

Bulk standard model in the Randall-Sundrum background

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(Received 27 December 1999; published 26 September 2000)

We discuss issues in an attempt to put the standard model (SM) in five-dimensional anti-de Sitter spacetime compactified on S^1/Z_2 . The recently proposed approach to the gauge hierarchy problem by using this background geometry, with the SM confined on a boundary, is extended to a situation where (some of) the SM particles reside in the five-dimensional bulk. In particular, we find a localization of zero modes of bulk fermions near the boundary with a negative tension. Unlike the compactification with the flat metric, these fermion zero modes couple to Kaluza-Klein (KK) excitations of the SM gauge bosons. Interestingly, only low-lying modes of such KK gauge bosons have non-negligible couplings. Current electroweak precision data give a constraint that the first KK mode be heavier than 9 TeV. We also argue that at least the Higgs field should be confined on the brane to utilize the Randall-Sundrum background as a solution to the gauge hierarchy.

PACS number(s): 04.50.+h, 11.10.Kk, 12.60.-i

I. INTRODUCTION

There have recently been new proposals to the gauge hierarchy problem by using geometry of extra dimension(s). The first of such proposals in Ref. [1] was that extra dimensions with large radii can account for the weakness of the gravitational interactions in four dimensions, even if the fundamental scale is close to the electroweak scale (see also Refs. [2,3] for earlier attempts).

More recently Randall and Sundrum [4,5] proposed another approach to the gauge hierarchy by utilizing a warped extra dimension. In this approach, the spacetime is five dimensional, with one extra dimension compactified on S^1/Z_2 . The metric in the Randall-Sundrum (RS) model is

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad (1)$$

where $y = x^5$ is a coordinate of the fifth dimension with identifications $y \sim y + 2\pi r_c$ and $y \sim -y$, and $\sigma(y) = k|y|$ with a curvature scale k determined by the negative cosmological constant $\Lambda < 0$ in the five-dimensional bulk. At each boundary $y = y_i$ ($y_0 = 0$ and $y_1 = \pi r_c$), there locates a set of branes, whose tension (vacuum energy) V_i has to be fine tuned to realize four-dimensional Poincaré invariance:

$$k^2 \equiv \frac{-\Lambda}{24M_{5d}^3}, \quad \frac{V_0}{24M_{5d}^3} = k = \frac{-V_1}{24M_{5d}^3}. \quad (2)$$

It was then argued that the Planck mass M_{pl} in the effective four-dimensional theory is related to the ‘‘fundamental’’ scale M_{5d} in five dimensions by

$$M_{\text{pl}}^2 = \frac{M_{5d}^3}{k} (1 - e^{-2\pi k r_c}). \quad (3)$$

In the following we assume that both M_{5d} and k are of the order M_{pl} (with $k \lesssim M_{5d}$).

The warp factor $e^{-\sigma(y)}$ represents an energy scale of physics phenomena at the position y as measured by the four-dimensional flat metric. Thus the electroweak scale is naturally realized on the distant brane at $y = \pi r_c$, with $V_1 < 0$ if one appropriately adjusts the length of the extra dimension to get $k e^{-\pi k r_c} \sim 100\text{--}1000$ GeV. In fact, in the proposal of Ref. [4], all the standard model (SM) particles are assumed to be confined on this brane.

Various aspects of this model and its extensions [6,7] have been studied in the literature [8–12]. Among other things, Goldberger and Wise pointed out in Ref. [13] that the physics scale of a scalar field is characterized by the warp factor at the distant brane, even if it resides in the whole bulk. This leads one to imagine that the Higgs field can naturally be embedded in the bulk of the five-dimensional spacetime. Furthermore the authors of Refs. [14,15] considered the gauge bosons in the bulk while the leptons and quarks are on the brane.

In this paper, we would like to pursue this line further, and in particular consider a situation that fermions as well as the gauge bosons reside in the bulk. We will show in Sec. II that zero modes of the bulk fermions, which we identify as quarks and leptons in the SM, are localized near the brane at $y = \pi r_c$. This explains why the RS solution to the gauge hierarchy problem applies also for the bulk SM even if we are not assuming from the start that the SM fields are confined on ‘‘our’’ brane; put differently, the gravity is automatically weak for the matter fields in the bulk SM. It turns out, however, that such fermion zero modes couple to Kaluza-Klein (KK) modes of the SM gauge bosons. Thus the theory is severely constrained by the electroweak measurement because the exchange of the KK modes generates four

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Fermi type interactions as we will describe in Sec. IV. This is in contrast to the case with the flat metric for the extra dimension, where the KK modes of the gauge bosons decouple from the zero mode fermions at the tree level [16].

Finally in Sec. V, we will discuss the Higgs mechanism and how the gauge bosons and the fermions acquire masses. We will mainly examine the simplest case in which the Higgs field also lives in the bulk and develops a *constant* vacuum expectation value (VEV). Then, as is shown in the Appendix, the gauge boson masses naturally become of the order of the energy scale of our brane, which is forced to be much higher than the weak scale by the constraint from the current precision experiments, unless we make an extreme fine tuning for the Higgs boson mass. In this case the gauge hierarchy problem would be back, and thus the bulk Higgs case should be virtually excluded. This leaves the case where the Higgs is confined on our brane.

