Divergence of the quantum stress tensor on the Cauchy horizon in 2D dust collapse

Sukratu Barve* and T. P. Singh†

Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400 005, India

Cenalo Vaz‡

Unidade de Ciencias Exactas e Humanas, Universidade do Algarve, Faro, Portugal

We prove that the quantum stress tensor for a massless scalar field in two dimensional non-self-similar Tolman-Bondi dust collapse and Vaidya radiation collapse models diverges on the Cauchy horizon, if the latter exists. The two dimensional model is obtained by suppressing angular coordinates in the corresponding four dimensional spherical model.

PACS number(s): 04.62.+v, 04.70.Dy

I. INTRODUCTION

If a classical model of gravitational collapse results in the formation of a naked singularity, quantum effects can be expected to play a significant role during the final stages of the collapse. One way to study these effects is the quantization of test matter fields in the background spacetime provided by the collapsing classical matter. The semiclassical approximation is expected to be valid up to Planck scales. In particular, one is interested in the behavior of the stress tensor of the quantized field in the approach to the Cauchy horizon. The divergence of the vacuum expectation value of the quantized stress tensor on the Cauchy horizon signals an instability of the horizon, and suggests that back reaction will prevent the naked singularity from forming.

Two well-known examples of the formation of naked singularities are the spherical collapse of inhomogeneous dust (the Tolman-Bondi model) and the spherical collapse of null dust (the Vaidya model). It is known for both these models that for certain initial data the collapse ends in a black hole and for other initial data it ends in a naked singularity [1–3]. The Penrose diagrams in Fig. 1 exhibit the formation of a black hole and a naked singularity.

The stress-tensor of a quantized scalar field on these background spacetimes has been investigated, by specializing to the case of 2D self-similar collapse. The restriction to 2D is similar to the geometric optics approximation in 4D—the latter amounts to keeping only the $l=0$ modes. The 2D spacetime is obtained by suppressing the angular coordinates in the 4D spherical model. For a two dimensional model, explicit expressions for the vacuum expectation value of the stress-tensor can be obtained from the trace anomaly, by imposing conservation of the stress tensor. The assumption of self-similarity allows double null coordinates to be constructed explicitly. Using these coordinates it has been shown that the outgoing quantum flux diverges on the Cauchy horizon, for self-similar Vaidya collapse [2], as well as for self-similar Tolman-Bondi collapse [4].

A priori, it may be the case that the divergence in the 2D model could be because of the assumption of self-similarity. In this paper, we prove that this assumption can be relaxed, and that the outgoing quantum flux will diverge on the Cauchy horizon, for all initial conditions for which a naked singularity forms in the 2D Vaidya and Tolman-Bondi models.

The outline of the proof is as follows. Consider the collapse of a classical non-self-similar spherical dust cloud and choose the initial conditions to be such that the collapse results in a naked singularity. We now construct a new initial distribution by replacing a spherical region by a self-similar distribution. The new distribution is hence a self-similar spherical region surrounded by part of the original distribution. The free parameter of the self-similar distribution is fixed by requiring the first and second fundamental forms to match at the boundary between the self-similar region and the original distribution. It is then shown that if the evolution of the original distribution results in a naked (covered) singularity, the evolution of the modified distribution also results in a naked (covered) singularity. We show that, in general, the density of the cloud will change discontinuously at the boundary, but this change will be finite, and not infinite.

We next consider the quantum stress tensor for a massless scalar field on the classical background dust spacetime. As has been shown earlier, the stress tensor diverges on the Cauchy horizon for a self-similar model. Consider now the modified distribution (self-similar region surrounded by non-
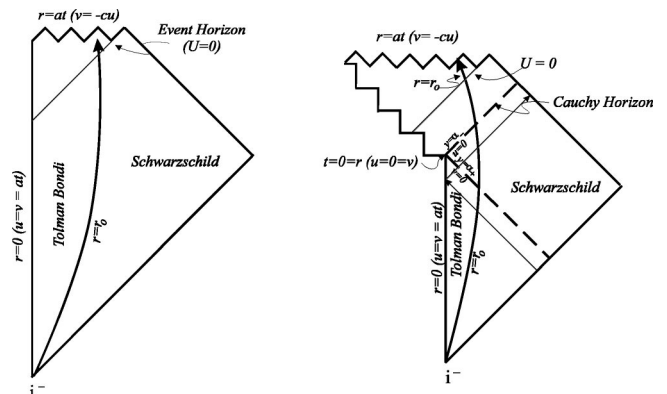
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*Email address: sukkoo@relativity.tifr.res.in

