

Gaussian superpositions in scalar-tensor quantum cosmological models

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A free scalar field minimally coupled to gravity model is quantized and the Wheeler-DeWitt equation in minisuperspace is solved analytically, exhibiting positive and negative frequency modes. The analysis is performed for positive, negative and zero values of the curvature of the spatial section. Gaussian superpositions of the modes are constructed, and the quantum Bohmian trajectories are determined in the framework of the Bohm–de Broglie interpretation of quantum cosmology. Oscillating universes appear in all cases, but with a characteristic scale of the order of the Planck scale. Bouncing regular solutions emerge for the flat curvature case. They contract classically from infinity until a minimum size, where quantum effects become important, acting as repulsive forces avoiding the singularity and creating an inflationary phase, expanding afterwards to an infinite size, approaching the classical expansion as long as the scale factor increases. These are nonsingular solutions which are viable models to describe the early Universe.

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I. INTRODUCTION

The existence of an initial singularity is one of the major drawbacks of classical cosmology. In spite of the fact that the standard cosmological model, based in the classical general relativity theory, has been successfully tested until the nucleosynthesis era (around $t \sim 1$ s), the extrapolation of this model to higher energies leads to a breakdown of the geometry in a finite cosmic time. This breakdown of the geometry may indicate that the classical theory must be replaced by a quantum theory of gravitation: quantum effects may avoid the presence of the singularity, leading to a complete regular cosmological model.

The quantization of gravity is plagued with many conceptual and technical problems, and when it is applied to the whole universe new issues appear. In the Dirac quantization approach, a functional equation for the wave function of the Universe is obtained, the Wheeler-DeWitt equation [1], which is the basic equation of quantum cosmology. It is formulated in so-called superspace, the space of all possible three-dimensional spatial geometries. It is very hard to find exact solutions of the full Wheeler-DeWitt equation, but solutions may be found in minisuperspaces where all but a finite number of degrees of freedom are frozen.

Among the fundamental questions that come from the quantization of the universe as a whole, one of the most important concerns the interpretation of the wave function coming from the Wheeler-DeWitt equation. In order to ex-

tract predictions from the wave function of the Universe, the Bohm–de Broglie ontological interpretation of quantum mechanics [2,3] has been proposed [4–6], since it avoids many conceptual difficulties that follow from the application of the standard Copenhagen interpretation to a unique system that contains everything. In opposition to the latter one, the ontological interpretation does not need a classical domain outside the quantized system to generate the physical facts out of potentialities (the facts are there *ab initio*), and hence it can be applied to the universe as a whole.¹ With this interpretation in hand, one can ask if the quantum scenario predicted by the Wheeler–DeWitt equation is free of singularities and which type of classical universe emerges from the quantum phase.

In a preceding work [8], we have applied this proposal to a free scalar-tensor model with minimal coupling in Friedmann-Robertson-Walker geometry, which can be obtained from a nonminimal scalar-tensor theory through a conformal transformation. Free scalar fields are good candidates to describe the material content of the early Universe because of their simplicity and because they represent stiff matter, the type of fluid advocated by Zel’dovich [9] to be relevant at early stages of cosmic evolution. Only positive curvature spatial sections have been studied. The Bohmian trajectories in configuration space revealed an unexpected scenario: they behaved as the classical solutions for small values of the scale factor, but display quantum behavior when the scale factor is big. As a consequence, the initial

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¹Other alternative interpretations can be used in quantum cosmology such as the many worlds interpretation of quantum mechanics [7].

singularity is still present in this quantum model.

The Wheeler-DeWitt solutions for this scalar-tensor model contain positive and negative frequency modes, the first leading to an expanding universe, and the second to a contracting universe, near the singularity. Inspired by this observation, we constructed in Ref. [8] some particular superpositions mixing negative and positive models. In this way, we found nonsingular quantum solutions which were, however, of Planckian size and hence they could not be a model for our real Universe.

The aim of the present work is to explore further the possibilities of the minisuperspace model of Ref. [8]. First, we will not restrict ourselves to positive curvature spatial sections and second, we will explore more suitable superpositions of negative and positive modes, namely, the Gaussian superposition. For the case the spatial section is flat, it is possible to solve analytically the expressions for the phase of the wave function, and to reduce the equations for the Bohmian trajectories to a dynamical system. The critical points are calculated, and they are identified as center or nodes points. This leads to the existence of three kind of scenarios: periodic solutions representing oscillating universes, bouncing universes, models with a big-bang followed by a big crunch. The bouncing universes contract classically from infinity until a minimum size, where quantum effects become important acting as repulsive forces avoiding the singularity, expanding afterwards to an infinite size, approaching the classical expansion as long as the scale factor increases. These are nonsingular solutions which are viable models to describe the Universe we live in. For closed and open spatial sections, all calculations must be performed numerically, and the trajectories obtained in the configuration space reveal again the presence of oscillating universes in addition to those with a big-bang followed by a big-crunch. In all three cases, the oscillating universes have a characteristic scale of the order of the Planck length, except for very special Gaussians in the case of zero spatial curvature. Hence, the most interesting scenarios emerge from the flat case, where we have succeeded to obtain a viable nonsingular model.