II. BULK FERMION AND LOCALIZATION OF ZERO MODE

The five-dimensional Lagrangian for a free massless fermion $\Psi(x, y)$ can be written as¹

$$e^{-1} \mathcal{L}_{\text{fermion}} = \bar{\Psi} i \Gamma_{\underline{A}}^A \left(\partial_A + \frac{1}{8} \omega_A^{BC} [\Gamma_{\underline{B}}, \Gamma_{\underline{C}}] \right) \Psi, \quad (4)$$

where e_A^A is the inverse of the fünfbein, and the gamma matrices in five dimensions are given by $\Gamma_M = (\gamma_\mu, i\gamma_5)$, satisfying $\{\Gamma_M, \Gamma_N\} = 2\eta_{MN} = 2\text{diag}(+, -, -, -, -)$. In the RS background, Eq. (1), which respects the four-dimensional Poincaré invariance, only the nonvanishing component of the spin connection ω_A^{BC} is given by

$$\omega_{\underline{\mu}}^{\nu 5} = -e_{\underline{\mu}}^{\nu} e^{\underline{5} 5} \partial_5 \sigma + e^{-\sigma} \sigma' \delta_{\underline{\mu}}^{\nu}, \quad (5)$$

where $\sigma' = \partial_5 \sigma$. Therefore we obtain

$$\begin{aligned} \mathcal{L}_{\text{fermion}} &= e^{-3\sigma} \bar{\Psi} [i \gamma^\mu \partial_\mu - \gamma_5 e^{-\sigma} (\partial_5 - 2\sigma')] \Psi \\ &= e^{-(3/2)\sigma} \bar{\Psi} \left[i \gamma^\mu \partial_\mu - \gamma_5 e^{-\sigma} \left(\partial_5 - \frac{1}{2} \sigma' \right) \right] e^{-(3/2)\sigma} \Psi. \end{aligned} \quad (6)$$

Interestingly, the mass operator $\gamma_5 e^{-\sigma} (\partial_5 - 2\sigma')$ for Ψ receives such a piece from the spin connection that has a *kink profile with a gap*

$$\Delta \sigma'_i \equiv \sigma'(y_i + 0) - \sigma'(y_i - 0) = \frac{2V_i}{24M_{5d}^3}, \quad (7)$$

¹The term containing the spin connection cancels if one partially integrates the action into the form that is manifestly invariant under charge conjugation. But this is not done here in order to make it clear that such a term is present in the Dirac equation that follows from the action.

where V_i is a tension of the brane located at $y = y_i$. To pursue an analogy with the domain wall fermion [17] is another motivation to consider the bulk fermions in the RS background.

Before going into any detail, let us first consider the fermion zero mode $\Psi(x, y) = \Psi_0(x) e^{3\sigma(y)/2} \hat{\zeta}(y)$ with $i \gamma^\mu \partial_\mu \Psi_0(x) = 0$, where a factor $e^{3\sigma(y)/2}$ brings the kinetic term in Eq. (6) into the canonical form. By solving the five-dimensional Dirac equation, we find that the zero mode is localized near the brane with a *negative* tension $V_1 < 0$:

$$\hat{\zeta}(y) = \hat{\zeta}(\pi r_c) e^{-(k/2)|\pi r_c - y|}. \quad (8)$$

We should remark that our mechanism for localizing fermion zero modes resembles many earlier attempts [18, 2, 17, 19] which utilizes a kink background induced by a topological defect or scalar field, except that *it is automatic*; our kink mass term in Eq. (6) appears not by hand, but as a consequence of the gravitational background in the manner of Randall and Sundrum. One may regard the RS background as generated by the scalar potential in gauged supergravity [20], but the point we stress here is that one and the same mechanism is responsible for the generation of the gauge hierarchy and the localization of fermions.

In the simplest setting we are describing, the chiral nature of fermions results from the compactification on S^1/Z_2 by imposing the Z_2 projection.² For the bulk fermion, we impose that $\Psi(x, y)$ is even under five-dimensional parity:

$$\gamma_5 \Psi(x, -y) = + \Psi(x, y). \quad (9)$$

Then there remains only one zero mode with positive chirality (right-handed fermion), as we will see shortly. If we consider the opposite condition $\gamma_5 \Psi(x, -y) = - \Psi(x, y)$, we will have a left-handed fermion as the zero mode.