†Email address: tpsingh@tifr.res.in

‡Email address: cvaz@ualg.pt

FIG. 1. Formation of a black hole and a naked singularity.

self-similar region) mentioned above. We show that the nature of the divergence in the self-similar region is such that it implies a divergence in the outer non-self-similar region as well. Then, by considering a family of initial distributions, the size of the self-similar region is shrunk to zero—for each distribution in the family there is a divergence on the Cauchy horizon. The limiting distribution, in which the self-similar region disappears entirely, is the original distribution, and this also has a divergence on the Cauchy horizon.

The plan of the paper is as follows. In Sec. II we construct the modified distribution which includes a self-similar spherical region in the interior. In Sec. III we obtain the quantum stress tensor for a test scalar field on this modified distribution and prove that it diverges on the Cauchy horizon for a non-self-similar dust cloud.

II. THE MODIFIED DISTRIBUTION

In this section, we show how the modified initial distribution is constructed, for the cases of marginally bound and non-marginally bound dust collapse, as well as for the Vaidya model.

A. Marginally bound dust collapse

The collapse of a spherical dust cloud is described by the Tolman-Bondi line-element, using comoving coordinates (t,r,θ,ϕ):

$$ds^2 = -dt^2 + \frac{R'^2}{1+f(r)}dr^2 + R^2(t)d\Omega^2, \qquad (1)$$

where $R(t,r)$ is the area radius at time t of the shell labeled r, and $f(r)$ is a free function, satisfying $f > -1$. The marginally bound solution is one for which $f(r) = 0$. The only nonzero component of the energy-momentum tensor is the energy density $\rho(t,r)$, which satisfies the Einstein equation

$$\rho = \frac{F'(r)}{R^2 R'}, \qquad (2)$$

where $F(r)$ is another free function, and has the interpretation of being the mass to the interior of the shell r. The only other Einstein equation is

$$\dot{R}^2 = \frac{F(r)}{R} + f(r). \qquad (3)$$

Let us consider the collapse of marginally bound dust cloud, starting at a time t_i and having an initial density distribution $\rho_0(r)$, for $r \leq r_b$, where r_b is the boundary of the star. Integrating (3) gives the solution

$$R^{3/2}(t,r) = \frac{3}{2}\sqrt{F(r)}(t_0(r) - t) \qquad (4)$$

for the evolution of the area radius of the shell r. $t_0(r)$ is a function of integration, to be determined by choosing an initial scaling, $R(t_i,r)$, at the start of collapse. The area shrinks to zero at $t = t_0(r)$, resulting in the formation of a curvature

singularity. It is the singularity at $r = 0$, the central singularity, which is of interest to us, as this has been shown to be naked for some initial conditions. Specifically, it has been shown [1] that if the initial density distribution $\rho_0(R)$ has a Taylor expansion

$$\rho_0(R) = \rho_0 + \rho_1 R + \frac{1}{2}\rho_2 R^2 + \frac{1}{6}\rho_3 R^3 + \cdots, \qquad (5)$$

then the singularity is at least locally naked if one of the following conditions is satisfied: (i) $\rho_1 < 0$, or (ii) $\rho_1 = 0, \rho_2 < 0$, or (iii) $\rho_1 = \rho_2 = 0, \rho_3 < 0$ and $\xi = \sqrt{3}\rho_3/4\rho_0^{5/2}$ is less than or equal to -25.9904. The singularity may or may not be globally naked. We are interested in showing that in either case the outgoing quantum flux diverges on the Cauchy horizon.

