), \quad \kappa=\kappa_0\cos^{1/2}\xi+o(\kappa_0), \tag{40}$$

that is,

$$w_s=h_0^{-1}[\sin\xi\cos^{1/2}\xi+o(1)], \quad h_s=h_0\cos^{-1/2}\xi+o(h_0). \tag{41}$$

Thus, when h_0 and ϵ are sufficiently small $w_s(1)$ becomes greater than 1 and so does w_n for (at least) a few consecutive values of n. It follows then [see Eq. (16)] that such n_0 exists that

$$\xi_{n_0}=1+O(\epsilon), \quad m_{n_0}=-1, \quad {}^+w_{(n_0)}>0. \tag{42}$$

VI. CONSTRUCTION OF THE WORMHOLE

Let $\tilde{y}$ be a function of the kind considered in the previous subsection [that is a solution of Eq. (9) with h_0 and ϵ_0 so small that both Eqs. (6) and (42) are satisfied] and let y be the function defined by

$$y(\xi)\equiv\tilde{y}(\xi) \quad \text{at } \xi\leqslant\xi_{n_0}, \qquad {}^+y_{(n_0)}(m)\equiv\zeta(m)\,{}^+\tilde{y}_{(n_0)}(m). \tag{43}$$

Here ζ is a smooth function subjecting to the following requirements:

$$\zeta \quad \text{is convex}, \quad \zeta(0)=0, \quad \zeta(m<m_*)=1, \tag{44a}$$

and m_* (i.e., the point at which y begins to deviate from $\tilde{y}$, see Fig. 2) satisfies

$$|m_*|<\frac{1}{2}w_*y_* \tag{44b}$$

[for any z we denote ${}^+z_{(n_0)}(m_*)$ briefly by z_* and all ϵ, h, ν, etc., in this section are, in fact, ϵ_{n_0}, h_{n_0}, $\nu_{(n_0)}$, etc.].

We claim that the metric (1) with $\Omega(\xi)$ defined [see Eq. (10)] by y,ϵ_0,h_0 describes a desired wormhole. To prove this we must show (see Sec. IV) that such $\Omega(\xi)$ (A) has appropriate [see conditions (i),(iii)] asymptotic behavior at $\xi\to\infty$, (B) satisfies the system (5)–(7), and (C) provides the large throat to the wormhole.

A. Large ξ

We know from Eq. (42) that $w_*>0$. On the other hand, due to Eq. (44) for any $m\in[m_*,0]$

$$|w(m)-w_*|<\max_{(m_*,m)}|w_{,m}|=(\min_{(m_*,m)}|y/m|)^{-1} <(y_*\min_{(m_*,m)}|\zeta/m|)^{-1}=\left|\frac{y_*}{m_*}\right|^{-1}<\frac{1}{2}w_*. \tag{45}$$

So,

$$w \quad \text{is positive and bounded on} [m_*,0]. \tag{46}$$

By Eq. (12) $\xi\to\infty$ when $m\to 0$ (recall that y is smooth in 0). Also

$$E=\exp\left\{2\epsilon\int_{-1}^{m}\frac{[w_\infty+O(m)]dm}{y}\right\}=c_E e^{2hw_\infty\xi}e^{O(m)}, \tag{47}$$

where c_E, $w_\infty\equiv w(0)$, and c_m (in the next formula) are some positive constants. Thus the resulting metric differs from that considered in Sec. II, only by the conformal factor $e^{O(m)}$. But it is easy to see from Eq. (12) that

$$m=-c_m e^{y_{,m}(0)\epsilon^{-1}h\xi}[1+O(m)]. \tag{48}$$

E is proportional to r^2 and hence comparing Eq. (47) with Eq. (48) we see that, at large r,

$$m\sim -c_E^{-1}c_m r^{y_{,m}(0)\epsilon^{-1}w_\infty^{-1}}.$$

So [recall that $y_{,m}(0)<0$], when ϵ is small enough, the factor differs from 1 by a function falling (at least) exponentially with r and hence Eq. (1) still describes a wormhole.