We make a mode expansion with respect to the fifth dimension:

$$\Psi(x, y) = \sum_n [\psi_L^{(n)}(x) \xi_n(y) + \psi_R^{(n)}(x) \eta_n(y)], \quad (10)$$

where $\gamma_5 \psi_{L/R}^{(n)} = \mp \psi_{L/R}^{(n)}$. Using this expansion in Eq. (6) and integrating over y , we get the four-dimensional effective theory

$$\begin{aligned} \mathcal{L}_{\text{fermion}}^{(4\text{dim})} &= \sum_n [\bar{\psi}_L^{(n)} i \gamma^\mu \partial_\mu \psi_L^{(n)} + \bar{\psi}_R^{(n)} i \gamma^\mu \partial_\mu \psi_R^{(n)} \\ &\quad - (m_n \bar{\psi}_L^{(n)} \psi_R^{(n)} + \text{H.c.})]. \end{aligned} \quad (11)$$

Here the mode functions satisfy the eigenvalue equations

$$-e^{-\sigma} (\partial_y - 2\sigma') \xi_n(y) = m_n \eta_n(y), \quad (12)$$

²The chiral asymmetry could be produced if we introduced a suitable Dirac mass term for our bulk fermion. In fact the five-dimensional parity invariance forbids us from introducing such a bare mass term, and both chirality of zero modes are localized near the same brane.

$$+ e^{-\sigma}(\partial_y - 2\sigma')\eta_n(y) = m_n \xi_n(y) \quad (13)$$

with the normalizations

$$\int_0^{\pi r_c} dy e^{-3\sigma} \xi_n(y) \xi_m(y) = \int_0^{\pi r_c} dy e^{-3\sigma} \eta_n(y) \eta_m(y) = \delta_{mn}. \quad (14)$$

Since condition (9) is translated into $\xi_n(y) = -\xi_n(-y)$ and $\eta_n(y) = +\eta_n(-y)$, the Z_2 projection and the periodicity $\Psi(x, y + 2\pi r_c) = \Psi(x, y)$ give the boundary conditions

$$\xi_n(y = y_i) = 0 = \partial_y \eta_n(y = y_i) \quad (15)$$

at $y_0 = 0$ and $y_1 = \pi r_c$. With these conditions, one can easily find the explicit solution for the mode functions. We present the result for $\hat{\xi}_n(y) \equiv e^{-3\sigma(y)/2} \xi_n(y)$ and $\hat{\eta}_n(y) \equiv e^{-3\sigma(y)/2} \eta_n(y)$, for which a physical picture is most transparent [since the normalization condition (14) becomes the canonical ones³]:

$$\begin{aligned} \hat{\xi}_n(y) &= \sqrt{\frac{2k}{1 - e^{-\pi k r_c}}} e^{-k/2 |\pi r_c - y|} \sin \frac{m_n}{k} (e^{\sigma(y)} - 1), \\ \hat{\eta}_n(y) &= \sqrt{\frac{2k}{1 - e^{-\pi k r_c}}} e^{-k/2 |\pi r_c - y|} \cos \frac{m_n}{k} (e^{\sigma(y)} - 1) \end{aligned} \quad (16)$$

for $m_n = n\pi k / (e^{\pi k r_c} - 1) \neq 0$. For the zero mode $m_0 = 0$,

$$\hat{\xi}_0(y) = 0, \quad \hat{\eta}_0(y) = \sqrt{\frac{k}{1 - e^{-\pi k r_c}}} e^{-k/2 |\pi r_c - y|}. \quad (17)$$

This clearly shows that the right-handed fermion zero mode is localized near the orientifold plane at $y = \pi r_c$, while the left-handed zero mode is projected out.

As mentioned above, the left-handed zero mode can be obtained by the opposite projection. One expects that, as in the SM, these fermion zero modes will acquire their masses through Yukawa couplings to the Higgs field. To realize this in our model, we prepare an $SU(2)$ doublet $\Psi_L(x, y)$ and a singlet $\Psi_R(x, y)$, and impose the Z_2 -projection condition $\gamma_5 \Psi_{L/R}(x, -y) = \mp \Psi_{L/R}(x, y)$. Then the Z_2 -invariant operators are given by $\bar{\Psi}_R \Psi_L H$. As for the Higgs field H , there are two distinct possibilities that H also lives in the bulk, or it is confined on the brane at $y = \pi r_c$. Which of these two cases leads to a viable model is the subject of the subsequent sections.

³This is similar to the rescaling that was discussed in Ref. [21], but it is not exactly the same because we are considering the effective theory after integrating over the fifth dimension, not that on the brane.