The article is organized as follows. In Sec. II, we describe the classical model and the corresponding Wheeler-DeWitt equation in the minisuperspace. Section III is devoted to the study of the Gaussian superposition of the quantum solutions found before, and their corresponding analysis. In Sec. IV we present our conclusions.

II. THE CLASSICAL AND QUANTUM MINISUPERSPACE MODELS

Let us take the Lagrangian

$$L = \sqrt{-g} e^{-\phi} (R - w \phi_{, \rho} \phi^{, \rho}). \quad (1)$$

For $w = -1$ we have effective string theory without the Kalb-Ramond field. For $w = -3/2$ we have a conformally coupled scalar field. Performing the conformal transformation $g_{\mu\nu} = e^{\phi} \bar{g}_{\mu\nu}$ we obtain the following Lagrangian:

$$L = \sqrt{-g} \left[R - \left(\omega + \frac{3}{2} \right) \phi_{, \rho} \phi^{, \rho} \right], \quad (2)$$

where the bars have been omitted. We will define $C_w \equiv (\omega + \frac{3}{2})$, which we will consider, from now on, to be strictly positive in order not to violate any of the energy conditions, at least classically.

We will consider the Robertson-Walker metric

$$ds^2 = -N^2 dt^2 + \frac{a(t)^2}{1 + (\epsilon/4)r^2} \{ dr^2 + r^2 [d\theta^2 + \sin^2(\theta) d\varphi^2] \}, \quad (3)$$

where the spatial curvature ϵ takes the values $0, 1, -1$. Inserting this line element into the Lagrangian (2), and using the units where $\hbar = c = 1$, we obtain the following action:

$$S = \frac{3V}{4\pi l_p^2} \int \frac{Na^3}{2} \left(\frac{-\dot{a}^2}{N^2 a^2} + C_w \frac{\dot{\phi}^2}{6N^2} + \frac{\epsilon}{a^2} \right) dt, \quad (4)$$

where V is the total volume divided by a^3 of the spacelike hypersurfaces, which are supposed to be closed, and l_p is the Planck length. V depends on the value of ϵ and on the topology of the hypersurfaces. For $\epsilon = 0$, V can have any value because the fundamental polyhedra of $\epsilon = 0$ hypersurfaces can have arbitrary size (see Ref. [10]). In the case of $\epsilon = 1$ and topology S^3 , $V = 2\pi^2$. Defining $\beta^2 = 4\pi l_p^2/3V$, $\bar{\phi} \equiv \sqrt{C_w}/6\phi$, and omitting again the bars, the Hamiltonian reads

$$H = N \left(-\beta^2 \frac{p_a^2}{2a} + \beta^2 \frac{p_\phi^2}{2a^3} - \epsilon \frac{a}{2\beta^2} \right), \quad (5)$$

where

$$p_a = -\frac{a\dot{a}}{\beta^2 N}, \quad (6)$$

$$p_\phi = \frac{a^3 \dot{\phi}}{\beta^2 N}. \quad (7)$$

Usually, the scale factor has dimensions of length because we use angular coordinates in closed spaces. Hence we will define a dimensionless scale factor $\tilde{a} \equiv a/\beta$. In that case the Hamiltonian becomes, omitting the tilde,

$$H = \frac{N}{2\beta} \left(-\frac{p_a^2}{a} + \frac{p_\phi^2}{a^3} - \epsilon a \right). \quad (8)$$

As β appears as an overall multiplicative constant in the Hamiltonian, we can set it equal to one without any loss of generality, keeping in mind that the scale factor which appears in the metric is βa , not a . We can further simplify the Hamiltonian by defining $\alpha \equiv \ln(a)$ obtaining

$$H = \frac{N}{2 \exp(3\alpha)} [-p_\alpha^2 + p_\phi^2 - \epsilon \exp(4\alpha)], \quad (9)$$

where

$$p_\alpha = -\frac{e^{3\alpha} \dot{\alpha}}{N}, \quad (10)$$

$$p_\phi = \frac{e^{3\alpha} \dot{\phi}}{N}. \quad (11)$$

The momentum p_ϕ is a constant of motion which we will call \bar{k} . The classical solutions are, in the gauge $N=1$, the following.

(1) $\epsilon=0$:

$$\phi = \pm \alpha + c_1, \quad (12)$$

where c_1 is an integration constant. In terms of cosmic time they read

$$a = e^\alpha = 3\bar{k}t^{1/3}, \quad (13)$$

$$\phi = \frac{\ln(t)}{3} + c_2. \quad (14)$$

The solutions contract or expand forever from a singularity, depending on the sign of \bar{k} , without any inflationary epoch.