The self-similar solution, i.e. one for which the spacetime of the collapsing cloud possesses a homothetic Killing vector field, is a special case of the marginally bound solution. If we choose the scaling in such a way that $t_0(r) = r$, then the mass function of the self-similar solution is of the form $F_{ss}(r) = \lambda r$, with λ a non-negative constant. All dimensionless quantities are functions of t/r. The central singularity forms at $t = 0$, and is known to be (globally) naked for $\lambda \leq \lambda_c = 0.1809$ [5]. It can also be shown that the initial density distribution for a self-similar cloud is of the form

$$\rho_0(R) = \rho_0 + \rho_3 R^3 + \rho_6 R^6 + \rho_9 R^9 + \cdots. \qquad (6)$$

In order to construct the modified initial density distribution, we start with the original distribution $\rho_0(r)$ and replace a central region, up to some $r = r_c$, by the self-similar solution (with $r_c < r_b$). For $r_c < r < r_b$, the distribution continues to be the original distribution $\rho_0(r)$. There is only one free parameter in the self-similar solution, namely λ, and this is determined by requiring that the total mass in the self-similar region is equal to the total mass contained in the original distribution, up to $r = r_c$. That is, $\lambda r_c = F(r_c)$.

We would now like to ensure that the determined value of λ is such that in the limit r_c going to zero, the modified distribution admits a naked singularity if and only if the original distribution $\rho_0(r)$ admits a naked singularity. This is essential because we are interested in examining properties of the Cauchy horizon in the original distribution. Hence it is natural to demand that the modified distribution possess a Cauchy horizon if and only if the original distribution does. The limiting value λ_0, of λ, is clearly $dF(r)/dr|_{r=0}$, and should satisfy the condition $\lambda \leq \lambda_c$ if and only if the original distribution $\rho_0(r)$ admits a naked central singularity.

To check this, we go back to the solution (4) and assume, for simplicity, the scaling to be such that $t_0(r) = r$, so that the solution can be written as

$$R^3 = \frac{9}{4}F(r)(t-r)^2. \qquad (7)$$

We substitute this in Eq. (2) and assuming the collapse to begin at $t = t_{in}$, find the initial density $\rho_0(r)$ near the center, to be

$$\rho_0(r) = \frac{4}{3t_{in}^2}\left[1 + \frac{2F(r)}{F'(r)\,t_{in}}\right]. \qquad (8)$$

We compare this with the form (5) for the initial density, after using Eq. (7) with $t = t_{in}$ in Eq. (5). This comparison allows us to deduce the form of $F(r)$ near $r=0$, from which the limiting value $\lambda_0 = dF(r)/dr|_{r=0}$ can be worked out. We find that for two of the naked cases, (i) $\rho_1 < 0$, and (ii) $\rho_1 = 0, \rho_2 < 0$, we get $\lambda_0 = 0$, which is a special case of the naked self-similar model. For the case (iii) $\rho_1 = \rho_2 = 0, \rho_3 < 0$ we get $\lambda_0^{3/2} = -8\rho_0^{5/2}/\sqrt{3}\rho_3$, which implies that the chosen self-similar distribution is naked if and only if the original distribution of type (iii) is naked. For the covered case, which is $\rho_1 = \rho_2 = \rho_3 = 0$, we get that λ_0 is infinite, which is a covered self-similar distribution. Hence it is shown that the modified distribution admits a naked singularity if and only if the original distribution does.

Next, we discuss the question of the matching of the self-similar region and the non-self-similar region, at the boundary r_c. By comparing the metrics of the two spacetimes at this boundary we find that the area radii in the self-similar metric and in the non-self-similar one will be equal if the two mass functions are equal. Since the masses have been chosen to be equal, that ensures the matching of the first fundamental form (i.e. the line element on the boundary). The second fundamental form (i.e., the extrinsic curvature) is defined to be

$$K_{\mu\nu} = -\frac{1}{2}(\xi_{\alpha;\beta} + \zeta_{\beta;\alpha})h_\mu^\alpha h_\nu^\beta, \qquad (9)$$

where ξ_α is a unit normal to the boundary, and $h_\beta^\alpha = \delta_\beta^\alpha - \xi^\alpha\xi_\beta$ is the projection tensor. The only non-zero component of the extrinsic curvature is $K_\theta^\theta = K_\phi^\phi = -1/R$, which matches for the two metrics, since the mass functions have been chosen to be equal. Hence we are assured that the first and second fundamental forms match on the boundary.