III. GAUGE BOSONS IN THE BULK

Let us now proceed to the bulk gauge bosons, which were recently discussed in Refs. [14,15]. Here we briefly discuss the abelian case for simplicity. The Lagrangian for a bulk gauge field in the RS background, Eq. (1), is given by

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}(F_{\mu\nu})^2 + e^{-2\sigma} \left[\frac{1}{2}(\partial_5 A_\mu)^2 - \partial_5 A_\mu \partial^\mu A_5 + \frac{1}{2}(\partial_\mu A_5)^2 \right], \quad (18)$$

where the contraction by using the flat metric should be understood. The action principle requires a gauge-invariant boundary condition $0 = F_{5\mu} = \partial_5 A_\mu - \partial_\mu A_5$ at $y_0 = 0$ and $y_1 = \pi r_c$, but Z_2 -orbifold projection implies stronger conditions

$$\partial_5 A_\mu(x, y = y_i) = 0 = A_5(x, y = y_i). \quad (19)$$

That is, Z_2 projection implies the Neumann (Dirichlet)-type boundary condition for A_μ (A_5). Although we can proceed in a gauge covariant manner,⁴ let us take $A_5 = 0$ gauge [14] for simplicity. Then after integrating by parts, the Lagrangian reduces to

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{2}A^\mu \partial_5 (e^{-2\sigma} \partial_5 A_\mu), \quad (20)$$

supplemented with Gauss law constraint $0 \approx \partial_5 (\partial^\mu A_\mu)$.

Let us expand A_μ into the KK modes as

$$A_\mu(x, y) = \sum_n A_\mu^{(n)}(x) \hat{\chi}_n(y), \quad (21)$$

by using mode functions $\hat{\chi}_n(y)$ specified by the conditions

$$-\partial_y (e^{-2\sigma} \partial_y \hat{\chi}_n(y)) = M_n^2 \hat{\chi}_n(y), \quad (22)$$

$$\int_0^{\pi r_c} dy \hat{\chi}_n(y) \hat{\chi}_m(y) = \delta_{mn}, \quad (23)$$

as well as the Neumann-type boundary condition $\partial_y \hat{\chi}_n(y_i) = 0$ at $y_0 = 0$ and $y_1 = \pi r_c$. Substituting this expansion into Eq. (20) and integrating over y gives the four-dimensional effective theory

$$\mathcal{L}_{\text{gauge}}^{(4\text{dim})} = \sum_n \left[-\frac{1}{4}(F_{\mu\nu}^{(n)})^2 + \frac{1}{2}M_n^2 A_\mu^{(n)} A^{(n)\mu} \right]. \quad (24)$$

The explicit form of $\hat{\chi}_n(y)$ is given by the Bessel functions of the order $\nu = 1$;

⁴In this case, the ‘‘Nambu-Goldstone’’ field $A_5(x, y)$ should be mode expanded by using the Bessel functions of the order $\nu = 0$ to diagonalize the mixing between $A_\mu^{(m)}(x)$ and $A_5^{(n)}(x)$. Note also that the zero mode of A_5 is projected out.

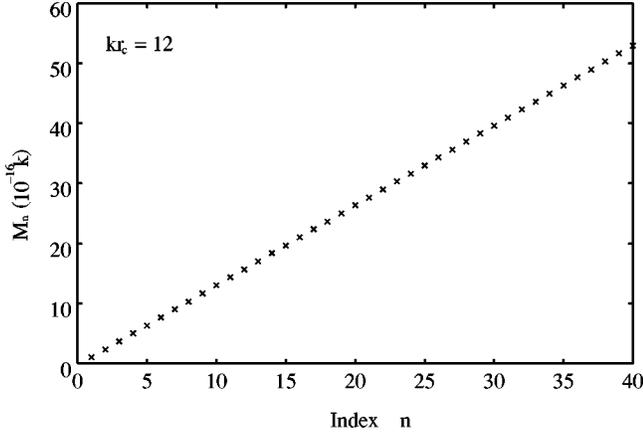


FIG. 1. Plot of the masses of the n th Kaluza-Klein modes of the gauge bosons in units of $10^{-16}k$. We take $kr_c = 12$.

$$\hat{\chi}_n(y) = \frac{\sqrt{2k}e^{\sigma(y)}}{N_n} [J_1(\lambda_n e^{\sigma(y)}) + c_n Y_1(\lambda_n e^{\sigma(y)})], \quad (25)$$

where $\lambda_n \equiv M_n/k \neq 0$, and by denoting $z_c \equiv e^{\pi k r_c}$,

$$\begin{aligned} c_n &= -\frac{J_0(\lambda_n)}{Y_0(\lambda_n)}, \quad N_n^2 = \int_1^{z_c} 2z dz [J_1(\lambda_n z) + c_n Y_1(\lambda_n z)]^2 \\ &= z^2 [J_1(\lambda_n z) + c_n Y_1(\lambda_n z)]^2 \Big|_1^{z_c}. \end{aligned} \quad (26)$$

The mass eigenvalues are determined by the condition $\partial_y \hat{\chi}_n(\pi r_c) = 0$;

$$J_0(\lambda_n) Y_0(\lambda_n e^{\pi k r_c}) = Y_0(\lambda_n) J_0(\lambda_n e^{\pi k r_c}). \quad (27)$$

The behavior of the mass eigenvalues M_n is depicted in Fig. 1, where we plot the values of $(M_n/k) \exp[\sigma(\pi r_c)]$ for $n = 1, \dots, 40$. Asymptotically at higher mass level $n \gg 1$, the mode functions behave like

$$\hat{\chi}_n(y) \sim \sqrt{\frac{2k}{1 - e^{-\pi k r_c}}} e^{-k/2 |\pi r_c - y|} \cos\left(n \pi \frac{e^{\sigma(y)} - 1}{e^{\pi k r_c} - 1}\right) \quad (28)$$

with the same mass eigenvalues $M_n \sim m_n$ as the KK fermion masses. The zero mode is flat in the extra dimension, $\hat{\chi}_0(y) = 1/\sqrt{\pi r_c}$, and the KK gauge bosons show the universal behavior of localizing near the brane at $y_1 = \pi r_c$ as in other bulk fields.