(2) $\epsilon=1$:

$$a = e^\alpha = \frac{\bar{k}}{\cosh(2\phi - c_1)}, \quad (15)$$

where c_1 is an integration constant, and from the conservation of p_ϕ we get

$$\bar{k} = e^{3\alpha} \dot{\phi}. \quad (16)$$

The cosmic time dependence is complicated and we will not write it here. These solutions describe universes expanding from a singularity till a maximum size and contracting again to a big crunch. Near the singularity, these solutions behave as in the flat case. There is no inflation.

(3) $\epsilon=-1$:

$$a = e^\alpha = \frac{\bar{k}}{|\sinh(2\phi - c_1)|}, \quad (17)$$

where c_1 is an integration constant, and again, from the conservation of p_ϕ we get

$$\bar{k} = e^{3\alpha} \dot{\phi}. \quad (18)$$

As before, the cosmic time dependence is complicated and we will not write it here. These solutions describe universes

contracting forever to or expanding forever from a singularity. Near the singularity, these solutions behave as in the flat case. There is no inflation.²

Let us now quantize the model. The Wheeler-DeWitt equation is obtained through the Dirac quantization procedure where the wave function must be annihilated by the operator version of the constraint in Eq. (9). With the choice of factor ordering which makes it covariant through field redefinitions, it reads

$$-\frac{\partial^2 \Psi}{\partial \alpha^2} + \frac{\partial^2 \Psi}{\partial \phi^2} + \epsilon e^{4\alpha} \Psi = 0. \quad (19)$$

Employing the separation of variables method, we obtain the general solution

$$\Psi(\alpha, \phi) = \int F(k) A_k(\alpha) B_k(\phi) dk, \quad (20)$$

where k is a separation constant and $F(k)$ is an arbitrary function of k ,

$$B_k(\phi) = b_1 \exp(ik\phi) + b_2 \exp(-ik\phi), \quad (21)$$

and for $\epsilon=0$

$$A_k(\alpha) = a_1 \exp(ik\alpha) + a_2 \exp(-ik\alpha), \quad (22)$$

for $\epsilon=1$

$$A_k(\alpha) = a_1 I_{ik/2}(e^{2\alpha/2}) + a_2 K_{ik/2}(e^{2\alpha/2}), \quad (23)$$

and for $\epsilon=-1$

$$A_k(\alpha) = a_1 J_{ik/2}(e^{2\alpha/2}) + a_2 N_{ik/2}(e^{2\alpha/2}). \quad (24)$$

The functions J, N, I, K are Bessel and modified Bessel functions of first and second kind.

The Bohm-de Broglie interpretation of homogeneous minisuperspace models goes as follows: in general, the minisuperspace Wheeler-DeWitt equation is

$$\mathcal{H}[\hat{p}^\alpha(t), \hat{q}_\alpha(t)] \Psi(q) = 0. \quad (25)$$

Writing $\Psi = R \exp(iS/\hbar)$, and substituting it into Eq. (25), we obtain the following equation:

$$\frac{1}{2} f_{\alpha\beta}(q_\mu) \frac{\partial S}{\partial q_\alpha} \frac{\partial S}{\partial q_\beta} + U(q_\mu) + Q(q_\mu) = 0, \quad (26)$$

where the quantum potential is

$$Q(q_\mu) = -\frac{1}{2R} f_{\alpha\beta} \frac{\partial^2 R}{\partial q_\alpha \partial q_\beta}. \quad (27)$$

²In the case $\epsilon=-1$ there are classical solutions with $C_w < 0$. Qualitatively, they represent universes contracting from an infinite to a minimum size and then expanding again to infinity.

The Bohm–de Broglie interpretation applied to quantum cosmology states that the trajectories $q_\alpha(t)$ are real, independently of any observations. Equation (26) is the Hamilton–Jacobi equation for them, which is the classical one amended with a quantum potential term (27), responsible for the quantum effects. This suggests that we define

$$p^\alpha = \frac{\partial S}{\partial q_\alpha}, \quad (28)$$

where the momenta are related to the velocities in the usual way:

$$p^\alpha = f^{\alpha\beta} \frac{1}{N} \frac{\partial q_\beta}{\partial t}. \quad (29)$$

To obtain the quantum trajectories we have to solve the following system of first order differential equations, called the guidance relations:

$$\frac{\partial S(q_\alpha)}{\partial q_\alpha} = f^{\alpha\beta} \frac{1}{N} \dot{q}_\beta. \quad (30)$$

In the present case of the Hamiltonian (9), the quantum potential (27) becomes

$$Q(\alpha, \phi) = \frac{e^{3\alpha}}{2R} \left[\frac{\partial^2 R}{\partial \alpha^2} - \frac{\partial^2 R}{\partial \phi^2} \right], \quad (31)$$

and the guidance relations (30) read

$$\frac{\partial S}{\partial \alpha} = - \frac{e^{3\alpha} \dot{\alpha}}{N}, \quad (32)$$

$$\frac{\partial S}{\partial \phi} = \frac{e^{3\alpha} \dot{\phi}}{N}. \quad (33)$$

Equations (30) are invariant under time reparametrization. Hence, even at the quantum level, different choices of $N(t)$ yield the same spacetime geometry for a given nonclassical solution $q_\alpha(t)$. There is no problem of time in the Bohm–de Broglie interpretation of minisuperspace quantum cosmology.³ Let us then apply this interpretation to our minisuperspace models and choose the gauge $N=1$.