We note that the derivative $F'(r)$ will in general not match at the boundary, when calculated for the self-similar region, and for the non-self-similar region. As a result, the density function $\rho(t,r)$, given by Eq. (2), will in general be discontinuous at the boundary. However, it is important for our purposes to note that this discontinuity will be finite, since $F'(r)$ will be finite.

B. Non-marginally bound dust collapse

The metric function $f(r)$ which appears in Eq. (1) is now non-zero. Equation (3) can be solved exactly, for given $F(r)$ and $f(r)$. Whereas $F(r)$ is determined from the initial density distribution, $f(r)$ gets determined from the velocity distribution $\dot{R}$ at the onset of collapse.

Given an initial distribution of density and velocity which is non-marginal, one cannot straightaway match it to a self-similar region at a boundary r_c, because for the former distribution we have $f(r) \neq 0$, and for the latter we have $f(r) = 0$. Matching of the line-elements requires that both $F(r)$ and $f(r)$ match on the boundary between the two regions.

Thus, given the original non-marginal distribution $[\rho_0(r), f(r)]$ we construct the modified distribution as follows. Replace the original model by a self-similar one, as before, for $r \leqslant r_c$. For some $r_* < r_b$, introduce a non-marginal dust distribution $[\rho_0(r), f_s(r)]$ in the region $r_c < r < r_*$, which has the property that $\rho_0(r)$ [and hence $F(r)$] is the same as in the original distribution; $f_s(r_c) = 0$ and $f_s(r_*) = f(r_*)$. As before, the self-similar model is chosen such that $\lambda r_c = F(r_c)$. The introduction of this sandwiched region ensures that the first and second fundamental forms are matched at the boundaries r_c and r_*. The limiting value of λ, as r_c goes to zero, is again given by $\lambda_0 = dF(r)/dr|_{r=0}$, and is calculated by letting both r_c and r_* go to zero, while always keeping $r_* > r_c$. By carrying out an analysis similar to the marginal case, it can be shown that λ_0 is such that the introduced self-similar model is naked if and only if the original distribution is naked.

C. The Vaidya model

The collapse of a null dust cloud is described by the Vaidya metric

$$ds^2 = -\left(1 - \frac{2m(v)}{R}\right)dv^2 + 2\,dv\,dR + r^2 d\Omega^2, \qquad (10)$$

where the mass function $m(v)$ depends on the advanced time coordinate v. The mass function is zero for $v < 0$, and constant for $v > v_b$. Thus the cloud is bounded in the region $0 < v < v_b$, and its exterior is Schwarzschild spacetime. A curvature singularity forms when the inner boundary of the cloud hits $R = 0$. This singularity is known to be naked for $dm(v)/dv|_{v=0} \leqslant 1/16$ [3] and covered for $dm(v)/dv|_{v=0} > 1/16$. The special case of a self-similar model is described by the linear mass function, $m(v) = \mu v$, $0 < v < v_b$.

Given a non-self-similar model $m(v)$, we construct the modified distribution by replacing the original model by a self-similar one in the region $0 < v < v_c$, with $v_c < v_b$. The free parameter μ in the self-similar model is fixed by demanding $\mu v_c = m(v_c)$. The limiting value of μ, as v_c goes to zero, is $dm(v)/dv|_{v=0}$, and it is apparent that the introduced self-similar distribution is naked if and only if the original distribution $m(v)$ is naked.

Since the boundary v_c is a null hypersurface, the matching of the spacetimes at the boundary is done by comparing, not the line elements, but the affine parameter for outgoing or ingoing null geodesics. We now show that the matching of the affine parameters on the boundary v_c implies that the two mass functions should be equal. For this purpose, it is convenient to restrict to the 2D line element obtained from Eq. (10) by suppressing the angular coordinates, and to write it in double null coordinates u, v, i.e.

$$ds^2 = C^2(u,v)\,du\,dv. \qquad (11)$$

The function $C(u,v)$ satisfies the differential equation

$$\frac{C^2_{,v}}{C^4}\left(1 - \frac{2m(v)}{R}\right) + \frac{2m(v)}{R^2 C^2} + 2\frac{C^2_{,v}}{C^4} = 0. \qquad (12)$$

By integrating the geodesic equation for the metric (11) it is shown that the affine parameter along ingoing and outgoing null geodesics is of the form

$$p = a \int C^2 du + b, \quad q = c \int C^2 dv + d. \quad (13)$$

Matching of the affine parameter at the boundary v_c hence requires that $C_a^2 du = C_b^2 du$, and the use of this equality in the differential equation (12) gives the result that the mass functions of the two regions (self-similar and non-self-similar) must match. Further, it may be shown, as in the dust case, that the extrinsic curvatures match at the boundary between the two regions.