From Eqs. (16) and (41) it is evident that $\omega(\xi)$ is bounded (say, $|\omega|<2$ when h_0 is small) on $(0,\xi_{n_0})$, and from Eq. (45) it is clear that the same is true for all ξ. The derivatives μ, μ', μ'' of ω obviously are also bounded. It follows [to verify note that the left hand sides of Eqs. (5)–(7) are the left hand sides of Eq. (3) multiplied by Ω^4] that the components of both T^C and T^Q fall off at infinity (at least as Ω^{-2} and $\Omega^{-4}\ln\Omega$, correspondingly). Thus condition (iii) of Sec. IV is fulfilled.

B. The WEC

At $0\leqslant\xi\leqslant\xi_*$ the function y satisfies Eq. (8) [since being equal to $\tilde y$ on this segment it satisfies Eq. (13)]. So, it only remains to check that ${}^+y_{(n_0)}(m>m_*)$ satisfies it too, for which it would suffice [see Eq. (13)] to prove that the inequality

$$\Upsilon[yy_{,m}+Em-\epsilon[E(w^2-\lambda)+y(4w-2k)-2m^2]]\leqslant 0, \tag{49}$$

holds, where we have introduced a new symbol Υ: for any expression $Q(y)$ we denote by $\Upsilon[Q]$ the difference

$$\Upsilon[Q]\equiv Q(y)-Q(\tilde y).$$

It is easy to show that as $m\to m_*+0$

$$\Upsilon[y]=(\zeta-1)y,$$

$$\Upsilon[yy_{,m}]\sim y_*^2\zeta_{,m},$$

$$\Upsilon[w],\Upsilon[E],\Upsilon[\lambda],\Upsilon[k]=o(1-\zeta).$$

These assessments are uniform by ϵ. Thus on some segment $m_*\leqslant m\leqslant m_{**}$ the inequality (49) holds when ϵ is sufficiently small (when ϵ is smaller than some ε).

Further, for sufficiently small $|m|$ (say, $m_{***}<m\leqslant 0$) both $\lambda=0$ and $k=0$. Therefore, if m_{***} is chosen appropriately, the term in the inner brackets in Eq. (49) is positive on $(m_{***},0)$ (since E is positive and both y and m tend to zero while w [by Eq. (46)] and E do not). Hence for any $\epsilon<\varepsilon'$ Eq. (49) holds on $(m_{***},0)$, too.

Finally, for $m\in(m_{**},m_{***})$.

$$\Upsilon[yy_{,m}]=\tilde y^2\zeta_{,m}\zeta-(\zeta^2+1)\tilde y_{,m}\tilde y<0 \quad\text{and}\quad \Upsilon[E]>0. \tag{50}$$

So (again, when ϵ is small enough)

$$m\in(m_{**},m_{***}),\quad \Upsilon[yy_{,m}+Em]<c<0,\quad\text{where}$$
$$c\neq c(\epsilon).$$

Summarizing, when ϵ_0 is sufficiently small inequality (49) holds for any m, and hence [see Eq. (13)] y satisfies Eq. (8).

We have proved that Ω satisfies Eq. (6). Now let us verify that by choosing appropriate K_0, h_0 and ϵ_0 (the two latter still being ''sufficiently small'') the remaining inequalities (5),(7) can be satisfied as well. Indeed,

$$\begin{aligned}\text{LHS}(5)-\text{LHS}(6)=&\frac{1}{8\pi}\Omega^2[K^{-2}-3\omega^2-4\varkappa\omega-\varkappa^2]-T_1\\&-L_4(K^{-2},\varkappa,\nu^{(l)})\ln\Omega-2\gamma yh^3\epsilon^{-2}\\&\times(2\varkappa+3\omega)\\&-P_4(K^{-2},h^2\epsilon^{-1}m,\omega,\varkappa,\nu^{(l)}).\end{aligned} \tag{51}$$

As noted above $\omega(\xi)$ is bounded. So, let us choose K_0 so that

$$K_0^{-2}\gg\varkappa^2,|\nu|,\omega^2 \tag{52}$$

[we increase K_0 leaving $\varkappa$ fixed, so Eq. (52) also means that $K^{-2}\gg\varkappa^2,|\nu|$]. This enables us to neglect all terms in the brackets in Eq. (51) but the first. What thus remains of the first term $[(1/8\pi)\Omega^2K^{-2}]$ grows as ϵ^{-2} and hence we can neglect the two next terms.