IV. BULK PHENOMENOLOGY

In this section we will examine phenomenological constraints on the bulk gauge bosons and fermions. For the moment we assume that a Higgs mechanism takes place and the zero modes corresponding to the W and Z bosons acquire tiny masses of the weak scale. We will discuss the details of the mechanism in Sec. V.

In the case that both fermions and gauge bosons are living in the bulk, the gauge coupling of the bulk fermion to the

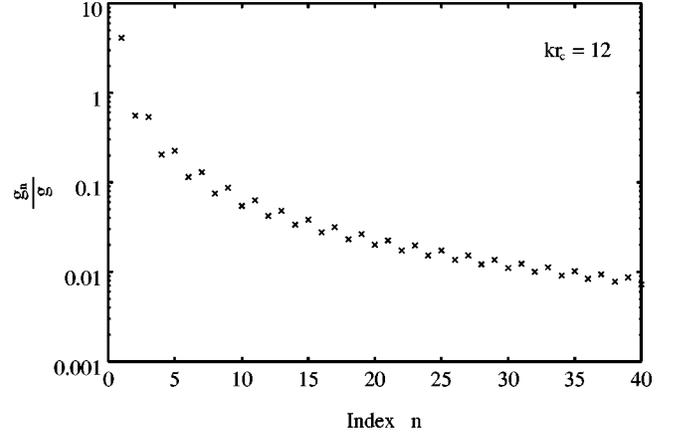


FIG. 2. Plot of the couplings g_n of the n th Kaluza-Klein modes to the bilinear of the zero modes of the bulk fermions relative to the gauge coupling constant g .

bulk gauge boson is written as

$$e^{-1} \mathcal{L}_{\text{coupling}} = g_{5d} \bar{\Psi}(x, y) i \Gamma^M \underline{e}_M^{\underline{M}}(y) A_M(x, y) \Psi(x, y). \quad (29)$$

Using the results given above, we find that the coupling constant of a KK mode of the gauge boson to the massless (zero-mode) fermion bilinear is given by

$$g_n = g \frac{\sqrt{2\pi k r_c}}{N_n} \int_1^{z_c} \frac{z dz}{z_c - 1} [J_1(\lambda_n z) + c_n Y_1(\lambda_n z)], \quad (30)$$

where $z_c = e^{\pi k r_c}$, and $g = g_{5d}/\sqrt{\pi r_c}$ is the four-dimensional gauge coupling constant. In Fig. 2, we plot the values of g_n/g . We found that the KK modes of the gauge boson have nonvanishing couplings to the bilinear of the zero-mode fermions. This is in sharp contrast to the flat metric case (or the factorizable extra dimension), where the conservation of the fifth-dimensional momentum prohibits these couplings. Another interesting point to be stressed is that *only the first KK mode of the gauge boson strongly couples to the fermion zero mode*. We find

$$\frac{g_1}{g} \simeq 4.1, \quad \frac{g_2}{g} \simeq 0.55, \quad \frac{g_3}{g} \simeq 0.54, \quad (31)$$

and $g_n \ll 1$ for higher n . Physically this suppression for higher KK modes is understood by the oscillating behavior, Eq. (28), of the Bessel functions. Thus one may expect that the high energy behavior of this model is rather moderate. Note that this is quite different from the case of the brane fermion where the coupling is determined by the wave function at the brane and turns out to be universal, i.e., $g_n/g = \sqrt{2\pi k r_c} \simeq 8.4$ for all KK modes.

Phenomenologically the existence of the nonvanishing couplings, Eq. (30), plays an important role [22,23] because the exchange of the KK modes of the gauge bosons induces four Fermi interactions. For the weak boson case, following Ref. [14], it is convenient to define

$$V \equiv \frac{\sum g_n^2/M_n^2}{g^2/m_W^2} = \sum_{n=1}^{\infty} \left(\frac{g_n}{g} \right)^2 \frac{m_W^2}{M_n^2}. \quad (32)$$

Using Eq. (31), we approximate the above equation to

$$V_{\text{bulk}} \approx 4.1^2 \frac{m_W^2}{M_1^2} \approx 17 \frac{m_W^2}{M_1^2}. \quad (33)$$

Here it is interesting to compare it with the case of the brane fermion. In this case, as we mentioned, $g_n/g = \sqrt{2\pi k r_c} \approx 8.4$ for all n , and $\sum M_1^2/M_n^2 \approx 1.5$, we find

$$V_{\text{brane}} \approx 8.4^2 \times 1.5 \frac{m_W^2}{M_1^2} \approx 100 \frac{m_W^2}{M_1^2}. \quad (34)$$

Comparison between Eqs. (33) and (34) implies that, for a given experimental constraint on V , the bound on the first excited mode in the bulk fermion case is weaker than that in the brane fermion case by a factor $\sqrt{100/17} \sim 2.5$.