We have now completed the construction of the modified classical distribution, which will be used to prove the divergence of the outgoing quantum flux on the Cauchy horizon.

III. THE QUANTUM STRESS TENSOR

We restrict attention to the two dimensional spacetime (Tolman-Bondi or Vaidya) obtained by suppressing angular coordinates in the four-dimensional spherical spacetime. The expectation value $\langle 0_{in} | T_{\mu\nu} | 0_{in} \rangle$ of the energy-momentum tensor of a quantized scalar field in the Minkowski vacuum $|0_{in}\rangle$ can be calculated from the trace anomaly, and Wald's axioms. The trace anomaly is equal to $\mathcal{R}/24\pi$, where $\mathcal{R}$ is the Ricci scalar for the background spacetime [6]. The two dimensional spacetime can be expressed in terms of global null coordinates $\hat{u}$ and $\hat{v}$ as

$$ds^2 = C^2(\hat{u}, \hat{v}) \, d\hat{u} \, d\hat{v}. \quad (14)$$

These coordinates are chosen such that the initial quantum state of the scalar field, which is the standard Minkowski vacuum $|0_{in}\rangle$ on $\mathcal{I}^-$, is the vacuum with respect to the normal modes of the scalar wave equation in $\hat{u}, \hat{v}$ coordinates. The components of $\langle T_{\mu\nu} \rangle$ are given by

$$\langle T_{\hat{u}\hat{u}} \rangle = -\frac{1}{12\pi} C \left(\frac{1}{C} \right)_{,\hat{u},\hat{u}}, \quad (15)$$

$$\langle T_{\hat{v}\hat{v}} \rangle = -\frac{1}{12\pi} C \left(\frac{1}{C} \right)_{,\hat{v},\hat{v}}, \quad (16)$$

$$\langle T_{\hat{u}\hat{v}} \rangle = \frac{\mathcal{R} C^2}{96\pi}. \quad (17)$$

Consider a situation where two solutions for the background metric are matched across a hypersurface, like a geodesic, for instance, which in fact will be the case for our models. Let C_1^2 and C_2^2 be the metrics in the two regions. In either of the regions where the solutions are given, expressions (15)–(17) are analytic. Now consider the hypersurface (boundary) where the matching has taken place. Assuming the line element on the boundary and the second fundamental form of the boundary being calculated in both of the regions to be identical respectively, one obtains $C_1 = C_2 = C$ at the

boundary. The difference in the quantum stress tensor components can be expressed at the boundary [e.g. from Eq. (1)]:

$$\langle T_{1\hat{u}\hat{u}} \rangle - \langle T_{2\hat{u}\hat{u}} \rangle = -1/12\pi C (1/C_1 - 1/C_2)_{,\hat{u},\hat{u}}. \quad (18)$$

Since both the metrics are analytic in their respective regions (we only require their second order partial derivatives to be finite) and if one extends each one of them smoothly across the boundary, one finds that the expression above is certainly finite. So, the discontinuous change in $\langle T_{\hat{u}\hat{u}} \rangle$ across the boundary will be finite. The same can be said of the rest of the components.

We next introduce double null coordinates in the various regions of the modified distribution under consideration. Consider first the case of marginally bound dust. The various regions are the introduced self-similar region (henceforth labeled 2) in the coordinate range $0 < r < r_c$, the original distribution in the region (henceforth labeled 1) $r_c < r < r_b$, and the Schwarzschild region (henceforth labeled 0) $r > r_b$. We write the line-elements in these regions as

$$ds^2 = A^2(u_2, v_2) du_2 dv_2 \quad (19)$$

in region 2, as

$$ds^2 = B^2(u_1, v_1) du_1 dv_1 \quad (20)$$

in region 1, and finally, as

$$ds^2 = D^2(u, v) du dv \quad (21)$$

in region 0.