III. BOHM INTERPRETATION OF GAUSSIAN SUPERPOSITIONS

We will now make Gaussian superpositions of these solutions and interpret the results using the Bohm–de Broglie interpretation of quantum mechanics. We will begin by the case $\epsilon=0$, which is simpler, and it is the one to which the others reduce when $\alpha \rightarrow -\infty$.

A. Hypersurfaces with $\epsilon=0$

This case can be solved analytically. We choose the arbitrary function $F(k)$ as the Gaussian

$$F(k) = \exp \left[- \frac{(k-d)^2}{\sigma^2} \right]. \quad (34)$$

We can study two types of wave function

$$\Psi_1(\alpha, \phi) = \int F(k) B_k(\phi) [A_k(\alpha) + A_{-k}(\alpha)] dk \quad (35)$$

and

$$\Psi_2(\alpha, \phi) = \int F(k) A_k(\alpha) [B_k(\phi) + B_{-k}(\phi)] dk, \quad (36)$$

both with $a_2 = b_2 = 0$. We will restrict ourselves to Ψ_1 because it yields the most interesting results. The results coming from Ψ_2 can be obtained from the first by changing α with ϕ .

Performing the integration in k we obtain, for Ψ_1 ,

$$\Psi_1 = \sigma \sqrt{\pi} \left\{ \exp \left[- \frac{(\alpha + \phi)^2 \sigma^2}{4} \right] \exp[id(\alpha + \phi)] + \exp \left[- \frac{(\alpha - \phi)^2 \sigma^2}{4} \right] \exp[-id(\alpha - \phi)] \right\}. \quad (37)$$

In order to obtain the Bohmian trajectories, we have to calculate the phase S of the above wave function and substitute it into the guidance formula (32),(33), working in the gauge $N=1$. These equations constitute a planar system which can be easily studied:

$$\dot{\alpha} = \frac{[\phi \sigma^2 \sin(2d\alpha) + 2d \sinh(\sigma^2 \alpha \phi)]}{\exp(3\alpha) \{2[\cos(2d\alpha) + \cosh(\sigma^2 \alpha \phi)]\}}, \quad (38)$$

$$\dot{\phi} = \frac{[-\alpha \sigma^2 \sin(2d\alpha) + 2d \cos(2d\alpha) + 2d \cosh(\sigma^2 \alpha \phi)]}{\exp(3\alpha) \{2[\cos(2d\alpha) + \cosh(\sigma^2 \alpha \phi)]\}}. \quad (39)$$

The line $\alpha=0$ divides configuration space in two symmetric regions. The line $\phi=0$ contains all singular points of this system, which are nodes and centers. The nodes appear when the denominator of the above equations, which is proportional to the norm of the wave function, is zero. No trajectory can pass through these points. They happen when $\phi=0$ and $\cos(d\alpha)=0$, or $\alpha=(2n+1)\pi/2d$, n an integer, with separation π/d . The center points appear when the numerators are zero. They are given by $\phi=0$ and $\alpha=2d[\cot(d\alpha)]/\sigma^2$. They are intercalated with the node points, and their separations cannot exceed π/d . As $|\alpha| \rightarrow \infty$ these points tend to $n\pi/d$. As one can see from the above system, the classical solutions ($a(t) \propto t^{1/3}$) are recovered when $|\alpha| \rightarrow \infty$ or $|\phi| \rightarrow \infty$, the other being different from zero.

A field plot of this planar system is shown in Fig. 1, for $\sigma=d=1$. We can see plenty of different possibilities, depending on the initial conditions. Near the center points we

³This is not the case, however, for the full superspace (see Ref. [5]).