Using the data of the electroweak precision measurements, Ref. [14] gives the constraint $V < 0.0013$ at 95% C.L. In our case of the bulk fermion, this gives the following bound on the mass of the first KK excitation of the W boson:

$$M_1 \gtrsim 9 \text{ TeV}. \quad (35)$$

Note that this bound is certainly weaker than the case of Ref. [14], though it still exceeds the electroweak scale.⁵

Another stringent bound comes from photon and gluon. The KK modes of the photon and gluon will effectively generate contact interactions

$$\mathcal{L}_{\text{eff}} = \frac{2\pi}{\Lambda^2} J^\mu J_\mu. \quad (36)$$

Experimentally Λ is constrained to be higher than 2–4 TeV [24], with detail depending on which current one considers. Note that the coupling of the first KK mode is enhanced by $g_1 = 4.1g$. Thus this constraint alone will raise the bound on the first excited state well above 1 TeV.

In passing, some remarks are in order. First, the reason for having such stringent constraints is that the first KK mode couples to fermions more strongly than the massless gauge boson; recalling Eq. (26), we can approximate Eq. (30) for a large $z_c = e^{\pi k r_c}$ to

$$\frac{g_1}{g} \approx \sqrt{2\pi k r_c} \int_1^{z_c} \frac{z dz}{z_c^2} \frac{J_1(\lambda_1 z)}{J_1(\lambda_1 z_c)} \sim \frac{\sqrt{2\pi k r_c}}{2}. \quad (37)$$

This fact can be understood by noting that although the zero mode of the gauge boson is flat in the fifth dimension, the first KK mode is localized (without oscillating) near the TeV brane where fermion zero modes are also localized. Second, we comment on how the constraint could be changed when we consider the massive gauge bosons. In that case, as we describe in Sec. V, the lowest mode of a massive bulk gauge boson has the mass of the order M_1 (unless we make an extreme fine tuning of the bulk gauge boson mass). Given that, one might wonder whether the gauge coupling of our W boson should be identified by g_1 , not g of the zero mode, and it would be M_2 and not M_1 that should be constrained as the mass of the first KK mode. If this were the case, the constraint discussed above would have further been relaxed by a factor $g_1/g_2 \sim 7.5$; $M_2/m_W \approx M_2/M_1 \sim (g_2/g_1)/\sqrt{V} \approx 3.7$. Unfortunately, however, this is actually ruled out from another constraint coming from the KK photon and gluons.

V. HIGGS MECHANISM AND GAUGE BOSON MASS

Now, we would like to discuss the mechanisms to generate the gauge boson mass. Let us first consider the case where the Higgs boson is also in the bulk. If we assume that the potential of the five-dimensional Higgs field takes the form

$$V(H) = -\mu^2 H^\dagger H + \frac{\lambda_{5d}}{2} (H^\dagger H)^2, \quad (38)$$

with a negative mass squared, the Higgs field develops the constant VEV in the bulk⁶ $\sim \sqrt{\mu^2/\lambda_{5d}}$, which generates the bulk mass term m for the gauge boson. Then the mode functions are expressed like in Eq. (25) but with the order $\nu = \sqrt{1+m^2/k^2}$. With the constraint $ke^{-\pi k r_c}$ of the order 10 TeV or higher, one has to take a small mass parameter m to realize the gauge boson mass of 100 GeV.

One might naively expect that a moderate fine tuning of $m/k \sim 10^{-2}$ would be enough to realize the correct gauge boson mass since there would be an approximate zero mode for a small bulk mass m . In fact, as we show explicitly in the Appendix, the lowest mass eigenvalue M'_1 for a very small m is proportional to m

$$M'_1{}^2 \approx \frac{m^2}{2 \ln e^{\sigma(y_1)}} = \frac{m^2}{2\pi k r_c}, \quad (39)$$

but there is no suppression by a warp factor!

The absence of a warp factor and the fate of the zero mode may be understood by regarding the small bulk mass m

⁵Reference [15] considered the case where the fermions are localized at $y=0$. In this case the bound on the KK excitations from the electroweak measurement is relaxed due to the small couplings of the KK modes of the gauge bosons to the boundary fermions at $y=0$. The energy scale at this brane does not contain the small warp factor so that one needs to invoke another mechanism to solve the hierarchy problem.