The relationship among these coordinates can be given, following the procedure described in Birrel and Davies [7]. Because of matching the first fundamental form at various boundaries, we have

$$u_1 = \alpha_1(u), \quad v = \beta_1(v_1), \quad (22)$$

and

$$u_2 = \alpha_2(u_1), \quad v_1 = \beta_2(v_2). \quad (23)$$

The center of the cloud being given by both

$$\hat{u} = \hat{v} \quad (24)$$

and

$$u_2 = v_2 - 2R_0, \quad (25)$$

in a manner similar to Birrell and Davies, we obtain a relation between $\hat{u}$ and u_1

$$\hat{u} = \beta_1 [\beta_2(\alpha_2(u_1) + 2R_0)], \quad (26)$$

as well as a relation between $\hat{u}$ and u

$$\hat{u} = \beta_1 \{\beta_2 [\alpha_2(\alpha_1(u)) + 2R_0]\}. \quad (27)$$

Starting from expressions (15)–(17) for the expectation value of the stress-tensor, and using the coordinate relation-

ships given above, we obtain the following expressions for the $T_{\mu\nu}$ in various regions. In region 2,

$$\langle T_{u_2 u_2}\rangle = -F_{u_2}(A^2) + F_{u_2}\left(\frac{d\hat{u}}{du_2}\right), \tag{28}$$

$$\langle T_{v_2 v_2}\rangle = -F_{v_2}(A^2) + F_{v_2}\left(\frac{d\hat{v}}{dv_2}\right), \tag{29}$$

$$\langle T_{u_2 v_2}\rangle = \frac{1}{6\pi A^2}\left(\frac{d\hat{u}}{du_2}\frac{d\hat{v}}{dv_2}\right)^2\left[\frac{A^2_{,u_2,v_2}}{A^2} - \frac{A^2_{,u_2}A^2_{,v_2}}{A^4}\right]. \tag{30}$$

The function $F_x(y)$ is defined as

$$F_x(y) = \frac{1}{12\pi}\sqrt{y}\left(\frac{1}{\sqrt{y}}\right)_{,x,x}. \tag{31}$$

In region 1,

$$\langle T_{u_1 u_1}\rangle = -F_{u_1}(B^2) + F_{u_1}\left(\frac{d\hat{u}}{du_1}\right), \tag{32}$$

$$\langle T_{v_1 v_1}\rangle = -F_{v_1}(B^2) + F_{v_1}\left(\frac{d\hat{v}}{dv_1}\right), \tag{33}$$

$$\langle T_{u_1 v_1}\rangle = \frac{1}{6\pi B^2}\left(\frac{d\hat{u}}{du_1}\frac{d\hat{v}}{dv_1}\right)^2\left[\frac{B^2_{,u_1,v_1}}{B^2} - \frac{B^2_{,u_1}B^2_{,v_1}}{B^4}\right]. \tag{34}$$

Similarly in region 0,

$$\langle T_{uu}\rangle = F_u(D^2) + \alpha' F_{u_2}(\beta') + F_u(\alpha'), \tag{35}$$

$$\langle T_{vv}\rangle = -F_v(D^2), \tag{36}$$

$$\langle T_{uv}\rangle = -1/24\pi(\ln(D^2))_{,u,v}, \tag{37}$$

where $'$ indicates the first derivative with respect to the argument, and

$$\alpha(u) = \alpha_2(\alpha_1(u)), \quad \beta(v_2) = \beta_1(\beta_2(v_2)). \tag{38}$$

Let us first consider region 2; we have the self-similar metric there according to the procedure adopted. Consider a parameter value λ for which the collapse results in a naked central singularity. From Barve *et al.* [4] we know that the component $\langle T_{u_2 u_2}\rangle$ diverges on the entire Cauchy horizon in the region (the other components remaining finite). It is obvious that $\langle T_{\hat{u}\hat{u}}\rangle$ will also diverge there. As pointed out above, it is important now to note that the discontinuity in the quantum stress tensor, at the boundary r_c, is finite at the boundary between regions 2 and 1. This is because the various expectation values are determined by the trace anomaly and derivatives (up to second derivative) of the metric. The metric derivatives are finite on the Cauchy horizon, for r

>0. Hence the quantum stress tensor component $\langle T_{u_1 u_1}\rangle$ at the boundary, in the limit of approach from region 1, will be divergent.