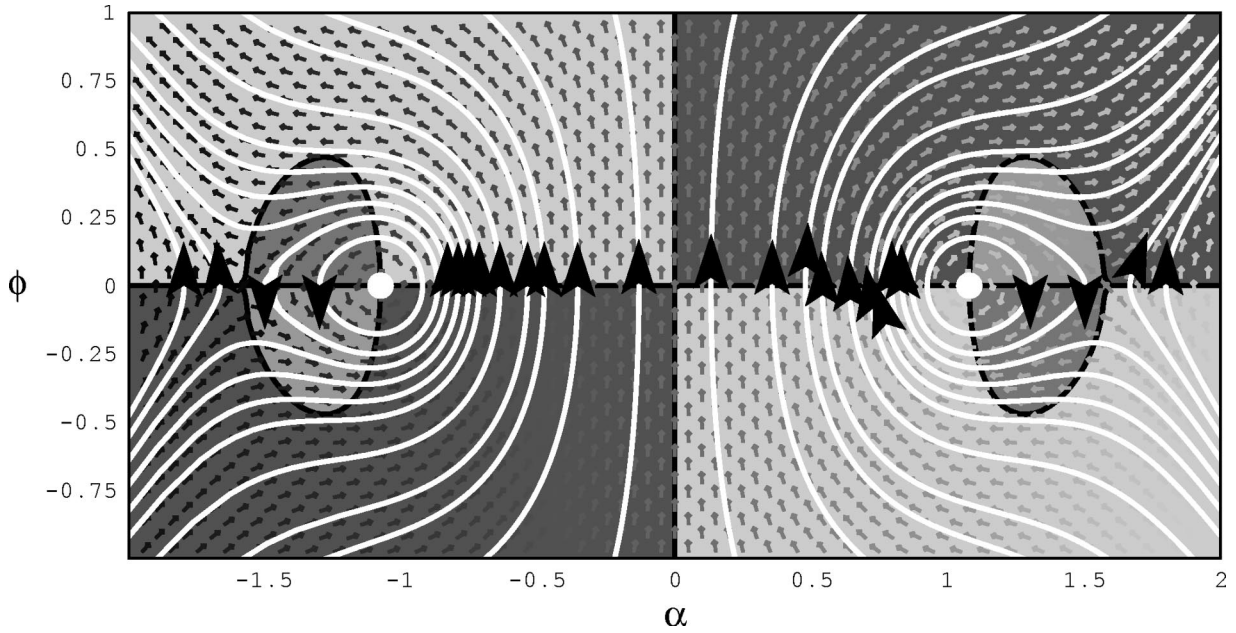


FIG. 1. Field plot of the system of planar equations (38), (39) for $\sigma=d=1$, which uses the Bohm-de Broglie interpretation with the wave function Ψ_1 , Eq. (37). Each arrow of the vector field is shaded according to its true length, black representing short vectors and white, long ones. The four shades of gray show the regions where the vector field is pointing to northeast, northwest, southeast, or southwest. The black curves are the nullcline curves that separate these regions. The white points are the center points whose neighborhoods have oscillating trajectories. The trajectories are the white curves with direction arrows.

can have oscillating universes without singularities and with amplitude of oscillation of order 1.⁴ For negative values of α , the universe arises classically from a singularity but quantum effects become important forcing it to recollapse to another singularity, recovering classical behavior near it. For positive values of α , the universe contracts classically but when ϕ and α are small enough, quantum effects become important creating an inflationary phase which avoids the singularity. The universe contracts to a minimum size and after reaching this point it expands forever, recovering the classical limit when α becomes sufficiently large. These are models which can represent the early Universe. We can see that for α negative we have classical limit for small scale factor while for α positive we have classical limit for big scale factor.

For the wave function Ψ_2 , the analysis goes in the same way but we have to interchange α with ϕ . In this case we also have periodic solutions but the others are universes arising classically from a singularity, experiencing quantum effects in the middle of their expansion, and recovering their classical behavior for large values of α . There are no further possibilities.

We will now pass to the cases with curved spatial sections. One can immediately notice an important difference. The case $\epsilon=0$ has a symmetry $\alpha \rightarrow -\alpha$ which is present not only in the Wheeler-DeWitt equation (19) but also in the solution (37). The cases $\epsilon \neq 0$ do not possess this symmetry

(the potential $\epsilon e^{4\alpha}$ in the Wheeler-DeWitt equation breaks it), and one should not expect to obtain the $\alpha > 0$ part of the field plots in these cases from the $\alpha < 0$ part through a reflection, as in the case $\epsilon=0$ (see Fig. 1).

B. Hypersurfaces with $\epsilon=1$

The Wheeler-DeWitt equation (19) for $\epsilon=1$, in the case we neglect the $\partial_{\phi\phi}\Psi$, is analogous to a stationary Schrödinger equation with $E=0$ and $V=e^{4\alpha}$. Hence, one should make superpositions involving only the parts of $A_k(\alpha)$ which goes to zero as α goes to infinity, which are the Bessel functions $K_{ik/2}(e^{2\alpha/2})$. Consequently, we will take the following superposition:

$$\Psi_3(\alpha, \phi) = \int \exp\left[-\frac{(k-d)^2}{\sigma^2}\right] K_{ik/2}(e^{2\alpha/2}) B_k(\phi) dk. \quad (40)$$

The limit $\alpha \rightarrow -\infty$ does not give the preceding results for $\epsilon=0$ because the Bessel function K reduces in this limit to

$$K_{ik/2}(e^{2\alpha/2}) \approx \frac{i}{k} \left[\exp\{ik[\alpha - \ln(2)]\} \Gamma\left(1 - \frac{ik}{2}\right) - \exp\{-ik[\alpha - \ln(2)]\} \Gamma\left(1 + \frac{ik}{2}\right) \right], \quad (41)$$

and the presence of the gamma functions spoils their similarity.