⁶This VEV should be sufficiently smaller than the curvature scale k not to disturb the background; otherwise, it could be an origin of the bulk vacuum energy Λ .

as a perturbation; evaluating the bulk mass term by using the zero mode eigenfunction $\hat{\chi}_0(y) = 1/\sqrt{\pi r_c}$ in the massless case, we find

$$\begin{aligned} M_1'^2 &\simeq m^2 \int_0^{\pi r_c} dy e^{-2\sigma(y)} \hat{\chi}_0(y) \hat{\chi}_0(y) \\ &= \frac{m^2}{2\pi k r_c} (1 - e^{-2\pi k r_c}). \end{aligned} \quad (40)$$

This will be a good approximation to the exact mass eigenvalue M_1' as far as the mixings between the ‘‘zero mode’’ $A_\mu^{(0)}(x)$ and ‘‘nonzero modes’’ $A_\mu^{(n)}(x)$ are small

$$M_{0n}^2 = m^2 \int_0^{\pi r_c} dy e^{-2\sigma(y)} \hat{\chi}_0(y) \hat{\chi}_n(y) \ll M_1^2. \quad (41)$$

When the bulk mass (and thus the mixings) goes up and becomes comparable to M_1 , then the perturbation breaks down and we will find that the lowest mass eigenvalue M_1' smoothly goes up and eventually becomes of the same order as M_1 of the first excited state in the massless case. Apparently the (approximate) zero mode disappears even for, say, $m/k \sim 10^{-10}$.

Therefore the mass parameter m itself must be much smaller than $k \sim M_{\text{pl}}$, whereas the natural value for m would be of the order k . Since the constraints discussed in Sec. V push the energy scale $ke^{-\pi k r_c}$ of our brane well above 1 TeV, the mass parameter m must be chosen to be the electroweak scale. This small m parameter for the gauge boson requires a hierarchically small μ parameter in the Higgs potential. This is nothing but the conventional fine tuning of the Higgs boson mass in nonsupersymmetric theories and the gauge hierarchy is not solved at all. Thus we should discard the model with the bulk Higgs mechanism.

This leaves the case where the Higgs is confined on our brane.⁷ In this case, the energy scale of the brane is already reduced to be $ke^{-\pi k r_c} \sim 10$ TeV. Thus to realize the electroweak scale, the Higgs mass parameter should be tuned just by 10^2 . This should be compared with the previous case of the bulk Higgs mechanism where we need the conventional 10^{16} . In fact, the brane Higgs seems to be the only choice we can take to avoid the extreme fine tuning of the Higgs mass in the ‘‘bulk SM’’ approach.

VI. CONCLUSIONS

We have discussed various issues in an attempt to construct a bulk standard model in the RS background geometry. In particular, by solving the Dirac equation in this background geometry, we observed the localization of the bulk fermion due to the kink profile of the spin connection. Since the localization takes place near the brane with a negative

tension where the gravity is weak, the bulk SM makes the RS approach to the hierarchy problem more attractive. The chiral nature of the fermion zero mode is realized by the Z_2 -orbifold projection in the present model.

We have also found that the couplings of fermion zero modes to the (oscillating) KK modes of the gauge boson are suppressed compared with the brane fermion case. This relaxes the phenomenological constraint, but not enough. In fact the first KK mode of the W gauge boson must be heavier than 9 TeV, which implies that the energy scale of the distant brane itself must exceed the TeV scale.

With this phenomenological constraint, the bulk SM suffers from a fine-tuning problem. In particular, when the whole SM is put in the bulk as we discussed in Sec. V, the hierarchy problem is not solved at all and we need an extreme fine tuning to realize the electroweak scale. In this case the RS background has nothing to do with the hierarchy problem, and we need another mechanism completely, for instance supersymmetry, to realize the idea of the bulk SM.

If we want to keep the advantage of the RS setting as a solution to the hierarchy problem, we have to confine the Higgs field on the TeV brane. In this case, the VEV of the Higgs boson localized at the brane will give contributions to the masses of the gauge bosons and fermions. We can easily construct a viable model that contains the SM particles as the lowest modes once we accept a moderate fine tuning of $1/100$. Of course, some care should be taken to ensure the proton stability; higher-dimensional operators will be suppressed only by the mass scale of the TeV brane and should be forbidden by some symmetry reasons for instance. In this respect, the bulk SM in the simplest formulation suffers from similar problem as in models with large extra dimensions.

Besides phenomenological implications, the present formulation of bulk fermion in the RS background (and its generalization) deserves further study. Among others, an interesting application would be to formulate chiral fermions on a lattice.

While we completed our manuscript, interesting preprints appeared [25,26]; the former deals with bulk fermions including right-handed neutrinos, and our result here is consistent with theirs. The latter discusses the dynamical Higgs scenario in the extra dimension(s).

ACKNOWLEDGMENTS

We thank the Summer Institute at Yamanashi, Japan, where the present work was initiated. S.C., H.N. and M.Y. also acknowledge KEK for its kind hospitality where a part of the work was done. H.N. thanks T. Kugo and H. So for discussions. This work was supported in part by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports, and Culture of Japan, on Priority Area 707 ‘‘Supersymmetry and Unified Theory of Elementary Particles’’ (J.H. and M.Y.), and by Grant-in-Aid No. 10740133 (J.H.), No. 11640246 (M.Y.), and No. 98270 (S.C.). S.C. and N.O. thank the Japan Society for the Promotion of Science for financial support.