We now examine the expression for $\langle T_{u_1 u_1}\rangle$, i.e., Eq. (32). The first term in this expression is finite, because we demand the metric component to be at least C^2. This term is finite not only at the event on the boundary but all over the null ray (Cauchy horizon) emanating from that event (in fact, all over the region 1).

Now, the second statement above implies that the second term in Eq. (32) diverges at the intersection of the boundary with the null ray (Cauchy horizon). This term is a function of only the retarded null coordinate u_1. Since it diverges at one event (on the boundary), it diverges all along the outgoing null ray ($u_1 = $const). The first term, as mentioned before, is finite there. Hence the tensor component $\langle T_{u_1 u_1}\rangle$ diverges all along the Cauchy horizon in region 1 as well.

Finally, we consider a family of modified distributions, each with a successively smaller value of the boundary coordinate r_c, so that in the limiting case, r_c tends to zero. For each family in the distribution, there is a divergence of the outgoing flux, in region 1. In the limit that r_c tends to zero, we recover the original non-self-similar distribution $\rho_0(r)$ which admits a naked singularity, and by virtue of the construction given here, has a divergence of $\langle T_{u_1 u_1}\rangle$ on the Cauchy horizon.

We can make a similar argument at the second boundary, between region 1 and the Schwarzschild region 0. Thus, the divergence occurs all over the Cauchy horizon, to whichever extent it exists. In particular, if we have a globally naked singularity, there is a divergence at the intersection of the Cauchy horizon with $\mathcal{I}^+$.

Also, we note that this argument works for any number of regions with different metric solutions matched at the boundaries. This is important from the point of view of the extra shell we needed to introduce in the non-marginally bound Tolman-Bondi case. The finiteness of the rest of the tensor components can be argued on similar lines.

As regards the Vaidya metric, the entire argument is the same except for the fact that the boundaries between the regions are ingoing null geodesics. In fact, $\beta()$ being the identity function [2], the calculation is simplified considerably.

IV. CONCLUSIONS

We have shown that the quantum stress tensor diverges on the Cauchy horizon, if it exists, in non-self-similar Tolman-Bondi dust and Vaidya radiation collapse in two dimensions. There is no direct way of deducing this by analytical calculations of the expressions in the general case. However, we employ a limiting process of approaching the required spacetime metric via patching up tractable solutions. From this technique, it appears that the divergence technically results from the metric rendered non-invertible at the Cauchy horizon in the self-similar portion. This does not seem to be the ultimate reason for the divergence, for we see that the divergence persists even after the limit to the actual

metric is taken. Needless to say, the divergence does not seem to be ultimately a result of the self-similar nature although that does help in making the divergence evident in the calculations.

Finally, we would like to comment on two related issues. One is the ''infinite blueshift'' effect and the other is the role of the boundary conditions in the occurrence of the divergence. It is known, for instance, in the case of a charged spherical dust shell collapsing to form a Reissner-Nordström black hole, that infalling classical waves suffer an infinite blueshift on the Cauchy horizon. Also, the quantum stress tensor for a test scalar field in this spacetime diverges on the Cauchy horizon [7]. One might be led to ask if the quantum divergence on the horizon is necessarily correlated with an infinite blueshift of classical waves. However, this does not appear to be the case. In our present analysis, infalling classical scalar waves do not necessarily suffer an infinite blueshift on the Cauchy horizon, yet the quantum stress tensor diverges there. The difference from the Reissner-Nordström case probably results from the two spacetimes having different conformal geometries.

As regards boundary conditions, we note that expressions (15)–(17) for the quantum stress tensor are derived by imposing boundary conditions on the center and on $\mathcal{I}^-$. This is sufficient, because we are physically interested in the tensor only in the region before the Cauchy horizon. No boundary conditions have been imposed to the future of the Cauchy horizon. It is true though that the boundary condition imposed at the center is assumed to continue to hold at the intersection of the center with the Cauchy horizon and the first point of the naked singularity (see Fig. 1). This however is not expected to affect the occurrence of the divergence, since it is clear that the outgoing quantum flux grows in an unbounded way in the approach to the Cauchy horizon.

ACKNOWLEDGMENT

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