⁴As discussed above, these amplitudes can be very large as long as d becomes very small because the separation of the center points are of the order of $1/d$.

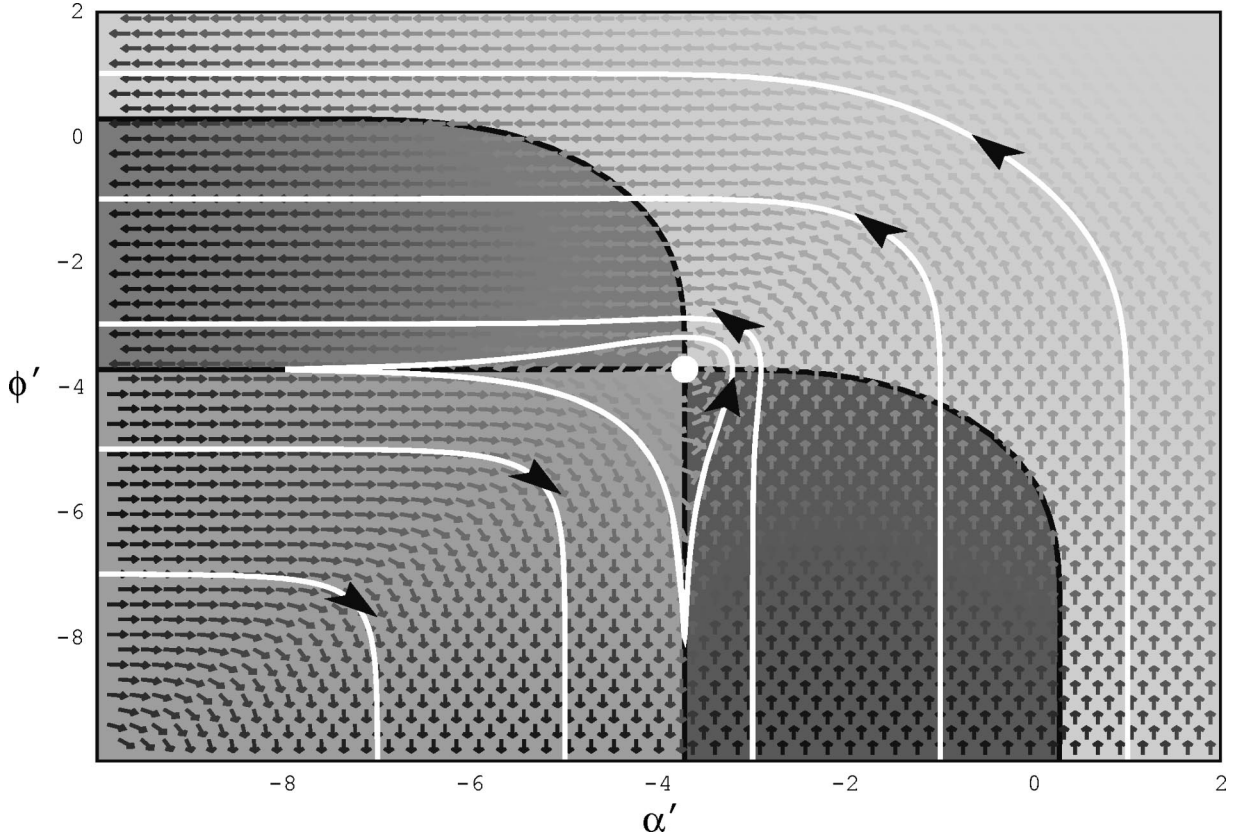


FIG. 2. Field plot of the numerical solution of the system of planar equations (44),(45) using the Bohm–de Broglie interpretation with the wave function Ψ_3 , Eq. (40), for $\sigma=d=1$. Each arrow of the vector field is shaded according to its true length, black representing short vectors and white, long ones. The four shades of gray show the regions where the vector field is pointing to northeast, northwest, southeast, or southwest. The black curves are the nullcline curves that separate these regions. The white point is the center point whose neighborhood has oscillating trajectories. The trajectories are the white curves with direction arrows.