The two last terms of Eq. (51) contain ϵ^{-2} but only in combination with the factor h^3. So, for small enough h_0 they also can be neglected. Thus

$$\text{LHS}(5)-\text{LHS}(6)>0.$$

In the same manner it can be proved that

$$\text{LHS}(7)>0.$$

C. The width of the throat

Three specifiable parameters were used in constructing the wormhole—ϵ_0, h_0, and K_0. All we required of them so far is that

$$K_0^{-1}\gg\varkappa,|\nu|^{1/2},\quad h_0,\epsilon_0,\epsilon_0^{3/2}h_0^{-4}\quad\text{be sufficiently small.} \tag{53}$$

Clearly these conditions can be easily satisfied at once by choosing K_0 appropriately small to satisfy the first one, putting, say,

$$\epsilon_0 = h_0^3$$

and finally choosing h_0 appropriately small to satisfy the remaining. Obviously for any R_0 without spoiling this procedure we can add the requirement

$$K_0\Omega_0 = \sqrt{8\pi\gamma}K_0 h_0^{-2} > R_0$$

thus making the radius of the throat, arbitrarily large.

VII. CONCLUSION

In this paper we have considered the class of traversable wormholes constituted by the space-times (1) with Ω and K subject to condition (i) of Sec. IV. In the total stress-energy tensor of the matter filling a wormhole we separated out the contribution of the conformal (neutrino, electromagnetic, or massless scalar) quantum field T^Q so that the Einstein equations took the form $T^C = 1/(8\pi)G - T^Q$. That enabled us to express $T^C \equiv T_{\text{total}} - T^Q$ in terms of Ω and K (T^Q can be expressed so due to Page's formula [see Eq. (4)] and to the fact that for macroscopic wormholes (large Ω) the nongeometric term $\Omega^{-4}\mathring{T}^Q$ can be neglected). Thus we were able to reformulate (in Sec. IV) the physical problem under study (existence of traversable wormholes with T^C obeying the Weak energy condition) as a mathematical problem [existence of Ω and K satisfying the set of conditions (i)–(iv)]. In Secs. V and VI we analyzed this problem and proved that the desired solutions exist. Thus we conclude that (at the moment) there are no theoretical grounds to believe that static macroscopic wormholes are impossible.

Regarding experimental tests, the situation is not hopeless. Existence of macroscopic wormholes (born, say, in the big bang era) can lead to observable effects. One such effect is gravitational microlensing of background bright sources (say, quasars). Wormhole microlensing can differ considerably from microlensing related to ordinary compact objects (stars) due to the difference in their gravitational fields [16]. Some bounds on possible abundance of the wormholes has already been obtained in Ref. [17] by analyzing from that angle the microlensing experimental data. These bounds, however, are highly model-dependent as is pointed out in Ref. [16]. In particular, the gravitational field of a wormhole is assumed to be that of a clump of exotic matter with a stellar-scale negative mass (that is just the Schwarzschild metric with $M \sim -M_\odot$). This does not hold, for example, for the wormhole considered in this paper.

Yet another observable (in principle) effect is brought about by the fact that due to certain mechanisms (see Ref. [19], Sec. 18, for details and references) a typical wormhole (this time we mean a tunnel connecting distant regions of "the same" universe) is inclined to evolve towards formation of a time machine. Whether the time machine will actually appear, or not (which is an open question) such evolution inevitably gives rise to some "dangerous null geodesics." These geodesics are the worldlines of the photons that pass through the wormhole infinitely many times (within a finite period of time) before the wormhole converts into a time machine. A real photon (it can be, say, a relic photon that happened to fly into the wormhole) will sooner or later come off the dangerous trajectory and miss the inlet mouth of the wormhole, but by this moment its energy will increase (each time the photon passes through the tunnel it experiences some blueshift) [18]. The closer its trajectory was to a dangerous null geodesic the greater is the increase in its energy. Thus a wormhole at some stage of its evolution can generate a well-collimated beam of high-energy photons. If such a beam fall on the Earth we shall observe a flash (γ-ray burst?).

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