⁷Another logical possibility would be to consider the bulk Higgs field with a positive mass squared and to expect that some dynamics (in four-dimensional effective theory) would drive the mass squared of its lowest mode negative.

APPENDIX: FATE OF THE BOSON ZERO MODES

Here we discuss how the masses of the lowest modes for spin 0 and 1 particles behave when they have a nonzero bulk mass m .

As usual, the n th mode of a bosonic bulk field is expressed in terms of Bessel functions as

$$\chi_n(y) = \frac{\sqrt{2k}}{N_n} \left(\frac{x_n}{2}\right)^a [J_\nu(x_n) + \alpha_n J_{-\nu}(x_n)], \quad (\text{A1})$$

where N_n and α_n are proper normalization constants and $x_n = (M'_n/k)e^{\sigma(y)}$. The order ν is given by

$$\nu = \sqrt{a^2 + \frac{m^2}{k^2}} \approx a + \Delta\nu, \quad (\text{A2})$$

where $a=2$ for scalar and $a=1$ for vector boson, and $\Delta\nu=0$ corresponds to the vanishing bulk mass.

We are interested in the mass eigenvalue $\lambda_1 \equiv M'_1/k$ of the lowest mode $\chi_1(y)$. Let us consider the situation in which the bulk mass m is small enough that the resulting mass is tiny $(M'_1/k)e^{\pi k r_c} \ll 1$. Then we can make the approximation for the Bessel functions near the origin; for $x_1 \ll 1$

$$J_\nu(x_1) \approx \left(\frac{x_1}{2}\right)^\nu \frac{1}{\Gamma(1+\nu)},$$

$$J_{-\nu}(x_1) \approx \left(\frac{x_1}{2}\right)^{-\nu} \left[\frac{1}{\Gamma(1-\nu)} - \frac{1}{\Gamma(2-\nu)} \left(\frac{x_1}{2}\right)^2 \right].$$

At $y=0$, $x_1 = \lambda_1$ and the boundary condition gives

$$\begin{aligned} -\frac{1}{\alpha_1} &= \frac{\frac{d}{dx}[(x/2)^a J_{-\nu}(x)]}{\frac{d}{dx}[(x/2)^a J_\nu(x)]} \Bigg|_{x=\lambda_1} \\ &\approx \left(\frac{\lambda_1}{2}\right)^{-2\nu} \frac{\Gamma(1+\nu)}{a+\nu} \left[\frac{a-\nu}{\Gamma(1-\nu)} \right. \\ &\quad \left. - \frac{a+2-\nu}{\Gamma(2-\nu)} \left(\frac{\lambda_1}{2}\right)^2 \right]. \end{aligned} \quad (\text{A3})$$

The boundary condition at the other boundary $y = \pi r_c$ gives

$$\begin{aligned} -\frac{1}{\alpha_1} &\approx \left(\frac{\lambda_1 z_c}{2}\right)^{-2\nu} \frac{\Gamma(1+\nu)}{a+\nu} \left[\frac{a-\nu}{\Gamma(1-\nu)} \right. \\ &\quad \left. - \frac{a+2-\nu}{\Gamma(2-\nu)} \left(\frac{\lambda_1 z_c}{2}\right)^2 \right], \end{aligned} \quad (\text{A4})$$

where $z_c = e^{\pi k r_c}$. These two equations can be summarized as

$$\frac{a+2-\nu}{\Gamma(2-\nu)} \left(\frac{\lambda_1}{2}\right)^2 = \frac{a-\nu}{\Gamma(1-\nu)} \frac{1-z_c^{-2\nu}}{1-z_c^{2(1-\nu)}},$$

which leads a relation

$$\lambda_1^2 \approx \frac{2(\nu-1)}{1-z_c^{2(1-\nu)}} \Delta\nu. \quad (\text{A5})$$

For the scalar case, $\nu \approx a=2$, this correctly reproduces the result in Ref. [13]

$$\lambda_1 = \frac{M'_1}{k} \approx \sqrt{2\Delta\nu} \approx \frac{1}{\sqrt{2}} \frac{m}{k}. \quad (\text{A6})$$

On the other hand, for the vector boson case, $\nu \approx a=1$, Eq. (A5) reduces to

$$\lambda_1^2 \approx \frac{2\Delta\nu(\nu-1)}{1-(1+2(1-\nu)\ln z_c)} \approx \frac{\Delta\nu}{\ln z_c}, \quad (\text{A7})$$

which gives the announced relation, Eq. (39)

$$\frac{M'_1}{k} \approx \frac{1}{\sqrt{2\pi k r_c}} \frac{m}{k}. \quad (\text{A8})$$

We note again that these results, Eqs. (A6) and (A8), with no suppression by a warp factor, are valid only for a sufficiently small bulk mass m .

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