This case must be studied numerically, and the transformation

$$\alpha' = \alpha - \phi, \quad (42)$$

$$\phi' = \phi + \alpha, \quad (43)$$

eases this task. The guidance relations (32)–(33) become

$$\dot{\alpha}' = -2 \exp\left[\frac{-3(\alpha' + \phi')}{2}\right] \frac{\partial S}{\partial \phi'}, \quad (44)$$

$$\dot{\phi}' = -2 \exp\left[\frac{-3(\alpha' + \phi')}{2}\right] \frac{\partial S}{\partial \alpha'}, \quad (45)$$

and Fig. 2 shows the field plot of this transformed planar system, using $\sigma=d=1$. There are periodic solutions without singularities which happen when the Bohmian trajectories cross the lines $(\alpha' = -3.73, \phi' < -3.73)$, or $(\phi' = -3.73, \alpha' < -3.73)$ or, equivalently, $(\alpha = |\phi| - 3.73, \alpha < -3.73)$. These oscillating trajectories can reach very negative values of α but their maximum size cannot exceed $\alpha=0$, or $a \approx l_{pl}$. Another behavior is related to the trajectories shown in Fig. 2 which are exclusively in the light gray region. They begin classically from a singularity, expand to a maximum

value of α , and then return classically to a singularity. Concluding, we have two types of trajectories in this case. One which is periodic due to quantum effects, and the other which exhibit the pattern of classical behavior: expansion from a singularity until a maximum size followed by a contraction to a big crunch. The periodic solutions have maximum size around $\alpha=0$, or $a \approx l_{pl}$ and they cannot represent the Universe we live in.

C. Hypersurfaces with $\epsilon=-1$

In this case we will choose as $A_k(\alpha)$ the combination

$$A_k(\alpha) = \left[\Gamma\left(1 + \frac{ik}{2}\right) J_{ik/2}(e^{2\alpha/2}) + \Gamma\left(1 - \frac{ik}{2}\right) J_{-ik/2}(e^{2\alpha/2}) \right] \quad (46)$$

in order to get rid of the gamma functions and obtain the preceding results for $\epsilon=0$ when α is very negative because the Bessel function J reduces in this limit to

$$J_{ik/2}(e^{2\alpha/2}) \approx \frac{\exp\{ik[\alpha - \ln(2)]\}}{\Gamma(1 + ik/2)}. \quad (47)$$

Taking this choice of $A_k(\alpha)$, Eq. (46), into the Gaussian superposition

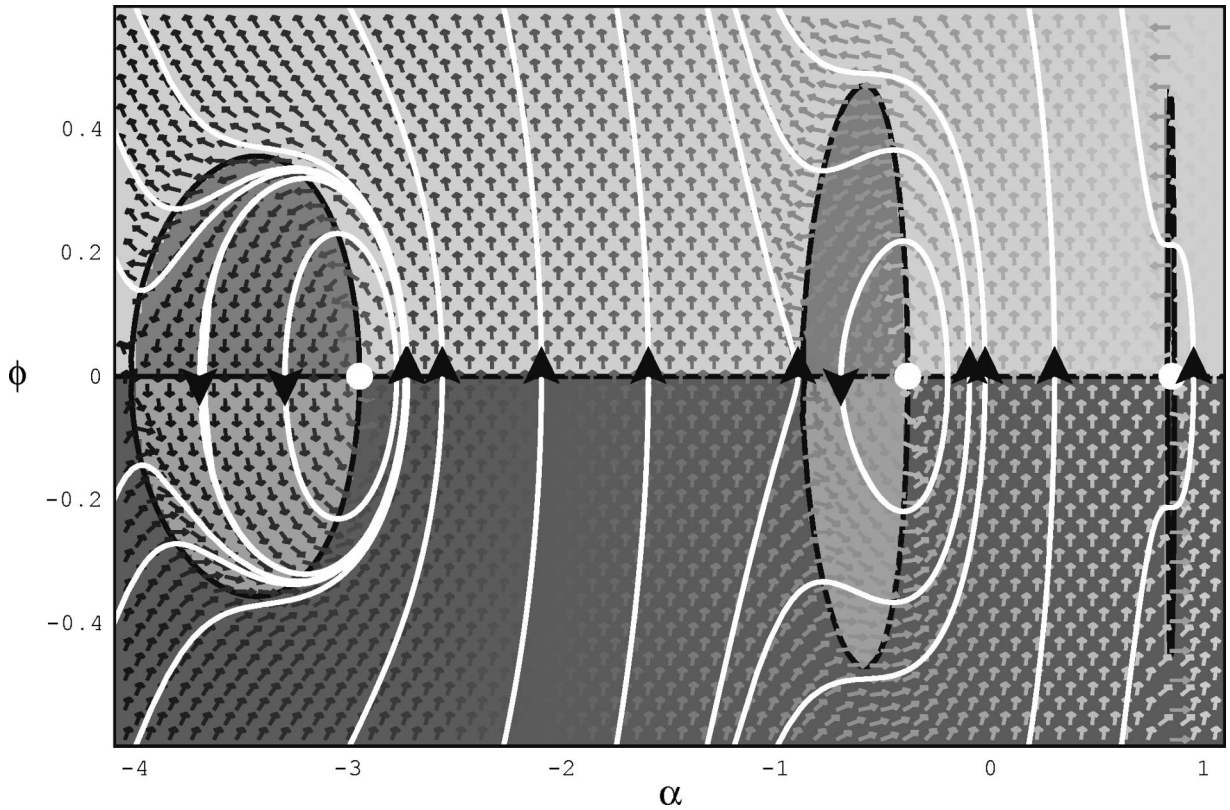


FIG. 3. Field plot of the numerical solution of the system of planar equations (32),(33) using the Bohm-de Broglie interpretation with the wave function Ψ_4 , Eq. (48), for $\sigma=d=1$. Each arrow of the vector field is shaded according to its true length, black representing short vectors and white, long ones. The four shades of gray show the regions where the vector field is pointing to northeast, northwest, southeast, or southwest. The black curves are the nullcline curves that separate these regions. The white points are the center points whose neighborhoods have oscillating trajectories. The trajectories are the white curves with direction arrows.

$$\Psi_4(\alpha, \phi) = \int \exp\left[-\frac{(k-d)^2}{\sigma^2}\right] A_k(\alpha) B_k(\phi) dk, \quad (48)$$

the numerical calculations with respect to the $\epsilon=-1$ case show that the behavior for very negative values of α is similar to the $\epsilon=0$ case, as one can see by comparing Fig. 3 with Fig. 1. As α increases the regions with oscillating universes are squeezed and their separation decreases monotonically. Like the $\epsilon=0$ case, there are periodic solutions without singularities and with amplitude of oscillation of order 1. The other behavior is described by trajectories that arise classically from a singularity, experiment a quantum halt at some maximum value of the scale factor, and then classically contracts to a big crunch, contrary to the classical solutions of Eq. (17) which contract forever to or expand forever from a singularity.

IV. CONCLUSION

The quantization of a scalar-tensor model in the minisuperspace leads to a separable partial differential equation, admitting analytical solutions, with positive and negative frequencies. In this work, we have studied Gaussian superpositions of these different modes and the corresponding Bohmian trajectories. Such analysis was performed for zero, positive and negative curvature spatial sections, which are

considered to be compact. The Bohmian trajectories in configuration space were calculated numerically, except for the flat case, where it is possible to reduce the equations for the Bohmian trajectories to a two-dimensional dynamical system.

The comparison of the trajectories in the configuration space of the variables a and ϕ , which are the dynamical degrees of freedom of the minisuperspace, with the classical ones, allows one to identify the classical and quantum phases for the scalar-tensor cosmological models. For all three different values of the curvature of the spatial sections, the configuration space of the quantum solutions displays oscillating universes. However, these oscillating universes remain at the Planck scale and they cannot be considered as candidates for the description of the early Universe (they are more similar to baby universes), except for the unnatural choice $|d| \ll 1$ in the $\epsilon=0$ case. There are also trajectories which correspond to universes which begin and end in singular states. Only for the flat case is it possible to have bouncing models.

In the bouncing models of the flat spatial section case, the scale factor has an infinite initial and final values, near which it behaves classically. As it approaches the singularity, the repulsive quantum effects lead to the bounce, avoiding the singularity. Such a scenario can be a candidate for the description of our early Universe, since it is free from the initial singularity and behaves classically in the asymptotic limit of

large values for the scale factor. It is worth noting that this classical asymptotic limit corresponds to the stiff matter Friedmann universe which, according to Zel'dovich [9], is the most promising one to describe the very early Universe.

The free scalar field model considered in this paper, on the other hand, can be connected to a nonminimal coupled scalar field, with a coupling parameter ω , as in the Brans-Dicke theory, by a conformal transformation. In Ref. [11] a quantum analysis of these nonminimal models was performed, and it was shown that nonsingular scenarios can be obtained when the parameter ω is negative. In fact, all quantum analysis performed here can be connected with a similar analysis in the nonminimal case through a conformal transformation, namely, $\alpha_{\text{NMC}} = \alpha + \phi$ and $\phi_{\text{NMC}} = \phi$. One can verify that when the minimal model displays singularities, it is possible to have nonsingular solutions in the corresponding nonminimal case; but the nonsingular solutions in the minimal case must also be nonsingular in the corresponding nonminimal models.

An important generalization of the model studied here would be to consider self-interacting scalar fields. It was

shown in Ref. [12] that to each perfect fluid barotropic equation of state, it is possible to construct a self-interacting scalar field model leading to the same classical description in minisuperspace. We may argue if this correspondence remains at the quantum level. Note that in Ref. [6] we have obtained bouncing universes for radiation fields with $\epsilon=0$ and $\epsilon=-1$ with the same qualitative behavior as the bouncing universes found here. They are also viable models for the early Universe. One should investigate if a scalar field model with a potential corresponding to the radiation fluid would give similar results.

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